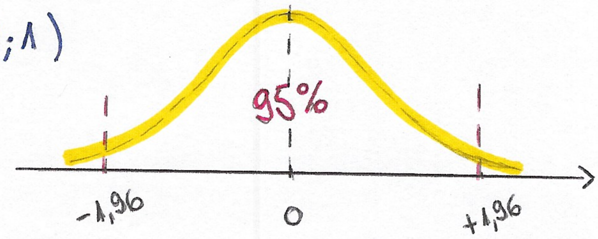
Capítulo 3 - O modelo de Regressão Linear Múltipla

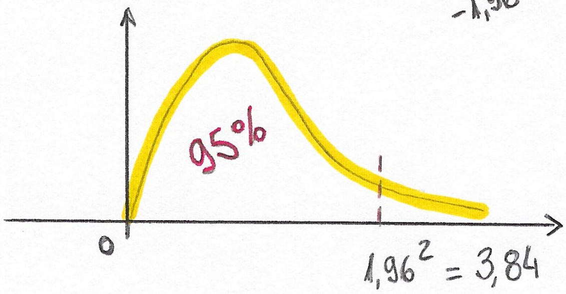
01/02/2020

$Y \sim N(\mu; \sigma^2)$ $\qquad \dfrac{Y-\mu}{\sigma} \sim N(0;1)$



uma propriedade interessante:

$$\left(\frac{Y-\mu}{\sigma}\right)^2 \sim \chi^2_1$$



$Y \sim \text{Normal}(\mu; \Sigma)$: $E(Y) = \mu$ $\quad Cov(Y) = \Sigma$

$$(Y-\mu)^T \Sigma^{-1} (Y-\mu) \sim \chi^2_n \quad \text{onde } Y_{n \times 1}$$

Exemplo: Vamos supor $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$, $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$, $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$

$Cov(Y_1, Y_2) = 0$

$$(Y-\mu)^T \Sigma^{-1} (Y-\mu) = \begin{bmatrix} Y_1-\mu_1 & Y_2-\mu_2 \end{bmatrix} \begin{bmatrix} 1/\sigma_1^2 & 0 \\ 0 & 1/\sigma_2^2 \end{bmatrix} \begin{bmatrix} Y_1-\mu_1 \\ Y_2-\mu_2 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{Y_1-\mu_1}{\sigma_1^2} & \dfrac{Y_2-\mu_2}{\sigma_2^2} \end{bmatrix} \begin{bmatrix} Y_1-\mu_1 \\ Y_2-\mu_2 \end{bmatrix}$$

$$= \left(\frac{Y_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{Y_2-\mu_2}{\sigma_2}\right)^2 \sim \chi^2_2$$

assumindo $Y \sim N(\mu; \Sigma)$

Na forma matricial, trabalhamos com a forma $(Y-\mu)^T \Sigma^{-1} (Y-\mu)$

Generalizando:

$$\hat{\beta} \sim \text{Normal}\left(\beta\,;\,\underbrace{\sigma^2 (X^T X)^{-1}}_{\Sigma}\right) \quad \text{então} \quad (\hat{\beta}-\beta)^T \left[\sigma^2 (X^T X)^{-1}\right]^{-1} (\hat{\beta}-\beta) \sim \chi^2_p$$

ou

$$\frac{(\hat{\beta}-\beta)^T (X^T X)(\hat{\beta}-\beta)}{\sigma^2} \sim \chi^2_p$$

$$\frac{SQ_{Res}}{\sigma^2} \sim \chi^2_{n-p}$$

ver demonstração na pg. 61

Pg. 65: Intervalos de confiança utilizando as propriedades do vetor $\hat{\beta}$

$$\frac{\dfrac{(\hat{\beta}-\beta)^T X^T X (\hat{\beta}-\beta)}{p \cdot \sigma^2}}{\dfrac{SQ_{Res}}{\sigma^2 (n-p)}} \sim \frac{\dfrac{\chi^2_p}{p}}{\dfrac{\chi^2_{n-p}}{n-p}} \sim F_{p,\,n-p}$$

3.11 Colinearidade e multicolinearidade (uma demonstração prática)

Detalhes sobre Transformações de Variáveis Aleatórias:

$X \sim N(\mu; \sigma^2)$: $\dfrac{X-\mu}{\sigma} = Z \sim N(0,1)$

$Z^2 \sim \chi^2_1$ $\qquad \sum_{i=1}^{n} Z_i^2 \sim \chi^2_n$ $\qquad \sum_{i=1}^{p} Z_i \sim \chi^2_p$

$$\frac{\dfrac{\chi^2_p}{p}}{\dfrac{\chi^2_n}{n}} \sim F_{p,n}.$$