Graphical Representation of Causal Effects

November 10, 2016

Lord's Paradox: Observed Data

	Covariates (X)	June weight		
Students	Sex, Sept. weight	Y(0)	Y(1)	Impact
1	X_1	?	$Y_1(1)$	?
2	X_2	?	$Y_2(1)$	?
3	X_3	?	$Y_3(1)$	?
$\vdots$	$\vdots$	$\vdots$		
N	X_N	?	$Y_N(1)$	?

Units: Students; Covariates: Sex, September Weight;
Potential Outcomes: June Weight under Treatment and Control;
Treatment = University diet; Control = ??

Statistician 1: June weight under control = September weight
Statistician 2: June weight under control = a linear function of September weight, i.e.

$$E[Y(0)] = \beta_0 + \beta_1 Sex + \beta_2 Weight_{sep}$$

Wainer H and Brown L (2007). Three Statistical Paradoxes in the Interpretation of Group Differences: Illustrated with Medical School Admission and Licsencing Data. *Handbook of Statistics*.

Assignment Mechanism

- Determines which units receive treatment, which receive control
- $P(\,T \mid X, Y(0), Y(1)\,)$
- Known for randomized trials; unknown for observational studies
- Model for assignment mechanism necessary (sometimes sufficient)
  - Model of "science", $P(\,Y(0), Y(1) \mid X\,)$ not necessary if one knows the assignment mechanism, e.g., randomized trials
- So, what's wrong with the assignment mechanism in Lord's Paradox?

Key Property of Randomized Trials

- Treatment assignment is “unconfounded”, also known as “conditional exchangeability”
  - $P(T \mid X, Y(0), Y(1)) = P(T \mid X)$
  - Assignment does not depend on potential outcomes
  - Removes confounding of all variables
  - Crucial for observational studies, but usually as an unverifiable assumption
- Positivity: each unit has a positive probability of receiving each treatment
  - $0 < P(T \mid X) < 1$ for all X
  - Everyone in the study relevant for comparisons
- Study must be designed without the use of the knowledge of outcomes

Randomization ensures balance of covariates.

Example: Truth vs Observation

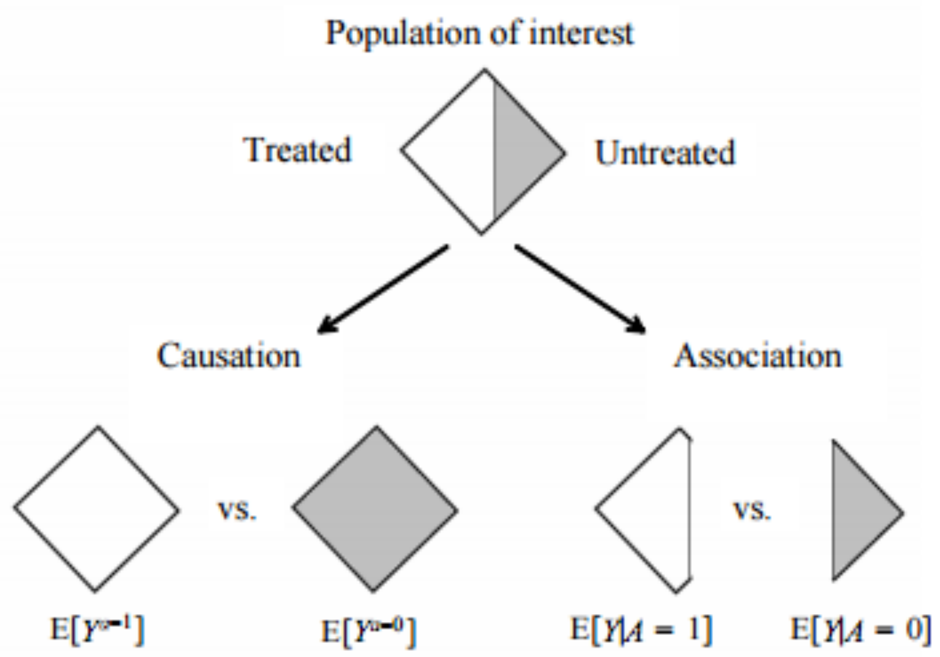
	A	Y	Y^0	Y^1
Rheia	0	0	0	?
Kronos	0	1	1	?
Demeter	0	0	0	?
Hades	0	0	0	?
Hestia	1	0	?	0
Poseidon	1	0	?	0
Hera	1	0	?	0
Zeus	1	1	?	1
Artemis	0	1	1	?
Apollo	0	1	1	?
Leto	0	0	0	?
Ares	1	1	?	1
Athena	1	1	?	1
Hephaestus	1	1	?	1
Aphrodite	1	1	?	1
Cyclope	1	1	?	1
Persephone	1	1	?	1
Hermes	1	0	?	0
Hebe	1	0	?	0
Dionysus	1	0	?	0

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Causal Diagram

- *Causal* Directed Acyclic Graph
- Can represent both association and causation
- Absence of an arrow from A to Y means no individual in the population has that causal effect
- Presence of an arrow from A to Y means there is at least one individual in the population having the causal effect
- Common causes to the treatment and the outcome must be represented in the graph

Association vs Causation



Assignment Mechanism

- Unconditional Randomization

- Conditional Randomization

Exchangeability

- Unconditional Exchangeability

- Conditional Exchangeability

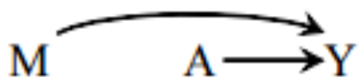
Causal Diagram for Structural Representation of Biases under the Null

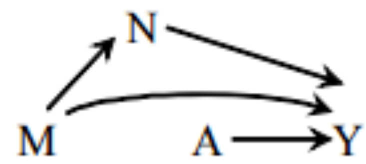
- Common causes for treatment A and outcome Y

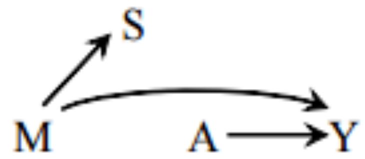
- Common effect for treatment A and outcome Y

- Measurement error on the nodes

Causal Diagram for Effect Modification (with causal effect on outcome)







Causal Diagram for Effect Modification (without causal effect on outcome)