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O estimador Ridge - Regression

ou Penalização do tipo ℓ_2

$$\tilde{\beta} = \arg\min_{\beta} (Y - X\beta)^T (Y - X\beta)$$

sujeito a : $\sum_{i=1}^{p} \beta_i^2 \leq c$ ou $\beta^T \beta \leq c$

$$J(\beta, \lambda) = (Y - X\beta)^T (Y - X\beta) + \lambda(\beta^T I \beta - c)$$

$$\frac{\partial J}{\partial \beta} = -2X^T(Y - X\tilde{\beta}) + 2\lambda\tilde{\beta} = 0$$

$$\tilde{\beta} = (X^T X + \lambda I)^{-1} X^T Y$$

Propriedades :

a) $E(\tilde{\beta}) = (X^T X + \lambda I)^{-1} X^T X \beta$

b) $Var(\tilde{\beta}) = \alpha^2 (X^T X + \lambda I)^{-1} X^T X (X^T X + \lambda I)^{-1}$

c) $H(\lambda) = X(X^T X + \lambda I)^{-1} X^T$

$$PRESS = \sum_{i=1}^{n} (y_i - \hat{y}_{(i)})^2 = \sum_{i=1}^{n} \left(\frac{r_i}{1 - h_{ii}} \right)^2$$

então, podemos encontrar um estimador pontual p/ λ na forma :

$$\hat{\lambda} = \arg\min_{\lambda} \sum_{i=1}^{r} \left(\frac{r_i(\lambda)}{1 - h_{ii}(\lambda)} \right)$$

vamos ver se funciona ...

Estimador LASSO :

$$\boxed{\text{Sujeito a : } \sum_{i=1}^{p} |\beta_i| \leq c}$$

spoiler ...