Variance of Nuclear Recoil Ionization

Thesis Proposal

Univeristy of Colorado Denver

Mitchell Matheny

23 July 2018

Contents

Chapter 1

Introduction

1.1 Evidence for Dark Matter

Dark matter is one of the most mysterious problems cosmologists are faced with today. There is an overwhelming amount of observational evidence for the existence of this elusive substance. One such observation is found looking at galactic rotation curves. Classical mechanics tells us that the expected rotational velocity is defended by:

$$V(r) \approx \sqrt{\frac{GM(r)}{r}} \tag{1.1}$$

giving the assumption that the velocity should drop off as $\frac{1}{r^{1/2}}$. As shown in figure 1, it can be seen that this is not the case. The rotational velocity of the stars remain constant as the distance from the center of the galaxy increases indicating that the mass varies as a function of r as we move outside the visible edges of the galaxy.[1]

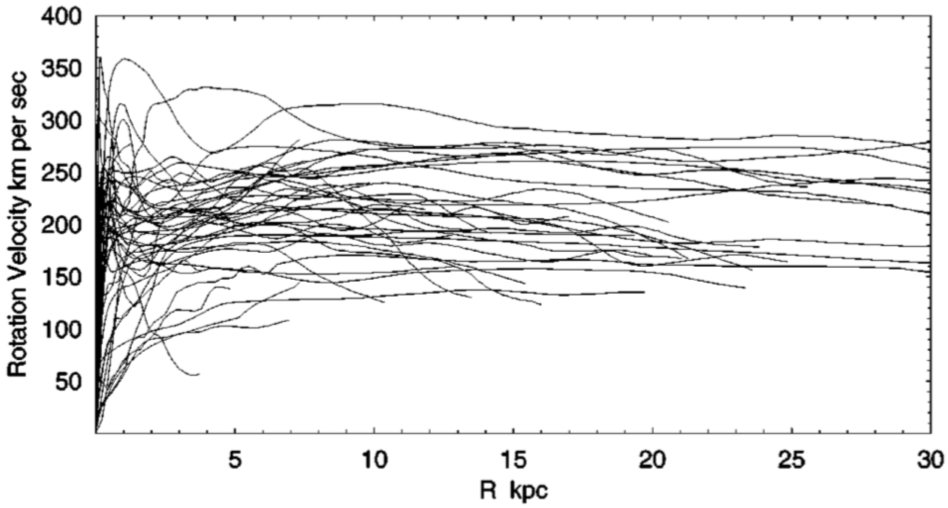

Figure 1.1: Galactic Velocity Distribution.[1]

1.2 Direct Detection

Weakly Interaction Massive Particles (WIMPS) are a favorite candidate for particle dark matter. WIMPS may elastically scatter off nuclei with enough energy to be detected by current laboratory detectors. Measurements of the nuclear-recoil energy spectrum by these experiments can constrain the properties of WIMP dark matter as measurements below $1keVNr$ are required.[2] Measuring a nuclear recoil at low energies does come with its difficulties. The measurements are constrained by the ability to subtract a background signal and the amount of exposure the detector gets as the expected WIMP-nucleon cross-section is only a few events per year. [3] The sensitivity is also limited by the ability to distinguish between electron recoil and nuclear recoil events. **this needs reworking / more info.**

1.3 Motivation and Preparation

With the parameter space for WIMP dark matter candidates being pushed to lower energies, it has become important to accurately determine the deposited energy within the detector and the amount of charge liberated in each event. The fraction of charge liberated in an event is called the ionization yield. Specifically this fraction of charge is liberated in the form of electron-hole pairs. To calculate the variance in the ionization yield one needs to take into account a few phenomenon. Not all of the energy in a scattering event goes into creating electron hole pairs. There is a coupling mechanism to where some of the energy is deposited into the phonon system and the lower the particle energy, the more this factor plays a role.[4] Variation in the amount of electron-hole pairs created from the energy deposited into the electronic system is also important, as a multiple events with the same energy don't always create the same amount of electron hole pairs.

To date, these phenomenon have been accounted for, (see equation 3.1) but when comparing the current model to experimental data there is a significant difference in the spread of the data.[5] There are a few reasons that this might be. One, the model that is used to find the nuclear recoil energies, the Lindhard model, has an energy dependent calculation that is not included in the variance calculation.[6] Two, similar to electron recoils, there exists a nuclear "fano factor" that is also not included. The question is then this: Does including a nuclear recoil fano factor explain the difference in variance between the model and the data? If so, is there a energy dependent functional form to this fano factor?

The Masters of Integrated Sciences program has prepared me for researching the content detailed in this research proposal. With my emphasis in mathematics, i have taken data analysis and statistical analysis. Theses courses will allow me to competently explain and understand the information necessary to determine the nuclear fano factor, as it will involve statistical modeling and error analysis. For my second emphasis, physics, i am taking graduate quantum mechanics. The combination of quantum and my class in numerical partial differential equations, I am confident that I can understand the physics necessary to look beyond the computational aspect of this project.

Chapter 2

Literature Review

2.1 Lindhard Model

When a particle interacts within a material it has the potential to excite electron-hole pairs into the conduction band, yielding the question: For a particle of a given energy, how much energy is given to the electrons, and how much energy loss is due to atomic motion? In the dark matter community, the most widely accepted model to answer this question is the Lindhard model. The Lindhard model predicts the percentage, or yield, of electron-hole pairs created in an interaction. A particle traversing a material has a high probability of scattering in multiple ways, meaning that the crosssections for interacting with the electrons or the nuclei compete for the dominant interaction. The total interaction is described by the following: [6]

$$k\epsilon^{1/2}\nu'(\epsilon) = \int_0^{\epsilon^2} \frac{dt}{2t^{\frac{3}{2}}} f(t^{1/2})(\nu(\epsilon - \frac{t}{\epsilon}) - \nu(\epsilon) + \nu(\frac{t}{\epsilon})) \tag{2.1}$$

Where

$$\epsilon = E\frac{a}{2Z^2e^2} \tag{2.2}$$

is dimensionless energy, ν is the energy loss due to atomic motion, and t is a variable representing the solid angle for a scatter at a given energy. Solving this equation for a non-screening potential gives an analytical form for the energy loss due to atomic motion:[3]

$$\nu(\epsilon) = \frac{\epsilon}{1 + kg(\epsilon)} \tag{2.3}$$

g is a parameterization dependent upon the energy of , and k is a constant that is determined by the material of interest. For Germanium k = 0.157. Since the quantity of interest is the amount of energy given to the electrons, $\nu(\epsilon)$ can be from the reduced energy to find the ionization yield for nuclear recoils.

$$f_n = \frac{\epsilon - \nu(\epsilon)}{\epsilon} = \frac{kg(\epsilon)}{1 + kg(\epsilon)} \tag{2.4}$$

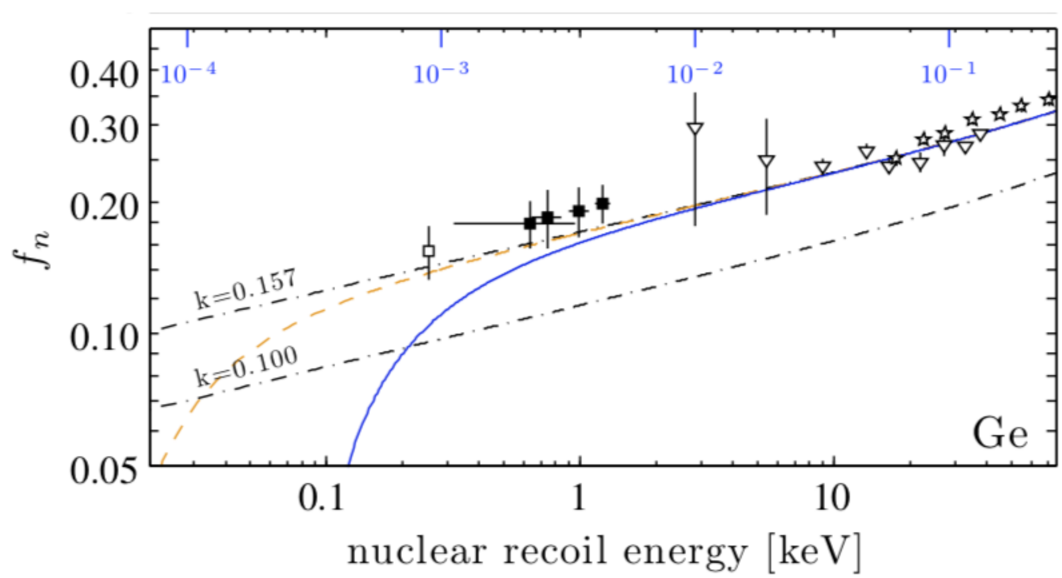

Figure 2.1: Lindhard Yield

Figure 3.1 shows the Lindhard yield as a function of energy. The two dashed lines represent functions for two different materials. $k = 0.157$ is the line for Germanium and $k = 0.100$ is the line for silicon. The blue line represents a modification made to the Lindhard model that might be explored later. As shown, the model predicts energy given to the electronic system down to 0keV.

2.2 Event Discrimination

An atomic recoil results in the creation of electron-hole pairs and phonons. The distribution of energy from the recoil depends on the interacting particle and it's method of interaction, i.e whether the recoil is a nuclear recoil or an electronic recoil. By measuring the ionization yield mentioned in the previous section, it is possible to discriminate between events.

When the incoming particle interacts with the electronic system, the majority of the recoiling electrons energy goes into creating highly energetic electron-hole pairs. These electron-hole pairs lose their energy by the creation of phonons and other electron-hole pairs. The phonons created also have the ability to create further charge carriers and his process continues until there is not enough energy to create phonons or liberate more e-h pairs.[1] The result is an abundance of free e-h pairs that will be collected by their oppositely charged electrodes.

A recoiling neutron can create electron-hole pairs in the same manner as mentioned above, but can also lose some of it's energy by exciting other neutrons in the crystal lattice. A low energy recoiling nucleus has a lower probability of losing energy through the excitation of another nucleus or creating charge carriers. The excited nucleus will then lose all of its energy through the liberation of phonons. In the CDMSlite experiment, these phonons are what is measured. The phonon signal is measured through transition edge sensors, which are superconducting tungsten-aluminum sensors that readout signal by taking advantage of quasiparticle trapping and electrothermal feedback.[7] This mechanism consequently reduces the intermediate ionization signal, as it takes more energy from a nuclear recoil to liberate the same amount of charge carriers as an electron recoil.[4]

When discriminating between electron and nuclear recoils, Luke-phonon production must be accounted for.[Carrier Thesis] Charge carriers that are accelerated across an electric potential will spontaneously emit phonons. Like the primary phonons created during the nuclear recoil, these phonons will be read by the TES and add to the overall measured signal. This combined signal must, therefore, be accounted for in the total measured energy signal.

$$E_{measured} = E_{Recoil} + E_{Luke} = E_{Recoil}(1 + \frac{q\Delta V}{\epsilon}) \tag{2.5}$$

As shown in figure 3.2, using the lindhard yield defined in equation 3.4 to compair the yield and the measured ionization energy can clearly show the discrimination between electron and nuclear recoils as the he different events seperate into distinct bands. The top band fitted in red is the electron recoil band and the bottom band fitted in blue is the nuclear recoil band. The red and blue fits are 2 σ bands centered about the mean. [8]

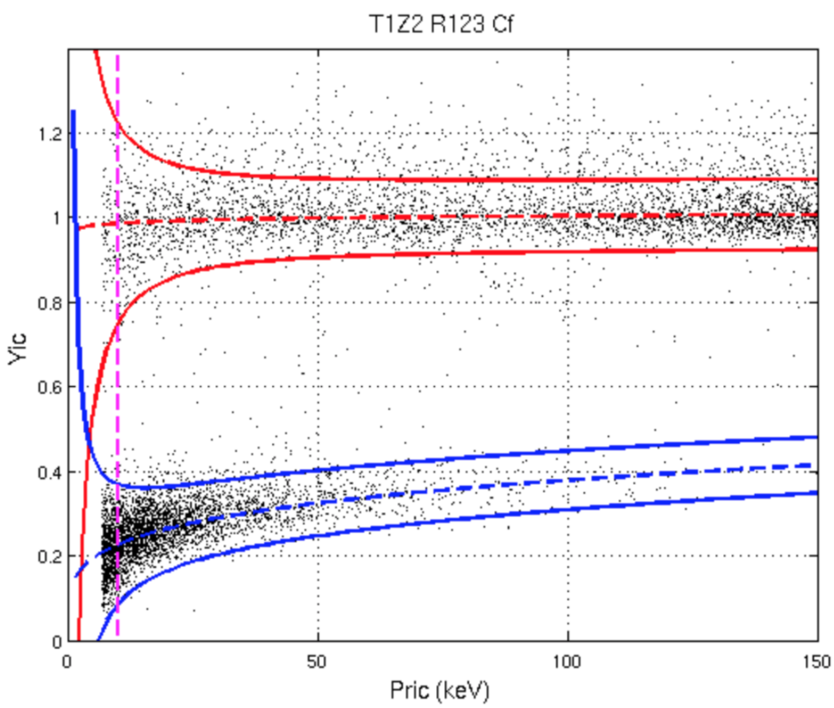

Figure 2.2: Atomic Recoil Bands

2.3 "Fano Factor"

The difference in width of the nuclear recoil bands is something that hasn't been looked at all that much. A possible explanation might be the need for a nuclear "fano factor." The quotes are needed as this is not the normal definition of a fano factor. A fano factor or constant of variation is defined as the measure of dispersion of a probability distribution, or the ratio of the variance to the mean, which is a purely statistical phenomenon. For example, if we assume a Poisson distribution, the Fano factor is equal to one as the variance is equal to the mean. In this case, the assumption is the "fano factor" might not be purely statistical and directly depend upon the energy of the recoiling nuclei. One approach is seen in [9], where they relate the variance in the energy bands to the normal definition of the fano factor, making it it energy dependent.

$$\begin{gathered}\sigma = \sqrt{NF} \\ \sigma = \sqrt{\frac{E_0}{\epsilon_i}} * \sqrt{\frac{E_x}{E_i}(\frac{\epsilon_i}{E_i} - 1)}\end{gathered} \tag{2.6}$$

Therefore

$$F = \sqrt{\frac{E_x}{E_i}(\frac{\epsilon_i}{E_i} - 1)} \tag{2.7}$$

Here E_x is the phonon energy, E_i is the energy of the charge pairs, and ϵ_i is the energy required to create one election hole pair. The derivation of 3.10 is found in equation 5 in [9]. Assuming Germanium with $\epsilon = 3.32eV$, this method yields a fano factor of 0.16. This does lessen the difference in bandwidth but lacks experimental constraints **Talk Amy and Anthony more about this and need to add Dougherty info**

Lindhard also makes a prediction for the width of the bands in terms of mean squared deviation Ω^2, which is the variance with a reference point of 0.[6]

Figure 3.4 shows the relative mean squared variance for the energy loss due to atomic motion $\nu(\epsilon)$ for three different values of k. The meaning for Ω^2 is straightforward if the probability of $\nu(\epsilon)$ is normally distributed,

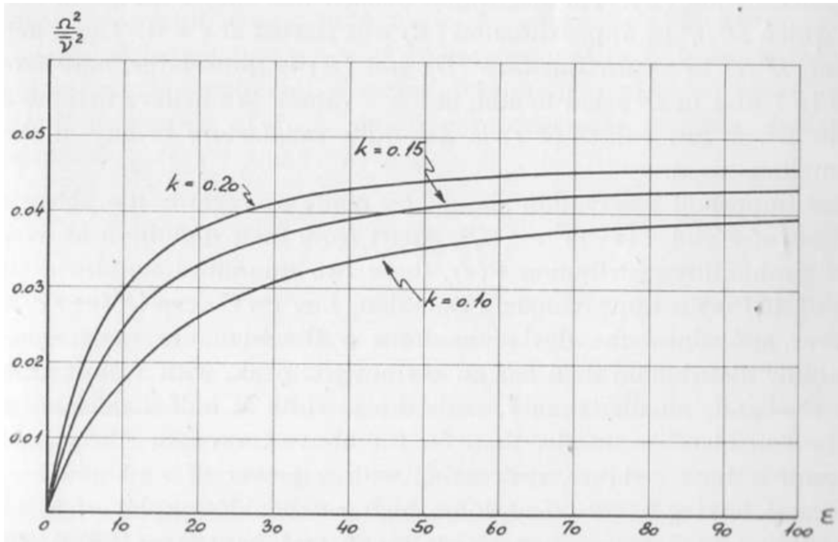

Figure 2.3: Relative Mean Squared Deviation

$$P(\nu) = Ce^{\frac{-(\nu-\bar{\nu}^2)}{2*\Omega^2}}$$
$$\Omega^2 =< (\nu - \bar{\nu})^2 > \tag{2.8}$$

and is useful if particles are collected individually. For many particles in a solid, it is useful to consider a Gaussian with mean $N\bar{\nu}$ and average square fluctuation of $N\Omega^2$.

Chapter 3

Research Plan

3.1 Modeling Recoil Bands

Using the process described below, i will model the atomic recoil bands. Modeling the yield in atomic recoil events requires a knowledge of the charge and phonon resolution [kennedy thesis]. The resolution for electron and nuclear recoils were found by D. Jardin on CDMS iZIP data to be,

$$\sigma_E = \sqrt{\alpha + \beta E + \gamma E^2} \tag{3.1}$$

where α is a constant from electronic noise, βE represents statistical variation, and γE^2 is related to the contribution from noise and incomplete charge. The values for the constants are determined experimentally for the two different recoil types. E is the energy of the recoil. The initial recoil energies are found through Monte Carlo, where the electron recoil energies are randomly drawn from an exponential distribution, and the nuclear recoil energies are drawn from a uniform distribution,

$$\begin{aligned} f(E_{er}, \lambda) &= \lambda e^{E_{er}\lambda} \\ f(E_{nr}) &= \frac{1}{b-a} \end{aligned} \tag{3.2}$$

where λ is the rate parameter and a and b form the energy interval of interest. With the random energies drawn from the distributions above and the ionization yield calculated in equation 2.4, the total charge and phonon energies can be calculated.

$$\begin{aligned} E_Q &= YE_R \\ E_T &= [1 + Y\frac{eV}{\epsilon}]E_R \end{aligned} \tag{3.3}$$

The energies are used to determine the resolution defined by equation 2.6. A normal distribution is formed with the newly defined resolution and randomly sampled. This randomly sampled energy is added to the previously calculated energy. The recoil energies calculated are what is expected to be measured. Using the expected energies, the expected ionization yield need to be calculated.

$$Y = \frac{E_Q}{E_T - \frac{eV}{\epsilon}E_Q} \tag{3.4}$$

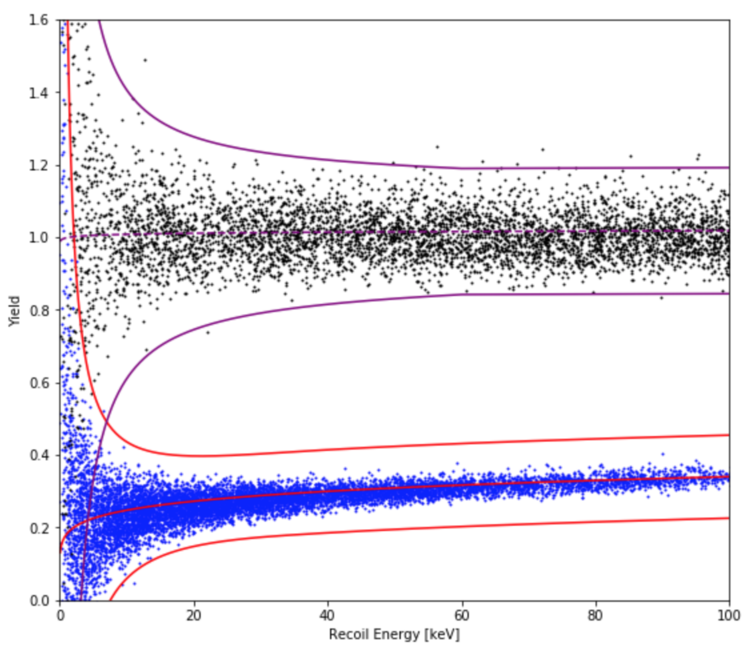

Figure 3.1: Simultaed Recoil Bands

Plotting the expected recoil energies versus the expected ionization yield creates the recoil bands shown in figure 3.1. Comparing figure 2.2 with figure 3.1, there is a distinct difference in the width of the bands, with the more extreme case shown in the nuclear recoil band pictured in blue.

The purpose of the process outlined above is to recreate the electronic and nuclear recoil bands with an added ”fano factor” to the variance. The fano factor will be added in a fashion similar to equation 2.7, but instead of a functional form dependent upon the energy, it will be a random constant. Once the bands with a fano factor are simulated, I will write a program to extract the fano factor from the width of the data. Extracting the fano factor will be done by breaking the data into logarithmically spaced bins and fitting the data in the bins to determine the width. The data in each bin is assumed to be approximately normally distributed. Successfully extracting the fano factor added to the simulated data will allow the extraction of a fano factor from real data, which will help give a more accurate determination of a fano factor that should be added to the simulation.

When a comfortable value for a the fano factor is determined. I will try to accurately model californium data. Doing so should give a good idea of how well a fano factor accurately explains the difference in the variance from data to model. The factor is not going to just be one number. This is due to the possible energy dependence. This should allow me to give a good ”bound” on what the possible fano factors can be.

Something that i would like to do is to determine a functional form of the fano factor from the data. This should give a better understanding of what the actualence of the variance is; as shown above there a different expected fucntional forfunctional variance. I would then like to compare this to what Lindhard predicts for the variance.

3.2 Lindhard Variance Calculation

The Lindhard variance is not something that is easily calculated. Equation 2.8 is the definition of variance, but involves first solving equation 2.1. Equation 2.1 is what is called a ” non-linear integral differential equation.” These equations cannot be solved analytically and therefor must be solved using a numerical quadrature algorithm combined with an accurate and efficient ODE solver. [10] Once solved, the variance equation needs to be solved using the same method with the result from solving equation 2.1.

$$
\begin{aligned}
k\epsilon^{1/2}\frac{d\Omega^2}{\epsilon}(\epsilon) = \int_0^{\epsilon^2} \frac{dt}{2t^{\frac{3}{2}}} f(t^{1/2})(\Omega^2(\epsilon - \frac{t}{\epsilon}) - \Omega^2(\epsilon) + \Omega^2(\frac{t}{\epsilon})) + \ldots \\
\int_0^{\epsilon^2} \frac{dt}{2t^{\frac{3}{2}}} f(t^{1/2})(\nu(\epsilon - \frac{t}{\epsilon}) - \nu(\epsilon) + \nu(\frac{t}{\epsilon}))
\end{aligned}
\tag{3.5}
$$

Once the method to solve this is figured out, the variance can be added to the simulation and then be compared to the data, and the fano factor method.

I am still uncertain how i am going to compared to data. Perhaps i should mention that is what i need to figure out? Or is that straight forward?

Bibliography

[1] A. T. Phipps, “Ionization collection in detectors of the cryogenic dark matter search.”

[2] A. et al. [CDMS Collaboration], “CDMSlite: A search for low-mass WIMPs using voltage-assisted calorimetric ionization detection in the SuperCDMS experiment,” vol. 112, no. 4.

[3] P. Sorensen, “Atomic limits in the search for galactic dark matter,” vol. 91, no. 8, 2015.

[4] M. Lundstrom, *Fundamentals of Carrier Transport.* Cambridge, 2009.

[5] A. Kennedy, *SuperCDMS Prototype Detector Design and Testing.* PhD thesis, Universitiy of Minnesota, 2017.

[6] M. J.Lindhard, V.Nielson, “Integral equations governing radiation effects,” vol. 33, no. 10, 1963.

[7] S. K.Irwin, “A quasiparticle-trap-assisted transition-edge sensor for phonon-mediated particle detection,” vol. 66, no. 11, 1995.

[8] D. Jardin, “Charge and phonon resolution,” vol. Internal Note, 2016.

[9] L. D.Mei, W.Wei, “Impact of low-energy response to nuclear recoils in dark matter detectors,” 2015.

[10] H. Jorquera, “Idsolver: A general purpose solver for th-order integro-differential equations,” vol. 185, 2014.