Modelos e algoritmos

Problemas de roteamento

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Definição:

Dados um conjunto de cidades e as respectivas distâncias de deslocamento entre cada par delas, o problema consiste em achar a rota de menor comprimento que visite cada cidade e retorne ao ponto de origem.

Histórico: Maine (1925)

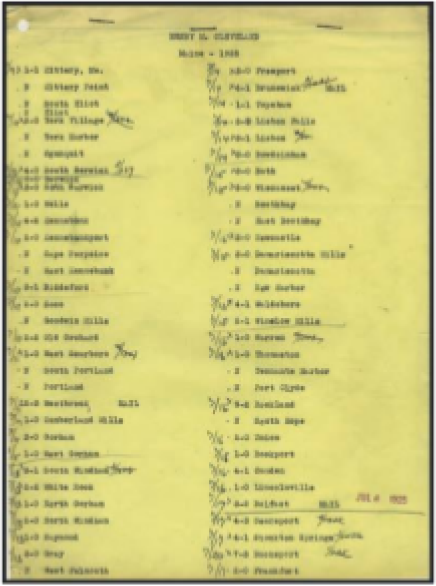
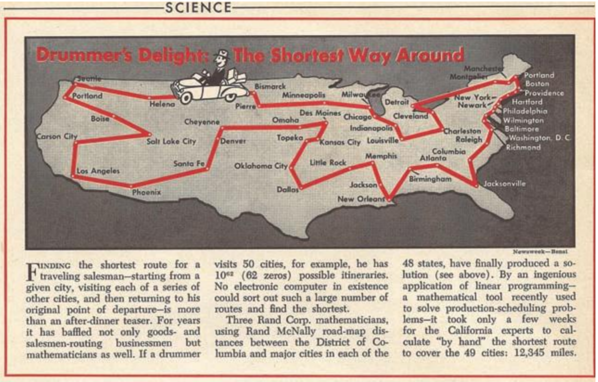


Figure 2.3

Rand McNally map cabinet and pin map. *Secretarial Studies*, 1922.

Desafio Procter & Gamble (1962)



FINDING the shortest route for a traveling salesman—starting from a given city, visiting each of a series of other cities, and then returning to his original point of departure—is more than an after-dinner teaser. For years it has baffled not only goods- and salesmen-routing businessmen but mathematicians as well. If a drummer visits 50 cities, for example, he has 10^{62} (62 zeros) possible itineraries. No electronic computer in existence could sort out such a large number of routes and find the shortest.

Three Rand Corp. mathematicians, using Rand McNally road-map distances between the District of Columbia and major cities in each of the 48 states, have finally produced a solution (see above). By an ingenious application of linear programming—a mathematical tool recently used to solve production-scheduling problems—it took only a few weeks for the California experts to calculate "by hand" the shortest route to cover the 49 cities: 12,345 miles.

Cálculos feitos a mão, duraram 17 anos até serem suplantados.

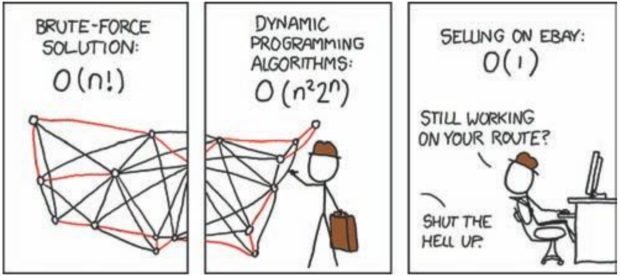
- *'It seems very likely that quite a different approach from any yet used may be required for successful treatment of the problem. In fact, there may well be no general method for treating the problem and impossibility results would also be valuable.'* —Merrill Flood, 1956.
- *'I conjecture that there is no good algorithm for the traveling salesman problem.'* —Jack Edmonds, 1967.
- *'In this paper we give theorems which strongly suggest, but do not imply, that these problems, as well as many others, will remain intractable perpetually.'* —Richard Karp, 1972.

- n : número de cidades
- $\frac{(n-1)!}{2}$: número de rotas com n cidades
  - $\frac{n}{1^o} \times \frac{(n-1)}{2^o} \times \ldots \times \frac{1}{n^o} = n!$
  - rotas idênticas (Ex: $n = 3$, $\{1,2,3\} = \{2,3,1\} = \{3,1,2\}$)
- $n^2 2^n$: melhor algoritmo teórico que resolve o problema (1962)

Tabela: Tempo de execução num computador com 10^9 operações por segundo

n	10	25	50	100
n^3	0.000001s	0.00002s	0.0001s	0.001s
2^n	0.000001s	0.03s	13 dias	40 trilhões de anos

Ainda há tempo!



De 49 a 1.904.711 cidades

- 49 : Dantzig, Fulkerson, e Johnson (1954) (manual)
- 64 : Michael Held e Richard Karp (1971)
- 80 : Panagiotis Miliotis (1975) (variante do Dantzig et al.)
- 120 : Martin Grötschel e Manfred Padberg (1977) (cidades da Alemanha)
- 318 : Harlan Crowder e Manfred Padberg (1978) (perfuração de placas eletrônicas)
- 3.038: Vašek Chvátal, William Cook, David Applegate, e Robert Bixby (1992) (perfuração de placas eletrônicas)
  - 13,509: cidades dos Estados Unidos (1998)
  - 24,978: cidades da Suécia (2004)
  - 85,900: perfuração de placa eletrônica (2006)
  - Solução computacional: Concorde (disponível na Internet)
- 1.904.711 : Keld Helsgaun (2010) (cidades do mundo) (comprimento 7.515.790.345m GAP 0.0476%)

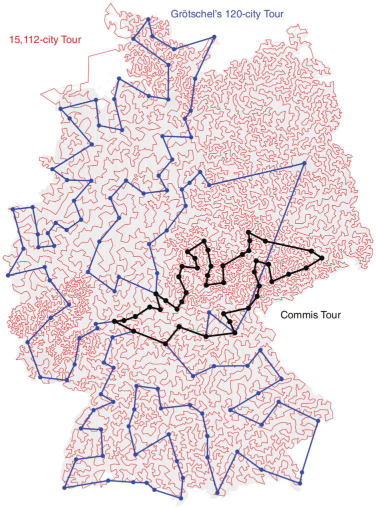


Figura: 15.122 cidades da Alemanha

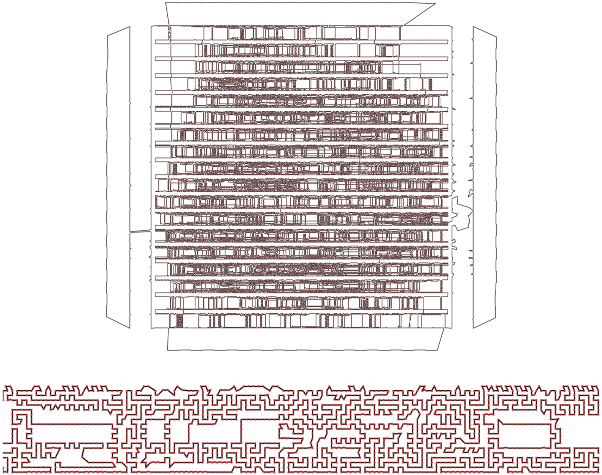


Figura: 85.900 pontos - chip de computador

Figura: 100.000 pontos - GAP de 0.003% - Prêmio de US$1.000,00

De 49 a 1.904.711 cidades

Figura: 1.904.711 cidades

De 49 a 1.904.711 cidades

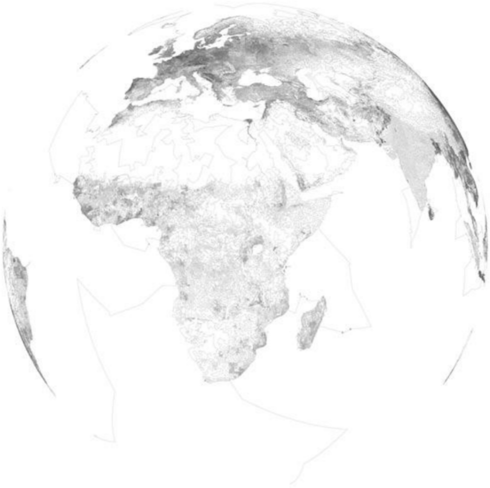


Figura: 1.904.711 cidades

William Cook

"...keep in mind that the real goal of problem-by-problem challenges is to gather ideas for use in general solution methods for the salesman, and beyond to application areas well outside the TSP. New avenues of attack are the name of the game. TSP remains essentially open. A new point of view could be just what is needed to dramatically alter our ability to tackle the salesman."

Outras aplicações do problema

- mapeamento de genomas
- alinhamento de telescópios, raio-x de alta precisão, lasers
- placas eletrônicas
- sequenciamento de máquinas