SGR Questions

Mathematics club

September 2024

1 Introduction

Question 1

Two players play a game where they have a pile of 22 coins. Player 1 begins and is allowed to draw a minimum of 1 and a maximum of 3 coins. Then player 2 is allowed to draw a minimum of 1 and maximum of 4 coins. In the next turn player 1 is allowed up to 5 coins, player 2 up to 6 and so on. The player who draws the last coin wins the game. If both players play perfectly, which player will win (mark the player number)?

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Question 2

3 friends take turns taking from a jar of 1000 candies to share. For example, the 1st friend could take 500 candies, then the 2nd friend could take 400, and the 3rd friend would take the remaining 100.

No one wants to be seen as greedy, but no one wants to end up with the fewest candies either. As such, their goals are (in order of preference):

Do not end up with the most candies, nor the fewest candies (a tie for most or fewest also fails this condition) End up with as many candies as possible All of them are logical, rational, know each other's goals, but cannot communicate before or during sharing. How many candies should each friend end up with?

Question 3

There are 9 cards numbered from 1 to 9. Atreya and Prad take turns picking up cards(Atreya starts first, then Prad picks a card and so on). The first player with a set of 3 cards that add up to 15 wins the game.

Can Atreya guarantee a win?

And does it really matter who starts first? If yes, then how

Question 4

1. Pratyaksh and Deena play a game. First, Pratyaksh chooses a non-zero real number a and announces it. Then Deena chooses a non-zero real number b and announces it. Then Pratyaksh chooses a non-zero real number c and announces it. Finally, Deena chooses a quadratic polynomial

whose three coefficients are a, b, c in some order. (a) Suppose that Deena wins if the quadratic polynomial has a real root and Pratyaksh wins otherwise. Determine which player has a winning strategy. (b) Suppose that Pratyaksh wins if the quadratic polynomial has a real root and Deena wins otherwise. Determine which player has a winning strategy

variants:

a. Is it possible to choose a value for a_1 in the polynomial

$$7x^4 + 9x^3 - 12x^2 + a_1x + 4$$

such that the polynomial has no rational roots? If so, what conditions must a_1 satisfy?

Question 5

There is an island with 100 perfect logicians, each with green eyes. None of them know their own eye color, but they can see everyone else's eyes. They are not allowed to communicate with each other, except for observing others. Every night at midnight, if a logician figures out their own eye color, they must leave the island by crossing a bridge to the mainland. There is a rule that if someone knows their eye color, they must leave that very night.

One day, an outsider comes to the island and makes a single announcement: "At least one of you has green eyes." The outsider leaves, and the islanders continue with their daily routine. What happens?

Question 6

You are one of a 100 prisoners locked in individual cells. You have no sense of day or night. At random intervals, the warden summons one of the prisoners to a room with two levers. Each prisoner must flick exactly one switch in every visit. The warden may call any prisoner any number of times and no prisoner knows which other prisoners have been called before him/her. This process continues indefinitely. It can be assumed that at one point all prisoners are called.

At any point, if the called prisoner claims that all prisoners have visited the room at least once, they are all freed. Come up with a strategy to escape the prison given that the prisoners are allowed to strategize initially. (Assume the initial state of the switches is known to he prisoners.)

2 Allotment

Q1 -Deena Pratyaksh
Q2 - QP
Q3 - Achintya KK Prasanna
Q4 - Madhav Shivanshu
Q5 - QP
Chocy - KV Navin
GoofSpeil - Enthu People (Location: OAT)