Junction Tree Algorithm Examples

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Junction Tree Algorithm

- Moralize (if starting from a directed graphical model)
- Triangulate (make it chordal)
- Construct a junction tree (maximum cardinality search)
- Define potentials on maximal cliques
- Introduce evidence (if any)
- Propagate probabilities

CHILD Example from Spiegelhalter et al (1993) Statistical Science

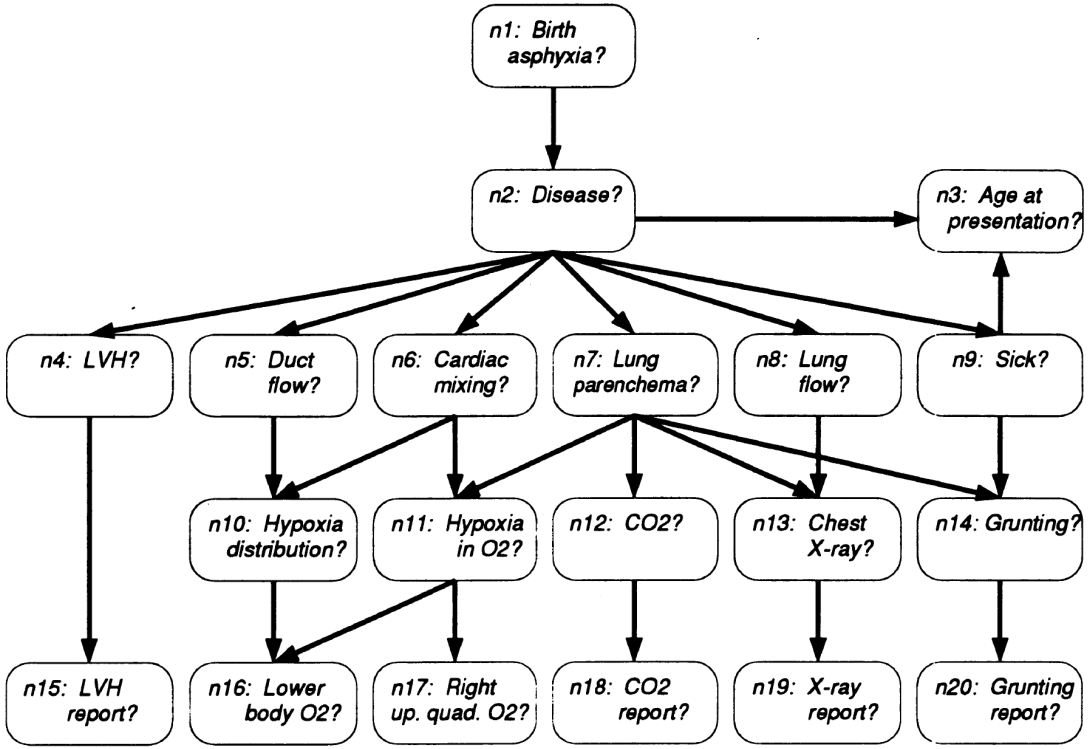


FIG. 2. *Directed acyclic graph representing the incidence and presentation of six possible diseases that would lead to a "blue" baby. LVH, left ventricular hypertrophy.*

Conditional Probability Tables

TABLE 1
Subjective assessments of conditional probability tables assessed by expert for links n2 → n4 and n4 → n15

	n4: LVH?	
n2: Disease?	Yes	No
PFC	0.10	0.90
TGA	0.10	0.90
Fallot	0.10	0.90
PAIVS	0.90	0.10
TAPVD	0.05	0.95
Lung	0.10	0.90

	n15: LVH-report?	
n4: LVH?	Yes	No
Yes	0.90	0.10
No	0.05	0.95

Visualization of Updated Beliefs on Every Node

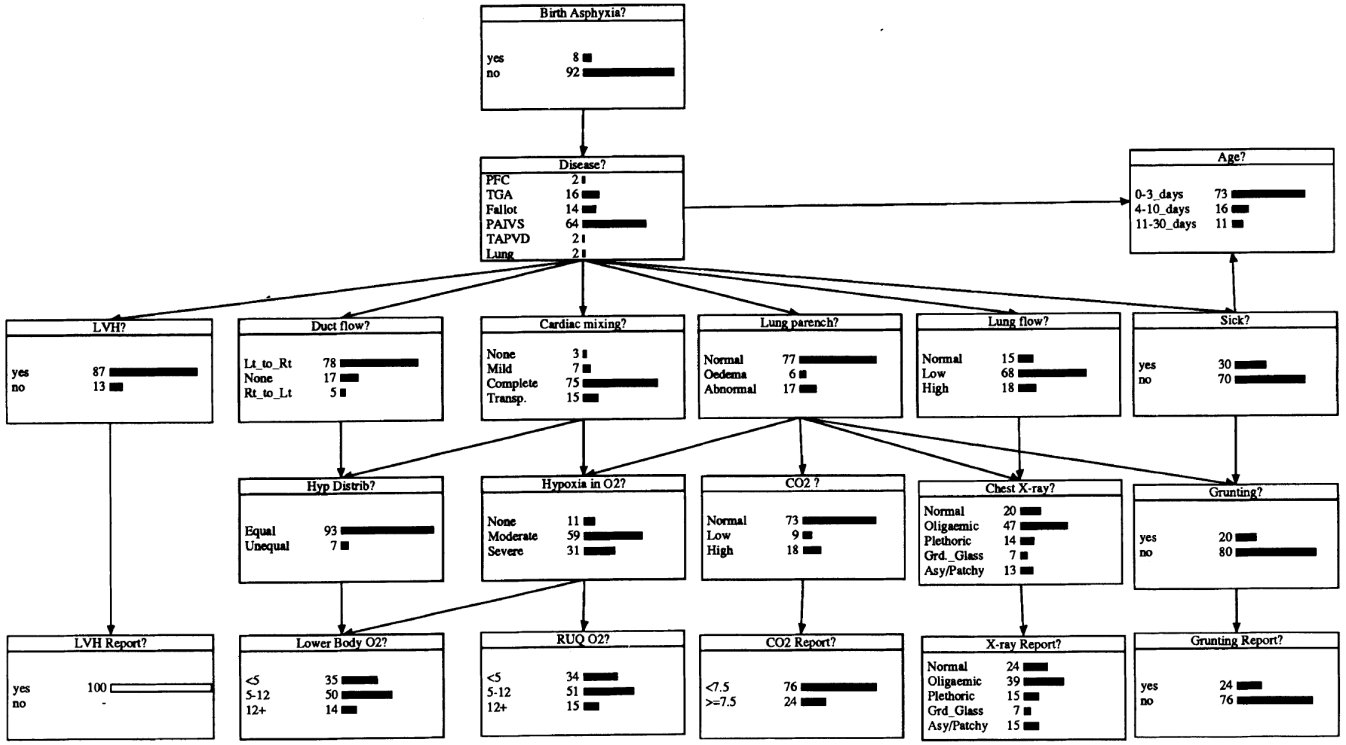


FIG. 3. *Conditional probability distributions on all nodes after propagation of evidence LVH-report = yes. The numbers and the length of the bars represent the current probability: for example, 64% belief that PAIVS is the true diagnosis, compared to a prior 22% belief. For observed evidence, that is, LVH-report = yes, the bar is hollow.*

Visualization of Updated Beliefs on Every Node

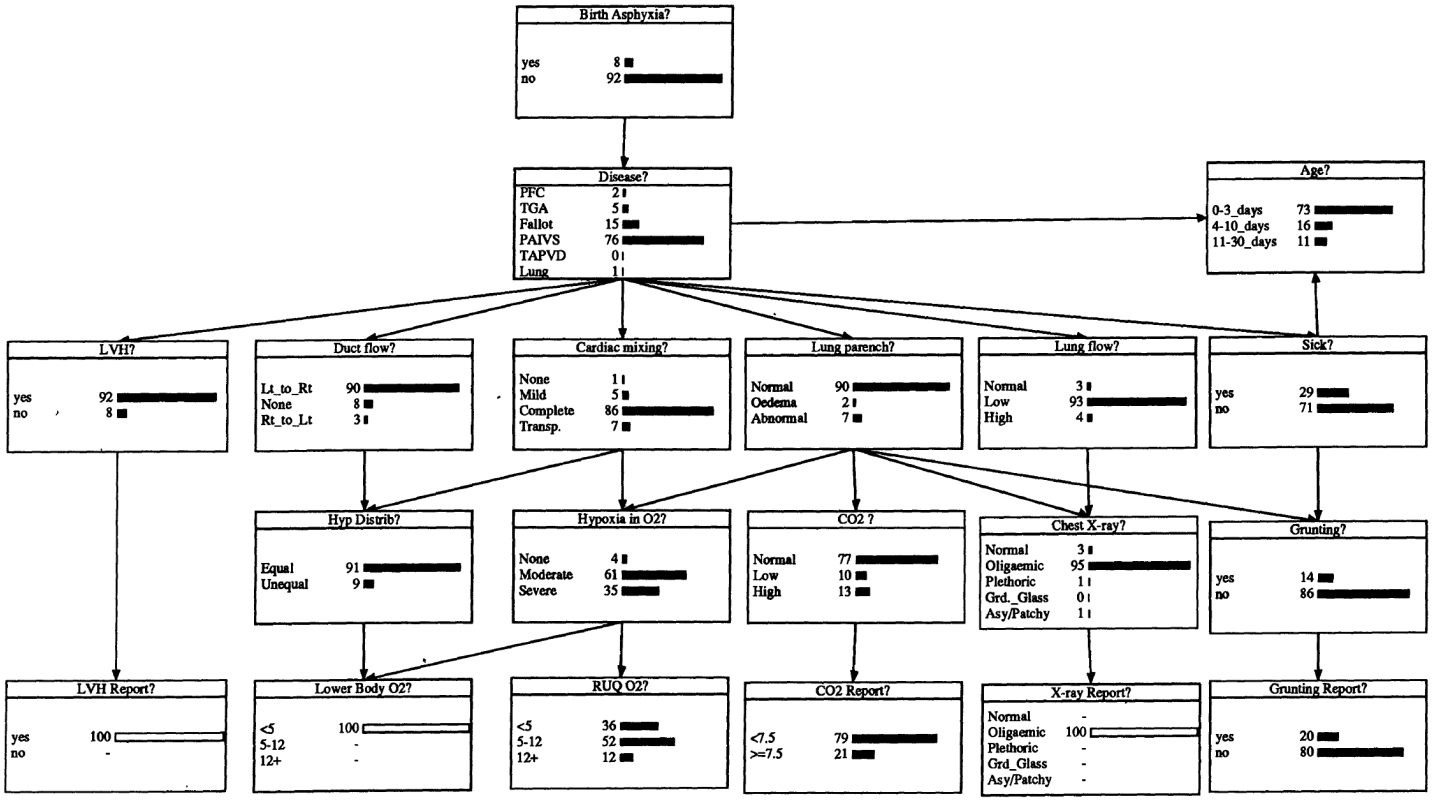


FIG. 4. *Status after propagation of additional evidence X-ray report = oligaemic and Lower body $O_2 < 5$.*

Moralize

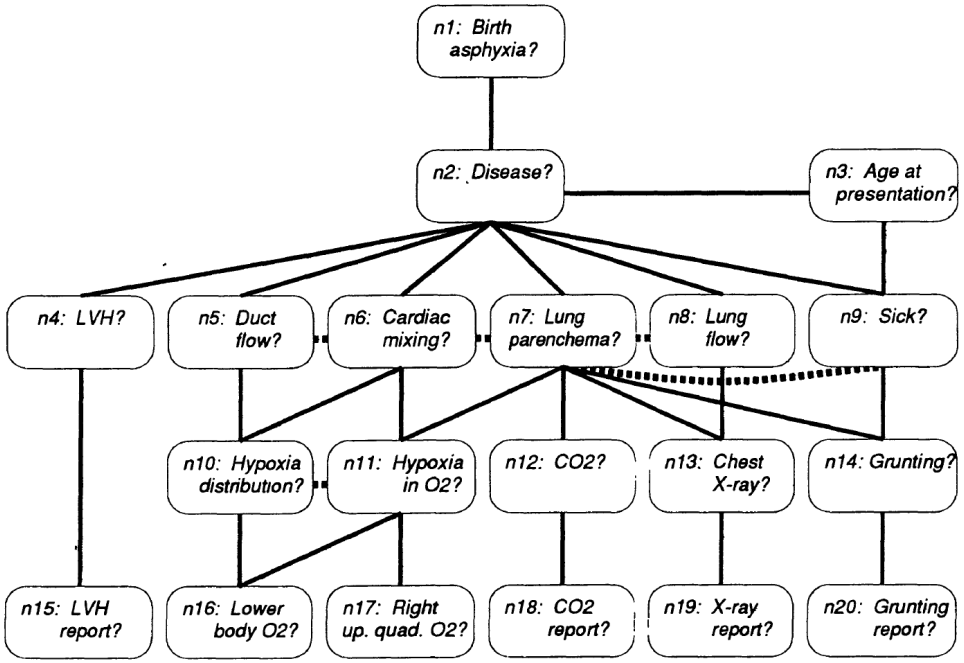


FIG. 9. *Moral graph formed from CHILD network by joining unconnected parents and dropping directions. The joint distribution of the variables is Markov with respect to this graph.*

Triangulation and Maximum Cardinality Search

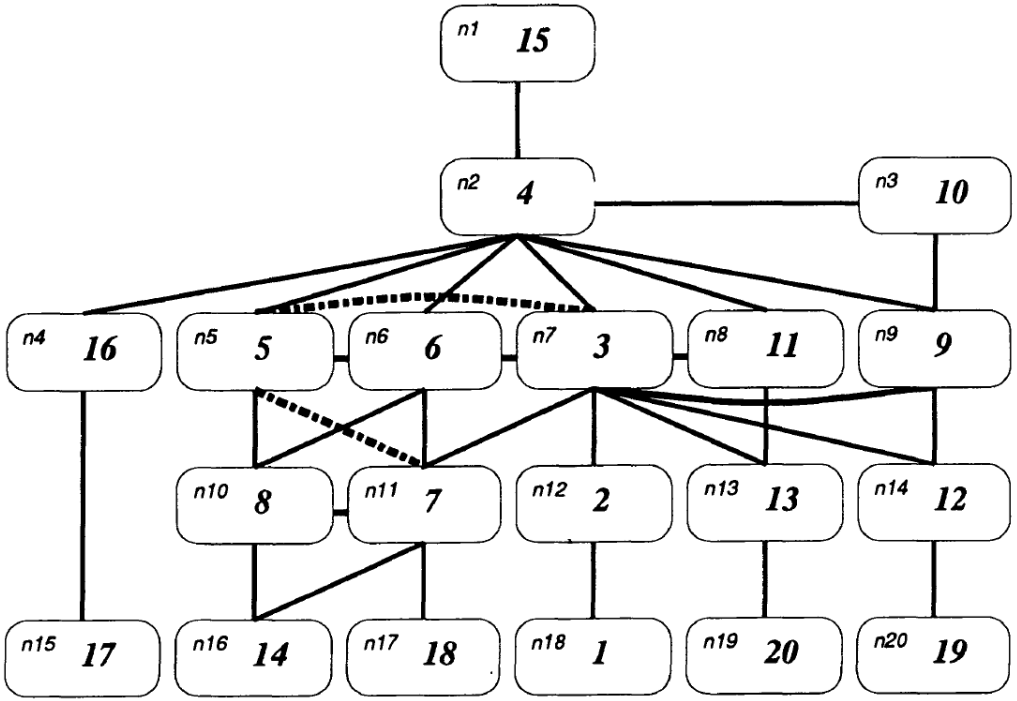


FIG. 11. *A perfect ordering of the nodes in CHILD arising from maximum cardinality search.*

Construct Junction Tree and Define Potentials on Maximal Cliques

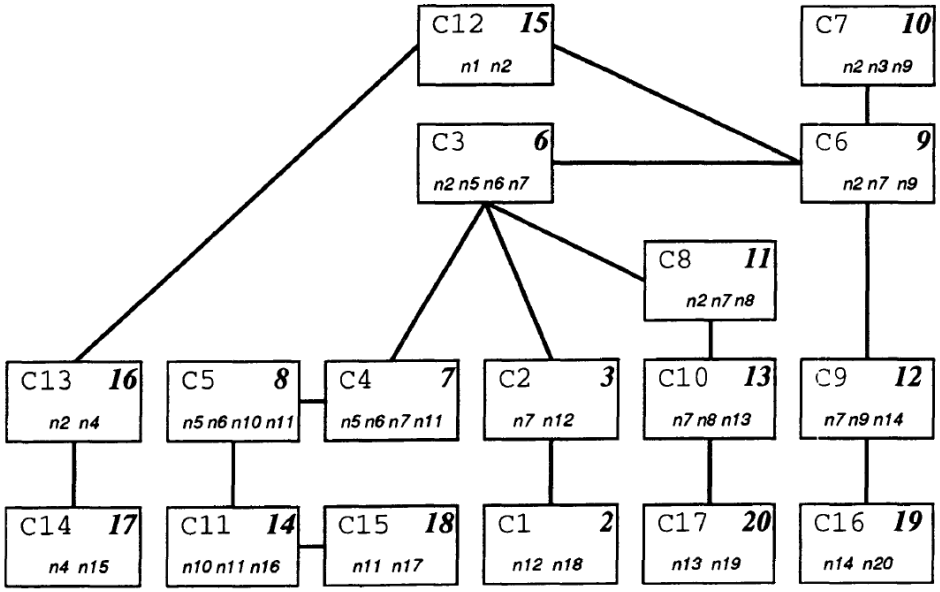
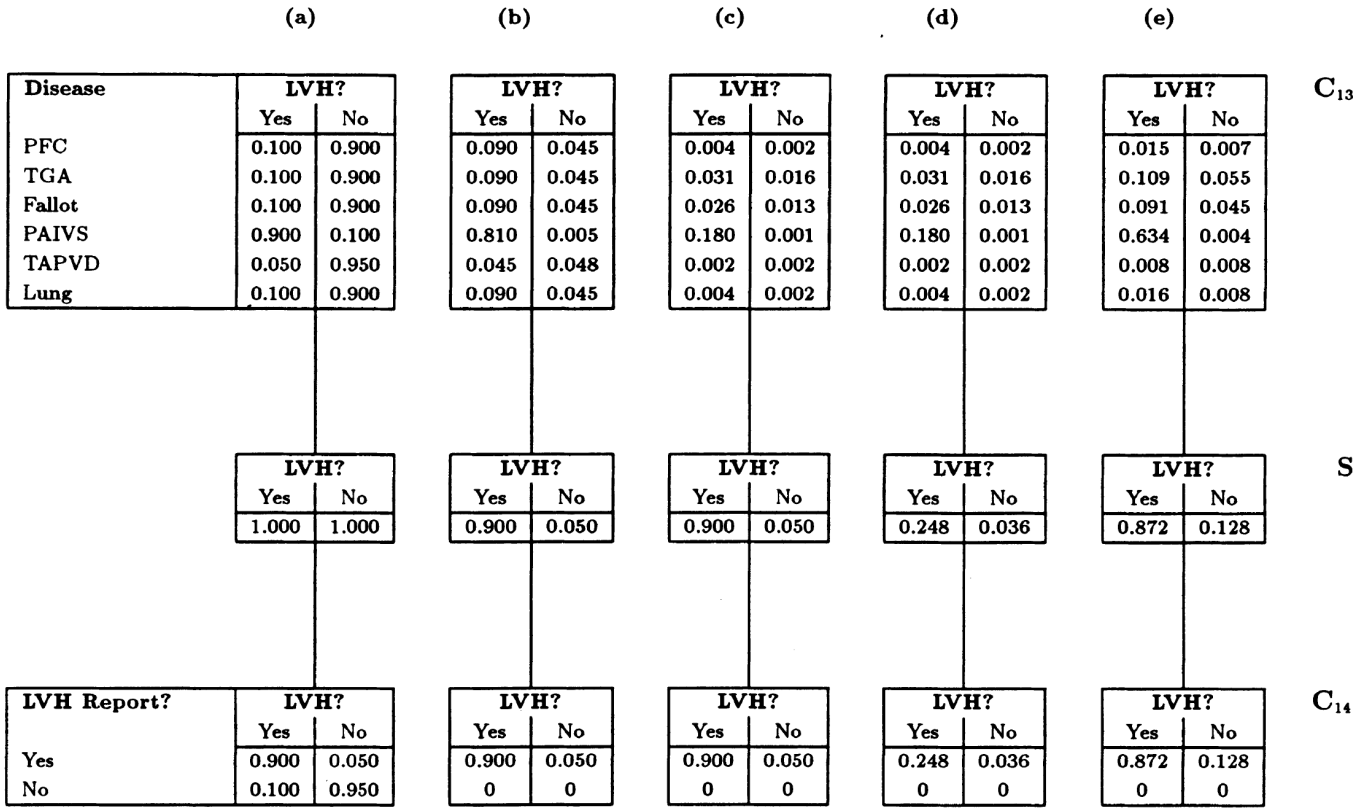


FIG. 12. *Junction tree of cliques derived from perfect ordering of the CHILD nodes. The members of each clique are shown, the highest label among the members is shown in the top right-hand corner, while the corresponding ordering of the cliques is shown in the top left-hand corner.*

Introduce Evidence and Propagate Probabilities



	(a)		(b)		(c)		(d)		(e)		
Disease	**LVH?**		**LVH?**		**LVH?**		**LVH?**		**LVH?**		C_{13}
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	
PFC	0.100	0.900	0.090	0.045	0.004	0.002	0.004	0.002	0.015	0.007	
TGA	0.100	0.900	0.090	0.045	0.031	0.016	0.031	0.016	0.109	0.055	
Fallot	0.100	0.900	0.090	0.045	0.026	0.013	0.026	0.013	0.091	0.045	
PAIVS	0.900	0.100	0.810	0.005	0.180	0.001	0.180	0.001	0.634	0.004	
TAPVD	0.050	0.950	0.045	0.048	0.002	0.002	0.002	0.002	0.008	0.008	
Lung	0.100	0.900	0.090	0.045	0.004	0.002	0.004	0.002	0.016	0.008	
	LVH?		**LVH?**		**LVH?**		**LVH?**		**LVH?**		S
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	
	1.000	1.000	0.900	0.050	0.900	0.050	0.248	0.036	0.872	0.128	
LVH Report?	**LVH?**		**LVH?**		**LVH?**		**LVH?**		**LVH?**		C_{14}
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	
Yes	0.900	0.050	0.900	0.050	0.900	0.050	0.248	0.036	0.872	0.128	
No	0.100	0.950	0	0	0	0	0	0	0	0	

FIG. 13. *Propagation of evidence through cliques C_{13} and C_{14} of junction tree*: (a) *initial potentials*, (b) *after incorporation of evidence LVH-report = yes*, (c) *after propagation through rest of network and back to C_{13}*, (d) *final potentials*, (e) *marginal tables after normalisation.*