

## Contents

<b>1</b>	<b>Graph</b>	<b>2</b>
1.1	2SAT	2
1.2	BFS Zero One	3
1.3	Binary Lifting	4
1.4	Boruvka MST	5
1.5	Center Of A Tree	6
1.6	Diameter And Center Of A Tree	6
1.7	Dijkstra	7
1.8	Dinic	7
1.9	Erdos Gallai Theorem	8
1.10	Eulirian Path	8
1.11	Floyd Sucessor Graph	9
1.12	Ford Fulkerson	9
1.13	Fully Dynamic Connectivity Check If Two Vertices Are In The Same Component	10
1.14	Fully Dynamic Connectivity Count Conected Components	11
1.15	Hopcroft Karp	13
1.16	K Short Paths	13
1.17	Knapsack Dijkstra	14
1.18	Kuhn MCBM	14
1.19	Lca With Square Root Decomposition	15
1.20	Lca With Tree Linearization And Segment Tree	15
1.21	Lca With Tree Linearization And Sparse Table	16
1.22	Longest And Shortest Path In DAG	17
1.23	MCE MinimumEdgeCover	17
1.24	MPC MinimumPathCover	18
1.25	MVC MinimumVertexCover	18
1.26	Maximum Clique	19
1.27	Min Cost Max Flow	19
1.28	Prim	20
1.29	Tree Isomorfism	21
<b>2</b>	<b>Data Structures</b>	<b>22</b>
2.1	BIT 1D	22
2.2	BIT 2D	22
2.3	BIT Range Sum And Range Update	23
2.4	Centroid Decomposition	23
2.5	Color Update	24
2.6	DSU With Partial Persistence	25
2.7	Dynamic Segment Tree With Lazy Propagation	25
2.8	Dynamic Segment Tree With Vector	26
2.9	Heavy Light Decomposition Path And Subtree Queries	27
2.10	Heavy Light Decomposition	29
2.11	Implicit Treap	31
2.12	LiChao Tree	32
2.13	Max Queue	33
2.14	Merge Sort Tree Iterative	34
2.15	Merge Sort Tree Range Order Statistics Queries	34
2.16	Merge Sort Tree With Set	35
2.17	Merge Sort Tree	35
2.18	Ordered Set With BIT	36
2.19	PBDS	37
2.20	Persistent Segment Tree Without Vector	37
2.21	Persistent Segment Tree	38
2.22	Segment Tree Iterative	39
2.23	Segment Tree Tree 2D	40
2.24	Segment Tree With Lazy Propagation	41
2.25	Sparse Table RMQ	41
2.26	Treap	42
2.27	TreelsomorfismWithMap	43
2.28	TreelsomorfismWithPolynomialHashing	44
2.29	Two Stacks Trick	45
2.30	Wavelet Tree	46
<b>3</b>	<b>Dynamic Programming</b>	<b>47</b>
3.1	Coin Problem Topdown Dp	47
3.2	Digit DP Sum Of Digits In Range	47
3.3	Edit Distance With DP	48
3.4	Kadane 2D	48
3.5	Kadane 3D	48
3.6	Knapsack With Copies SqrtN Memory	49
3.7	Knapsack With Repetition Recurrence	50
3.8	Knapsack Zero One Without Value	50
3.9	Knapsack0-kSemValor	51
3.10	KnapsackErrichto	51
3.11	KnapsackWithCopies	51
3.12	KnapsackwithPDtopdown	52
3.13	Longest Common Subsequece And Edit Distance	52
3.14	Longest Increasing Subsequence	52
3.15	Subset Sum	53
3.16	Traveling Salesman Problem Bottom Up Dp	53
3.17	Traveling Salesman Problem Topdown Dp	54
<b>4</b>	<b>String</b>	<b>54</b>
4.1	Aho Corasick	54
4.2	Dynamic Trie	55
4.3	KMP	56
4.4	LIS LDS	57
4.5	Longest Common Substring	57
4.6	Manacher	57
4.7	SA	58
4.8	Suffix Array And Applications	60
4.9	Suffix Array	62
4.10	Trie Static	63
4.11	Trie With Vector	64
4.12	Trie	64
4.13	Z Function	65
<b>5</b>	<b>Math</b>	<b>65</b>
5.1	Baby Step Giant Step	65
5.2	Catalan Numbers	66
5.3	Chinese Remainder Theorem	66
5.4	Conversion Base	67
5.5	Counting Number Of Times That A Digit Appears Until N	67
5.6	Fast Fourier Transform	68
5.7	Gaussian Elimination For Max Subset Xor	72
5.8	Karatsuba	72
5.9	Matrix Exponentiation	73
5.10	Miller Rabin	74
5.11	Mobius	74
5.12	Mod Gaussian Elimination	75
5.13	Mod Integer	75
5.14	Modular Arithmetic	76
5.15	Mulmod Trick	77
5.16	Pollard Rho	77
<b>6</b>	<b>Geometry</b>	<b>77</b>
6.1	Andrew Algorithm Convex Hull	77
6.2	Build Two Lines That Go Through All Points Of A Set	78
6.3	Check If A Point Is Inside A Convex Polygon	78
6.4	Convex Hull Trick	79
6.5	Distance Between Nearest Pair Of Points	80
6.6	Dynamic Convex Hull Trick	81
6.7	Enclosing Circle R2	82
6.8	Enclosing Circle R3	82
6.9	Geometry Stan	83
6.10	Graham Scan	84
6.11	Maximum Dot Product	85
6.12	Radial Sort	86
6.13	Segment Intersection	87
<b>7</b>	<b>Miscellaneous</b>	<b>87</b>
7.1	Big Num Product	87
7.2	Count Divisors	87
7.3	Count Sort	88

7.4	Counting Different Elements In A Path With Mo	88
7.5	Custom Hash Function Unordered Map Or Set	90
7.6	Divide Conquer Optimization	90
7.7	FastIO	91
7.8	Fence Problem With Max Flow	91
7.9	Gen Random Tree	93
7.10	Histogram	93
7.11	Inclusion Exclusion	93
7.12	Index Compression	94
7.13	Karp Rabin	94
7.14	Knapsack With Backtracking	95
7.15	Knuth Optimization	95
7.16	Longest Substring That Is A Correct Bracket Sequence	96
7.17	Maximum Subarray XOR	97
7.18	Mo	98
7.19	Odd Rectangles Area	100
7.20	Quick Sort And Select	102
7.21	Rectangles Union Area	102
7.22	Small To Large	103
7.23	Square Root Decomposition	106
7.24	String Matching Hash Sqrtdecomp	106
8	Useful Scripts	107
8.1	Brute	107
8.2	Gen Tree2	107
8.3	Gen Tree	108
8.4	Gen	108
8.5	Makefile	108
8.6	Sol	108
8.7	Stress Test Animated	108
8.8	Stress With Checker	108
8.9	Stress	109

# 1 Graph

## 1.1 2SAT

```
// Os vertices pares indicam as proposicoes falsas
// Os vertices impares indicam as proposicoes verdadeiras
// Achar qual proposicao relativa a cada vertice, eh so dividir
// vertice/2
// tamG = quantidade_proposicoes*2
```

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e3;

int n, m, tamG;
vector<int> G[MAX], G_t[MAX], C[MAX];
stack<int> sta;
bool cor[MAX];
int componente[MAX], comp;
```

```
void preenche(int v)
{
    cor[v] = true;
    for(const int &u : G_t[v])
        if(!cor[u])
            preenche(u);
    sta.push(v);
}
```

```
void dfs(int v, int comp)
```

```
{
    componente[v] = comp;
    C[comp].push_back(v);
    for(const int &u : G[v])
        if(!componente[u])
            dfs(u, comp);
}

void kosaraju()
{
    memset(cor, false, sizeof(cor));
    for(int i = 0; i < tamG; i++)
        if(!cor[i])
            preenche(i);
    memset(cor, false, sizeof(cor));
    comp = 1;
    while(!sta.empty())
    {
        int u = sta.top();
        sta.pop();
        if(componente[u]) continue;
        dfs(u, comp);
        comp++;
    }
}
```

```
// Id no grafo que representa a proposicao de numero P como verdadeira
```

```
int idTrue(int p)
{
    return (p << 1) + 1;
}
```

```
// Id no grafo que representa a proposicao de numero P como falsa.
```

```
int idFalse(int p)
{
    return (p << 1);
}
```

```
bool twoSat()
{
    kosaraju();
    for(int i = 0; i < tamG; i+=2)
    {
        // Todo par de proposicoes(proposicao falsa, proposicao verdadeira)
        // Nao podem estar no mesmo componente
        if(componente[i] == componente[i + 1])
            return false;
    }
    return true;
}
```

```
int addEdge(int u, int v)
{
    G[idFalse(u)].push_back(idTrue(v));
    G[idFalse(v)].push_back(idTrue(u));
    G[idTrue(u)].push_back(idFalse(v));
    G[idTrue(v)].push_back(idFalse(u));
    // montar grafo transposto para kosaraju nessa
```

```

// aplicacao o grafo G sera igual ao transposto
G_t[idFalse(u)].push_back(idTrue(v));
G_t[idFalse(v)].push_back(idTrue(u));
G_t[idTrue(u)].push_back(idFalse(v));
G_t[idTrue(v)].push_back(idFalse(u));
}

vector<int> g[MAX];
vector<int> ts;
int value[MAX];

void topSort(int v)
{
    cor[v] = true;
    for(int &u : G[v])
        if(!cor[u])
            topSort(u);
    ts.push_back(v);
}

void mountDAG()
{
    for(int v = 0; v < tamG; v++)
        for(int &u : G[v])
            if(componente[v] != componente[u])
                g[componente[v]].push_back(componente[u]);
    memset(cor, false, sizeof(cor));
    for(int v = 1; v < comp; v++)
        if(!cor[v])
            topSort(v);
    // nao inverter ts, pois precisamos da ordenacao
    // topologica ao contrario
}

// encontrar uma atribuicao (TREU ou FALSE) para as proposicoes
void assignment()
{
    if(!twoSat()) return;
    mountDAG();
    memset(value, -1, sizeof(value));
    for(int &v : ts)
        for(int &u : C[v])
            if(value[u >> 1] == -1) // u / 2 eh a proposicao
                value[u >> 1] = (u & 1 ? 1 : 0);
    for(int i = 0; i < (tamG >> 1); i++)
        cout << value[i] << ' ';
    puts("");
}

int main()
{
    cin >> n >> m;
    tamG = 2 * n;
    for(int i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        addEdge(u, v);
    }
}

```

```

cout << twoSat() << '\n';
assignment();

return 0;
}

```

## 1.2 BFS Zero One

```

// o peso das arestas eh 0 ou 1

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;
const int OO = 0x3f3f3f3f;
typedef pair<int, int> ii;

int n, m;
vector<ii> G[MAX];
int dist[MAX];
deque<int> dq;

void zeroOneBfs(int v)
{
    memset(dist, 63, sizeof(dist));
    dist[v] = 0;
    dq.push_back(v);
    while(!dq.empty())
    {
        int u = dq.front();
        dq.pop_front();
        for(int i = 0; i < G[u].size(); i++)
        {
            int w = G[u][i].first, d = G[u][i].second;
            if(dist[w] > dist[u] + d)
            {
                dist[w] = dist[u] + d;
                if(!d) dq.push_front(w);
                else dq.push_back(w);
            }
        }
    }
    for(int i = 0; i < n; i++)
        cout << dist[i] << ' ';
    puts("");
}

int main()
{
    cin >> n >> m;
    while(m--)
    {
        int u, v, w;
        cin >> u >> v >> w; u--; v--;
        G[u].push_back({v, w});
        G[v].push_back({u, w});
    }
    zeroOneBfs(0);

    return 0;
}

```

## 1.3 Binary Lifting

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

int n, m, nivel[MAX], anc[MAX][30], MAX_LOG;
vector<int> G[MAX];

void dfs(int v, int p, int d)
{
    anc[v][0] = p;
    nivel[v] = d;
    if(d) MAX_LOG = max(MAX_LOG, (int)log2(d));
    for(const int &u : G[v])
        if(u != p)
            dfs(u, v, d + 1);
}

int walk(int v, int k)
{
    while(k) v = anc[v][(int)log2(k&-k)], k -= k&-k;;
    return v;
}

int lca(int u, int v)
{
    if(nivel[u] < nivel[v]) v = walk(v, nivel[v]-nivel[u]);
    if(nivel[u] > nivel[v]) u = walk(u, nivel[u]-nivel[v]);
    if(u == v) return u;
    for(int i = MAX_LOG; i >= 0; i--)
        if(anc[u][i] != anc[v][i])
        {
            u = anc[u][i];
            v = anc[v][i];
        }
    return anc[u][0];
}

void build()
{
    memset(anc, -1, sizeof anc);
    nivel[0] = 0;
    dfs(0, -1, 0);
    for(int j = 1; j <= MAX_LOG; j++)
        for(int i = 1; i <= n; i++)
            if(anc[i][j-1] != -1)
                anc[i][j] = anc[anc[i][j-1]][j-1];
}

int main()
{
    int u, v;

    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v;
        u--; v--;
        G[u].push_back(v);
    }
}
```

```
G[v].push_back(u);
}
build();
cin >> u >> v;
cout << lca(u-1, v-1)+1 << '\n';

return 0;
}

////////////////////////////////////

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;
typedef pair<int, int> ii;

int n, m, max_log;
vector<ii> G[MAX];
int anc[MAX][30], min_edge[MAX][30], depth[MAX];

void dfs(int v, int d, int p, int a)
{
    anc[v][0] = p;
    depth[v] = d;
    if(d) max_log = max(max_log, (int)log2(d));
    if(p != -1) min_edge[v][0] = a;
    for(int i = 0; i < G[v].size(); i++)
    {
        int u = G[v][i].second, w = G[v][i].first;
        if(u != p)
            dfs(u, d + 1, v, w);
    }
}

void build()
{
    memset(anc, -1, sizeof(anc));
    memset(min_edge, 63, sizeof(min_edge));
    dfs(0, 0, -1, -1);
    for(int j = 1; j <= max_log; j++)
        for(int i = 0; i < n; i++)
        {
            if(anc[i][j-1] != -1)
            {
                anc[i][j] = anc[anc[i][j-1]][j-1];
                min_edge[i][j] = min(min_edge[i][j-1], min_edge[anc[i][j-1]][j-1]);
            }
        }
}

int walk(int v, int k)
{
    while(k) v = anc[v][(int)log2(k&-k)], k -= k&-k;
    return v;
}

int lca(int u, int v)
{
}
```

```

    if(depth[u] > depth[v]) u = walk(u, depth[u]-depth[v]);
    if(depth[u] < depth[v]) v = walk(v, depth[v]-depth[u]);
    if(u == v) return u;
    for(int i = max_log; i >= 0; i--)
        if(anc[u][i] != anc[v][i])
        {
            u = anc[u][i];
            v = anc[v][i];
        }
    return anc[u][0];
}

int queryMinEdge(int u, int v)
{
    int LCA = lca(u, v);
    int ans = INT_MAX;
    int k = depth[u]-depth[LCA];
    while(k)
    {
        ans = min(ans, min_edge[u][ (int)log2(k&-k) ]);
        u = walk(u, k&-k);
        k -= k&-k;
    }
    k = depth[v]-depth[LCA];
    while(k)
    {
        ans = min(ans, min_edge[v][ (int)log2(k&-k) ]);
        v = walk(v, k&-k);
        k -= k&-k;
    }
    return ans;
}

int main()
{
    int u, v, w;

    cin >> n;
    for(int i = 0; i < n-1; i++)
    {
        cin >> u >> v >> w;
        u--; v--;
        G[u].push_back({w, v});
        G[v].push_back({w, u});
    }
    cin >> u >> v;
    build();
    cout << lca(u-1, v-1)+1 << '\n';
    cout << queryMinEdge(u-1, v-1) << '\n';

    return 0;
}

```

## 1.4 Boruvka MST

```

#include <bits/stdc++.h>
using namespace std;

int n, m;
vector<array<int, 3>> edge;

```

```

int pai[100100], sz[100100];

int find(int x)
{
    return pai[x] == x ? x : pai[x] = find(pai[x]);
}

void join(int x, int y)
{
    x = find(x);
    y = find(y);
    if(x == y) return;
    if(sz[x] > sz[y]) swap(x, y);
    pai[x] = y;
    sz[y] += sz[x];
}

int main()
{
    scanf(" %d %d", &n, &m);
    for(int i = 0; i < m; i++)
    {
        int u, v, w;
        scanf(" %d %d %d", &u, &v, &w); u--; v--;
        edge.push_back({w, v, u});
    }
    for(int i = 0; i < n; i++)
        pai[i] = i, sz[i] = 1;
    int mst_cost = 0;
    bool fl = true;
    while(fl)
    {
        fl = false;
        vector<int> aux(n, -1);
        for(int i = 0; i < m; i++)
        {
            int u = find(edge[i][1]), v = find(edge[i][2]), w = edge[i][0];
            if(u == v) continue;
            if(aux[u] == -1) aux[u] = i;
            else if(edge[aux[u]][0] > w) aux[u] = i;
            if(aux[v] == -1) aux[v] = i;
            else if(edge[aux[v]][0] > w) aux[v] = i;
        }
        for(int i = 0; i < n; i++)
        {
            if(aux[i] == -1) continue;
            int u = find(edge[aux[i]][1]), v = find(edge[aux[i]][2]);
            if(u == v) continue;
            // add edge edge[aux[i]][1] --- edge[aux[i]][2] in the MST
            join(u, v);
            mst_cost += edge[aux[i]][0];
            fl = true;
        }
    }
    cout << mst_cost << '\n';

    return 0;
}

```

## 1.5 Center Of A Tree

```
#include <bits/stdc++.h>
const int MAX = 1e5;
using namespace std;

int n, degree[MAX];
vector<int> G[MAX];
bool vis[MAX];

int findCenter()
{
    queue<int> fila[2];
    for(int i = 0; i < n; i++)
        if(degree[i] == 1)
            fila[0].push(i);
    int cnt = 0, turn = 0;
    while(cnt + 2 < n)
    {
        while(!fila[turn].empty())
        {
            int u = fila[turn].front(); fila[turn].pop();
            vis[u] = true;
            cnt++;
            for(int i = 0; i < G[u].size(); i++)
                if(!vis[G[u][i]])
                {
                    degree[G[u][i]]--;
                    if(degree[G[u][i]] == 1)
                        fila[1-turn].push(G[u][i]);
                }
        }
        turn ^= 1;
    }
    cout << "the set of central vertices\n";
    for(int i = 0; i < n; i++)
        if(!vis[i])
            cout << i + 1 << '\n';
}

int main()
{
    cin >> n;
    for(int i = 1; i < n; i++)
    {
        int u, v;
        scanf("%d %d", &u, &v); u--; v--;
        G[u].push_back(v);
        G[v].push_back(u);
        degree[u]++;
        degree[v]++;
    }
    findCenter();

    return 0;
}
```

## 1.6 Diameter And Center Of A Tree

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

int n;
vector<int> G[MAX];

int bfs(int v, vector<int> &dist)
{
    queue<int> q;
    q.push(v);
    dist[v] = 0;
    int lgv = -1;
    while(!q.empty())
    {
        int u = q.front(); q.pop();
        lgv = u;
        for(int &w : G[u])
            if(dist[w] == -1)
            {
                dist[w] = dist[u] + 1;
                q.push(w);
            }
    }
    return lgv;
}

void findCenterAndDiameter(int w)
{
    vector<int> dist1(n + 1, -1);
    vector<int> dist2(n + 1, -1);
    int v = bfs(w, dist1);
    int u = bfs(v, dist2);
    int d = dist2[u];
    dist1.assign(n + 1, -1);
    u = bfs(u, dist1);
    cout << "center ";
    for(int i = 0; i < n; i++)
    {
        int d1 = dist1[i], d2 = dist2[i];
        if(d1 == d / 2 and d2 == d - d / 2 or d2 == d / 2 and d1 == d - d / 2)
            cout << i + 1 << ' ';
    }
    cout << "\ndiameter " << d << '\n';
}

int main()
{
    cin >> n;
    for(int i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        G[u].push_back(v);
        G[v].push_back(u);
    }
    findCenterAndDiameter(0);
}
```

```
return 0;
```

```
}
```

## 1.7 Dijkstra

```
#include <bits/stdc++.h>
using namespace std;
const int OO = 0x3f3f3f3f;
const int MAX = 1e5;

typedef pair<int, int> ii;

int n, m;
int dist[MAX];
vector<ii> G[MAX];

int dijkstra(int v, int z)
{
    memset(dist, 63, sizeof(dist));
    dist[v] = 0;
    priority_queue<ii> pq;
    pq.push({0, v});
    while(!pq.empty())
    {
        int u = pq.top().second;
        int d = -pq.top().first;
        pq.pop();
        if(d > dist[u]) continue;
        if(u == z) return d;
        for(int i = 0; i < G[u].size(); i++)
        {
            int w = G[u][i].second, _d = G[u][i].first;
            if(dist[w] > d + _d)
            {
                dist[w] = d + _d;
                pq.push({-dist[w], w});
            }
        }
    }
    return OO;
}

int main()
{
    int u, v, w;

    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v >> w;
        u--; v--;
        G[u].push_back({w, v});
        G[v].push_back({w, u});
    }
    cin >> u >> v;
    cout << dijkstra(u-1, v-1) << '\n';

    return 0;
}
```

## 1.8 Dinic

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e4;
const int OO = 0x3f3f3f3f;

struct edge
{
    int v, f, c;
    edge(){}
    edge(int _v, int _f, int _c)
    {
        v = _v, f = _f, c = _c;
    }
};

vector<edge> edges;
vector<int> G[MAX];
int dist[MAX], work[MAX];

void add_edge(int u, int v, int cp, int rc) {
    edges.push_back(edge(v, 0, cp));
    G[u].push_back(edges.size()-1);
    edges.push_back(edge(u, 0, rc));
    G[v].push_back(edges.size()-1);
}

bool bfs(int s, int t)
{
    memset(dist, -1, sizeof(dist));
    dist[s] = 0;
    queue<int> q;
    q.push(s);
    while(!q.empty())
    {
        int u = q.front();
        q.pop();
        for(int e : G[u])
            if(dist[edges[e].v] == -1 and edges[e].c-edges[e].f > 0)
            {
                q.push(edges[e].v);
                dist[edges[e].v] = dist[u] + 1;
            }
    }
    return dist[t] != -1;
}

int dfs(int s, int t, int f)
{
    if(s == t) return f;
    for(int &i = work[s]; i < G[s].size(); i++)
    {
        int e = G[s][i];
        if(dist[edges[e].v] == dist[s] + 1 and edges[e].c-edges[e].f > 0)
        {
            int a = dfs(edges[e].v, t, min(f, edges[e].c-edges[e].f));
            edges[e].f += a;
        }
    }
}
```

```

        edges[e^1].f -= a;
        return a;
    }
}
return 0;
}

int MaxFlow(int s, int t)
{
    int mf = 0;
    while(bfs(s, t))
    {
        memset(work, 0, sizeof(work));
        while(int a = dfs(s, t, 00))
            mf += a;
    }
    return mf;
}

int main()
{
    int n, m, u, v, w;

    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v >> w;
        add_edge(u-1, v-1, w, 0);
    }
    cin >> u >> v;
    cout << MaxFlow(u-1, v-1) << '\n';

    return 0;
}

```

## 1.9 Erdos Gallai Theorem

```

#include <bits/stdc++.h>
using namespace std;
#define int long long

int32_t main()
{
    int n;
    while(~scanf("%lld", &n))
    {
        vector<int> degree(n), pref(n + 2);
        for(int &i : degree)
            scanf("%lld", &i);
        sort(degree.begin(), degree.end(), greater<int>());
        for(int i = 0; i < n; i++)
            pref[i + 1] = pref[i] + degree[i];
        bool fl = true;
        if(pref[n] & 1)
            fl = false;
        int j = n;
        for(int k = 1; k <= n and fl; k++)
        {
            int L = pref[k];
            int R = k * (k - 1);

```

```

            while(j > 0 and degree[j - 1] < k)
                j--;
            int pos = max(j, k);
            R += pref[n] - pref[pos] + (pos - k) * k;
            if(L > R) fl = false;
        }
        puts(fl ? "possivel" : "impossivel");
    }
    return 0;
}

```

## 1.10 Eulirian Path

```

#include <bits/stdc++.h>
using namespace std;

int32_t main() {
    int n, m;

    cin >> n >> m;
    vector<vector<int>> g(n);
    vector<int> deg_in(n), deg_out(n);

    for(int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v; u--; v--;
        g[u].push_back(v);
        deg_in[v]++;
        deg_out[u]++;
    }

    int s = -1, f = -1;
    for(int i = 0; i < n; ++i) {
        if(deg_in[i] - deg_out[i] == 0) continue;

        if(s == -1 and deg_out[i] - deg_in[i] == 1) s = i;
        else if(f == -1 and deg_in[i] - deg_out[i] == 1) f = i;
        else return cout << "NO\n", 0;
    }

    if(s == -1 and f == -1) s = 0;
    else if(s != -1 and f == -1 or s == -1 and f != -1) return cout <<
        "NO\n", 0;

    stack<int> st;
    st.push(s);
    vector<int> res;

    while(!st.empty()) {
        int v = st.top();
        if(g[v].empty()) {
            res.push_back(v);
            st.pop();
        } else {
            int u = g[v].back();
            g[v].pop_back();
            st.push(u);
        }
    }
}

```



```

for(int i = 0; i < n; i++)
    if(g[i].empty() == false)
        return cout << "NO\n", 0;

reverse(res.begin(), res.end());

for(int w : res)
    cout << w + 1 << ' ';
cout << endl;

return 0;
}

```

## 1.11 Floyd Sucessor Graph

```

#include <bits/stdc++.h>
using namespace std;

int n;
int table[10000][20];

//table[i][j] armazena o sucessor de distancia 2^j do vertice i
void build()
{
    for(int j = 1; (1 << j) <= n; j++)
        for(int i = 0; i < n; i++)
            if(table[i][j-1] != -1)
                table[i][j] = table[table[i][j-1]][j-1];
}

int succ(int u, int k)
{
    while(k)
    {
        u = table[u][ (int)log2(k&-k) ];
        if(u == -1)
            return -1; // nao existe
        k -= k&-k;
    }
    return u;
}

//algoritmo de Floyd para encontrar o tamanho de um ciclo
//alcançado a partir de um vertice u em um grafo sucessor
int Floyd(int u)
{
    int a = succ(u, 1);
    int b = succ(u, 2);

    //encontra um vertice no ciclo
    while(a != b)
    {
        a = succ(a, 1);
        b = succ(b, 2);
        if(a == -1 or b == -1)
            return -1; // nao existe ciclo
    }
}

```

```

//a e b vao ficar posicionados no inicio do ciclo
a = u;
while(a != b)
{
    a = succ(a, 1);
    b = succ(b, 1);
}

//percorre todo o ciclo contando o seu tamanho
b = succ(a, 1);
int lenght = 1;
while(a != b)
{
    b = succ(b, 1);
    lenght++;
}
return lenght;
}

////////////////////////////////////

int main()
{
    int u, v, m;

    cin >> n >> m;
    memset(table, -1, sizeof(table));
    for(int i = 0; i < m; i++)
    {
        cin >> u >> v; u--; v--;
        table[u][0] = v;
    }
    build();
    cin >> u >> v;
    cout << "O sucessor de " << u << " com " << v <<
    " unidades a frente eh " << succ(u-1, v)+1 << '\n';

    cout << '\n';

    cin >> u;
    cout << "tamanho do ciclo iniciando em " << u <<
    ": " << Floyd(u-1) << '\n';

    return 0;
}

```

## 1.12 Ford Fulkerson

```

#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> ii;
const int OO = 0x3f3f3f3f;
const int MAX = 1e4;

struct edge
{
    int v, f, c;
    edge() {}
    edge(int _v, int _f, int _c)

```

```

    {
        v = _v, f = _f, c = _c;
    }
};

vector<edge> edges;
vector<int> G[MAX];
int tempo = 1, cor[MAX];

void add_edge(int u, int v, int cp, int rc)
{
    edges.push_back(edge(v, 0, cp));
    G[u].push_back(edges.size()-1);
    edges.push_back(edge(u, 0, rc));
    G[v].push_back(edges.size()-1);
}

int dfs(int s, int t, int f)
{
    if(s == t) return f;
    cor[s] = tempo;
    for(int e : G[s])
        if(cor[edges[e].v] < tempo and edges[e].c-edges[e].f > 0)
            if(int a = dfs(edges[e].v, t, min(f, edges[e].c-edges[e].f)))
                {
                    edges[e].f += a;
                    edges[e^1].f -= a;
                    return a;
                }
    return 0;
}

int MaxFlow(int s, int t)
{
    int mf = 0;
    while(int a = dfs(s, t, 0))
        mf += a, tempo++;
    return mf;
}

int main()
{
    int n, m, w, u, v;

    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v >> w;
        add_edge(u-1, v-1, w, 0);
    }
    cin >> u >> v;
    cout << MaxFlow(u-1, v-1) << '\n';

    return 0;
}

```

## 1.13 Fully Dynamic Connectivity Check If Two Vertices Are In The Same Component

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 131072;
const int OO = 0x3f3f3f3f;
const double EPS = 1e-9;

#define bug(x) cout << #x << " = " << x << '\n'
#define FOR(i, a, n) for(int i = a; i < n; i++)
#define REP(i, n) FOR(i, 0, n)
#define fi first
#define se second
#define pb push_back
#define mt make_tuple
#define mp make_pair
#define all(vetor) vetor.begin(), vetor.end()
#define X real()
#define Y imag()
// #define gc getchar_unlocked

typedef long long ll;
typedef long double ld;
// typedef pair<int, int> ii;
// typedef pair<int, ii> iii;
typedef complex<ll> Pll;
typedef complex<ld> Pld;

typedef pair<int, int> edge;

vector<edge> tree[4 * MAX], query[4 * MAX];;
int pai[MAX], sz[MAX];
stack<pair<int, int>> stk, size;
vector<int> ans;

int find(int x)
{
    if(pai[x] == x)
        return x;
    stk.push(mp(x, pai[x]));
    size.push(mp(x, sz[x]));
    return pai[x] = find(pai[x]);
}

void join(int x, int y)
{
    x = find(x);
    y = find(y);
    if(x == y) return;
    if(sz[x] > sz[y])
        swap(x, y);
    stk.push(mp(x, pai[x]));
    size.push(mp(y, sz[y]));
    sz[y] += sz[x];
    pai[x] = y;
}

```

```

void rollback(int rollback_to)//desfaz todas as alteracoes no DSU,
{//O(k) onde k eh a quantidade de operacoes realizadas
    while(rollback_to < stk.size())
    {
        pai[stk.top().fi] = stk.top().se;
        stk.pop();
        sz[size.top().fi] = size.top().se;
        size.pop();
    }
}

void add_edge(int node, int start, int end, int l, int r, edge e)
{
    if(start == l and end == r)
    {
        tree[node].push_back(e);
        return;
    }
    if(l >= r)
        return;
    int mid = (start + end) / 2;
    add_edge(2*node, start, mid, l, min(mid, r), e);
    add_edge(2*node + 1, mid + 1, end, max(l, mid + 1), r, e);
}

void add_query(int node, int start, int end, int idx, edge e)
{
    if(start == end)
        query[node].push_back(e);
    else
    {
        int mid = (start + end) / 2;
        if(idx <= mid)
            add_query(2*node, start, mid, idx, e);
        else
            add_query(2*node + 1, mid + 1, end, idx, e);
    }
}

void processar(int node)
{
    for(auto it : tree[node])
        join(it.first, it.second);
}

void dfs(int node, int start, int end)
{
    int rollback_to = stk.size();
    processar(node);
    if(start == end)
    {
        for(auto v : query[node])
        {
            bool rep = (find(v.first) == find(v.second));
            ans.push_back(rep);
        }
    }
    else
    {
        int mid = (start + end) / 2;

```

```

        dfs(2*node, start, mid);
        dfs(2*node + 1, mid + 1, end);
    }
    rollback(rollback_to);
}

int main()
{
    int n, q, o, u, v;

    cin >> n >> q;
    for(int i = 0; i <= n; i++)
        sz[i] = 1, pai[i] = i;
    int cur = 0;
    map<pair<int, int>, int> mapa;
    while(q--)
    {
        cin >> o >> u >> v; u--; v--;
        if(u > v) swap(u, v);
        if(o == 1) // adicionar aresta
            mapa[mp(u, v)] = cur++;
        else if(o == 2) // remover aresta
        {
            add_edge(1, 0, MAX-1, mapa[mp(u, v)], cur++, mp(u, v));
            mapa.erase(mp(u, v));
        }
        else // verificar se dois vertices estao na mesma componente
            add_query(1, 0, MAX-1, cur++, mp(u, v));
    }
    cur++;
    for(auto it : mapa)
        add_edge(1, 0, MAX-1, it.second, cur, it.first);
    dfs(1, 0, MAX-1);
    for(int i = 0; i < ans.size(); i++)
        cout << (ans[i] ? "Yes\n" : "No\n");

    return 0;
}

```

## 1.14 Fully Dynamic Connectivity Count Conected Components

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 131072;
const int OO = 0x3f3f3f3f;
const double EPS = 1e-9;

#define bug(x) cout << #x << " = " << x << '\n'
#define FOR(i, a, n) for(int i = a; i < n; i++)
#define REP(i, n) FOR(i, 0, n)
#define fi first
#define se second
#define pb push_back
#define mt make_tuple
#define mp make_pair
#define all(vetor) vetor.begin(), vetor.end()
#define X real()

```

```

#define Y imag()
//#define gc getchar_unlocked

typedef long long ll;
typedef long double ld;
//typedef pair<int, int> ii;
//typedef pair<int, ii> iii;
typedef complex<ll> Pll;
typedef complex<ld> Pld;

typedef pair<int, int> edge;

vector<edge> tree[4 * MAX];
vector<int> query[4 * MAX];
int pai[MAX], sz[MAX];
stack<pair<int, int>> stk, size;
stack<int> qtd;
vector<int> ans;
int rep;

int find(int x)
{
    if(pai[x] == x)
        return x;
    stk.push(mp(x, pai[x]));
    size.push(mp(x, sz[x]));
    qtd.push(rep);
    return pai[x] = find(pai[x]);
}

void join(int x, int y)
{
    x = find(x);
    y = find(y);
    if(x == y) return;
    if(sz[x] > sz[y])
        swap(x, y);
    qtd.push(rep);
    stk.push(mp(x, pai[x]));
    size.push(mp(y, sz[y]));
    sz[y] += sz[x];
    pai[x] = y;
    rep--;
}

void rollback(int rollback_to) //desfaz todas as alteracoes no DSU,
//O(k) onde k eh a quantidade de operacoes realizadas
{
    while(rollback_to < stk.size())
    {
        pai[stk.top().fi] = stk.top().se;
        stk.pop();
        sz[size.top().fi] = size.top().se;
        size.pop();
        rep = qtd.top();
        qtd.pop();
    }
}

void add_edge(int node, int start, int end, int l, int r, edge e)
{

```

```

    if(start == 1 and end == r)
    {
        tree[node].push_back(e);
        return;
    }
    if(l >= r)
        return;
    int mid = (start + end) / 2;
    add_edge(2*node, start, mid, l, min(mid, r), e);
    add_edge(2*node + 1, mid + 1, end, max(l, mid + 1), r, e);
}

void add_query(int node, int start, int end, int idx, int e)
{
    if(start == end)
        query[node].push_back(e);
    else
    {
        int mid = (start + end) / 2;
        if(idx <= mid)
            add_query(2*node, start, mid, idx, e);
        else
            add_query(2*node + 1, mid + 1, end, idx, e);
    }
}

void processar(int node)
{
    for(auto it : tree[node])
        join(it.first, it.second);
}

void dfs(int node, int start, int end)
{
    int rollback_to = stk.size();
    processar(node);
    if(start == end)
    {
        for(auto v : query[node])
            ans.push_back(rep);
    }
    else
    {
        int mid = (start + end) / 2;
        dfs(2*node, start, mid);
        dfs(2*node + 1, mid + 1, end);
    }
    rollback(rollback_to);
}

int main()
{
    int n, q, u, v;
    cin >> n >> q;
    rep = n;
    for(int i = 0; i <= n; i++)
        sz[i] = 1, pai[i] = i;
    int cur = 0;
    map<pair<int, int>, int> mapa;
    while(q--)
    {

```

```

char o;
cin >> o;
if(o != '?')
{
    cin >> u >> v; u--; v--;
    if(u > v) swap(u, v);
    if(o == '+') // adicionar aresta
        mapa[mp(u, v)] = cur++;
    else if(o == '-') // remover aresta
    {
        add_edge(1, 0, MAX-1, mapa[mp(u, v)], cur++, mp(u, v));
        mapa.erase(mp(u, v));
    }
    else // verificar se dois vertices estao na mesma componente
        add_query(1, 0, MAX-1, cur++, 1);
}
cur++;
for(auto it : mapa)
    add_edge(1, 0, MAX-1, it.second, cur, it.first);
dfs(1, 0, MAX-1);
for(int i = 0; i < ans.size(); i++)
    cout << ans[i] << '\n';

return 0;
}

```

## 1.15 Hopcroft Karp

```

#include <bits/stdc++.h>
using namespace std;
const int OO = 0x3f3f3f3f;

int n, m;
vector<int> G[10000];
queue<int> q;
int pairU[10000], pairV[10000], dist[10000];

bool bfs()
{
    for(int u = 1; u <= m; u++)
        if(!pairU[u])
        {
            dist[u] = 0;
            q.push(u);
        }
    else dist[u] = OO;
    dist[0] = OO;
    while(!q.empty())
    {
        int u = q.front();
        q.pop();
        if(dist[u] < dist[0])
            for(const int &v : G[u])
                if(dist[pairV[v]] == OO)
                {
                    dist[pairV[v]] = dist[u] + 1;
                    q.push(pairV[v]);
                }
    }
}

```

```

    }
    return (dist[0] != OO);
}

bool dfs(int u)
{
    if(u)
    {
        for(const int &v : G[u])
            if(dist[pairV[v]] == dist[u]+1)
                if(dfs(pairV[v]))
                {
                    pairV[v] = u;
                    pairU[u] = v;
                    return true;
                }
        dist[u] = OO;
        return false;
    }
    return true;
}

int hopcroftKarp()
{
    memset(pairU, 0, sizeof(pairU));
    memset(pairV, 0, sizeof(pairV));
    int result = 0;
    while(bfs())
        for(int u = 1; u <= m; u++)
            if(!pairU[u] and dfs(u))
                result++;
    return result;
}

int main()
{
    n = m = 4;

    G[1].push_back(2);
    G[2].push_back(1);
    G[1].push_back(3);
    G[3].push_back(1);
    G[2].push_back(1);
    G[1].push_back(2);
    G[3].push_back(2);
    G[2].push_back(3);
    G[4].push_back(2);
    G[2].push_back(4);
    G[4].push_back(4);
    G[4].push_back(4);

    cout << hopcroftKarp() << '\n';

    return 0;
}

```

## 1.16 K Short Paths

```

#include <bits/stdc++.h>
using namespace std;

```

```

#define int long long
const int OO = 0x3f3f3f3f3f3f3f3f;
const int MAX = 2000000;

typedef pair<int, int> ii;

int n, m, k;
vector<ii> G[MAX];
int cnt[MAX];

void dijkstra(int v) {
    priority_queue<ii> pq;
    pq.push({0, v});
    int c = 0;
    while(!pq.empty()) {
        int u = -pq.top().second;
        int d = -pq.top().first;
        pq.pop();
        cnt[u]++;
        if(cnt[u] > k) continue;
        if(u == n - 1) {
            cout << d << ' ';
            if(++c == k) { cout << '\n'; return; }
        }
        for(auto [_d, w] : G[u])
            if(cnt[w] < k)
                pq.push({-(d + _d), -w});
    }
}

int32_t main() {
    cin >> n >> m >> k;
    while(m--) {
        int u, v, w;
        cin >> u >> v >> w; u--; v--;
        G[u].push_back({w, v});
    }
    dijkstra(0);

    return 0;
}

```

## 1.17 Knapsack Dijkstra

```

#include <bits/stdc++.h>
using namespace std;
const int OO = 0x3f3f3f3f;
#define ii pair<int, int>
#define fi first
#define se second

vector<ii> G[105];
int dist[100000008];
vector<int> peso;

void dijkstra()
{
    memset(dist, 63, sizeof(dist));
    dist[0] = 0;
    priority_queue<ii> pq;

```

```

pq.push({0, 0});
while(!pq.empty())
{
    int u = pq.top().se;
    int d = -pq.top().fi;
    pq.pop();
    if(d > dist[u]) continue;
    for(int i = 0; i < G[u].size(); i++)
    {
        int w = G[u][i].fi, dd = G[u][i].se;
        if(dist[w] > dist[u] + dd)
        {
            dist[w] = dist[u] + dd;
            pq.push({-dist[w], w});
        }
    }
}
dist[0] = peso[0];
}

int32_t main()
{
    int n, e, d;
    cin >> n >> d >> e;
    // ler pesos
    peso = vector<int>{d, 2 * d, 5 * d, 10 * d, 20 * d, 50 * d, 100 * d,
        5 * e, 10 * e, 20 * e, 50 * e, 100 * e, 200 * e};
    // ordena pra pegar o menor valor
    sort(peso.begin(), peso.end());
    //montar grafo
    for(int i = 0; i < peso[0]; i++)
        for(int j = 0; j < peso.size(); j++)
        {
            int x = (i + peso[j]) % peso[0];
            G[i].push_back({x, peso[j]});
        }
    /*
    dist[i] eh o menor numero que eu consigo formar usando
    os meus objetos tal que dist[i] % peso[0] == i
    */
    dijkstra();

    /*
    se dist[X % peso[0]] <= X eh possivel gerar um valor X
    utilizando os valores do array peso
    OBS: cada valor pode ser usado infinitas vezes
    */
    return 0;
}

```

## 1.18 Kuhn MCBM

```

#include <bits/stdc++.h>
using namespace std;

int na, nb, m, tempo = 1;
int b[105];
int cor[105];
vector<int> G[105];

```

```

bool kuhn(int u)
{
    if(cor[u] == tempo)
        return 0;
    cor[u] = tempo;
    //random_shuffle(G[u].begin(), G[u].end(), [](int x){ return rand() %
    x; });
    for(const int &v : G[u])
        if(!b[v] or kuhn(b[v]))
            return b[v] = u;
    return 0;
}

int main()
{
    //srand(time(NULL));
    cin >> na >> nb >> m;
    while(m--)
    {
        int u, v;
        cin >> u >> v;
        G[u].push_back(v + na);
    }
    tempo = 1;
    int ans = 0;
    for(int i = 1; i <= na; i++)
        ans += kuhn(i), tempo++;
    cout << "MCBM = " << ans << '\n';
    for(int i = nb + 1; i <= na + nb; i++)
        if(b[i])
            cout << b[i] << ' ' << i - na << '\n';

    return 0;
}

```

## 1.19 Lca With Square Root Decomposition

```

#include <bits/stdc++.h>
const int MAX = 50500;

using namespace std;

vector<int> G[MAX];
int nivel[MAX], pai[MAX], jump[MAX], n, blk_sz;

void dfs(int v, int d, int p)
{
    pai[v] = p;
    nivel[v] = d;
    (nivel[v] % blk_sz == 0) ? jump[v] = pai[v] : jump[v] = jump[p];
    for(const int &u : G[v])
        if(u != p)
            dfs(u, d + 1, v);
}

int lcaTrivial(int u, int v)
{
    while(u != v)
        (nivel[u] > nivel[v]) ? u = pai[u] : v = pai[v];
    return u;
}

```

```

}

int lca(int u, int v)
{
    while(jump[u] != jump[v])
        (nivel[u] > nivel[v]) ? u = jump[u] : v = jump[v];
    return lcaTrivial(u, v);
}

void build()
{
    blk_sz = sqrt(n);
    dfs(0, 0, 0);
}

int main()
{
    int x, y;
    cin >> n;
    for(int i = 0; i < n-1; i++)
    {
        cin >> x >> y;
        G[x-1].push_back(y-1);
        G[y-1].push_back(x-1);
    }
    build();
    cin >> x >> y;
    cout << lca(x-1, y-1) + 1 << '\n';

    return 0;
}

```

## 1.20 Lca With Tree Linearization And Segment Tree

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

int pos[MAX];
int deft[MAX];
int segtree[5*MAX];
vector<int> tl;
vector<int> G[MAX];

void tree_linearization(int v, int p, int d)
{
    deft[v] = d;
    pos[v] = tl.size();
    tl.push_back(v);
    for(const int &u : G[v])
        if(u != p)
        {
            tree_linearization(u, v, d + 1);
            tl.push_back(v);
        }
}

void build(int node, int start, int end)
{
}

```

```

if(start == end)
    segtree[node] = tl[start];
else
{
    int mid = (start+end)/2;
    build(2*node, start, mid);
    build(2*node+1, mid+1, end);
    if(deft[segtree[2*node]] < deft[segtree[2*node+1]])
        segtree[node] = segtree[2*node];
    else
        segtree[node] = segtree[2*node+1];
}
}

int lca(int node, int start, int end, int l, int r)
{
    if(l > end or r < start)
        return -1;
    if(l <= start and end <= r)
        return segtree[node];
    int mid = (start+end)/2;
    int p1 = lca(2*node, start, mid, l, r);
    int p2 = lca(2*node+1, mid+1, end, l, r);
    if(p1 == -1) return p2;
    if(p2 == -1) return p1;
    return deft[p1] < deft[p2] ? p1 : p2;
}

/*int _lca(int a, int b)
{
    int ancestor = a, nivel = 0x3f3f3f3f;
    for(int i = pos[a]; i <= pos[b]; i++)
        if(deft[tl[i]] < nivel)
        {
            ancestor = tl[i];
            nivel = deft[tl[i]];
        }
    return ancestor;
}*/

int main()
{
    int n, u, v;

    cin >> n;
    for(int i = 0; i < n-1; i++)
    {
        cin >> u >> v;
        G[u].push_back(v);
        G[v].push_back(u);
    }

    tree_linearization(1, -1, 0);
    build(1, 0, tl.size()-1);

    /*
    for(int i = 1; i <= n; i++)
        cout << deft[i] << ' ';
    cout << '\n';

```

```

for(int i = 1; i <= n; i++)
    cout << pos[i] << ' ';
cout << '\n';
for(const int &p : tl)
    cout << p << ' ';
cout << '\n';
for(int i = 1; i <= 4*n; i++)
    cout << segtree[i] << ' ';
cout << '\n';*/

while(cin >> u >> v)
    cout << /*_lca(u, v) << ' ' <<*/
        lca(1, 0, tl.size()-1, pos[u], pos[v]) << '\n';

return 0;
}

```

## 1.21 Lca With Tree Linearization And Sparse Table

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5 + 10;

int n, m;
vector<int> G[MAX], tl;
int deft[MAX], SpT[27][MAX], pos[MAX];

void tree_linearization(int v, int p, int d)
{
    deft[v] = d;
    pos[v] = tl.size();
    tl.push_back(v);
    for(int &u : G[v])
    {
        if(u != p)
        {
            tree_linearization(u, v, d + 1);
            tl.push_back(v);
        }
    }
}

void build(int tam)
{
    for(int i = 0; (1 << i) <= tam; i++)
    {
        for(int j = 0; j + (1 << i) <= tam; j++)
            if(!i)
                SpT[i][j] = tl[j];
            else if(deft[SpT[i-1][j]] < deft[SpT[i-1][j+(1<<(i-1))]])
                SpT[i][j] = SpT[i-1][j];
            else
                SpT[i][j] = SpT[i-1][j+(1<<(i-1))];
    }
}

int lca(int i, int j)
{
    int k = log2(j-i+1);
    if(deft[SpT[k][i]] < deft[SpT[k][j+1-(1<<k)]])

```



```

    return SpT[k][i];
else
    return SpT[k][j+1-(1<<k)];
}

int main()
{
    int u, v, q;

    cin >> n >> m;
    for(int i = 0; i < m; i++)
    {
        cin >> u >> v;
        G[u].push_back(v);
        G[v].push_back(u);
    }

    tree_linearization(1, -1, 0);
    build(tl.size());

    cin >> q;
    while(q--)
    {
        cin >> u >> v;
        cout << lca(min(pos[u], pos[v]), max(pos[u], pos[v])) << '\n';
    }

    return 0;
}

```

## 1.22 Longest And Shortest Path In DAG

```

#include <bits/stdc++.h>

const int OO = 0x3f3f3f3f;
const int MAX = 1e6;

using namespace std;

int n, m;
vector<pair<int, int>> G[MAX];
int dist1[MAX], dist2[MAX];
vector<int> ts;
bool cor[MAX];

void dfs(int v)
{
    cor[v] = true;
    for(pair<int, int> &w : G[v])
        if(!cor[w.first])
            dfs(w.first);
    ts.push_back(v);
}

// caminho de 0 a n-1
pair<int, int> longestAndShortestPathInDAG()
{
    for(int i = 0; i <= n; i++)
        dist1[i] = -OO, dist2[i] = OO;
    dist1[0] = dist2[0] = 0;
}

```

```

int p = 0;
while(p < (int)ts.size())
{
    int v = ts[p++];
    if(dist1[v] != -OO)
        for(int i = 0; i < (int)G[v].size(); i++)
        {
            int u = G[v][i].first, d = G[v][i].second;
            if(dist1[u] < dist1[v] + d)
                dist1[u] = dist1[v] + d;
        }
    if(dist2[v] != OO)
        for(int i = 0; i < (int)G[v].size(); i++)
        {
            int u = G[v][i].first, d = G[v][i].second;
            if(dist2[u] > dist2[v] + d)
                dist2[u] = dist2[v] + d;
        }
}
return {dist1[n-1], dist2[n-1]};
}

int main()
{
    cin >> n >> m;
    for(int i = 0; i < m; i++)
    {
        int u, v, w;
        cin >> u >> v >> w; u--; v--;
        G[u].push_back({v, w});
    }
    for(int i = 0; i < n; i++)
        if(!cor[i])
            dfs(i);
    reverse(ts.begin(), ts.end());
    pair<int, int> ans = longestAndShortestPathInDAG();
    cout << "Longest Path " << ans.first << '\n';
    cout << "Shortest Path " << ans.second << '\n';

    return 0;
}

```

## 1.23 MCE MinimumEdgeCover

```

#include <bits/stdc++.h>
using namespace std;

vector<int> G[1000];
int b[1000], vis[1000], tempo;

bool kuhn(int v)
{
    if(vis[v] == tempo)
        return 0;
    vis[v] = tempo;
    for(const int &u : G[v])
        if(!b[u] or kuhn(b[u]))
            return b[u] = v;
    return 0;
}

```

```

int main()
{
    int n, m, e;
    cin >> n >> m >> e;
    while(e--)
    {
        int u, v;
        cin >> u >> v;
        G[u].push_back(v + n);
    }
    int ans = 0;
    tempo = 1;
    for(int i = 1; i <= n; i++)
        ans += kuhn(i), tempo++;

    //encontrar as arestas do Minimum Edge Cover
    vector<bool> covered(n + m + 10, false);
    vector<pair<int, int>> cover;
    for(int i = n + 1; i <= n + m; i++)
        if(b[i])
        {
            covered[b[i]] = covered[i] = true;
            cover.push_back({b[i], i - n});
        }
    for(int i = 1; i <= n; i++)
    {
        bool is_covered = covered[i];
        for(const int &u : G[i])
            if(!covered[u])
            {
                is_covered = true;
                cover.push_back({i, u - n});
                covered[i] = covered[u] = true;
            }
        if(!is_covered and !G[i].empty())
            cover.push_back({i, G[i].front() - n});
    }
    cout << "MEC = " << cover.size() << '\n';
    for(int i = 0; i < cover.size(); i++)
        cout << cover[i].first << ' ' << cover[i].second << '\n';

    return 0;
}

```

## 1.24 MPC MinimumPathCover

```

#include <bits/stdc++.h>
using namespace std;

int n, m;
vector<int> G[1000], bip[1000], ts;
int vis[1000], b[1000], go[1000], tempo = 1;

bool kuhn(int v)
{
    if(vis[v] == tempo)
        return 0;
    vis[v] = tempo;

```

```

    for(const int &u : bip[v])
        if(!b[u] or kuhn(b[u]))
        {
            go[v] = u - n;
            return b[u] = v;
        }
    return 0;
}

void topological_sort(int v)
{
    vis[v] = tempo;
    for(const int &u : G[v])
        if(vis[u] != tempo)
            topological_sort(u);
    ts.push_back(v);
}

int main()
{
    cin >> n >> m;
    while(m--)
    {
        int u, v;
        cin >> u >> v;
        G[u].push_back(v);
        bip[u].push_back(v + n);
    }
    int ans = 0;
    for(int i = 1; i <= n; i++)
        ans += kuhn(i), tempo++;
    for(int i = 1; i <= n; i++)
        if(vis[i] != tempo)
            topological_sort(i);
    reverse(ts.begin(), ts.end());
    tempo++;
    cout << n - ans << '\n';
    for(int i = 0; i < n; i++)
    {
        int u = ts[i];
        if(vis[u] != tempo)
        {
            while(u)
            {
                vis[u] = tempo;
                cout << u << ' ';
                u = go[u];
            }
            puts("");
        }
    }

    return 0;
}

```

## 1.25 MVC MinimumVertexCover

```

#include <bits/stdc++.h>
using namespace std;

```

```

vector<int> G[1000];
int b[1000], vis[1000], tempo;
bool be[1000];
set<int> r0, r1;

bool kuhn(int v)
{
    if(vis[v] == tempo)
        return 0;
    vis[v] = tempo;
    for(const int &u : G[v])
        if(!b[u] or kuhn(b[u]))
            return b[u] = v;
    return 0;
}

void MVC(int v)
{
    if(vis[v] == tempo)
        return;
    vis[v] = tempo;
    for(const int u : G[v])
        if(b[u] != v and b[u])
        {
            r1.insert(u);
            vis[b[u]] = tempo;
        }
}

int main()
{
    int n, m, e;
    cin >> n >> m >> e;
    while(e--)
    {
        int u, v;
        cin >> u >> v;
        G[u].push_back(v);
    }
    int ans = 0;
    tempo = 1;
    for(int i = 1; i <= n; i++)
        ans += kuhn(i), tempo++;
    for(int i = n + 1; i <= n + m; i++)
        if(b[i])
            be[i - n] = be[b[i]] = true;
    for(int i = 1; i <= n; i++)
        if(!be[i])
            MVC(i);
    for(int i = 1 ; i <= n; i++)
        if(vis[i] < tempo)
            r0.insert(i);
    cout << "MVC = "<< ans << '\n';
    cout << "tamanho lado esquerdo " << r0.size() << '\n';
    for(auto it = r0.begin(); it != r0.end(); it++)
        cout << *it << ' ';
    puts("");
    cout << "tamanho lado direito " << r1.size() << '\n';
    for(auto it = r1.begin(); it != r1.end(); it++)
        cout << *it - n << ' ';
    puts("");
}

```

```

        return 0;
    }
}

```

## 1.26 Maximum Clique

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int MAX = 43, C = 20;

int n, m, dp[1 << C];
ll G[MAX];

int maxClique()
{
    for(int i = 1; i < (1 << max(0, n - C)); i++)
    {
        int x = i;
        for(int j = 0; j < max(0, n - C); j++)
            if((i >> j) & 1)
                x &= G[j + C] >> C;
        if(x == i) dp[i] = __builtin_popcount(i);
    }
    for(int i = 1; i < (1 << max(0, n - C)); i++)
        for(int j = 0; j < max(0, n - C); j++)
            if((i >> j) & 1)
                dp[i] = max(dp[i], dp[i ^ (1 << j)]);
    int ans = 0;
    for(int i = 0; i < (1 << min(C, n)); i++){
        int x = i, y = (1 << max(0, n - C)) - 1;
        for(int j = 0; j < min(C, n); j++)
            if((i >> j) & 1)
                x &= G[j], y &= G[j] >> C;
        if(x == i)
            ans = max(ans, __builtin_popcount(i) + dp[y]);
    }
    return ans;
}

int main()
{
    cin >> n >> m;
    while(m--)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        G[u] |= (1LL << v);
        G[v] |= (1LL << u);
    }
    for(int i = 0; i < n; i++)
        G[i] |= (1LL << i);
    cout << maxClique() << '\n';

    return 0;
}

```

## 1.27 Min Cost Max Flow

```

/*
 * from IME Library
 */

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e3;
const int OO = 0x3f3f3f3f;

struct edge {int v, f, w, c; };

// flw_lmt eh a quantidade de de fluxo que posso passar
// no maximo, alterar se necessario
// node_count eh o valor do maior vertice no grafo...
// inicializar node_count com numero de vertices no inicio...
int node_count, flw_lmt = OO, p[MAX];
vector<edge> edges;
vector<int> G[MAX];

// u--->v, custo w e capacidade c
void add_edge(int u, int v, int w, int c)
{
    int k = edges.size();
    node_count = max(node_count, u+1);
    node_count = max(node_count, v+1);
    G[u].push_back(k);
    G[v].push_back(k+1);
    edges.push_back({ v, 0, w, c });
    edges.push_back({ u, 0, -w, 0 });
}

void clear()
{
    flw_lmt = OO;
    for(int i = 0; i < node_count; ++i) G[i].clear();
    edges.clear();
    node_count = 0;
}

bool SPFA(int s, int t)
{
    vector<int> dist(node_count, OO);
    vector<int> et(node_count, 0);
    deque<int> q;
    q.push_back(s), dist[s] = 0;
    while (!q.empty())
    {
        int u = q.front(); q.pop_front();
        et[u] = 2;
        for(int i : G[u])
        {
            edge &e = edges[i];
            int v = e.v;
            if (e.f < e.c and dist[v] > dist[u] + e.w)
            {
                dist[v] = dist[u] + e.w;
                if (et[v] == 0) q.push_back(v);
                else if (et[v] == 2) q.push_front(v);
                et[v] = 1;
                p[v] = i;
            }
        }
    }
}

```

```

    }
    return dist[t] != OO;
}

int min_cost_max_flow(int s, int t)
{
    int mf = 0, cost = 0;
    while(SPFA(s, t) and mf < flw_lmt)
    {
        int inc = flw_lmt - mf;
        for (int u = t; u != s; u = edges[p[u]^1].v)
        {
            edge &e = edges[p[u]];
            inc = min(inc, e.c - e.f);
        }
        for (int u = t; u != s; u = edges[p[u]^1].v)
        {
            edge &e = edges[p[u]], &rev = edges[p[u]^1].v;
            e.f += inc;
            rev.f -= inc;
            cost += inc * e.w;
        }
        if (!inc) break;
        mf += inc;
    }
    cout << "Max Flow " << mf << '\n';
    cout << "Min Cost " << cost << '\n';
    return cost;
}

int main()
{
    return 0;
}

```

## 1.28 Prim

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;
const int OO = 0x3f3f3f3f;
typedef pair<int, int> ii;
typedef pair<int, ii> iii;

int n, m;
vector<ii> G[MAX];
int dist[MAX], edge[MAX];
bool visit[MAX];

void prim(int s)
{
    memset(visit, 0, sizeof(visit));
    memset(dist, 63, sizeof(dist));
    dist[s] = 0;
    priority_queue<ii> pq;
    pq.push({0, s});
    while(!pq.empty())

```

```

{
    int u = pq.top().second;
    pq.pop();
    if(visit[u]) continue;
    for(int i = 0; i < G[u].size(); i++)
    {
        int v = G[u][i].second, d = G[u][i].first;
        if(!visit[v] and dist[v] > d)
        {
            dist[v] = d;
            edge[v] = u;
            pq.push({-d, v});
        }
    }
    visit[u] = true;
}
int ans = 0;
edge[s] = -2;
for(int i = 0; i < n; i++)
{
    cout << edge[i]+1 << ' ';
    ans += dist[i];
}
cout << '\n';
cout << ans << '\n';
}

int main()
{
    int u, v, w;

    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v >> w; u--; v--;
        G[u].push_back({w, v});
        G[v].push_back({w, u});
    }
    prim(0);

    return 0;
}

```

## 1.29 Tree Isomorphism

```

#include <bits/stdc++.h>
using namespace std;

const int ms = 100100;

int degree[ms], vis[ms];
int size[ms];
int n;

bool cmp(int a, int b)
{
    return size[a] < size[b];
}

void pre(vector<vector<int>> &edges, int on = 0)

```

```

{
    size[on] = 1;
    for(auto to : edges[on])
    {
        pre(edges, to);
        size[on] += size[to];
    }
    sort(edges[on].begin(), edges[on].end(), cmp);
}

void solve(vector<vector<int>> &edges, string &str, int on = 0)
{
    str += 'D';
    for(int l = 0, r = 0; l < edges[on].size(); l = r) {
        while(r < edges[on].size() &&
            size[edges[on][l]] == size[edges[on][r]]) r++;
        if(r == l + 1)
            solve(edges, str, edges[on][l]);
        else
        {
            priority_queue<string> hp;
            for(int i = l; i < r; i++) {
                string temp;
                solve(edges, temp, edges[on][i]);
                hp.push(temp);
            }
            while(!hp.empty())
            {
                str += hp.top();
                hp.pop();
            }
        }
    }
    str += 'U';
}

// enraizar arvore
void mount(vector<vector<int>> &graph,
    vector<vector<int>> &G, int v = 0, int p = -1)
{
    for(int &u : G[v])
        if(u != p)
        {
            graph[v].push_back(u);
            mount(graph, G, u, v);
        }
}

// achar centro da arvore e enraizar no centro
void findCenterAndComputeStr(vector<vector<int>> &graph,
    vector<vector<int>> &G, string *str)
{
    memset(vis, 0, sizeof(vis));
    queue<int> fila[2];
    for(int i = 0; i < n; i++)
        if(degree[i] == 1)
            fila[0].push(i);
    int cnt = 0, turn = 0;
    while(cnt + 2 < n)
    {
        while(!fila[turn].empty())

```

```

{
    int u = fila[turn].front(); fila[turn].pop();
    vis[u] = true;
    cnt++;
    for(int i = 0; i < G[u].size(); i++)
        if(!vis[G[u][i]])
        {
            degree[G[u][i]]--;
            if(degree[G[u][i]] == 1)
                fila[1-turn].push(G[u][i]);
        }
    }
    turn ^= 1;
}
int k = 0;
for(int i = 0; i < n; i++)
{
    if(vis[i]) continue;
    graph.clear();
    graph.resize(n + 1);
    mount(graph, G, i);
    pre(graph, i);
    solve(graph, str[k], i);
    k++;
}
}

int main()
{
    while(cin >> n)
    {
        string str[2][2];
        for(int i = 0; i < 2; i++)
        {
            vector<vector<int>> graph, G;
            G.resize(n + 1);
            memset(degree, 0, sizeof(degree));
            for(int j = 1; j < n; j++)
            {
                int u, v;
                scanf(" %d %d", &u, &v); v--; u--;

                G[v].push_back(u);
                G[u].push_back(v);
                degree[v]++;
                degree[u]++;
            }
            findCenterAndComputeStr(graph, G, str[i]);
        }
        bool f1 = (str[0][0] == str[1][0]) or (str[0][0] == str[1][1]);
        f1 |= ((str[0][1] == str[1][0]) or (str[0][0] == str[1][1]));
        puts(f1 ? "S" : "N");
    }
}

return 0;
}

```

## 2 Data Structures

### 2.1 BIT 1D

```

#include <bits/stdc++.h>
using namespace std;

int aux, n, arr[1000], BIT[1000];

// construir uma BIT a partir de um array em O(N)
void build() {
    for(int i = 1; i <= n; i++) {
        BIT[i] += arr[i];
        if(i + (i & -i) <= n)
            BIT[i + (i & -i)] += BIT[i];
    }
}

// construir o array que gera a BIT a partir de uma BIT em O(N)
void buildArray() {
    for(int i = n; i >= 1; i--)
        if(i + (i & -i) <= n)
            BIT[i + (i & -i)] -= BIT[i];
}

int sum(int x) {
    int s = 0;
    while(x) s += BIT[x], x -= x & -x;
    return s;
}

void update(int x, int value) {
    while(x <= n) BIT[x] += value, x += x & -x;
}

int main() {
    cin >> n;
    for(int i = 1; i <= n; i++) {
        cin >> aux;
        update(i, aux);
    }
    int a, b;
    cin >> a >> b;
    cout << sum(b) - sum(a-1) << '\n';

    return 0;
}

```

### 2.2 BIT 2D

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e3;

int n, aux, BIT[MAX][MAX];

void update(int x, int y, int value)

```

```

{
    for(int i = x; i <= n; i += i&-i)
        for(int j = y; j <= n; j += j&-j)
            BIT[i][j] += value;
}

int query(int x, int y)
{
    int sum = 0;
    for(int i = x; i > 0; i -= i&-i)
        for(int j = y; j > 0; j -= j&-j)
            sum += BIT[i][j];
    return sum;
}

int queryInRectangle(int x1, int y1, int x2, int y2)
{
    int sum = 0;
    sum += query(max(x1, x2), max(y1, y2));
    sum -= query(max(x1, x2), min(y1, y2) - 1);
    sum -= query(min(x1, x2) - 1, max(y1, y2));
    sum += query(min(x1, x2) - 1, min(y1, y2) - 1);
    return sum;
}

int main()
{
    cin >> n;
    for(int i = 1; i <= n; i++)
        for(int j = 1; j <= n; j++)
            cin >> aux, update(i, j, aux);
    int x1, y1, x2, y2;
    while(cin >> x1 >> y1 >> x2 >> y2)
        cout << queryInRectangle(x1, y1, x2, y2) << '\n';

    return 0;
}

```

## 2.3 BIT Range Sum And Range Update

```

#include <bits/stdc++.h>
const int MAX = 1e5;

using namespace std;

struct BIT {

    int N;
    int BIT1[MAX];
    int BIT2[MAX];

    BIT(int M) {
        N = M;
    }

    void add(int *b, int pos, int x) {
        while(pos <= N) b[pos] += x, pos += pos&-pos;
    }

    void range_add(int l, int r, int x) {

```

```

        add(BIT1, l, x);
        add(BIT1, r + 1, -x);
        add(BIT2, l, x * (1 - 1));
        add(BIT2, r + 1, -x * r);
    }

    int sum(int *b, int pos) {
        int s = 0;
        while(pos) s += b[pos], pos -= pos&-pos;
        return s;
    }

    int prefix_sum(int pos) {
        return sum(BIT1, pos) * pos - sum(BIT2, pos);
    }

    int range_sum(int l, int r) {
        return prefix_sum(r) - prefix_sum(l - 1);
    }
};

int main() {

    int n, q;
    cin >> n >> q;

    BIT B(n);

    while(q--) {
        int o, l, r, x;
        cin >> o >> l >> r;
        if(o == 1) {
            cout << B.range_sum(l, r) << '\n';
        } else {
            cin >> x;
            B.range_add(l, r, x);
        }
    }

    return 0;
}

```

## 2.4 Centroid Decomposition

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

int n, m, Centroid_Tree[MAX], Size[MAX];
vector<int> G[MAX], cTree[MAX];
bool cut[MAX];

int dfs(int v, int p)
{
    int s = 1;
    for(const int &u : G[v])
        if(!cut[u] and u != p)
            s += dfs(u, v);
    return Size[v] = s;
}

```

```

int Find_Centroid(int v, int p, int tot)
{
    int next, cnt = 0;
    for(const int &u : G[v])
        if(!cut[u] and u != p and cnt < Size[u])
        {
            cnt = Size[u];
            next = u;
        }
    if(cnt > tot/2) return Find_Centroid(next, v, tot);
    return v;
}

void build(int v, int p)
{
    dfs(v, -1);
    int u = Find_Centroid(v, -1, Size[v]);
    cut[u] = true;
    Centroid_Tree[u] = p;
    if(p != -1)
    {
        cTree[u].push_back(p);
        cTree[p].push_back(u);
    }
    for(const int &w : G[u])
        if(!cut[w])
            build(w, u);
}

int main()
{
    int u, v;
    memset(Centroid_Tree, -1, sizeof Centroid_Tree);
    cin >> n >> m;
    while(m--)
    {
        cin >> u >> v;
        u--; v--;
        G[u].push_back(v);
        G[v].push_back(u);
    }
    build(0, -1);
    for(int i = 0; i < n; i++)
    {
        cout << i+1 << ": ";
        for(int &w : cTree[i])
            cout << w+1 << ' ';
        cout << '\n';
    }

    return 0;
}

```

## 2.5 Color Update

```

#include <bits/stdc++.h>
using namespace std;

#define bug(x) cout << #x << " >>>>>> " << x << '\n'

```

```

#define _ << " , " <<
#define INF 0x3f3f3f3f

#define ii pair<int, int>
#define fi first
#define se second

struct Color {
    int x;
    Color(int _x) : x(_x) {}
    Color() : x(-1) {}
    bool operator<(const Color &c) const {
        return x < c.x;
    }
};

#define iic pair<pair<int, int>, Color>
#define default_color Color(-1)

struct ColorUpdate {
    set<iic> intervals;

    ColorUpdate(int begin = -INF, int end = INF) {
        intervals.insert( { {begin, end}, default_color } );
    }

    void paint(int l, int r, Color c = default_color) {
        if(l > r) return;

        auto a = prev(intervals.upper_bound({{l, INF}, INF}));
        auto b = prev(intervals.upper_bound({{r, INF}, INF}));

        int l1 = a->fi.fi, r1 = l - 1;
        Color c1 = a->se;
        int l2 = r + 1, r2 = b->fi.se;
        Color c2 = b->se;

        intervals.erase(a, next(b));

        if(l1 <= r1) intervals.insert({{l1, r1}, c1});
        if(l2 <= r2) intervals.insert({{l2, r2}, c2});
        if(l <= r) intervals.insert({{l, r}, c});

        // printall();
    }

    Color get_color_of(int x) {
        return prev(intervals.upper_bound({{x, INF}, INF}))->se;
    }

    // true if x is in some interval
    bool find(int x) {
        auto a = intervals.upper_bound({{x, INF}, INF});
        if(a == intervals.begin()) return false;
        a--;
        if(a->fi.fi > x or a->fi.se < x) return false;
        return true;
    }

    ii get_interval_of(int x) {
        if(!find(x)) return {-INF, INF};
    }
}

```



```

    return prev(intervals.upper_bound({{x, INF}, INF}))->fi;
}

ii get_interval_of(int l, int r) {
    if(!find(l)) return {-INF, INF};
    ii i = get_interval_of(l);
    if(i.fi <= r and r <= i.se) return i;
    return {-INF, INF};
}

// x will be on the left side
void cut_at(int x) {
    if(!find(x)) return;

    auto a = prev(intervals.upper_bound({{x, INF}, INF}));

    Color c = a->se;

    int l1 = a->fi.fi, r1 = x;
    int l2 = x + 1, r2 = a->fi.se;

    intervals.erase(a);

    if(l1 <= r1) intervals.insert({{l1, r1}, c});
    if(l2 <= r2) intervals.insert({{l2, r2}, c});
}

void remove_interval(int l, int r) {
    cut_at(l - 1);
    cut_at(r);

    auto a = prev(intervals.upper_bound({{l, INF}, INF}));
    auto b = prev(intervals.upper_bound({{r, INF}, INF}));

    intervals.erase(a, next(b));
}

void remove_at(int x) {
    remove_interval(x, x);
}

void p_interval(ii i) {
    cout << "elements from " << i.fi.fi << " to " << i.fi.se;
    cout << " have color " << i.se.x << endl;
}

void printall() {
    cout << "\n\nColor Of The Elements:\n";
    for(auto it : intervals)
        p_interval(it);
    cout << endl << endl;
}
};

int32_t main() {

    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    ColorUpdate C(1, 100);

```

```

    for(int i = 3; i <= 50; i += 10)
        C.cut_at(i);

    C.remove_interval(16, 57);

    C.remove_at(100);
    C.remove_at(1);
    C.remove_at(2);
    C.remove_at(3);

    C.cut_at(20);

    C.printall();

    return 0;
}

```

---

## 2.6 DSU With Partial Persistence

```

int n, pai[MAX], sz[MAX], his[MAX], tempo;

void init()
{
    tempo = 0;
    for(int i = 0; i < n; i++)
        pai[i] = i, sz[i] = 1, his[i] = 0;
}

int find(int x, int t)
{
    if(pai[x] == x) return x;
    if(his[x] > t) return x;
    return find(pai[x], t);
}

void join(int u, int v)
{
    tempo++;
    u = find(u, tempo);
    v = find(v, tempo);
    if(sz[u] > sz[v]) swap(u, v);
    pai[u] = v;
    his[u] = tempo;
    sz[v] += sz[u];
}

```

---

## 2.7 Dynamic Segment Tree With Lazy Propagation

```

#include <bits/stdc++.h>
using namespace std;

struct Node
{
    int l, r, value;
};

vector<Node> tree;
vector<int> lazy;

```

```

int init()
{
    tree.clear();
    lazy.clear();
    tree.emplace_back();
    lazy.push_back(0);
}

void createL(int node)
{
    tree[node].l = tree.size();
    tree.emplace_back();
    lazy.push_back(0);
}

void createR(int node)
{
    tree[node].r = tree.size();
    tree.emplace_back();
    lazy.push_back(0);
}

void calc(int node)
{
    tree[node].value = 0;
    if(tree[node].l) tree[node].value += tree[tree[node].l].value;
    if(tree[node].r) tree[node].value += tree[tree[node].r].value;
}

void push(int node, int start, int end)
{
    // +=
    tree[node].value = lazy[node] * (end - start + 1);
    if(start != end)
    {
        if(tree[node].l == 0) createL(node);
        if(tree[node].r == 0) createR(node);
        lazy[tree[node].l] = lazy[node]; // +=
        lazy[tree[node].r] = lazy[node]; // +=
    }
    lazy[node] = 0;
}

void update(int node, int start, int end, int l, int r, int value)
{
    if(lazy[node])
        push(node, start, end);

    if(start > r or l > end) return;

    if(l <= start and end <= r)
    {
        tree[node].value = value * (end - start + 1); // +=
        if(start != end)
        {
            if(tree[node].l == 0) createL(node);
            if(tree[node].r == 0) createR(node);
            lazy[tree[node].l] = value; // +=
            lazy[tree[node].r] = value; // +=
        }
    }
}

```

```

    }
    else
    {
        int mid = (start + end) / 2;
        if(tree[node].l == 0) createL(node);
        update(tree[node].l, start, mid, l, r, value);
        if(tree[node].r == 0) createR(node);
        update(tree[node].r, mid + 1, end, l, r, value);
        calc(node);
    }
}

int query(int node, int start, int end, int l, int r)
{
    if(lazy[node])
        push(node, start, end);

    if(start > r or l > end) return 0;

    if(l <= start and end <= r) return tree[node].value;

    int mid = (start + end) / 2, q1 = 0, q2 = 0;
    if(tree[node].l) q1 = query(tree[node].l, start, mid, l, r);
    if(tree[node].r) q2 = query(tree[node].r, mid + 1, end, l, r);
    return q1 + q2;
}

int main()
{
    int n, q;

    cin >> n >> q;
    init();
    for(int i = 0; i < n; i++)
    {
        int x;
        cin >> x;
        update(0, 0, n - 1, i, i, x);
    }
    while(q--)
    {
        int o, l, r, x;
        cin >> o >> l >> r;
        if(o == 1)
        {
            cin >> x;
            update(0, 0, n - 1, l - 1, r - 1, x);
        }
        else
            cout << query(0, 0, n - 1, l - 1, r - 1) << '\n';
    }

    return 0;
}

```

## 2.8 Dynamic Segment Tree With Vector

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

```

```

struct Node
{
    int l, r, value;
};

vector<Node> tree;

int init()
{
    tree.emplace_back();
}

void calc(int node)
{
    tree[node].value = 0;
    if(tree[node].l) tree[node].value += tree[tree[node].l].value;
    if(tree[node].r) tree[node].value += tree[tree[node].r].value;
}

void update(int node, int start, int end, int idx, int value)
{
    if(start == end)
        tree[node].value = value;
    else
    {
        int mid = (start + end) / 2;
        if(start <= idx and idx <= mid)
        {
            if(tree[node].l == 0)
            {
                tree[node].l = tree.size();
                tree.emplace_back();
            }
            update(tree[node].l, start, mid, idx, value);
        }
        else
        {
            if(tree[node].r == 0)
            {
                tree[node].r = tree.size();
                tree.emplace_back();
            }
            update(tree[node].r, mid + 1, end, idx, value);
        }
        calc(node);
    }
}

int query(int node, int start, int end, int l, int r)
{
    if(l > end or r < start) return 0;
    if(l <= start and end <= r) return tree[node].value;
    int mid = (start + end) / 2, q1 = 0, q2 = 0;
    if(tree[node].l) q1 = query(tree[node].l, start, mid, l, r);
    if(tree[node].r) q2 = query(tree[node].r, mid + 1, end, l, r);
    return q1 + q2;
}

int main()
{

```

```

    int n, q;

    cin >> n >> q;
    init();
    while(q--)
    {
        int o, l, r;
        cin >> o >> l >> r;
        if(o == 1) update(0, 0, n - 1, l - 1, r);
        else cout << query(0, 0, n - 1, l - 1, r - 1) << '\n';
    }

    return 0;
}

```

## 2.9 Heavy Light Decomposition Path And Subtree Queries

*/\*  
no intervalo [ in[v], out[v] ) do array A  
temos a subarvore de v. Para fazer consultas  
basta usar a segtree.*

*no intervalo [ in[nxt[v]], in[v] ] temos os vertices no  
caminho de nxt[v] ate v, Em que nxt[v] esta no inicio da  
cadeia da HLD. e o caminho de nxt[v] ate v faz parte da  
cadeia que comeca em nxt[v].*

*Assim, podemos processar queries rapidamente em caminhos  
e subarvores usando a mesma segment tree.*

*Bonus: para uma query de mudanca de raiz: se a raiz atual  
for v e a consulta for na subarvore de u, entao, se u for  
ancestral de v a resposta eh a consulta da arvore  
total menos a consulta da subarvore enraizada pelo filho  
de u que eh ancestral de v, caso contrario a consulta eh  
normal como se a raiz da arvore nunca tivesse mudado.  
obs: Para encontrar o filho de u que eh ancestral de v  
podemos usar binary lifting, da mesma forma que usamos  
para calcular lca.  
\*/*

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 5 * 1e5;

int n;
vector<int> Adj[MAX], G[MAX], A;
int in[MAX], out[MAX], rin[MAX], sz[MAX], nxt[MAX], arr[MAX], t = 0;
int st[MAX], lazy[MAX], val_vertex[MAX], depth[MAX], father[MAX];

void mount(int v = 0, int p = -1)
{
    for(int &u : Adj[v])
        if(u != p)
        {
            G[v].push_back(u);
            mount(u, v);
        }
}

```

```

void dfs_sz(int v = 0, int p = 0, int d = 0)
{
    sz[v] = 1;
    depth[v] = d;
    father[v] = p;
    for(int &u: G[v])
    {
        if(u == p) continue;
        dfs_sz(u, v, d + 1);
        sz[v] += sz[u];
        if(sz[u] > sz[G[v][0]])
            swap(u, G[v][0]);
    }
}

void dfs_hld(int v = 0, int p = -1)
{
    in[v] = t++;
    rin[in[v]] = v;
    A.push_back(val_vertex[v]);
    for(int u: G[v])
    {
        if(u == p) continue;
        nxt[u] = (u == G[v][0] ? nxt[v] : u);
        dfs_hld(u, v);
    }
    out[v] = t;
}

void build(int node, int start, int end)
{
    if(start == end)
        st[node] = A[start];
    else
    {
        int mid = (start + end) / 2;
        build(2 * node, start, mid);
        build(2 * node + 1, mid + 1, end);
        st[node] = st[2 * node] + st[2 * node + 1];
    }
}

void update(int node, int start, int end, int l, int r, int value)
{
    if(lazy[node])
    {
        st[node] += (end - start + 1) * lazy[node];
        if(start != end)
        {
            lazy[2 * node] += lazy[node];
            lazy[2 * node + 1] += lazy[node];
        }
        lazy[node] = 0;
    }
    if(l > end or start > r)
        return;
    if(l <= start and end <= r)
    {
        st[node] += value * (end - start + 1);
        if(start != end)
        {

```

```

            lazy[2 * node] += value;
            lazy[2 * node + 1] += value;
        }
        return;
    }
    int mid = (start + end) / 2;
    update(2 * node, start, mid, l, r, value);
    update(2 * node + 1, mid + 1, end, l, r, value);
    st[node] = st[2 * node] + st[2 * node + 1];
}

int query(int node, int start, int end, int l, int r)
{
    if(lazy[node])
    {
        st[node] += (end - start + 1) * lazy[node];
        if(start != end)
        {
            lazy[2 * node] += lazy[node];
            lazy[2 * node + 1] += lazy[node];
        }
        lazy[node] = 0;
    }
    if(l > end or start > r)
        return 0;
    if(l <= start and end <= r)
        return st[node];
    int mid = (start + end) / 2;
    int q1 = query(2 * node, start, mid, l, r);
    int q2 = query(2 * node + 1, mid + 1, end, l, r);
    return q1 + q2;
}

int lca(int u, int v)
{
    while(nxt[u] != nxt[v])
    {
        if(depth[nxt[u]] < depth[nxt[v]])
            v = father[nxt[v]];
        else
            u = father[nxt[u]];
    }
    return depth[u] < depth[v] ? u : v;
}

void update_up(int u, int l, int value)
{
    while(true)
    {
        if(nxt[u] == nxt[l])
        {
            update(1, 0, n - 1, in[l], in[u], value);
            break;
        }
        update(1, 0, n - 1, in[nxt[u]], in[u], value);
        u = father[nxt[u]];
    }
}

// atualiza o valor de cada vertice no caminho de
void updatePath(int u, int v, int value)//u para v.

```

```

{
    int l = lca(u, v);
    update_up(u, l, value);
    update_up(v, l, value);
    update(l, 0, n - 1, in[l], in[l], -value);
}

int query_up(int u, int l)
{
    int ans = 0;
    while(true)
    {
        if(nxt[u] == nxt[l])
        {
            ans += query(l, 0, n - 1, in[l], in[u]);
            break;
        }
        ans += query(l, 0, n - 1, in[nxt[u]], in[u]);
        u = father[nxt[u]];
    }
    return ans;
}

//consulta a soma do valor de cada vertice no caminho
int queryPath(int u, int v) // de u para v.
{
    int l = lca(u, v), ans = 0;
    ans += query_up(u, l);
    ans += query_up(v, l);
    ans -= query(l, 0, n - 1, in[l], in[l]);
    return ans;
}

int main()
{
    int q;
    scanf(" %d %d", &n, &q);
    for(int i = 1; i < n; i++)
    { // ler a arvore em qualquer ordem
        int u, v;
        scanf(" %d %d", &u, &v); u--; v--;
        Adj[u].push_back(v);
        Adj[v].push_back(u);
    }
    mount(); // montar a arvore direcionada

    dfs_sz();
    dfs_hld();

    // realizar consultas

    return 0;
}

```

## 2.10 Heavy Light Decomposition

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

```

```

// Heavy-Light Decomposition
vector<int> adj[MAX];
int par[MAX], h[MAX];

int chainno, chain[MAX], head[MAX], chainpos[MAX];
int chainsz[MAX], pos[MAX], arrsz;
int sc[MAX], sz[MAX];

void dfs(int u)
{
    sz[u] = 1, sc[u] = 0; // nodes 1-indexed (0-ind: sc[u]==-1)
    for(int v : adj[u])
        if(v != par[u])
        {
            par[v] = u, h[v] = h[u]+1, dfs(v);
            sz[u] += sz[v];
            if(sz[sc[u]] < sz[v]) sc[u] = v; // 1-indexed (0-ind: sc[u]
                                           ]<0 or ...)
        }
}

void hld(int u)
{
    if(!head[chainno]) head[chainno] = u; // 1-indexed
    chain[u] = chainno;
    chainpos[u] = chainsz[chainno];
    chainsz[chainno]++;
    pos[u] = ++arrsz;
    if(sc[u]) hld(sc[u]);
    for(int v : adj[u]) if(v != par[u] and v != sc[u]) chainno++, hld(
        v);
}

int lca(int u, int v)
{
    while(chain[u] != chain[v])
    {
        if(h[head[chain[u]]] < h[head[chain[v]]]) swap(u, v);
        u = par[head[chain[u]]];
    }
    if(h[u] > h[v]) swap(u, v);
    return u;
}

/*int query_up(int u, int v)
{
    if(u == v) return 0;
    int ans = -1;
    while(true)
    {
        if(chain[u] == chain[v])
        {
            if (u == v) break;
            ans = max(ans, query(1, 1, n, chainpos[v]+1, chainpos[u]));
            break;
        }
        ans = max(ans, query(1, 1, n, chainpos[head[chain[u]]],
            chainpos[u]));
        u = par[head[chain[u]]];
    }
}
*/

```

```

    return ans;
}

int query(int u, int v)
{
    int l = lca(u, v);
    return max(query_up(u, l), query_up(v, l));
}*/

int main()
{
    int n;
    cin >> n;
    for(int i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    dfs(1);
    hld(1);
    for(int i = 1; i <= n; i++)
        cout << chain[i] << ' ';
    puts("");
    for(int i = 1; i <= n; i++)
        cout << chainpos[i] << ' ';
    puts("");

    return 0;
}

////////////////////
//Heavy Light Decomposition para encontrar a maior aresta no
// caminho de u para v em uma arvore

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

// Heavy-Light Decomposition
vector<int> adj[MAX], W[MAX];
int par[MAX], h[MAX];
map<pair<int, int>, int> number_edge;
int chainno, chain[MAX], head[MAX], A[MAX], pos[MAX];
int sc[MAX], sz[MAX], weight[MAX], st[MAX], edge_counted, n;

void dfs(int u)
{
    sz[u] = 1, sc[u] = 0; // nodes 1-indexed (0-ind: sc[u]=-1)
    for(int i = 0; i < adj[u].size(); i++)
    {
        int v = adj[u][i], w = W[u][i];
        if(v != par[u])
        {
            weight[v] = w, par[v] = u, h[v] = h[u] + 1, dfs(v);
            sz[u] += sz[v];
            if(sz[sc[u]] < sz[v]) sc[u] = v; // 1-indexed (0-ind: sc[u]
            ]<0 or ...)
        }
    }
}

```

```

}

void hld(int u)
{
    if(!head[chainno]) head[chainno] = u; // 1-indexed
    chain[u] = chainno;
    pos[u] = edge_counted;
    A[edge_counted++] = weight[u];
    if(sc[u]) hld(sc[u]);
    for(int v : adj[u])
        if(v != par[u] and v != sc[u])
        {
            number_edge[{u, v}] = edge_counted;
            chainno++, hld(v);
        }
}

int lca(int u, int v)
{
    while(chain[u] != chain[v])
    {
        if(h[head[chain[u]]] < h[head[chain[v]]]) swap(u, v);
        u = par[head[chain[u]]];
    }
    if(h[u] > h[v]) swap(u, v);
    return u;
}

void build(int node, int start, int end)
{
    if(start == end)
        st[node] = A[start];
    else
    {
        int mid = (start + end) / 2;
        build(2 * node, start, mid);
        build(2 * node + 1, mid + 1, end);
        st[node] = max(st[2 * node], st[2 * node + 1]);
    }
}

int query(int node, int start, int end, int l, int r)
{
    if(l > end or start > r)
        return -1;
    if(l <= start and end <= r)
        return st[node];
    int mid = (start + end) / 2;
    int q1 = query(2 * node, start, mid, l, r);
    int q2 = query(2 * node + 1, mid + 1, end, l, r);
    return max(q1, q2);
}

void update(int node, int start, int end, int idx, int value)
{
    if(start == end)
        st[node] = A[idx] = value;
    else
    {
        int mid = (start + end) / 2;
        if(start <= idx and idx <= mid)

```

```

        update(2 * node, start, mid, idx, value);
    else
        update(2 * node + 1, mid + 1, end, idx, value);
    st[node] = max(st[2 * node], st[2 * node + 1]);
}

int query_up(int u, int v)
{
    if(u == v) return 0;
    int ans = -1;
    while(true)
    {
        if(chain[u] == chain[v])
        {
            if(u == v) break;
            ans = max(ans, query(1, 0, n-1, pos[v] + 1, pos[u]));
            break;
        }
        ans = max(ans, query(1, 0, n-1, pos[head[chain[u]]], pos[u]));
        u = par[head[chain[u]]];
    }
    return ans;
}

int queryMaxEdge(int u, int v)
{
    int l = lca(u, v);
    return max(query_up(u, l), query_up(v, l));
}

void updateEdge(int u, int v, int value)
{
    int idx;
    if(number_edge.find({u, v}) != number_edge.end())
        idx = number_edge[{u, v}];
    else
        idx = number_edge[{v, u}];
    update(1, 0, n-1, idx, value);
}

int main()
{
    cin >> n;
    for(int i = 1; i < n; i++)
    {
        int u, v, w;
        cin >> u >> v >> w;
        adj[u].push_back(v);
        adj[v].push_back(u);
        W[u].push_back(w);
        W[v].push_back(w);
    }
    weight[1] = -1;
    dfs(1);
    hld(1);
    build(1, 0, n-1);
    int x, y, o;
    while(cin >> o >> x >> y)
    {
        if(o == 1)

```

```

        cout << queryMaxEdge(x, y) << '\n';
    else
    {
        int w; cin >> w;
        updateEdge(x, y, w);
    }
}
return 0;
}

```

## 2.11 Implicit Treap

```

#include<bits/stdc++.h>
using namespace std;

struct Node
{
    int valor, priority, size, sum;
    Node *l, *r;
    bool rev;
    Node(int _valor) : rev(false), sum(_valor), valor(_valor),
        priority((rand() << 16) ^ rand()), size(1), l(nullptr), r(nullptr)
    {}
    ~Node() { delete l; delete r; }

    void recalc()
    {
        size = 1;
        sum = valor;
        if(l) size += l->size, sum += l->sum;
        if(r) size += r->size, sum += r->sum;
    }
};

struct Treap
{
    int size(Node* t) { return t ? t->size : 0; }

    int size() const { return root ? root->size : 0; }

    Node* propagate(Node* t)
    {
        if(t == nullptr) return t;
        if(t->rev)
        {
            swap(t->l, t->r);
            if(t->l != nullptr) t->l->rev ^= 1;
            if(t->r != nullptr) t->r->rev ^= 1;
            t->rev = 0;
        }
        t->recalc();
        return t;
    }

    int position(Node *t, int n)
    {
        //nao esta na treap, botar valor que noa esta no array...
        if(t == nullptr) return -1;
        propagate(t);
        if(n == size(t->l) + 1) return t->valor;
    }
}

```

```

    else if(n <= size(t->l)) return position(t->l, n);
    else return position(t->r, n - size(t->l) - 1);
}

int at(int n)
{
    return position(root, n);
}

Node* merge(Node *l, Node *r)
{
    l = propagate(l);
    r = propagate(r);
    if(!l or !r) return l ? l : r;
    if(l->priority < r->priority)
    {
        l->r = merge(l->r, r);
        l->recalc();
        return l;
    }
    else
    {
        r->l = merge(l, r->l);
        r->recalc();
        return r;
    }
}

void split(Node *v, int valor, Node *&l, Node *&r)
{
    v = propagate(v);
    l = r = nullptr;
    if(!v) return;
    if(size(v->l) < valor)
    {
        split(v->r, valor - size(v->l) - 1, v->r, r);
        l = v;
    } else
    {
        split(v->l, valor, l, v->l);
        r = v;
    }
    v->recalc();
}

Node * root;
Treap() : root(nullptr) {}
~Treap() { delete root; }

void insert(int valor, int pos)
{
    Node * l, * r;
    split(root, pos - 1, l, r);
    root = merge(merge(l, new Node(valor)), r);
}

void erase(int valor)
{
    Node * l, * m, * r;
    split(root, valor - 1, l, m);
    split(m, 1, m, r);

```

```

    delete m;
    root = merge(l, r);
}

void reverse(int l, int r)
{
    l--; r--;
    if(l > r) swap(l, r);
    Node *a, *b, *c, *d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    if(b) b->rev ^= 1;
    root = merge(a, merge(b, c));
}

int query(int l, int r)
{
    Node *a, *b, *c, *d;
    split(root, l - 1, a, b);
    split(b, r - l + 1, c, d);
    int ans = c->sum;
    root = merge(a, merge(c, d));
    return ans;
}

/*void emOrdem(Node *node)
{
    if(node == nullptr) return;
    emOrdem(node->l);
    printf("%d ", node->valor);
    emOrdem(node->r);
}*/

}treap;

int main()
{
    srand(time(0));

    for(int i = 1; i <= 6; i++)
    {
        int x; cin >> x;
        cout << x << ' ';
        treap.insert(x, i);
    }
    return 0;
}

```

## 2.12 LiChao Tree

```

#include <bits/stdc++.h>
using namespace std;
#define x real
#define y imag
typedef int ftype;
typedef complex<ftype> point;
const int OO = 0x3f3f3f3f;
const int maxn = 2e5;

point line[4 * maxn];

```



```

void init()
{
    for(int i = 0; i < 4 * maxn; i++)
        line[i] = point(0, 00);
}

ftype dot(point a, point b)
{
    return (conj(a) * b).x();
}

ftype f(point a, ftype x)
{
    return dot(a, {x, 1});
}

void add_line(point nw, int v = 1, int l = 0, int r = maxn)
{
    int m = (l + r) / 2;
    bool lef = f(nw, l) < f(line[v], l);
    bool mid = f(nw, m) < f(line[v], m);
    if(mid)
        swap(line[v], nw);
    if(r - l == 1)
        return;
    else if(lef != mid)
        add_line(nw, 2 * v, l, m);
    else
        add_line(nw, 2 * v + 1, m, r);
}

int get(int x, int v = 1, int l = 0, int r = maxn)
{
    int m = (l + r) / 2;
    if(r - l == 1)
        return f(line[v], x);
    else if(x < m)
        return min(f(line[v], x), get(x, 2 * v, l, m));
    else
        return min(f(line[v], x), get(x, 2 * v + 1, m, r));
}

int main()
{
    init();

    point a(2, 4);
    point b(1, 3);

    add_line(a);
    add_line(b);

    cout << get(2) << '\n';

    return 0;
}

```

```

#include <bits/stdc++.h>
using namespace std;

struct MaxQueue
{
    int plus = 0;
    deque<pair<int, int>> dq;

    bool empty()
    {
        return (int)dq.size() == 0;
    }

    void clear()
    {
        plus = 0;
        dq.clear();
    }

    void add(int x)
    { // somar x em cada elemento da fila
        plus += x;
    }

    int max()
    {
        return dq.begin()->first + plus;
    }

    void push(int x)
    {
        x -= plus;
        int amt = 0;
        while (dq.size() and dq.back().first <= x)
            amt += dq.back().second + 1, dq.pop_back();
        dq.push_back({ x, amt });
    }

    void pop()
    {
        if (dq.empty()) return;
        if (!dq.front().second) dq.pop_front();
        else dq.front().second--;
    }
};

int main()
{
    int n, aux;
    MaxQueue Q;
    cin >> n;
    for(int i = 0; i < n; i++)
    {
        int aux;
        cin >> aux;
        Q.push(aux);
        cout << "max " << Q.max() << '\n';
    }
    return 0;
}

```

## 2.13 Max Queue

## 2.14 Merge Sort Tree Iterative

```
#include <bits/stdc++.h>
using namespace std;
#define OO 0x3f3f3f3f

struct MergeSortTree
{
    int n;
    vector<vector<int>> tree;

    MergeSortTree(vector<int> &a)
    {
        n = a.size();
        tree.resize(n << 1);
        for(int i = 0; i < n; i++)
            tree[i + n] = vector<int>{a[i]};
        build();
    }

    void build()
    {
        for(int i = n - 1; i > 0; --i)
        {
            int L = i << 1;
            int R = (i << 1) | 1;
            int l = 0, r = 0, sz = tree[L].size() + tree[R].size();

            tree[i].resize(sz);

            tree[L].push_back(OO);
            tree[R].push_back(OO);

            for(int j = 0; j < sz; j++)
                if(tree[L][l] < tree[R][r])
                    tree[i][j] = tree[L][l++];
                else
                    tree[i][j] = tree[R][r++];

            tree[L].pop_back();
            tree[R].pop_back();
        }
    }

    int queryMax(int l, int r, int x)
    {
        if(l >= r) return 0;
        int res = 0;
        for(l += n, r += n; l < r; l >>= 1, r >>= 1)
        {
            if(l & 1)
            {
                auto it = upper_bound(tree[l].begin(), tree[l].end(), x);
                int p = it - tree[l].begin();
                if(it != tree[l].end())
                {
                    int p = it - tree[l].begin();
                    res += (int)tree[l].size() - p;
                }
            }
            return res;
        }
    }

    int queryMin(int l, int r, int x)
    {
        if(l >= r) return 0;
        int res = 0;
        for(l += n, r += n; l < r; l >>= 1, r >>= 1)
        {
            if(l & 1)
            {
                auto it = lower_bound(tree[l].begin(), tree[l].end(), x);
                if(it == tree[l].end()) res += tree[l].size();
                else res += it - tree[l].begin();
                l++;
            }
            if(r & 1)
            {
                r--;
                auto it = lower_bound(tree[r].begin(), tree[r].end(), x);
                if(it == tree[r].end()) res += tree[r].size();
                else res += it - tree[r].begin();
            }
            return res;
        }
    }
};

int32_t main()
{
    int n;
    scanf("%d", &n);
    vector<int> v(n);
    for(int &w : v) cin >> w;
    MergeSortTree T(v);
    // query(l, r, x)    [l, r)

    return 0;
}
```

```

    }
    if(r & 1)
    {
        r--;
        auto it = upper_bound(tree[r].begin(), tree[r].end(), x);
        if(it != tree[r].end())
        {
            int p = it - tree[r].begin();
            res += (int)tree[r].size() - p;
        }
    }
    return res;
}

int queryMin(int l, int r, int x)
{
    if(l >= r) return 0;
    int res = 0;
    for(l += n, r += n; l < r; l >>= 1, r >>= 1)
    {
        if(l & 1)
        {
            auto it = lower_bound(tree[l].begin(), tree[l].end(), x);
            if(it == tree[l].end()) res += tree[l].size();
            else res += it - tree[l].begin();
            l++;
        }
        if(r & 1)
        {
            r--;
            auto it = lower_bound(tree[r].begin(), tree[r].end(), x);
            if(it == tree[r].end()) res += tree[r].size();
            else res += it - tree[r].begin();
        }
    }
    return res;
}

int32_t main()
{
    int n;
    scanf("%d", &n);
    vector<int> v(n);
    for(int &w : v) cin >> w;
    MergeSortTree T(v);
    // query(l, r, x)    [l, r)

    return 0;
}
```

## 2.15 Merge Sort Tree Range Order Statistics Queries

```
#include <bits/stdc++.h>
using namespace std;

int n;
vector<int> tree[100000];
vector<pair<int, int>> arr;
```

```

void build(int node, int start, int end)
{
    if(start == end)
        tree[node].push_back(arr[start].second);
    else
    {
        int mid = (start + end) / 2;
        build(2 * node, start, mid);
        build(2 * node + 1, mid + 1, end);
        merge(tree[2 * node].begin(), tree[2 * node].end(),
              tree[2 * node + 1].begin(), tree[2 * node + 1].end(),
              back_inserter(tree[node]));
    }
}

int query(int node, int start, int end, int l, int r, int k)
{
    if(start == end)
        return arr[start].first;
    int M = upper_bound(tree[2 * node].begin(), tree[2 * node].end(),
                        r)
        - lower_bound(tree[2 * node].begin(), tree[2 * node].end(), l);
    int mid = (start + end) / 2;
    if(M >= k)
        return query(2 * node, start, mid, l, r, k);
    else
        return query(2 * node + 1, mid + 1, end, l, r, k - M);
}

int main()
{
    cin >> n;
    int aux;
    for(int i = 0; i < n; i++)
    {
        cin >> aux;
        arr.push_back({aux, i});
    }
    sort(arr.begin(), arr.end());
    build(1, 0, n-1);

    int l, r, k;
    while(cin >> l >> r >> k)
        cout << query(1, 0, n-1, l-1, r-1, k) << '\n';

    return 0;
}

```

## 2.16 Merge Sort Tree With Set

```

/*
encontra o menor numero na range [L, R] que eh maior
ou igual a K. build eh O(NlogNlogN) e query eh O(logNlogN)
e o erasee eh O(logNlogN).
*/

```

```

#define ii pair<int, int>
#define value first

```

```

#define index second
const int MAX = 1e6;

int n, m;
vector<array<int, 3>> B, A;

set<ii> tree[MAX];

void build(int node, int start, int end)
{
    if(start == end)
        tree[node] = set<ii>{ii(A[start][1], start)};
    else
    {
        int mid = (start + end) / 2;
        build(2*node, start, mid);
        build(2*node + 1, mid + 1, end);
        for(auto &it : tree[2 * node])
            tree[node].insert(it);
        for(auto &it : tree[2 * node + 1])
            tree[node].insert(it);
    }
}

ii query(int node, int start, int end, int l, int r, int k)
{
    if(start > r or end < l)
        return ii(-1, -1);
    if(l <= start and end <= r)
    {
        auto it = tree[node].upper_bound({k, n + 1});
        ii q = {-1, -1};
        if(it != tree[node].begin()) q = *--it;
        return q;
    }
    int mid = (start + end) / 2;
    ii p1 = query(2 * node, start, mid, l, r, k);
    ii p2 = query(2 * node + 1, mid + 1, end, l, r, k);
    return p1.value <= p2.value ? p2 : p1;
}

void erasee(int node, int start, int end, ii p)
{
    if(tree[node].count(p) == 0) return;
    if(start == end)
    {
        tree[node].erase(p);
        return;
    }
    int mid = (start + end) / 2;
    erasee(2 * node, start, mid, p);
    erasee(2 * node + 1, mid + 1, end, p);
    tree[node].erase(p);
}

```

## 2.17 Merge Sort Tree

```

/*
O nome eh Merge Sort Tree pois o array armazenado em
um Node da Segment Tree eh igual ao gerado pelo algoritmo

```

```

    de ordenacao Merge Sort no Node correspondente da
    arvore de recursao.
*/

#include <bits/stdc++.h>
using namespace std;

const int MAX = 131072;

vector<int> a[MAX], tree[MAX];

void build(int node, int start, int end)
{
    if(start == end)
        tree[node] = a[start];
    else
    {
        int mid = (start + end) / 2;
        build(2*node, start, mid);
        build(2*node + 1, mid + 1, end);
        merge(tree[2*node].begin(), tree[2*node].end(),
              tree[2*node + 1].begin(), tree[2*node + 1].end()
              , back_inserter(tree[node]));
    }
}

int query(int node, int start, int end, int l, int r, int k)
{
    if(start > r or end < l)
        return 0;
    if(l <= start and end <= r)
        return upper_bound(tree[node].begin(), tree[node].end(), k)
              - tree[node].begin();
    int mid = (start + end) / 2;
    int p1 = query(2*node, start, mid, l, r, k);
    int p2 = query(2*node + 1, mid + 1, end, l, r, k);
    return p1 + p2;
}

int main()
{
    int n, aux;

    cin >> n;
    for(int i = 0; i < n; i++)
    {
        cin >> aux;
        a[i].push_back(aux);
    }
    build(1, 0, n-1);
    int l, r, k;
    cin >> l >> r >> k;
    //quantidade de elementos menores ou iguais a k na range [l - r].
    cout << query(1, 0, n-1, l-1, r-1, k) << '\n';

    return 0;
}

```

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5+10;

int bit[MAX], arr[MAX];

int bitSearch(int v)
{
    int sum = 0, pos = 0, LOGN = log2(MAX - 2);
    for(int i = LOGN; i >= 0; i--)
        if(pos + (1 << i) < MAX and sum + bit[pos + (1 << i)] < v)
        {
            sum += bit[pos + (1 << i)];
            pos += (1 << i);
        }
    return pos + 1;
    // pos + 1, pq pos eh a maior posicao cuja soma do prefixo ate
    // ela eh menor que V
    // essa funcao retorna o indice J no array em que a soma do
    // prefixo [1, J] eh o lower_bound para V
    // inserir os elemento na BIT com add(i, arr[i]), para todo i em [1, n
    ]

int query(int idx) // soma de um prefixo
{
    int sum = 0;
    for(; idx > 0; idx -= idx&-idx) sum += bit[idx];
    return sum;
}

void add(int idx, int k)
{
    for(int i = idx; i < MAX; i += i&-i) bit[i] += k;
}

int smallerCount(int v)
{
    return query(v);
}

int count(int v)
{
    return query(v) - query(v - 1);
}

int greaterCount(int v)
{
    return query(MAX - 3) - query(v - 1);
}

int orderOfKey(int v)
{
    return smallerCount(v);
}

int kth(int k)
{
    return bitSearch(k);
}

```

```
int main()
{
    int n;

    cin >> n;
    for(int i = 1; i <= n; i++)
    {
        cin >> arr[i];
        add(arr[i], 1);
    }
    cout << smallerCount(3) << '\n';
    cout << count(3) << '\n';
    cout << greaterCount(3) << '\n';
    cout << kth(2) << '\n';
    cout << orderOfKey(4) << '\n';

    return 0;
}
```

## 2.19 PBDS

```
#include <bits/stdc++.h>
// Common file
#include <ext/pb_ds/assoc_container.hpp>
// Including tree_order_statistics_node_update
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

typedef tree<int, null_type, less<int>,
            rb_tree_tag, tree_order_statistics_node_update> ordered_set;

int main()
{
    ordered_set X;
    X.insert(2);
    X.insert(13);
    X.insert(5);
    X.insert(2);
    cout << *X.find_by_order(0) << '\n';
    cout << X.order_of_key(1) << '\n';

    return 0;
}
```

```
////////////////////////////////////
```

```
#include <bits/stdc++.h>
// Common file
#include <ext/pb_ds/assoc_container.hpp>
// Including tree_order_statistics_node_update
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;

#define vi vector<int>
#define var pair<int,int>
#define ordered_multiset tree<var, null_type, less<var>
```

```
, rb_tree_tag, tree_order_statistics_node_update>

int id = 0; map<int,vi> ids;

void insere(ordered_multiset &s, int x)
{
    s.insert({x, ++id});
    ids[x].push_back(id);
}

void apaga(ordered_multiset &s, int x)
{
    if(ids[x].empty()) return;
    s.erase({x, ids[x].back()});
    ids[x].pop_back();
}

int kth(ordered_multiset &s, int x)
{
    return s.find_by_order(x)->first;
}

int smallerCount(ordered_multiset &s, int x)
{
    return s.order_of_key({x, 0});
}

int count(ordered_multiset &s, int x)
{
    return smallerCount(s, x + 1) - smallerCount(s, x);
}

ordered_multiset::iterator find(ordered_multiset &s, int x)
{
    if(ids[x].empty())
        return s.end();
    return s.find({x, ids[x].back()});
}

int main()
{
    ordered_multiset X;

    // usar funcoes ...

    return 0;
}
```

## 2.20 Persistent Segment Tree Without Vector

```
#include <bits/stdc++.h>
using namespace std;

#define _ << " , " <<
#define bug(x) cout << #x << " >>>>>> " << x << endl;
#define fastio ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0);

const int MAX = 800100;
const int LOG_MAX = 30;
```

```

struct PersistentSegmentTree {
    int n; // the queries will be performed in the range [0, n - 1]
    int L[LOG_MAX * MAX]; // L[node] is the left child of node
    int R[LOG_MAX * MAX]; // R[node] is the right child of node
    int tree[LOG_MAX * MAX]; // tree[node] is the value stored in the
    node
    int root[MAX]; // stores the root of each version
    int next_vertex; // next index for a vertex
    int version_count; // number of different versions of the tree

    void calc(int node) {
        int sum = 0;
        if(L[node]) sum += tree[L[node]];
        if(R[node]) sum += tree[R[node]];
        tree[node] = sum;
    }
    void update(int prev, int node, int start, int end, int idx, int
    value) {
        if(start == end)
            tree[node] = tree[prev] + value;
        else {
            int mid = (start + end) / 2;
            if(start <= idx and idx <= mid) {
                R[node] = R[prev];
                if(L[node] == 0) L[node] = next_vertex++;
                update(L[prev], L[node], start, mid, idx, value);
            } else {
                L[node] = L[prev];
                if(R[node] == 0) R[node] = next_vertex++;
                update(R[prev], R[node], mid + 1, end, idx, value);
            }
            calc(node);
        }
    }
    int query(int node, int start, int end, int l, int r) {
        if(l > end or r < start) return 0;
        if(l <= start and end <= r) return tree[node];
        int mid = (start + end) / 2, q1 = 0, q2 = 0;
        if(L[node]) q1 = query(L[node], start, mid, l, r);
        if(R[node]) q2 = query(R[node], mid + 1, end, l, r);
        return q1 + q2;
    }
}

void init(int _n) {
    root[0] = 0;
    next_vertex = 1;
    version_count = 1;
    n = _n;
}

int update(int idx, int value, int prev_version = -1) {
    if(prev_version == -1) prev_version = version_count - 1;
    root[version_count] = next_vertex++;
    update(root[prev_version], root[version_count], 0, n - 1, idx,
    value);
    version_count++;
    return version_count - 1;
}

int query(int l, int r, int version = -1) {
    if(version == -1) version = version_count - 1;
    return query(root[version], 0, n - 1, l, r);
}

```

```

};

int n, q;
PersistentSegmentTree PST;

int32_t main() {
    fastio

    cin >> n >> q;

    PST.init(n);
    map<int, int> lst;
    vector<int> root(n);

    for(int i = 0; i < n; ++i) {
        int x;
        cin >> x;

        int l = -1;
        if(lst.count(x)) l = lst[x];

        if(l != -1) PST.update(l, -1);

        root[i] = PST.update(i, 1);

        lst[x] = i;
    }

    while(q--) {
        int l, r;
        cin >> l >> r; l--; r--;
        cout << PST.query(l, r, root[r]) << '\n';
    }

    return 0;
}

```

## 2.21 Persistent Segment Tree

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

struct Node
{
    int l, r, value;
};

vector<Node> tree;
vector<int> root;

void init()
{
    tree.emplace_back();
    root.push_back(0);
}

void calc(int node)
{
    tree[node].value = 0;
}

```

```

    if(tree[node].l) tree[node].value += tree[tree[node].l].value;
    if(tree[node].r) tree[node].value += tree[tree[node].r].value;
}

void update(int prev, int node, int start, int end, int idx, int value)
{
    if(start == end)
        tree[node].value = value;
    else
    {
        int mid = (start + end) / 2;
        if(start <= idx and idx <= mid)
        {
            tree[node].r = tree[prev].r;
            if(tree[node].l == 0)
            {
                tree[node].l = tree.size();
                tree.emplace_back();
            }
            update(tree[prev].l, tree[node].l, start, mid, idx, value);
        }
        else
        {
            tree[node].l = tree[prev].l;
            if(tree[node].r == 0)
            {
                tree[node].r = tree.size();
                tree.emplace_back();
            }
            update(tree[prev].r, tree[node].r, mid + 1, end, idx, value);
        }
        calc(node);
    }
}

int query(int node, int start, int end, int l, int r)
{
    if(l > end or r < start) return 0;
    if(l <= start and end <= r) return tree[node].value;
    int mid = (start + end) / 2, q1 = 0, q2 = 0;
    if(tree[node].l) q1 = query(tree[node].l, start, mid, l, r);
    if(tree[node].r) q2 = query(tree[node].r, mid + 1, end, l, r);
    return q1 + q2;
}

int main()
{
    int n, q;

    cin >> n >> q;
    init();
    while(q--)
    {
        int o, l, r;
        cin >> o >> l >> r;
        if(o == 1)
        {
            int prev = root.back();
            root.push_back(tree.size());
            tree.emplace_back();

            update(prev, root.back(), 0, n - 1, l - 1, r);
        }
        else
        {
            int version;
            scanf("%d", &version);
            cout << query(root[version], 0, n - 1, l - 1, r - 1) << '\n';
        }
    }

    return 0;
}

```

## 2.22 Segment Tree Iterative

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5 + 10;

int tree[MAX << 1], n;

void build()
{
    for(int i = n - 1; i > 0; --i) tree[i] = tree[i << 1] + tree[i << 1 | 1];
}

void update(int p, int value)
{
    for(tree[p += n] = value; p > 1; p >>= 1) tree[p >> 1] = tree[p] + tree[p ^ 1];
}

int query(int l, int r)
{
    int res = 0;
    for(l += n, r += n; l < r; l >>= 1, r >>= 1)
    {
        if(l & 1) res += tree[l++];
        if(r & 1) res += tree[--r];
    }
    return res;
}

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> tree[i + n];
    build();
    int q, l, r, o;
    cin >> q;
    while(q--)
    {
        cin >> o >> l >> r;
        if(o == 1)
        {
            cout << query(l - 1, r) << '\n'; // soma de [l, r]
        }
        else
        {
            update(l - 1, r); // atualiza a posicao l pra r
        }
    }
}

```

```

    return 0;
}

```

## 2.23 Segment Tree Tree 2D

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e3;

int n, leaf;
int arr[MAX][MAX];
int ST[4*MAX][4*MAX];

void buildLeaf(int k, int node, int l, int r)
{
    if(l == r)
        ST[k][node] = arr[leaf][l];
    else
    {
        int mid = (l + r) / 2;
        buildLeaf(k, 2*node, l, mid);
        buildLeaf(k, 2*node + 1, mid + 1, r);
        ST[k][node] = ST[k][2*node] + ST[k][2*node+1];
    }
}

void build(int node, int l, int r)
{
    if(l == r)
        buildLeaf(node, l, 0, n-1), leaf++;
    else
    {
        int mid = (l + r) / 2;
        build(2*node, l, mid);
        build(2*node + 1, mid + 1, r);
        for(int i = 1; i < 4*n; i++)
            ST[node][i] = ST[2*node][i] + ST[2*node+1][i];
    }
}

int queryNode(int k, int node, int l, int r, int cx, int cy)
{
    if(l > cy or r < cx)
        return 0;
    if(cx <= l and r <= cy)
        return ST[k][node];
    int mid = (l + r) / 2;
    int ans = queryNode(k, 2*node, l, mid, cx, cy);
    ans += queryNode(k, 2*node + 1, mid + 1, r, cx, cy);
    return ans;
}

int query(int node, int l, int r, int lx, int ly, int cx, int cy)
{
    if(l > ly or r < lx)
        return 0;
    if(lx <= l and r <= ly)
        return queryNode(node, l, 0, n-1, cx, cy);
    int mid = (l + r) / 2;

```

```

    int ans = query(2*node, l, mid, lx, ly, cx, cy);
    ans += query(2*node + 1, mid + 1, r, lx, ly, cx, cy);
    return ans;
}

```

```

void updateNode(int k, int node, int l, int r, int x, int y, int value)
{
    if(l == r)
        ST[k][node] = arr[x][y] = value;
    else
    {
        int mid = (l + r) / 2;
        if(l <= y and y <= mid)
            updateNode(k, 2*node, l, mid, x, y, value);
        else
            updateNode(k, 2*node + 1, mid + 1, r, x, y, value);
        ST[k][node] = ST[k][2*node] + ST[k][2*node + 1];
    }
}

```

```

void update(int node, int l, int r, int x, int y, int value)
{
    if(l == r)
        updateNode(node, l, 0, n-1, x, y, value);
    else
    {
        int mid = (l + r) / 2;
        if(l <= x and x <= mid)
            update(2*node, l, mid, x, y, value);
        else
            update(2*node + 1, mid + 1, r, x, y, value);
        for(int i = 1; i < 4*n; i++)
            ST[node][i] = ST[2*node][i] + ST[2*node+1][i];
    }
}

```

```

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
            cin >> arr[i][j];
    build(1, 0, n-1);
    int o, a, b, c, d;
    while(cin >> o)
    {
        cin >> a >> b >> c;
        if(o == 1)
            update(1, 0, n-1, a-1, b-1, c);
        else
        {
            cin >> d;
            cout << query(1, 0, n-1, a-1, b-1, c-1, d-1) << '\n';
        }
    }

    return 0;
}

```



## 2.24 Segment Tree With Lazy Propagation

```
#include <bits/stdc++.h>
using namespace std;

#define int long long

#define esq node << 1LL
#define dir (node << 1) | 1LL

struct SegmentTree {
    vector<int> tree, lazy;
    int size;

    void build(int node, int start, int end, vector<int> &a) {
        if(start == end) {
            tree[node] = a[start];
            return;
        }
        int mid = (start + end) >> 1;
        build(esq, start, mid, a);
        build(dir, mid + 1, end, a);
        tree[node] = tree[esq] + tree[dir];
    }

    SegmentTree() {}

    SegmentTree(int n) {
        size = n;
        tree.resize(size << 2);
        lazy.resize(size << 2);
    }

    void init(vector<int> &a) {
        build(1, 0, size - 1, a);
    }

    // += add in the interval
    void push(int node, int start, int end) {
        tree[node] += lazy[node] * (end - start + 1);
        if(start != end) {
            lazy[esq] += lazy[node];
            lazy[dir] += lazy[node];
        }
        lazy[node] = 0;
    }

    int query(int node, int start, int end, int l, int r) {
        if(lazy[node]) push(node, start, end);
        if(l > end or start > r or l > r) return 0;
        if(l <= start and end <= r) return tree[node];
        int mid = (start + end) >> 1;
        int q1 = query(esq, start, mid, l, r);
        int q2 = query(dir, mid + 1, end, l, r);
        return q1 + q2;
    }

    int query(int l, int r) {
        return query(1, 0, size - 1, l, r);
    }
}
```

```
void update(int node, int start, int end, int l, int r, int v) {
    if(lazy[node]) push(node, start, end);
    if(l > end or start > r or l > r) return;
    if(l <= start and end <= r) {
        lazy[node] += v;
        push(node, start, end);
        return;
    }
    int mid = (start + end) >> 1;
    update(esq, start, mid, l, r, v);
    update(dir, mid + 1, end, l, r, v);
    tree[node] = tree[esq] + tree[dir];
}

void update(int l, int r, int v) {
    update(1, 0, size - 1, l, r, v);
}

int32_t main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int n;
    cin >> n;
    SegmentTree T(n);

    return 0;
}
```

## 2.25 Sparse Table RMQ

```
#include <bits/stdc++.h>
#define maxn 100000
#define maxnlog 20

using namespace std;

const double EPS = 1e-6;

int n, q, Sparse_Table[maxnlog][maxn];

void build()
{
    for(int i = 1; (1 << i) <= n; i++)
        for(int j = 0; j + (1 << i) <= n; j++)
            Sparse_Table[i][j] = max(Sparse_Table[i-1][j],
                                     Sparse_Table[i-1][j+(1 << (i-1))]);
}

int range_query(int i, int j)
{
    int sz = log2(j-i+1);
    return max(Sparse_Table[sz][i], Sparse_Table[sz][j+1-(1 << sz)]);
}

int main()
```

```

{
    scanf("%d %d", &n, &q);
    for(int i = 0; i < n; i++)
        scanf("%d", &Sparse_Table[0][i]);
    build();
    for(int i = 0; i < q; i++)
    {
        int a, b;
        scanf("%d %d", &a, &b);
        cout << range_query(a,b) << endl;
    }

    return 0;
}

```

## 2.26 Treap

```

#include<bits/stdc++.h>

using namespace std;

struct Node
{
    int valor, priority, size, maior;
    Node *l, *r;

    Node(int _valor) : valor(_valor), priority((rand() << 16)
        ^ rand()), size(1), l(nullptr), r(nullptr), maior(_valor) {}
    ~Node() { delete l; delete r; }

    void recalc()
    {
        size = 1;
        maior = valor;
        if (l) size += l->size, maior = max(maior, l->maior);
        if (r) size += r->size, maior = max(maior, r->maior);
    }
};

struct Treap
{
    Node* merge(Node *l, Node *r)
    {
        if(!l or !r) return l ? l : r;
        // Se a prioridade esquerda eh menor.
        if(l->priority < r->priority)
        {
            l->r = merge(l->r, r);
            l->recalc();
            return l;
            // Se a prioridade direita eh maior ou igual.
        }
        else
        {
            r->l = merge(l, r->l);
            r->recalc();
            return r;
        }
    }
}

```

```

// Valores maiores ou iguais a "valor" ficarao no r, e os demais no l.
void split(Node *v, int valor, Node *&l, Node *&r)
{
    l = r = nullptr;
    if(!v) return;
    // Se o valor for maior, ir para direita
    if(v->valor < valor)
    {
        split(v->r, valor, v->r, r);
        l = v;
        // Se o valor for menor ou igual ir para esquerda.
    }else
    {
        split(v->l, valor, l, v->l);
        r = v;
    }
    v->recalc();
}

bool find(Node *v, int valor)
{
    if(!v) return false;
    if( v->valor == valor ) return true;
    if( v->valor < valor ) return find(v->r, valor);
    if( v->valor > valor ) return find(v->l, valor);
}

int smallestCount(Node *v, int valor)
{
    if(!v) return 0;
    // Se for menor ou igual adicionar + 1.
    if(v->valor == valor) return (v->l ? v->l->size : 0);
    if(v->valor < valor) return 1 + (v->l ? v->l->size : 0)
        + smallestCount(v->r, valor);
    if(v->valor > valor) return smallestCount(v->l, valor);
}

Node* kth(Node *v, int posicao)
{
    if(!v) return nullptr;
    int esquerda = (v->l ? v->l->size : 0);
    if(posicao-esquerda == 1) return v;
    if(posicao-esquerda > 1) return kth(v->r, posicao-esquerda-1);
    if(posicao-esquerda < 1) return kth(v->l, posicao);
}

// Sendo i e j os indices no array ordenado
// Talvez deh problemas de i e j estiverem fora do range.
int query(int i, int j)
{
    Node *l, *q, *r;
    split(root, kth(root, i+1)->valor, l, q);
    split(q, kth(q, j+1-i)->valor+1, q, r);
    int x = q->maior;
    root = merge(l, merge(q,r));
    return x;
}

Node * root;
Treap() : root(nullptr) {}

```

```

~Treap() { delete root; }

// Se existe um elemento com o valor
bool find(int valor)
{
    return find(root, valor);
}
// Quantidade de elementos menores que o valor
int smallestCount(int valor)
{
    return smallestCount(root, valor);
}
// Retorna o k-th menor elemento
Node * kth(int posicao){
    return kth(root, posicao);
}
// Insere o valor mesmo se ja exista outro com valor igual
void insert(int valor)
{
    Node * l, * r;
    split(root, valor, l, r);
    root = merge(merge(l, new Node(valor)), r);
}
// Apaga todos os elementos que possuem o valor.
void erase(int valor)
{
    Node * l, * m, * r;
    split(root, valor, l, m);
    split(m, valor + 1, m, r);
    delete m;
    root = merge(l, r);
}

// Quantos valores existem menor que "valor"
int menoresQue(int valor)
{
    Node * l, * r;
    split(root, valor, l, r);
    int res = (l ? l->size: 0);
    root = merge(l, r);
    return res;
}

// splitSmallest eh uma funcao que esta na implicit treap

// Retorna a consulta dos primeiros "quantidade" valor
int top(int quantidade)
{
    Node *l, *r;
    splitSmallest(root, quantidade, l, r);
    int valor = (l ? l->maior : 0);
    root = merge(l, r);
    return valor;
}
// Remover os d menores
void removeSmallest(int d)
{
    Node *l, *r;
    splitSmallest(root, d, l, r);
    root = r;
}

```

```

    if(l) delete l;
}
// Remover todos menos os d menores
void limit(int d)
{
    Node * l, * r;
    splitSmallest(root, d, l, r);
    root = l;
    if(r) delete r;
}
int size() const { return root ? root->size : 0; }
}treap;

int n, a;
char op;

int main()
{
    srand(time(0));

    cin >> n;
    for(int i = 0; i < n; i++)
    {
        cin >> op >> a;
        if(op == 'I')
        {
            if(!treap.find(a))
                treap.insert(a);
        }
        else if(op == 'D' )
            treap.erase(a);
        else if(op == 'C' )
            cout << treap.smallestCount(a) << '\n';
        else if(op == 'K' )
        {
            Node *v = treap.kth(a);
            if(v == nullptr) cout << "invalid" << '\n';
            else cout << v->valor << '\n';
        }
    }

    return 0;
}

```

## 2.27 TreeIsomorfismWithMap

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5 + 10;

int n, ID, degree[MAX];
map<map<int, int>, int> formato;
bool vis[MAX];

void init()
{
    formato.clear();
    ID = 1;
}

```

```

int dfs(int v, int p, vector<vector<int>> &G)
{
    if((int)G[v].size() == 1)
        return 1;
    map<int, int> ids;
    for(int &u : G[v])
    {
        if(u == p) continue;
        int x = dfs(u, v, G);
        ids[x]++;
    }
    if(formato.count(ids) <= 0)
        formato[ids] = ++ID;
    return formato[ids];
}

inline void findCenterAndComputeID(vector<vector<int>> &G, vector<int>
    &val)
{
    memset(vis, 0, sizeof(vis));
    queue<int> fila[2];
    for(int i = 0; i < n; i++)
        if(degree[i] == 1)
            fila[0].push(i);
    int cnt = 0, turn = 0;
    while(cnt + 2 < n)
    {
        while(!fila[turn].empty())
        {
            int u = fila[turn].front(); fila[turn].pop();
            vis[u] = true;
            cnt++;
            for(int i = 0; i < G[u].size(); i++)
                if(!vis[G[u][i]])
                {
                    degree[G[u][i]]--;
                    if(degree[G[u][i]] == 1) fila[1-turn].push(G[u][i]);
                }
        }
        turn ^= 1;
    }
    for(int i = 0; i < n; i++)
    {
        if(vis[i]) continue;
        val.push_back(dfs(i, -1, G));
    }
}

int32_t main()
{
    while(cin >> n)
    {
        init();
        vector<int> val[2];
        for(int j = 0; j < 2; j++)
        {
            memset(degree, 0, sizeof(degree));
            vector<vector<int>> G(n + 1);
            for(int i = 1; i < n; i++)
            {

```

```

                int u, v;
                scanf(" %d %d", &u, &v); u--; v--;
                G[u].push_back(v);
                G[v].push_back(u);
                degree[v]++;
                degree[u]++;
            }
            findCenterAndComputeID(G, val[j]);
        }
        bool fl = false;
        for(int &v0 : val[0])
            for(int &v1 : val[1])
                if(v0 == v1)
                    fl = true;
        puts(fl ? "S" : "N");
    }

    return 0;
}

```

## 2.28 TreeIsomorfismWithPolynomialHashing

```

#include <bits/stdc++.h>
using namespace std;
#define ll long long
const int MAX = 1e5 + 10;
const ll A = 911382323;
const ll B = 972663749;

int n, ID, degree[MAX];
map<ll, ll> formato;
bool vis[MAX];

inline ll norm(ll a)
{
    return a > B ? a = a % B : a;
}

inline ll add(ll a, ll b)
{
    a = norm(a); b = norm(b);
    return norm(a + b);
}

inline ll prod(ll a, ll b)
{
    a = norm(a); b = norm(b);
    return norm(a * b);
}

inline ll pol_hash(vector<ll> &v)
{
    ll p = 1, ans = 0;
    for(ll &w : v)
    {
        ans = add(ans, prod(p, w));
        p = prod(p, A);
    }
    return norm(ans);
}

```

```

11 dfs(int v, int p, vector<vector<int>> &G)
{
    if((int)G[v].size() == 1)
        return 1;
    vector<ll> ids;
    for(int &u : G[v])
    {
        if(u == p) continue;
        ll x = dfs(u, v, G);
        ids.push_back(x);
    }
    sort(ids.begin(), ids.end());
    ll ph = pol_hash(ids);
    if(formato.count(ph) <= 0) formato[ph] = ++ID;
    return formato[ph];
}

inline void findCenterAndComputeID(vector<vector<int>> &G, vector<ll>
    &val)
{
    memset(vis, 0, sizeof(vis));
    queue<int> fila[2];
    for(int i = 0; i < n; i++)
        if(degree[i] == 1)
            fila[0].push(i);
    int cnt = 0, turn = 0;
    while(cnt + 2 < n)
    {
        while(!fila[turn].empty())
        {
            int u = fila[turn].front(); fila[turn].pop();
            vis[u] = true;
            cnt++;
            for(int i = 0; i < G[u].size(); i++)
                if(!vis[G[u][i]])
                {
                    degree[G[u][i]]--;
                    if(degree[G[u][i]] == 1) fila[1-turn].push(G[u][i]);
                }
        }
        turn ^= 1;
    }
    for(int i = 0; i < n; i++)
    {
        if(vis[i]) continue;
        val.push_back(dfs(i, -1, G));
    }
}

int32_t main()
{
    while(cin >> n)
    {
        formato.clear();
        ID = 1;
        vector<ll> val[2];
        for(int j = 0; j < 2; j++)
        {
            memset(degree, 0, sizeof(degree));

```

```

        vector<vector<int>> G(n + 1);
        for(int i = 1; i < n; i++)
        {
            int u, v;
            scanf("%d %d", &u, &v); u--; v--;
            G[u].push_back(v);
            G[v].push_back(u);
            degree[v]++;
            degree[u]++;
        }
        findCenterAndComputeID(G, val[j]);
    }
    bool fl = false;
    for(ll &v0 : val[0])
        for(ll &v1 : val[1])
            if(v0 == v1)
                fl = true;
    puts(fl ? "S" : "N");
}

return 0;
}

```

## 2.29 Two Stacks Trick

```

const int OO = 0x3f3f3f3f;

struct Stack {
    vector<int> s, smax = {-OO}, smin = {OO};
    void push(int x) {
        s.push_back(x);
        smax.push_back(max(smax.back(), x));
        smin.push_back(min(smin.back(), x));
    }
    int pop() {
        int x = s.back();
        s.pop_back();
        smax.pop_back();
        smin.pop_back();
        return x;
    }
    int min_() {
        return smin.back();
    }
    int max_() {
        return smax.back();
    }
    bool empty() {
        return s.empty();
    }
};

Stack s1, s2;

void push(int x) {
    s2.push(x);
}

void pop() {
    if(s1.empty()) {

```

```

        while(!s2.empty())
            s1.push(s2.pop());
    }
    s1.pop();
}

int min_() {
    return min(s1.min_(), s2.min_());
}

int max_() {
    return max(s1.max_(), s2.max_());
}

```

## 2.30 Wavelet Tree

```

#include <bits/stdc++.h>
using namespace std;
const int N = 100100;
const int MAX = 30 * N;
// MAX = N * log(maxX - minX)
// Queries in O(log(maxX - minX))

struct WaveletTree
{
    int arr[N], aux[N];
    int lo[MAX], hi[MAX];
    vector<int> freq[MAX];
    int lef[MAX], rig[MAX];
    int nextNode;

    WaveletTree(vector<int> a, int minX, int maxX)
    {
        int sz = a.size();
        for(int i = 0; i < sz; i++)
            arr[i] = a[i];
        nextNode = 1;
        build(0, 0, sz, minX, maxX);
    }

    int stable_partition(int s, int e, int mid)
    {
        int pivot = 0;
        for(int i = s; i < e; i++)
            aux[i] = arr[i], pivot += (arr[i] <= mid);
        int l = s, r = s + pivot;
        for(int i = s; i < e; i++)
            if(aux[i] <= mid)
                arr[l++] = aux[i];
            else
                arr[r++] = aux[i];
        return l;
    }

    void build(int node, int s, int e, int minX, int maxX)
    {
        lo[node] = minX, hi[node] = maxX;
        if(lo[node] == hi[node] or s >= e) return;

        int mid = (minX + maxX - 1) / 2;

```

```

        freq[node].resize(e - s + 1);
        freq[node][0] = 0;

        for(int i = s; i < e; i++)
            freq[node][i - s + 1] = freq[node][i - s] + (arr[i] <= mid);

        int pivot = stable_partition(s, e, mid);

        lef[node] = nextNode++, rig[node] = nextNode++;

        build(lef[node], s, pivot, minX, mid);
        build(rig[node], pivot, e, mid + 1, maxX);
    }

    int went_right(int node, int i)
    {
        return i - freq[node][i];
    }

    // less than ou equal to x in range [l, r]
    int lte(int l, int r, int x, int node = 0)
    {
        if(l > r or x < lo[node]) return 0;
        if(hi[node] <= x) return r - l + 1;

        int l1 = freq[node][l - 1] + 1, r1 = freq[node][r];
        int l2 = went_right(node, l - 1) + 1, r2 = went_right(node, r);

        return lte(l1, r1, x, lef[node]) + lte(l2, r2, x, rig[node]);
    }

    // greater than ou equal to x in range [l, r]
    int gte(int l, int r, int x, int node = 0)
    {
        if(l > r or x > hi[node]) return 0;
        if(lo[node] >= x) return r - l + 1;

        int l1 = freq[node][l - 1] + 1, r1 = freq[node][r];
        int l2 = went_right(node, l - 1) + 1, r2 = went_right(node, r);

        return gte(l1, r1, x, lef[node]) + gte(l2, r2, x, rig[node]);
    }

    // counting numbers equal to x in range [l, r]
    int count(int l, int r, int x, int node = 0)
    {
        if(l > r or lo[node] > x or hi[node] < x) return 0;

        if(lo[node] == hi[node] and lo[node] == x) return r - l + 1;

        int l1 = freq[node][l - 1] + 1, r1 = freq[node][r];
        int l2 = went_right(node, l - 1) + 1, r2 = went_right(node, r);

        return count(l1, r1, x, lef[node]) + count(l2, r2, x, rig[node]);
    }

    // find kth number in range [l, r]
    int kth(int l, int r, int k, int node = 0)
    {
        if(l > r) return 0;

```

```

    if(lo[node] == hi[node]) return lo[node];

    int inLeft = freq[node][r] - freq[node][l - 1];
    int l1 = freq[node][l - 1] + 1, r1 = freq[node][r];

    if(k <= inLeft) return kth(l1, r1, k, lef[node]);

    int l2 = went_right(node, l - 1) + 1, r2 = went_right(node, r);

    return kth(l2, r2, k - inLeft, rig[node]);
}
};

int main()
{
    vector<int> a = {2, 5, 3, 2, 4, 2};

    WaveletTree T(a, 0, 9);

    cout << T.lte(3, 5, 3) << '\n';
    cout << T.gte(3, 5, 3) << '\n';
    cout << T.count(1, 6, 2) << '\n';
    cout << T.kth(1, 6, 5) << '\n';

    return 0;
}

```

## 3 Dynamic Programming

### 3.1 Coin Problem Topdown Dp

```

#include <bits/stdc++.h>
using namespace std;
using namespace std;

vector<int> coin;
int memo[1000000];

int solve(int troco)
{
    if(troco < 0)
        return (1 << 25);
    if(memo[troco] != -1)
        return memo[troco];
    if(troco == 0)
        return 0;
    int ans = (1 << 25);
    for(int i = 0; i < coin.size(); i++)
        ans = min(ans, 1 + solve(troco - coin[i]));
    return memo[troco] = ans;
}

void ans(int troco)
{
    if(troco < 0)
        return;
    if(troco == 0)
        return;

```

```

    for(int i = 0; i < coin.size(); i++)
        if(solve(troco - coin[i]) + 1 == memo[troco])
        {
            cout << coin[i] << ' ';
            ans(troco - coin[i]);
            break;
        }
    }

int main()
{
    memset(memo, -1, sizeof(memo));
    int n, troco;
    cin >> n >> troco;
    coin.resize(n);
    for(int &w : coin)
        cin >> w;
    cout << solve(troco) << '\n';
    puts("");

    return 0;
}

```

### 3.2 Digit DP Sum Of Digits In Range

```

#include "bits/stdc++.h"
using namespace std;

int dp[20][200][2];

int digitDP(int idx, int sum, int can, vector<int> &digit)
{
    // idx eh o indice atual, sum a soma dos digitos ate idx,
    // e can uma flag para indicar se pode colocar
    // qualquer valor a partir daqui
    if(idx == (int)digit.size())
        return sum;
    if(dp[idx][sum][can] != -1)
        return dp[idx][sum][can];
    int ans = 0;
    for(int i = 0; i < 10; i++)
        if(can or i <= digit[idx])
            ans += digitDP(idx + 1, sum + i,
                           can or i < digit[idx], digit);
    return dp[idx][sum][can] = ans;
}

int query(int x) // responde a consulta de 0 ate x
{
    memset(dp, -1, sizeof(dp));
    vector<int> digit;
    while(x)
    {
        digit.push_back(x%10);
        x /= 10;
    }
    reverse(digit.begin(), digit.end());
    return digitDP(0, 0, 0, digit);
}

```

```
int main()
{
    int q, a, b;
    cin >> q;
    while(q--)
    {
        cin >> a >> b;
        cout << query(b) - query(a - 1) << '\n';
    }
    return 0;
}
```

### 3.3 Edit Distance With DP

```
#include <bits/stdc++.h>
using namespace std;

string a, b;
int PD[2008][2008];

int solve(int i, int j)
{
    if(!i) return j;
    if(!j) return i;
    if(PD[i][j] != -1)
        return PD[i][j];
    // substituir um caracter se for preciso
    int ans1 = solve(i - 1, j - 1) + (a[i] != b[j]);
    //apagar o caracter da string i
    int ans2 = solve(i - 1, j) + 1;
    //apagar o caracter da string j
    int ans3 = solve(i, j - 1) + 1;
    return PD[i][j] = min(ans1, min(ans2, ans3));
}

int main()
{
    int q;
    cin >> q;
    while(q--)
    {
        memset(PD, -1, sizeof(PD));
        cin >> a >> b;
        a = "#" + a;
        b = "#" + b;
        cout << solve(a.size()-1, b.size()-1) << '\n';
    }

    return 0;
}
```

### 3.4 Kadane 2D

```
#include <bits/stdc++.h>
using namespace std;

int pd[100][100], A[100][100];
```

```
int main()
{
    int n, m;
    cin >> n >> m;
    for(int i = 1; i <= n; i++)
        for(int j = 1; j <= m; j++)
            cin >> A[i][j], pd[i][j] = pd[i][j - 1] + A[i][j];
    int ans = 0;
    for(int i = 1; i <= n; i++)
        for(int j = i + 1; j <= m; j++)
        {
            int sum = 0;
            for(int k = 1; k <= n; k++)
            {
                sum += pd[k][j] - pd[k][i - 1];
                if(sum < 0) sum = 0;
                ans = max(ans, sum);
            }
        }
    cout << ans << '\n';

    return 0;
}
```

### 3.5 Kadane 3D

```
#include <bits/stdc++.h>
using namespace std;

int A, B, C;
int par[22][22][22], pd[22][22][22];

int main()
{
    cin >> A >> B >> C;
    for(int i = 1; i <= A; i++)
        for(int j = 1; j <= B; j++)
            for(int k = 1; k <= C; k++)
                cin >> par[i][j][k];

    for(int i = 1; i <= A; i++)
        for(int j = 1; j <= B; j++)
            for(int k = 1; k <= C; k++)
                pd[i][j][k] = pd[i][j - 1][k] + pd[i][j][k - 1]
                    - pd[i][j - 1][k - 1] + par[i][j][k];

    int ans = -(1 << 25);
    for(int h1 = 1; h1 <= C; h1++)
        for(int h2 = h1; h2 <= C; h2++)
            for(int l1 = 1; l1 <= B; l1++)
                for(int l2 = l1; l2 <= B; l2++)
                {
                    int sum = -(1 << 25);
                    for(int i = 1; i <= A; i++)
                    {
                        int s = pd[i][l2][h2] - pd[i][l1 - 1][h2]
                            - pd[i][l2][h1 - 1] + pd[i][l1 - 1][h1 - 1];
                        sum = max(sum + s, s);
                        ans = max(ans, sum);
                    }
                }
}
```



```

    }
    cout << ans << '\n';

    return 0;
}

```

### 3.6 Knapsack With Copies SqrtN Memory

```

#include <bits/stdc++.h>
using namespace std;
#define bug(x) cout << #x << " >>>>>> " << x << '\n'
#define _ << " , " <<
// #define int long long
#define Max(a, b) (a > b ? a : b)
#define Min(a, b) (a < b ? a : b)
#define ii pair<int, int>
#define fi first
#define se second
#define UNTIL(t) while (clock() < (t) * CLOCKS_PER_SEC)
const int MAX = 20002; // 2 * 10^5
const int MOD = 1000000007; // 10^9 + 7
const int OO = 0x3f3f3f3f; // 0x3f3f3f3f
const double EPS = 1e-9; // 10^-9
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

int n, S;
int B[205];
int C[205];
int V[3500];
int W[3500];
int ID[3500];
int id = 1;
int dp[MAX];
int linha[70][MAX], T[70][MAX];
int L[70];

void add(int j)
{
    int sum = 0;
    for(int k = 0; sum + (1 << k) <= C[j]; sum += (1 << k), k++)
    {
        V[id] = B[j] * (1 << k);
        W[id] = (1 << k);
        ID[id] = j;
        id++;
    }
    int r = C[j] - sum;
    if(r > 0)
    {
        V[id] = B[j] * r;
        W[id] = r;
        ID[id] = j;
        id++;
    }
}

int32_t main()
{
    cin >> n;
    for(int i = 1; i <= n; i++) cin >> B[i];

```

```

    for(int i = 1; i <= n; i++) cin >> C[i];
    for(int i = 1; i <= n; i++) add(i);
    cin >> S;

    for(int j = 1; j <= S; j++)
        dp[j] = OO;

    int cnt = 0, k = -1, sq = max(10, (int)sqrt(id * 1.));
    for(int i = 1; i < id; i++)
    {
        if(cnt % sq == 0)
        {
            cnt = 0;
            k++;
            for(int j = 0; j <= S; j++)
                linha[k][j] = dp[j];
            L[k] = i - 1;
        }
        for(int j = S; j >= V[i]; j--)
            dp[j] = Min(dp[j - V[i]] + W[i], dp[j]);
        cnt++;
    }

    int last_raw = id - 1, s = S;

    vector<int> note(n + 1);

    while(last_raw >= 1)
    {
        int first_raw = last_raw - 1;
        while(first_raw > L[k])
            first_raw--;

        for(int j = 0; j <= S; j++)
            T[0][j] = linha[k][j];

        for(int i = 1; i <= last_raw - first_raw; i++)
            for(int j = 0; j <= S; j++)
                if(j >= V[i + first_raw])
                    T[i][j] = Min(T[i - 1][j], T[i - 1][j - V[i + first_raw]] + W[i + first_raw]);
                else
                    T[i][j] = T[i - 1][j];

        for(int i = last_raw - first_raw; i > 0; i--)
            if(T[i][s] != T[i - 1][s])
            {
                note[ID[i + first_raw]] += W[i + first_raw];
                s -= V[i + first_raw];
            }

        last_raw = first_raw;
        k--;
    }

    int number_of_notes = 0;
    for(int &w : note)
        number_of_notes += w;

    cout << number_of_notes << '\n';

```

```

for(int i = 1; i <= n; i++)
    cout << note[i] << ' ';
puts("");

return 0;
}

```

### 3.7 Knapsack With Repetition Recurrence

```

#include <bits/stdc++.h>
using namespace std;

#define Max(a, b) (a > b ? a : b)
#define Min(a, b) (a < b ? a : b)
#define ii pair<int, int>
#define fi first
#define se second

const int OO = 0x3f3f3f3f;

void solve(){
    int n, s;
    cin >> n >> s;
    vector<ii> items;
    for(int i = 0; i < n; ++i) {
        int w, c;
        cin >> w >> c;
        items.push_back({w, c});
    }
    sort(items.begin(), items.end());

    vector<int> maxi(n);
    for(int i = 0; i < n; ++i)
        if(i) maxi[i] = Max(maxi[i - 1], items[i].se);
        else maxi[i] = items[i].se;

    int l = s, r = s;
    vector<int> states = {s};
    for(int i = 0; i < 36; ++i) {
        l = (l / 2) - 505;
        r = (r / 2) + 505;
        for(int j = Max(l, 1); j <= Min(r, s); ++j)
            states.push_back(j);
    }

    sort(states.begin(), states.end());
    states.resize(unique(states.begin(), states.end()) - states.begin());

    unordered_map<int, long long> dp;

    for(int &w : states) {
        long long ans = 0;
        auto it = upper_bound(items.begin(), items.end(), ii(w, OO));
        if(it != items.begin()) {
            it--;
            int p = it - items.begin();
            ans = maxi[p];
        }
    }
}

```

```

l = Max((w / 2) - 505, 1);
r = Min((w / 2) + 505, s);

for(int j = 1; j <= r; ++j)
    ans = Max(ans, dp[j] + dp[w - j]);

dp[w] = ans;
}

cout << dp[s] << '\n';
}

int32_t main() {
    solve();
    return 0;
}

```

### 3.8 Knapsack Zero One Without Value

```

// Knapsack 0 - 1 sem valor em O((N*W) / word)
#include <bits/stdc++.h>
using namespace std;

int n, W, weight[10000];
bitset<10000> T[100];

bool knapsack()
{
    T[0][0] = 1;
    for(int i = 1; i <= n; i++)
        T[i] = ((T[i - 1] << weight[i - 1]) | T[i - 1]);
    return T[n][W];
}

void retrieve()
{
    vector<int> ans;
    for(int i = n; i > 0; i--)
        if(W >= weight[i - 1] and T[i - 1][W - weight[i - 1]])
        {
            ans.push_back(i - 1);
            W -= weight[i - 1];
        }
    for(int &w : ans) cout << weight[w] << ' '; puts("");
}

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> weight[i];
    cin >> W;
    cout << knapsack() << '\n';
    retrieve();

    return 0;
}

```

### 3.9 Knapsack0-kSemValor

```
// Knapsack 0 - k sem valor em O((N*W*LogK) / word)

#include <bits/stdc++.h>
using namespace std;

int n, W, weight[10000], K[10000];
bitset<10000> T[100];

bool knapsack()
{
    T[0][0] = 1;
    for(int i = 1; i <= n; i++)
    {
        int s = K[i - 1];
        T[i] = T[i - 1];
        for(int p = 1; p <= s; s -= p, p *= 2)
            T[i] |= ((T[i] << (weight[i - 1] * p)) | T[i]);
        if(s)
            T[i] |= ((T[i] << (weight[i - 1] * s)) | T[i]);
    }
    return T[n][W];
}

void retrieve()
{
    vector<pair<int, int>> ans;
    for(int i = n; i > 0; i--)
    {
        int s = K[i - 1], qtd = 0;
        for(int p = 1; p <= s; p *= 2)
            if(W >= weight[i - 1] * p and T[i - 1][W - weight[i - 1] * p])
                W -= weight[i - 1] * p, qtd += p, s -= p;
            if(W >= weight[i - 1] * s and T[i - 1][W - weight[i - 1] * s])
                W -= s * weight[i - 1], qtd += s;
            if(qtd) ans.push_back({qtd, i - 1});
    } //first eh q quantidade de pesos i - 1
    for(pair<int, int> &w : ans)
        cout << w.first << ' ' << weight[w.second] << '\n';
}

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> weight[i];
    for(int i = 0; i < n; i++)
        cin >> K[i];
    cin >> W;
    cout << (knapsack() ? "possible\n" : "impossible\n");
    retrieve();

    return 0;
}
```

### 3.10 KnapsackErrichto

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
const int OO = 0x3f3f3f3f3f3f3f3f;

int n, w, maxi;
int value[120], weight[120];
int dp[100040];

int32_t main()
{
    cin >> n >> w;
    for(int i = 0; i < n; i++)
        cin >> weight[i] >> value[i];
    // dp[i] = maximum total value of itens with total weight exactly i
    for(int item = 0; item < n; item++)
        for(int cur_wei = w - weight[item]; cur_wei >= 0; cur_wei--)
            dp[cur_wei + weight[item]] = max(dp[cur_wei + weight[item]],
                                              dp[cur_wei] + value[item]);

    cout << *max_element(dp, dp + w + 1) << '\n';

    return 0;
}
```

### 3.11 KnapsackWithCopies

```
// O( S * sqrt( SumKi ) )

#include <bits/stdc++.h>
using namespace std;
const int OO = 0x3f3f3f3f;

int freq[50];
vector<int> value, weight;
int memo[5000][5000];

// Decompor o numero em uma soma de potencias
//de 2 de tal forma que qualquer numero entre 0 e k
// pode ser formado usando os numeros da decomposicao.
void decomp(int k, int w, int v)
{
    int i = 1;
    freq[1] = k;
    while(true)
    {
        int m = (freq[i] - 1) / 2;
        if(freq[i] - m * 2 == 0) break;
        freq[i] -= 2 * m;
        freq[2 * i] += m;
        i++;
    }
    for(int i = 0; i < 32; i++)
    {
        while(freq[i]--)
        {
            value.push_back(i * v);
            weight.push_back(i * w);
        }
        freq[i] = 0;
    }
}
```

```

}

int solve(int id, int W)
{
    if(memo[id][W] != -1)
        return memo[id][W];
    if(id == value.size() or !W)
        return memo[id][W] = 0;
    int ans = 0;
    if(weight[id] > W)
        ans = solve(id + 1, W);
    else
        ans = max(value[id] + solve(id + 1, W
        - weight[id]), solve(id + 1, W));
    return memo[id][W] = ans;
}

int main()
{
    int n, w, v, k, S;
    memset(memo, -1, sizeof(memo));
    cin >> n >> S;
    for(int i = 0; i < n; i++)
    {
        cin >> v >> w >> k;
        decomp(k, w, v);
    }
    cout << solve(0, S) << '\n';

    return 0;
}

```

### 3.12 KnapsackwithPDtopdown

```

#include <bits/stdc++.h>
using namespace std;

int n, W, weight[2005], value[2005];
int memo[2005][2005];

int solve(int id, int W)
{
    if(memo[id][W] != -1)
        return memo[id][W];
    if(id == n or !W)
        return memo[id][W] = 0;
    int ans = 0;
    if(weight[id] > W)
        ans = solve(id + 1, W);
    else
        ans = max(value[id] + solve(id + 1,
        W - weight[id]), solve(id + 1, W));
    return memo[id][W] = ans;
}

void ans(int id, int W)
{
    if(id == n or !W)
        return;
    if(solve(id + 1, W) == memo[id][W])

```

```

        ans(id + 1, W);
    else
    {
        cout << id << ' ';
        ans(id + 1, W - weight[id]);
    }
}

int main()
{
    memset(memo, -1, sizeof(memo));
    cin >> n >> W;
    for(int i = 0; i < n; i++)
        cin >> weight[i] >> value[i];
    cout << solve(0, W) << '\n';
    cout << "Objetos escolhidos 0 - indexdos\n";
    ans(0, W);
    puts("");

    return 0;
}

```

### 3.13 Longest Common Subsequence And Edit Distance

```

#include <bits/stdc++.h>
using namespace std;

// longest common substring

int pd[1000][1000];

int LCS(string a, string b)
{
    for(int i = 1; i <= a.size(); i++)
        for(int j = 1; j <= b.size(); j++)
            if(a[i-1] == b[j-1])
                pd[i][j] = pd[i-1][j-1] + 1;
            else
                pd[i][j] = max(pd[i][j-1], pd[i-1][j]);
    return pd[a.size()][b.size()];
}

int main()
{
    string a, b;

    cin >> a >> b;
    int lcs = LCS(a, b);

    cout << lcs << '\n';
    cout << "Edit Distance: " << a.size()+b.size()-2*lcs << '\n';

    return 0;
}

```

### 3.14 Longest Increasing Subsequence

```

#include <bits/stdc++.h>

```

```

using namespace std;

void lis(vector<int> &arr)
{
    vector<int> pilha;
    int pai[1000], pos[1000];
    for(int i = 0; i < arr.size(); i++)
    {
        int p = int(upper_bound(pilha.begin(),
            pilha.end(), arr[i]) - pilha.begin());
        if(p == pilha.size())
            pilha.push_back(arr[i]);
        else
            pilha[p] = arr[i];
        pos[p] = i;
        if(!p)
            pai[i] = -1;
        else
            pai[i] = pos[p - 1];
    }
    vector<int> L;
    int aux = pos[pilha.size() - 1];
    cout << pilha.size() << '\n';
    while(aux != -1)
    {
        L.push_back(arr[aux]);
        aux = pai[aux];
    }
    reverse(L.begin(), L.end());
    for(const int &w : L)
        cout << w << ' ';
    cout << '\n';
}

int main()
{
    int n;
    cin >> n;
    vector<int> arr(n);
    for(int &w : arr)
        cin >> w;
    lis(arr);

    return 0;
}

```

### 3.15 Subset Sum

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e6 + 10;

int n, x, weight[1005];
bool pd[MAX];
int ans[MAX];

void printAns(int m)
{
    cout << ans[m] << ' ';
}

```

```

    if(m - ans[m] > 0)
        printAns(m - ans[m]);
}

int main()
{
    cin >> n >> x;
    int sum = 0;
    for(int i = 0; i < n; i++) cin >> weight[i], sum += weight[i];
    pd[0] = 1;
    for(int j = 0; j < n; j++)
        for(int i = sum; i >= 0; i--)
            if(pd[i] and !pd[i + weight[j]])
            {
                ans[i + weight[j]] = weight[j];
                pd[i + weight[j]] = 1;
            }

    printAns(x);
    puts("");

    return 0;
}

```

### 3.16 Traveling Salesman Problem Bottom Up Dp

```

#include <bits/stdc++.h>
using namespace std;
const int OO = 0x3f3f3f3f;

int n;
double dist[20][20];
double pd[1 << 17][20];

int tsp(int ori)
{
    memset(pd, 63, sizeof(pd));
    for(int i = 0; i < n; i++)
        if(i != ori)
            pd[1 << i][i] = dist[ori][i];
    for(int k = 0; k < (1 << n); k++)
        for(int i = 0; i < n; i++)
            if(k & (1 << i))
                for(int j = 0; j < n; j++)
                    if((k & (1 << j)) and i != j)
                        pd[k][j] = min(pd[k][j], pd[k ^ (1 << j)][i] + dist[i][j]);
    return pd[(1 << n) - 1][ori];
}

int main()
{
    // inicializar dist, dist[i][j] guarda a distancia de i para j no
    // grafo
    // chamar tsp

    return 0;
}

```

## 3.17 Traveling Salesman Problem Topdown Dp

```
#include <bits/stdc++.h>
using namespace std;

int dist[22][22], m;
int memo[20][1 << 20];

int solve(int id, int mask) {
    if((1 << m) - 1 == mask)
        return dist[id][0];
    if(memo[id][mask] != -1)
        return memo[id][mask];
    int ans = INT_MAX;
    for(int i = 0; i < m; i++)
        if((mask & (1 << i)) == 0)
            ans = min(ans, dist[id][i] + solve(i, mask | (1 << i)));
    return memo[id][mask] = ans;
}

int main() {
    memset(memo, -1, sizeof(memo));
    //inicializa a matriz dist com as distancias
    //de todo mundo pra todo mundo..
    cout << solve(0, 1) << '\n';
    return 0;
}
```

## 4 String

### 4.1 Aho Corasick

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e4;

int p[MAX], f[MAX], nxt[MAX][26], ch[MAX];
int tsz = 1; // size of the trie

int cnt[MAX]; // used to know number of matches

const int S = 2e3+5;
bitset<MAX> elem[S];
// S eh tamanho da maior das N strings que sao
// pradoes para buscar no texto

void init()
{
    tsz = 1;
    memset(f, 0, sizeof(f));
    memset(nxt, 0, sizeof(nxt));
    memset(cnt, 0, sizeof(cnt));
    for (int i = 0; i < MAX; i++)
        elem[i].reset();
}

void add(const string &s, int x)
```

```
{
    // the first element of the trie is the root
    int cur = 1;
    for(int i = 0; s[i]; ++i)
    {
        int j = s[i] - 'a';
        if(!nxt[cur][j])
        {
            tsz++;
            p[tsz] = cur;
            ch[tsz] = j;
            nxt[cur][j] = tsz;
        }
        cur = nxt[cur][j];
    }
    cnt[cur]++;
    elem[cur].set(x);
}

void build()
{
    queue<int> q;
    for(int i = 0; i < 26; ++i)
    {
        nxt[0][i] = 1;
        if(nxt[1][i])
            q.push(nxt[1][i]);
    }
    while(!q.empty())
    {
        int v = q.front(); q.pop();
        int u = f[p[v]];
        while(u and !nxt[u][ch[v]]) u = f[u];
        f[v] = nxt[u][ch[v]];
        cnt[v] += cnt[f[v]];
        for(int i = 0; i < 26; ++i)
            if(nxt[v][i])
                q.push(nxt[v][i]);
    }
}

bitset<MAX> match(const string &s)
{
    int ans = 0;
    // Numero de matches
    bitset<MAX> found;
    // Usado pra saber quais strings matches
    int x = 1;
    for(int i = 0; i < s.size(); ++i)
    {
        int t = s[i] - 'a';
        while(x and !nxt[x][t])
            x = f[x];
        x = nxt[x][t];
        ans += cnt[x];
        found |= elem[x];
    }
    return found;
}

int main()
```

```

{
    int n;
    string s;
    cin >> n;
    for(int i = 0; i < n; i++)
    {
        cin >> s;
        add(s, i);
    }
    build();
    cin >> s;
    bitset<MAX> ans = match(s);
    for(int i = 0; i < n; i++)
        cout << ans[i] << '\n';
    // 1 se a i-esima string lida
    // aparece no texto, 0 cc
    return 0;
}

//
#####

#include <bits/stdc++.h>
using namespace std;

const int K = 60;

struct Vertex {
    int next[K];
    bool leaf = false;
    int p = -1;
    char c;
    int link = -1;
    int go[K];
    bitset<1005> S;
    Vertex(int _p=-1, char _c='$') : p(_p), c(_c) {
        fill(begin(next), end(next), -1);
        fill(begin(go), end(go), -1);
    }
};

vector<Vertex> t;

void init() {
    t.clear();
    t.resize(1);
}

void add(string &s, int i) {
    int v = 0;
    for(char ch : s) {
        int c = ch - 'A';
        if(t[v].next[c] == -1) {
            t[v].next[c] = t.size();
            t.push_back(Vertex(v, ch));
        }
        v = t[v].next[c];
    }
    t[v].leaf = true;
    t[v].S[i] = 1;
}

}

int go(int v, char ch);

int get_link(int v) {
    if(t[v].link == -1) {
        if(v == 0 or t[v].p == 0)
            t[v].link = 0;
        else
            t[v].link = go(get_link(t[v].p), t[v].c);
    }
    return t[v].link;
}

int go(int v, char ch) {
    int c = ch - 'A';
    if(t[v].go[c] == -1) {
        if(t[v].next[c] != -1)
            t[v].go[c] = t[v].next[c];
        else
            t[v].go[c] = t[v].p == 0 ? 0 : go(get_link(v), ch);
    }
    return t[v].go[c];
}

int32_t main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);

    int caso;
    cin >> caso;
    while(caso--) {
        init();
        string s;
        int n;
        cin >> s >> n;
        bitset<1005> S;
        for(int i = 0; i < n; i++) {
            string a;
            cin >> a;
            add(a, i);
        }
        int v = 0;
        for(char &c : s) {
            v = go(v, c);
            S |= t[v].S;
        }
        for(int i = 0; i < n; i++)
            cout << (S[i] ? 'Y' : 'N') << '\n';
    }

    return 0;
}

#include <bits/stdc++.h>
using namespace std;

struct TrieNode

```

## 4.2 Dynamic Trie

```

#include <bits/stdc++.h>
using namespace std;

struct TrieNode

```

```

{
    map<int, TrieNode*> children;
    bool isLeaf;
    TrieNode()
    {
        isLeaf = false;
    }
};

void inserir(TrieNode *root, string s)
{
    TrieNode *node = root;
    for(int i = 0; i < s.size(); i++)
    {
        int index = s[i] - 'a';
        if(node->children.find(index) == node->children.end())
            node->children[index] = new TrieNode();
        node = node->children[index];
    }
    node->isLeaf = true;
}

bool buscar(TrieNode *root, string s)
{
    TrieNode *node = root;
    for(int i = 0; i < s.size(); i++)
    {
        int index = s[i] - 'a';
        if(node->children.find(index) == node->children.end())
            return false;
        node = node->children[index];
    }
    return node->isLeaf;
}

bool remover(TrieNode *node, string s, int level)
{
    if(node != nullptr)
        if(s.size() == level)
        {
            if(node->isLeaf)
            {
                node->isLeaf = false;
                return !node->children.size();
            }
        }
        else
        {
            int index = s[level] - 'a';
            if(remover(node->children[index], s, level+1))
            {
                delete node->children[index];
                node->children.erase(index);
                return !node->children.size();
            }
        }
    return false;
}

int main()
{

```

```

    TrieNode *root = new TrieNode();

    inserir(root, "abc");
    inserir(root, "abd");
    inserir(root, "cfa");
    remover(root, "abc", 0);
    printf(buscar(root, "abc") ? "yes\n" : "no\n");
    printf(buscar(root, "abd") ? "yes\n" : "no\n");

    return 0;
}

```

### 4.3 KMP

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e6;

int n, m;
// n eh o tamanho do texto e m eh o
// tamanho do padrao
int arr[MAX];
// array que guarda o tamanho do maior
// prefixo proprio que tambem eh sufixo
string t, p; // t eh o texto e p eh o padrao

void build() // KMP Preprocess
{
    int i = 0, j = 1;
    while(j < m)
    {
        if(p[i] == p[j])
            arr[j] = ++i;
        else
        {
            i = 0;
            if(p[i] == p[j])
                arr[j] = ++i;
        }
        j++;
    }
}

int matching() // KMP search
{
    int i = 0, j = 0;
    while(j < n)
    {
        if(p[i] == t[j]) i++, j++;
        else if(i == arr[i - 1])
            i = arr[i - 1];
        else j++;
        if(i == m)
            return j - m;
        // a substring P inicia na posicao j - m em T
    }
    return -1; // P nao eh substring de T
}

int main()
{

```



```

cin >> t >> p;
n = (int)t.size();
m = (int)p.size();
build();
cout << matching() << '\n';

return 0;
}

```

## 4.4 LIS LDS

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e4;

int n;
int LI[MAX], LD[MAX];
vector<int> arr;

int LIS() {
    for(int i = 0; i < n; i++)
        LI[i] = 1;
    for(int i = n - 1; i >= 0; i--)
        for(int j = 0; j < i; j++)
            if(arr[j] < arr[i])
                LI[j] = max(LI[j], LI[i] + 1);
}

int LDS() {
    reverse(arr.begin(), arr.end());
    for(int i = 0; i < n; i++)
        LD[i] = 1;
    vector<int> pilha;
    for(int i = 0; i < n; i++) {
        int p = (int)(lower_bound(pilha.begin(),
            pilha.end(), arr[i]) - pilha.begin());
        if(p == pilha.size())
            pilha.push_back(arr[i]);
        else
            pilha[p] = arr[i];
        LD[i] = p + 1;
    }
}

int main() {
    cin >> n; arr.resize(n);
    for(int i = 0; i < n; i++) cin >> arr[i];

    LIS();
    LDS();

    for(int i = 0; i < n; i++)
        cout << LI[i] << ' '; puts("");
    for(int i = 0; i < n; i++)
        cout << LD[n - i - 1] << ' '; puts("");

    return 0;
}

```

## 4.5 Longest Common Substring

```

#include <stdio.h>
#include <string.h>

int pd[10000][10000];

int max(int a, int b)
{
    return a > b ? a : b;
}

int solve(char *a, char *b)
{
    int i, j, ans = 0;
    int t1 = strlen(a), t2 = strlen(b);
    for(i = 1; i < t1; i++)
        for(j = 1; j < t2; j++)
            if(a[i-1] == b[j-1])
                pd[i][j] = pd[i-1][j-1] + 1,
                ans = max(ans, pd[i][j]);
            else
                pd[i][j] = 0;
    return ans;
}

int main()
{
    char s1[55], s2[55];

    while(fgets(s1, 54, stdin) != NULL
        && fgets(s2, 54, stdin) != NULL)
        printf("%d\n", solve(s1, s2));

    return 0;
}

```

## 4.6 Manacher

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 100;

int lps[2*MAX+5];
char s[MAX];

int manacher()
{
    int n = strlen(s);
    string p(2*n+3, '#');
    p[0] = '^';
    for(int i = 0; i < n; i++)
        p[2*(i+1)] = s[i];
    p[2*n+2] = '$';
    int k = 0, r = 0, m = 0;
    int l = p.length();
    for(int i = 1; i < l; i++)
    {

```

```

    int o = 2*k - i;
    lps[i] = (r > i) ? min(r-i, lps[o]) : 0;
    while(p[i + 1 + lps[i]] == p[i - 1 - lps[i]])
        lps[i]++;
    if(i + lps[i] > r) k = i, r = i + lps[i];
    m = max(m, lps[i]);
}
//for(int i = 1; i <= 2 * n + 1; i++)
//    cout << lps[i] << ' ';
puts("");*/
return m;
}

int main()
{
    cin >> s;
    cout << manacher() << '\n';
    return 0;
}

```

## 4.7 SA

```

/*
 * Code from Competitive Programming 3
 */

#include <algorithm>
#include <cstdio>
#include <cstring>
using namespace std;

typedef pair<int, int> ii;

#define MAX_N 100010 // second approach: O(n log n)
char T[MAX_N]; // the input string, up to 100K characters
int n; // the length of input string
int RA[MAX_N], tempRA[MAX_N]; // rank array and temporary rank array
int SA[MAX_N], tempSA[MAX_N]; // suffix array and temporary suffix array
int c[MAX_N]; // for counting/radix sort

char P[MAX_N]; // the pattern string (for string matching)
int m; // the length of pattern string

int Phi[MAX_N]; // for computing longest common prefix
int PLCP[MAX_N];
int LCP[MAX_N]; // LCP[i] stores the LCP between previous suffix T+SA[i-1]
                // and current suffix T+SA[i]

```

```

bool cmp(int a, int b) { return strcmp(T + a, T + b) < 0; } // compare

void constructSA_slow() { // cannot go beyond 1000 characters
    for (int i = 0; i < n; i++) SA[i] = i; // initial SA: {0, 1, 2, ..., n-1}
    sort(SA, SA + n, cmp); // sort: O(n log n) * compare: O(n) = O(n^2 log n)
}

void countingSort(int k) { // O(n)
    int i, sum, maxi = max(300, n); // up to 255 ASCII chars or length of n
    memset(c, 0, sizeof c); // clear frequency table
    for (i = 0; i < n; i++) // count the frequency of each integer rank
        c[i + k < n ? RA[i + k] : 0]++;
    for (i = sum = 0; i < maxi; i++) {
        int t = c[i]; c[i] = sum; sum += t;
    }
    for (i = 0; i < n; i++) // shuffle the suffix array if necessary
        tempSA[c[SA[i]+k < n ? RA[SA[i]+k] : 0]++] = SA[i];
    for (i = 0; i < n; i++) // update the suffix array SA
        SA[i] = tempSA[i];
}

void constructSA() { // this version can go up to 100000 characters
    int i, k, r;
    for (i = 0; i < n; i++) RA[i] = T[i]; // initial rankings
    for (i = 0; i < n; i++) SA[i] = i; // initial SA: {0, 1, 2, ..., n-1}
    for (k = 1; k < n; k <= 1) { // repeat sorting process log n times
        countingSort(k); // actually radix sort: sort based on the second item
        countingSort(0); // then (stable) sort based on the first item
        tempRA[SA[0]] = r = 0; // re-ranking; start from rank r = 0
        for (i = 1; i < n; i++) // compare adjacent suffixes
            tempRA[SA[i]] = // if same pair => same rank r; otherwise, increase r
                (RA[SA[i]] == RA[SA[i-1]] && RA[SA[i]+k] == RA[SA[i-1]+k]) ? r : ++r;
        for (i = 0; i < n; i++) // update the rank array RA
            RA[i] = tempRA[i];
        if (RA[SA[n-1]] == n-1) break; // nice optimization trick
    }
}

void computeLCP_slow() {
    LCP[0] = 0; // default
}

```

```

    value
    for (int i = 1; i < n; i++) {           // compute LCP by
        definition
        int L = 0;                         // always reset L
        to 0
        while (T[SA[i] + L] == T[SA[i-1] + L]) L++; // same L-th char
        , L++
        LCP[i] = L;
    } }

void computeLCP() {
    int i, L;
    Phi[SA[0]] = -1;                       // default
    value
    for (i = 1; i < n; i++)                 // compute Phi in
        O(n)
        Phi[SA[i]] = SA[i-1]; // remember which suffix is behind this
        suffix
    for (i = L = 0; i < n; i++) {           // compute Permuted LCP in
        O(n)
        if (Phi[i] == -1) { PLCP[i] = 0; continue; } // special
        case
        while (T[i + L] == T[Phi[i] + L]) L++; // L increased max n
        times
        PLCP[i] = L;
        L = max(L-1, 0); // L decreased max n
        times
    }
    for (i = 0; i < n; i++)                 // compute LCP in
        O(n)
        LCP[i] = PLCP[SA[i]]; // put the permuted LCP to the correct
        position
    }

ii stringMatching() {                     // string matching in O(m
    log n)
    int lo = 0, hi = n-1, mid = lo;        // valid matching =
    [0..n-1]
    while (lo < hi) {                      // find lower
        bound
        mid = (lo + hi) / 2; // this is round
        down
        int res = strcmp(T + SA[mid], P, m); // try to find P in suffix
        'mid'
        if (res >= 0) hi = mid; // prune upper half (notice the >=
        sign)
        else lo = mid + 1; // prune lower half
        including mid
    } // observe '=' in "res >= 0"
    above
    if (strcmp(T + SA[lo], P, m) != 0) return ii(-1, -1); // if not
    found
    ii ans; ans.first = lo;
    lo = 0; hi = n - 1; mid = lo;
    while (lo < hi) { // if lower bound is found, find upper
        bound
        mid = (lo + hi) / 2;
        int res = strcmp(T + SA[mid], P, m);
        if (res > 0) hi = mid; // prune upper
        half
        else lo = mid + 1; // prune lower half
    }
}

    including mid
    } // (notice the selected branch when res
    == 0)
    if (strcmp(T + SA[hi], P, m) != 0) hi--; // special
    case
    ans.second = hi;
    return ans;
} // return lower/upperbound as first/second item of the pair,
    respectively

ii LRS() { // returns a pair (the LRS length and its
    index)
    int i, idx = 0, maxLCP = -1;
    for (i = 1; i < n; i++) // O(n), start from
        i = 1
        if (LCP[i] > maxLCP)
            maxLCP = LCP[i], idx = i;
    return ii(maxLCP, idx);
}

int owner(int idx) { return (idx < n-m-1) ? 1 : 2; }

ii LCS() { // returns a pair (the LCS length and its
    index)
    int i, idx = 0, maxLCP = -1;
    for (i = 1; i < n; i++) // O(n), start from
        i = 1
        if (owner(SA[i]) != owner(SA[i-1]) && LCP[i] > maxLCP)
            maxLCP = LCP[i], idx = i;
    return ii(maxLCP, idx);
}

int main() {
    //printf("Enter a string T below, we will compute its Suffix Array:\n
    n");
    strcpy(T, "GATAGACA");
    n = (int)strlen(T);
    T[n++] = '$';
    // if '\n' is read, uncomment the next line
    //T[n-1] = '$'; T[n] = 0;

    constructSA_slow(); // O(n^2
    log n)
    printf("The Suffix Array of string T = '%s' is
    shown below (O(n^2 log n) version):\n", T);
    printf("i\tSA[i]\tSuffix\n");
    for (int i = 0; i < n; i++) printf("%2d\t%2d\t%s\n", i, SA[i], T +
    SA[i]);

    constructSA(); // O(n
    log n)
    printf("\nThe Suffix Array of string T = '%s' is
    shown below (O(n log n) version):\n", T);
    printf("i\tSA[i]\tSuffix\n");
    for (int i = 0; i < n; i++) printf("%2d\t%2d\t%s\n", i, SA[i], T +
    SA[i]);

    computeLCP(); //
    O(n)

    // LRS demo
}

```

```

ii ans = LRS(); // find the LRS of the first input
string
char lrsans[MAX_N];
strncpy(lrsans, T + SA[ans.second], ans.first);
printf("\nThe LRS is '%s' with length = %d\n\n", lrsans, ans.first);

// stringMatching demo
//printf("\nNow, enter a string P below, we will try to find P in T
: \n");
strcpy(P, "A");
m = (int)strlen(P);
// if '\n' is read, uncomment the next line
//P[m-1] = 0; m--;
ii pos = stringMatching();
if (pos.first != -1 && pos.second != -1) {
    printf("%s is found SA[%d..%d] of %s\n", P, pos.first, pos.second,
T);
    printf("They are:\n");
    for (int i = pos.first; i <= pos.second; i++)
        printf(" %s\n", T + SA[i]);
} else printf("%s is not found in %s\n", P, T);

// LCS demo
//printf("\nRemember, T = '%s'\nNow, enter another string P:\n", T);
// T already has '$' at the back
strcpy(P, "CATA");
m = (int)strlen(P);
// if '\n' is read, uncomment the next line
//P[m-1] = 0; m--;
strcat(T, P); //
append P // add '$' at the
strcat(T, "#"); back
n = (int)strlen(T); //
update n

// reconstruct SA of the combined strings
constructSA(); // O(n
log n)
computeLCP(); //
O(n)
printf("\nThe LCP information of 'T+P' = '%s':\n", T);
printf("i\tSA[i]\tLCP[i]\tOwner\tSuffix\n");
for (int i = 0; i < n; i++)
    printf("%2d\t%2d\t%2d\t%2d\t%2d\t%s\n", i, SA[i], LCP[i], owner(SA[i]),
T + SA[i]);

ans = LCS(); // find the longest common substring between T
and P
char lcsans[MAX_N];
strncpy(lcsans, T + SA[ans.second], ans.first);
printf("\nThe LCS is '%s' with length = %d\n", lcsans, ans.first);

return 0;
}

```

## 4.8 Suffix Array And Applications

```

#include <bits/stdc++.h>
using namespace std;

```

```

string s;
vector<int> sa, c, lcp;

// O(n)
void countSort()
{
    int n = sa.size();
    vector<int> buc(n), new_sa(n);
    for(int &w : sa)
        buc[c[w]]++;
    for(int i = 1; i < n; i++)
        buc[i] += buc[i - 1];
    for(int i = n - 1; i >= 0; i--)
        new_sa[ --buc[ c[sa[i]] ] ] = sa[i];
    sa = new_sa;
}

// O(|s| * log|s|)
void buildSuffixArray()
{
    int n = s.size();
    sa.resize(n);
    c.resize(n);
    for(int i = 0; i < n; i++)
        sa[i] = i, c[i] = s[i];
    sort(sa.begin(), sa.end(), [&](int a, int b)
    {
        return c[a] < c[b];
    });
    c[sa[0]] = 0;
    for(int i = 1; i < n; i++)
        if(s[sa[i - 1]] == s[sa[i]])
            c[sa[i]] = c[sa[i - 1]];
        else
            c[sa[i]] = c[sa[i - 1]] + 1;
    int k = 0;
    while((1 << k) < n)
    {
        for(int i = 0; i < n; i++)
            sa[i] = (sa[i] - (1 << k) + n) % n;
        countSort();
        vector<int> new_c(n);
        new_c[sa[0]] = 0;
        for(int i = 1; i < n; i++)
        {
            pair<int, int> prev = {c[sa[i - 1]], c[(sa[i - 1] + (1 << k)) %
n]};
            pair<int, int> cur = {c[sa[i]], c[(sa[i] + (1 << k)) % n]};
            if(prev == cur) new_c[sa[i]] = new_c[sa[i - 1]];
            else new_c[sa[i]] = new_c[sa[i - 1]] + 1;
        }
        c = new_c;
        k++;
    }
}

```

```

// 0: padrao esta no sufixo
// 1: o padrao eh lexicograficamente maior que o sufixo k
//-1: o padrao eh lexicograficamente menor que o sufixo k
// O(|p|)

```

```

int cmp(int k, string &p)
{
    for(int i = 0; i < p.size(); i++)
    {
        if(i + k >= s.size()) return 1;
        if(s[i + k] < p[i]) return 1;
        if(s[i + k] > p[i]) return -1;
    }
    return 0;
}

// posicao no suffix array do sufixo mais
// a esquerda que contem p como prefixo
// O(|p| * log|s|)
int lower_bound(string &p)
{
    int b = 0, e = (int)sa.size() - 1, ans = 0;
    while(b <= e)
    {
        int mid = (b + e) / 2;
        int r = cmp(sa[mid], p);
        if(!r)
            e = mid - 1, ans = mid;
        else if(r == -1)
            e = mid - 1;
        else
            b = mid + 1;
    }
    if(s.substr(sa[ans], p.size()) != p)
        return -1;
    return ans;
}

// posicao no suffix array do sufixo mais
// a direita que contem p como prefixo
// O(|p| * log|s|)
int upper_bound(string &p)
{
    int b = 0, e = (int)sa.size() - 1, ans = 0;
    while(b <= e)
    {
        int mid = (b + e) / 2;
        int r = cmp(sa[mid], p);
        if(!r)
            b = mid + 1, ans = mid;
        else if(r == -1)
            e = mid - 1;
        else
            b = mid + 1;
    }
    if(s.substr(sa[ans], p.size()) != p)
        return -1;
    return ans;
}

// numero de ocorrencias da string p
// como substring de s
// O(|p| * log|s|)
int count(string &p)
{
    int l = lower_bound(p);

```

```

    int u = upper_bound(p);
    if(l == -1 or u == -1)
        return 0;
    return u - l + 1;
}

// construcao do array lcp
// lcp[i] eh o maior prefixo comum
// aos sufixos i e i - 1 do suffix array
// O(n)
void buildLcp()
{
    int n = s.size();
    lcp.resize(n);
    int k = 0;
    for(int i = 0; i < n - 1; i++)
    {
        // pi eh a posicao no suffix array do
        // sufixo que comeca na posicao i da strig
        int pi = c[i];
        int j = sa[pi - 1];
        while(s[i + k] == s[j + k]) k++;
        lcp[pi] = k;
        k = max(k - 1, 0);
    }
}

// conta a quantidade de substrings
// diferentes na string s
// O(|s|)
long long numberOfDifSubStr()
{
    long long n = s.size();
    long long ans = n * (n - 1) / 2;
    for(int i = 0; i < n; i++)
        ans -= lcp[i];
    return ans;
}

// encontra a maior substring comum a s e p
// O(|s + p|) depois de construir suffix array
void longestCommonSubstring(string p)
{
    int n = s.size();
    int m = p.size();
    int ans = -1, j = 1;
    s = s + "$" + p + "#";

    buildSuffixArray();
    buildLcp();

    for(int i = 1; i < sa.size(); i++)
    {
        int p = sa[i - 1] < n ? 1 : -1;
        int q = sa[i] < n ? 1 : -1;
        if(p * q < 0 and ans < lcp[i])
            ans = lcp[i], j = i;
    }

    int a = sa[j];
    int b = sa[j - 1];

```

```

string lcs;

while(a < s.size() and b < s.size() and s[a] == s[b])
{
    lcs.push_back(s[a]);
    a++, b++;
}

cout << ans << '\n';
cout << lcs << '\n';
}

int32_t main()
{
    string p;
    cin >> s >> p;

    // o tamanho do maior prefixo comum entre dois
    // sufixos que estao nas posicoes a e b do
    // suffix array eh igual ao menor valor no
    // intervalo [a+1, b] do array lcp

    // descomentar as linhas abaixo para todas as
    // funcoes, exceto para longestCommonSubstring

    // s.push_back('$');
    // buildSuffixArray();
    // buildLcp();

    longestCommonSubstring(p);

    return 0;
}

```

## 4.9 Suffix Array

```

#include <bits/stdc++.h>
using namespace std;

string txt; // texto
string pat; // padrao
int n; // tamanho do texto
int chave[100]; // chave para comparacao dos sufixos
int vs[100]; // vetor de sufixos
int ord[100]; // ordem de um sufixo (qual classe ele pertence)
int lcp[100]; // maior prefixo comum

bool comp(int a, int b)
{
    return chave[a] < chave[b];
}

void constroi() // O(N*Log(N)*Log(N))
{
    for(int i = 0; i < n; i++)
    {
        vs[i] = i;
        chave[i] = txt[i] - 'a' + 1;
    }
    sort(vs, vs+n, comp);
}

```

```

for(int i = 0; ; i++)
{
    int classes = 0;
    for(int j = 0; j < n; j++)
        ord[vs[j]] = j > 0 and chave[vs[j]]
            == chave[vs[j-1]] ? ord[vs[j-1]] : ++classes;
    if(classes == n) break;
    for(int j = 0; j < n; j++)
    {
        chave[j] = ord[j]*(classes+1);
        chave[j] += j+(1<<i) < n ? ord[j+(1<<i)] : 0;
    }
    sort(vs, vs+n, comp);
}

int strcompara(int pos)
{
    // retorna 0 se o padrao esta no sufixo, maior que zero se o padrao
    // eh
    //lexicograficamente maior que o sufixo, e menor que 0 se o padrao
    //eh lexicograficamente menor que o sufixo
    for(int i = 0; i < pat.size(); i++)
        if(i+pos >= n)
            return 1;
        else if(pat[i] != txt[i+pos])
            return pat[i] - txt[i+pos];
    return 0;
}

bool search() // O(Size(Pat)*Log(N))
{
    int b = 0, e = n - 1, m, aux;
    while(b <= e)
    {
        m = (b + e) / 2;
        aux = strcompara(vs[m]);
        if(aux == 0)
            return true;
        else if(aux > 0)
            b = m + 1;
        else
            e = m - 1;
    }
    return false;
}

// numero de vezes que o padrao aparece no texto. O(Size(Pat)*Log(N))
int numberOfOcur()
{
    int b = 0, e = n - 1, m, aux, l = INT_MAX, r = INT_MIN;
    while(b <= e)
    {
        m = (b + e) / 2;
        aux = strcompara(vs[m]);
        if(aux == 0)
        {
            l = min(l, m);
            e = m - 1;
        }
        else if(aux > 0)
            b = m + 1;
    }
}

```

```

        else
            e = m - 1;
    }
    b = 0, e = n - 1;
    while(b <= e)
    {
        m = (b + e) / 2;
        aux = strcmpara(vs[m]);
        if(aux == 0)
        {
            r = max(r, m);
            b = m + 1;
        }
        else if(aux > 0)
            b = m + 1;
        else
            e = m - 1;
    }
    return abs(r-l+1);
}

// kasai em O(NlogN) pra construir o LCP (longest common prefix)
void kasai()
{
    vector<int> invSuff(n, 0);
    for(int i = 0; i < n; i++)
        invSuff[vs[i]] = i;
    int k = 0;
    for(int i = 0; i < n; i++)
    {
        if(invSuff[i] == n-1)
        {
            k = 0;
            continue;
        }
        int j = vs[invSuff[i]+1];
        while(i + k < n and j + k < n and txt[i + k] == txt[j + k])
            k++;
        lcp[invSuff[i]] = k;
        if(k > 0)
            k--;
    }
}

void printAll()
{
    cout << "Vetor de sufixos:\n";
    for(int i = 0; i < n; i++)
        cout << vs[i] << ' ';
    cout << '\n';
    cout << "Sufixos em ordem:\n";
    for(int i = 0; i < n; i++)
        cout << txt.substr(vs[i]) << '\n';
}

/*
Dado um array LCP onde LCP[i] armazena o tamanho do maior prefixo
em comum entre os sufixos i e i + 1 da suffix array
, entao para achar o maior prefixo em comum entre dois sufixos que
estao nas posicoes a e b da suffix array, corresponde a achar
o menor valor no intervalo [a, b-1] no LCP array.

```

Outra aplicacao eh dada uma string, para contar quantas substrings diferentes ela tem basta contar quantas tem no total ( $((n + 1) * n) / 2$  possiveis substrings) e remover todos os valores de LCP do total.

\*/

```

int main()
{
    cin >> txt;
    n = txt.size();
    constroi();
    printAll();

    cout << '\n';

    cin >> pat;
    cout << (search() ? "found " : "not found ") << numberOfOcur()
        << " vez(es)\n\n";

    kasai();
    cout << "LCP\n";
    for(int i = 0; i < n; i++)
        cout << lcp[i] << ' ';
    cout << '\n';

    return 0;
}

```

## 4.10 Trie Static

```

#include <bits/stdc++.h>
using namespace std;

// as posicoes de 0 ate 25 representam as letras
// de a ate z do alfabeto.
// a posicao 26 armazena quantas strings terminam
// nesse vertice.
// a posicao 27 armazena quantas strings passam
// nesse vertice.

int trie[8000000][30], CUR = 1;

// fl eh zero se for uma operacao de inserir
// fl eh um se for uma operacao de buscar
int add(string &s, int fl)
{
    int root = 0;
    for(char &c : s)
    {
        if(trie[root][c - 'a'] == 0)
        {
            if(fl) return 0;
            trie[root][c - 'a'] = CUR++;
        }
        if(!fl) trie[root][27]++;
        root = trie[root][c - 'a'];
    }
    if(fl) return trie[root][26];
    trie[root][26]++;
}

```

```

    return 1;
}

void sub(string &s)
{
    int root = 0;
    for(char &c : s)
        if(trie[root][c - 'a'] and trie[root][27])
        {
            trie[root][27]--;
            root = trie[root][c - 'a'];
        }
        trie[root][26]--;
}

int main()
{
    int q;

    cin >> q;

    while(q-->0)
    {
        int o;
        string s;
        cin >> o >> s;
        if(o == 1) add(s, 0);
        else if(o == 2) puts(add(s, 1) ? "existe" : "nao existe");
        else sub(s);
    }

    return 0;
}

```

## 4.11 Trie With Vector

```

#include <bits/stdc++.h>
using namespace std;

struct TrieNode
{
    int child[26], size, cnt;
    TrieNode()
    {
        memset(child, 0, sizeof(child));
        size = cnt = 0;
    }
};

vector<TrieNode> trie;

void init()
{
    trie.clear();
    trie.push_back(TrieNode());
}

void add(string s)
{
    int root = 0;

```

```

    for(int i = 0; i < (int)s.size(); i++)
    {
        int index = s[i] - 'a';
        if(trie[root].child[index] == 0)
        {
            trie[root].child[index] = trie.size();
            trie.push_back(TrieNode());
        }
        root = trie[root].child[index];
        trie[root].size++;
    }
    trie[root].cnt++;
}

void sub(string s)
{
    int root = 0;
    for(int i = 0; i < (int)s.size(); i++)
    {
        int index = s[i] - 'a';
        root = trie[root].child[index];
        trie[root].size--;
    }
    trie[root].cnt--;
}

int query(string s)
{
    int root = 0;
    for(int i = 0; i < (int)s.size(); i++)
    {
        int index = s[i] - 'a';
        if(!trie[trie[root].child[index]].size)
            return false;
        root = trie[root].child[index];
    }
    return trie[root].cnt;
}

int main()
{
    string s;
    int o;
    init();
    while(cin >> o >> s)
    {
        if(o == 1) add(s);
        else if(o == 2) sub(s);
        else cout << query(s) << '\n';
    }

    return 0;
}

```

## 4.12 Trie

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e6;

```



```

int trie[MAX][26], cnt[MAX], tsz = 1;
bool leaf[MAX];

void insert(string s)
{
    int cur = 1;
    cnt[cur]++;
    for(int i = 0; i < s.size(); i++)
    {
        int a = s[i] - 'a';
        if(!trie[cur][a]) trie[cur][a] = ++tsz;
        cur = trie[cur][a];
        cnt[cur]++;
    }
    leaf[cur] = true;
}

bool find(string s)
{
    int cur = 1;
    for(int i = 0; i < s.size(); i++)
    {
        int a = s[i] - 'a';
        if(!trie[cur][a] or !cnt[cur])
            return false;
        cur = trie[cur][a];
    }
    return leaf[cur] and cnt[cur];
}

int remove(string s)
{
    int cur = 1;
    for(int i = 0; i < s.size(); i++)
    {
        int a = s[i] - 'a';
        cnt[cur]--;
        cur = trie[cur][a];
    }
    leaf[cur] = false;
    cnt[cur]--;
}

int main()
{
    string s;
    int n, o;
    while(cin >> o >> s)
    {
        if(o == 1)
            cout << (find(s) ? "found\n" : "not found\n");
        else if(o == 2)
            insert(s);
        else
            remove(s);
    }

    return 0;
}

```

## 4.13 Z Function

```

#include "bits/stdc++.h"
#define Max(a, b) (a > b ? a : b)
#define Min(a, b) (a < b ? a : b)
using namespace std;
const int MAX = 1e5;

vector<int> Z(string &s)
{
    int n = s.size(), x = 0, y = 0;
    vector<int> z(n);
    for(int i = 1; i < n; i++)
    {
        z[i] = Max(0, Min(z[i - x], y - i + 1));
        while(i + z[i] < n and s[z[i]] == s[i + z[i]])
            x = i, y = i + z[i], z[i]++;
    }
    return z;
}

int main()
{
    string txt, pattern;

    cin >> txt >> pattern;
    string s = pattern + "#" + txt;
    vector<int> z = Z(s);
    for(int &w : z)
        cout << w << ' ';
    cout << '\n';

    return 0;
}

```

## 5 Math

### 5.1 Baby Step Giant Step

```

// a ^ kcong b mod m

int value[1000008];
int cor[1000008], tempo = 1;

// com vetor o modulo deve ser <= 10^7 fica O(sqrt(m))
inline int discreteLogarithm(int a, int b, int m) {
    tempo++;
    a %= m; b %= m;
    int n = (int)sqrt(m + .0) + 1, an = 1;
    for(int i = 1; i <= n; i++) an = (an * 1LL * a) % m;
    for(int i = 1, cur = an; i <= n; i++) {
        if(cor[cur] < tempo) value[cur] = i, cor[cur] = tempo;
        cur = (cur * 1LL * an) % m;
    }
    for(int j = 0, cur = b; j <= n; j++) {
        if(cor[cur] == tempo) {
            int ans = value[cur] * n - j;

```

```

        if(ans < m)
            return ans;
    }
    cur = (cur * 1LL * a) % m;
}
return -1;
}

// com mapa o modulo pode ser ateh <= 10^12 fica O(sqrt(m) * log(m))
int discreteLogarithm(int a, int b, int m)
{
    a %= m; b %= m;
    int n = (int)sqrt(m + .0) + 1, an = 1;
    for(int i = 1; i <= n; i++) an = (an * a) % m;
    unordered_map<int, int> value;
    for(int i = 1, cur = an; i <= n; i++) {
        if(!value.count(cur)) value[cur] = i;
        cur = (cur * an) % m;
    }
    for(int j = 0, cur = b; j <= n; j++) {
        if(value[cur]) {
            int ans = value[cur] * n - j;
            if(ans < m)
                return ans;
        }
        cur = (cur * a) % m;
    }
    return -1;
}

```

## 5.2 Catalan Numbers

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e5 + 10;
const long long MOD = 10000000000;

int catalan[MAX];

void init()
{
    catalan[0] = catalan[1] = 1;
    for(int i = 2; i <= 1000; i++)
        for(int j = 0; j < i; j++)
        {
            catalan[i] += (catalan[j] * catalan[i-j-1]) % MOD;
            if(catalan[i] >= MOD)
                catalan[i] -= MOD;
        }
}

int main()
{
    init();
    int n;

    while(cin >> n)
        printf("%d\n", catalan[n]);
}

```

```

return 0;
}

```

## 5.3 Chinese Remainder Theorem

```

//codar em Python para evitar problemas de overflow
// O(Tlog(lcm(n1*n2*...)))
//https://codeforces.com/blog/entry/61290

#include<bits/stdc++.h>
using namespace std;
const int MAX = 20;
#define ll long long

ll GCD(ll a, ll b) { return (b == 0) ? a : GCD(b, a % b); }

inline ll LCM(ll a, ll b) { return a / GCD(a, b) * b; }

inline ll normalize(ll x, ll mod) { x %= mod; if (x < 0) x += mod;
return x; }

struct GCD_type { ll x, y, d; };

GCD_type ex_GCD(ll a, ll b)
{
    if (b == 0) return {1, 0, a};
    GCD_type pom = ex_GCD(b, a % b);
    return {pom.y, pom.x - a / b * pom.y, pom.d};
}

int t;
ll a[MAX], n[MAX], ans, lcm;

int main()
{
    cin >> t;
    for(int i = 1; i <= t; i++)
        cin >> a[i] >> n[i], normalize(a[i], n[i]);
    ans = a[1];
    lcm = n[1];
    for(int i = 2; i <= t; i++)
    {
        auto pom = ex_GCD(lcm, n[i]);
        ll x1 = pom.x;
        ll d = pom.d;
        if((a[i] - ans) % d != 0) return cerr << "No solutions" <<
            endl, 0;
        ans = normalize(ans + x1 * (a[i] - ans) / d % (n[i] / d) * lcm
            ,
            lcm * n[i] / d);
        lcm = LCM(lcm, n[i]);
        // you can save time by replacing above lcm * n[i] / d
        // by lcm = lcm * n[i] / d
    }
    cout << ans << " " << lcm << endl;

    return 0;
}

```

## 5.4 Conversion Base

```
#include <bits/stdc++.h>
using namespace std;

int a,b; char sa[10000]; char sb[10000];

void rev(char s[])
{
    int l = strlen(s);
    for(int i = 0; i < l - 1 - i; i++)
        swap(s[i], s[l - 1 - i]);
}

void multi(char s[], int k)
{
    int i, c = 0, d;
    for(i=0; s[i]; i++)
    {
        d = (s[i] - '0') * k + c;
        c = d / b; d %= b;
        s[i] = '0' + d;
    }
    while(c)
    {
        s[i] = '0' + (c % b); i++;
        c /= b;
    }
    s[i] = '\0';
}

void add(char s[], int k)
{
    int i, c = k, d;
    for(i = 0; s[i]; i++)
    {
        d = (s[i] - '0') + c;
        c = d / b; d %= b;
        s[i] = '0' + d;
    }
    while(c)
    {
        s[i] = '0' + (c % b); i++;
        c /= b;
    }
    s[i] = '\0';
}

void trans(char s[])
{
    for(int i = 0; s[i]; i++)
    {
        char& c = s[i];
        if(c >= 'A' && c <= 'Z') c = '0' + 10 + (c - 'A');
        if(c >= 'a' && c <= 'z') c = '0' + 36 + (c - 'a');
    }
}

void itrans(char s[])
{

```

```
for(int i = 0; s[i]; i++)
{
    char& c = s[i]; int d = c - '0';
    if(d >= 10 && d <= 35) c = 'A' + (d - 10);
    if(d >= 36) c = 'a' + (d - 36);
}
}

int main()
{
    //digitos {0-9,A-Z,a-z}
    int q; cin>>q;
    int i,j;
    while(q)
    {
        q--;

        cin >> a >> b >> sa; sb[0] = '0'; sb[1] = '\0';
        // a e b sao dados na base 10
        // sa eh dado na base a
        // converter sa da base a pra base b
        cout << a << " " << sa << '\n';
        trans(sa);
        for(i = 0; sa[i]; i++)
        {
            multi(sb, a);
            add(sb, sa[i] - '0');
        }
        rev(sb);
        itrans(sb);
        // sb eh a na base b
        cout << b << " " << sb << '\n';
        puts("");
    }
    return 0;
}
```

## 5.5 Counting Number Of Times That A Digit Appears Until N

```
ll digits(int n, int d)
{
    ll res = 0, pot = 1, rem = 0;
    while (n)
    {
        int x = n%10;
        n /= 10;
        if (x > d) res += (n+1)*pot;
        else res += n*pot;
        if (x == d) res += rem+1;
        if (d == 0) res -= pot;
        rem += pot * x;
        pot *= 10;
    }
    return res;
}
```

## 5.6 Fast Fourier Transform

```
#include <bits/stdc++.h>

using namespace std;

typedef complex<double> ftype;
const double pi = acos(-1);
const int maxn = 1 << 22;
ftype w[maxn];

void init()
{
    for(int i = 0; i < maxn; i++)
        w[i] = polar(1., 2 * pi / maxn * i);
}

template<typename T>
void fft(T *in, ftype *out, int n, int k = 1)
{
    if(n == 1)
    {
        *out = *in;
        return;
    }
    int t = maxn / n;
    n >>= 1;
    fft(in, out, n, 2 * k);
    fft(in + k, out + n, n, 2 * k);
    for(int i = 0, j = 0; i < n; i++, j += t)
    {
        ftype t = w[j] * out[i + n];
        out[i + n] = out[i] - t;
        out[i] += t;
    }
}

vector<ftype> evaluate(vector<int> p)
{
    while(__builtin_popcount(p.size()) != 1)
        p.push_back(0);
    vector<ftype> res(p.size());
    fft(p.data(), res.data(), p.size());
    return res;
}

vector<int> interpolate(vector<ftype> p)
{
    int n = p.size();
    vector<ftype> inv(n);
    fft(p.data(), inv.data(), n);
    vector<int> res(n);
    for(int i = 0; i < n; i++)
        res[i] = round(real(inv[i]) / n);
    reverse(begin(res) + 1, end(res));
    return res;
}

void align(vector<int> &a, vector<int> &b)
{
```

```
    int n = a.size() + b.size() - 1;
    while(a.size() < n)
        a.push_back(0);
    while(b.size() < n)
        b.push_back(0);
}

vector<int> poly_multiply(vector<int> a, vector<int> b)
{
    align(a, b);
    auto A = evaluate(a);
    auto B = evaluate(b);
    for(int i = 0; i < A.size(); i++)
        A[i] *= B[i];
    return interpolate(A);
}

const int base = 10;
vector<int> normalize(vector<int> c)
{
    int carry = 0;
    for(auto &it: c)
    {
        it += carry;
        carry = it / base;
        it %= base;
    }
    while(carry)
    {
        c.push_back(carry % base);
        carry /= base;
    }
    return c;
}

vector<int> multiply(vector<int> a, vector<int> b)
{
    return normalize(poly_multiply(a, b));
}

vector<int> faz(string s)
{
    vector<int> ans;
    for(char &c : s)
        ans.push_back(c - '0');
    return ans;
}

string multAB(string s1, string s2)
{
    if(s1 == "0" or s2 == "0")
        return "0";
    bool sinall;
    if(s1[0] == '-' and s2[0] == '-' or s1[0] != '-' and s2[0] != '-')
        sinall = true;
    else
        sinall = false;
    if(s1[0] == '-') s1[0] = '0';
    if(s2[0] == '-') s2[0] = '0';
    vector<int> A = faz(s1), B = faz(s2);
    A = normalize(A);
```

```

B = normalize(B);
reverse(A.begin(), A.end());
reverse(B.begin(), B.end());
auto C = multiply(A, B);
while(C.back() == 0)
    C.pop_back();
reverse(C.begin(), C.end());
string ans;
ans += (!sinall ? "-" : "");
for(int &c: C)
    ans += char(c + '0');
return ans;
}

int main()
{
    int t;
    init();

    cin >> t;
    while(t--)
    {
        string s1, s2;
        cin >> s1 >> s2; // le os dois numeros como strings
        if(s1 == "0" or s2 == "0")
        {
            puts("0");
            continue;
        }
        vector<int> A = faz(s1), B = faz(s2);
        A = normalize(A);
        B = normalize(B);
        reverse(A.begin(), A.end());
        reverse(B.begin(), B.end());

        auto C = multiply(A, B);

        while(C.back() == 0)
            C.pop_back();
        reverse(C.begin(), C.end());
        for(int &c: C)
            cout << c;
        puts("");
    }

    /*
    init();
    vector<int> a = {3, 4}, b = {2, 3};
    auto C = poly_multiply(a, b);
    int k = int(a.size() + b.size()) - 1;
    for(int i = 0; i < k; i++)
        cout << C[i] << "X^" << k-i-1 << (i < k-1 ? " + " : "\n");
    */

    return 0;
}

```

```

////////////////////////////////////

```

```

#include <bits/stdc++.h>

using namespace std;

typedef long double ld;
const double PI = acos(-1);

struct T
{
    ld x, y;
    T() : x(0), y(0) {}
    T(ld a, ld b=0) : x(a), y(b) {}

    T operator/=(ld k) { x/=k; y/=k; return (*this); }
    T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
    T operator+(T a) const { return T(x+a.x, y+a.y); }
    T operator-(T a) const { return T(x-a.x, y-a.y); }
} a[1 << 23], b[1 << 23];

void fft(T* a, int n, int s)
{
    for (int i=0, j=0; i<n; i++)
    {
        if (i>j) swap(a[i], a[j]);
        for (int l=n/2; (j^=1) < 1; l>>=1);
    }
    for(int i = 1; (1<<i) <= n; i++)
    {
        int M = 1 << i;
        int K = M >> 1;
        T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
        for(int j = 0; j < n; j += M)
        {
            T w = T(1, 0);
            for(int l = j; l < K + j; ++l)
            {
                T t = w*a[l + K];
                a[l + K] = a[l]-t;
                a[l] = a[l] + t;
                w = wn*w;
            }
        }
    }
}

void multiply(T* a, T* b, int n)
{
    fft(a,n,1);
    fft(b,n,1);
    for (int i = 0; i < n; i++)
        a[i] = a[i]*b[i];
    fft(a,n,-1);
    for (int i = 0; i < n; i++)
        a[i] /= n;
}

int main()
{
    int n, na, nb, c;

```

```

cin >> na >> nb;
n = na + nb;
while(n & (n-1))
    n++;
for(int i = n - na; i < n; i++)
{
    cin >> c;
    a[i] = T(c);
}
for(int i = n - nb; i < n; i++)
{
    cin >> c;
    b[i] = T(c);
}
multiply(a, b, n);
for(int i = 0; i < n - 1; i++)
    cout << int(a[i].x + 0.5) << "X^"
        << n - 2 - i << (i < n - 2 ? " + " : "");
puts("");

/*
  3 2
  1 0 0
  2 3
  0X^6 + 0X^5 + 0X^4 + 2X^3 + 3X^2 + 0X^1 + 0X^0
*/

return 0;
}

////////////////////////////////////
//contar quantos subarrays de soma diferentes existem usando FFT

#include <bits/stdc++.h>

using namespace std;

typedef long double ld;
const long double PI = acos(-1);

struct T
{
    ld x, y;
    T() : x(0), y(0) {}
    T(ld a, ld b=0) : x(a), y(b) {}

    T operator/=(ld k) { x/=k; y/=k; return (*this); }
    T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
    T operator+(T a) const { return T(x+a.x, y+a.y); }
    T operator-(T a) const { return T(x-a.x, y-a.y); }
} a[16777219], b[16777219];

int pd[16777219];

void fft(T* a, int n, int s)
{
    for(int i=0, j=0; i<n; i++)
    {
        if (i>j) swap(a[i], a[j]);
        for (int l=n/2; (j^=1) < 1; l>=>1);
    }

```

```

    }
    for(int i = 1; (1<<i) <= n; i++)
    {
        int M = 1 << i;
        int K = M >> 1;
        T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
        for(int j = 0; j < n; j += M)
        {
            T w = T(1, 0);
            for(int l = j; l < K + j; ++l)
            {
                T t = w*a[l + K];
                a[l + K] = a[l]-t;
                a[l] = a[l] + t;
                w = wn*w;
            }
        }
    }
}

void multiply(T* a, T* b, int n)
{
    fft(a,n,1);
    fft(b,n,1);
    for (int i = 0; i < n; i++)
        a[i] = a[i]*b[i];
    fft(a,n,-1);
    for (int i = 0; i < n; i++)
        a[i] /= n;
}

int main()
{
    int k;
    cin >> k;
    for(int i = 1; i <= k; i++)
    {
        int aux;
        cin >> aux;
        pd[i] = pd[i - 1] + aux;
    }
    if(k >= 10000)
    {
        for(int i = 0; i <= k; i++)
        {
            a[pd[i] + pd[k]].x = 1;
            b[pd[k] - pd[i]].x = 1;
        }
        int n = pd[k] + pd[k];
        n = 2 * n;
        while(n & (n - 1))
            n++;
        multiply(a, b, n);
        int ans = 0;
        for(int i = 0; i <= n; i++)
            if(int(a[i].x + 0.5) > 0 and (i - 2 * pd[k]) > 0)
                ans++;
        cout << ans << '\n';
    }
    else
    {

```

```

    int cnt = 0;
    unordered_set<int> ans1;
    for(int i = 1; i <= k; i++)
        for(int j = i; j <= k; j++)
            if(ans1.find(pd[j] - pd[i - 1]) == ans1.end())
            {
                ans1.insert(pd[j] - pd[i - 1]);
                cnt++;
            }
    cout << cnt << '\n';
}
// quantidade de subarrays com soma diferente

return 0;
}

//
// //////////////////////////////////////

#include <bits/stdc++.h>

using namespace std;

typedef long double ld;
const double PI = acos(-1);

struct T
{
    ld x, y;
    T() : x(0), y(0) {}
    T(ld a, ld b=0) : x(a), y(b) {}

    T operator/=(ld k) { x/=k; y/=k; return (*this); }
    T operator*(T a) const { return T(x*a.x - y*a.y, x*a.y + y*a.x); }
    T operator+(T a) const { return T(x+a.x, y+a.y); }
    T operator-(T a) const { return T(x-a.x, y-a.y); }
} a[1 << 20], b[1 << 20];

void fft(T* a, int n, int s)
{
    for (int i=0, j=0; i<n; i++)
    {
        if (i>j) swap(a[i], a[j]);
        for (int l=n/2; (j^=l) < l; l>=>1);
    }
    for(int i = 1; (1<<i) <= n; i++)
    {
        int M = 1 << i;
        int K = M >> 1;
        T wn = T(cos(s*2*PI/M), sin(s*2*PI/M));
        for(int j = 0; j < n; j += M)
        {
            T w = T(1, 0);
            for(int l = j; l < K + j; ++l)
            {
                T t = w*a[l + K];
                a[l + K] = a[l]-t;
                a[l] = a[l] + t;
                w = wn*w;
            }
        }
    }

    void multiply(T* a, T* b, int n)
    {
        fft(a,n,1);
        fft(b,n,1);
        for (int i = 0; i < n; i++)
            a[i] = a[i]*b[i];
        fft(a,n,-1);
        for (int i = 0; i < n; i++)
            a[i] /= n;
    }

    const int base = 10;
    vector<int> normalize(vector<int> c)
    {
        int carry = 0;
        for(auto &it: c)
        {
            it += carry;
            carry = it / base;
            it %= base;
        }
        while(carry)
        {
            c.push_back(carry % base);
            carry /= base;
        }
        return c;
    }

    vector<int> faz(string s)
    {
        vector<int> ans;
        for(char &c : s)
            ans.push_back(c-'0');
        return ans;
    }

    string mul(string s1, string s2)
    {
        vector<int> A = normalize(faz(s1));
        vector<int> B = normalize(faz(s2));

        int na = A.size(), nb = B.size();
        int n = na + nb;

        while(n&(n-1))
            n++;

        reverse(A.begin(), A.end());
        reverse(B.begin(), B.end());

        while(A.size() < n) A.push_back(0);
        while(B.size() < n) B.push_back(0);

        reverse(A.begin(), A.end());

```

```

reverse(B.begin(), B.end());

for(int i = 0; i < n; i++)
    a[i] = T(A[i]);
for(int i = 0; i < n; i++)
    b[i] = T(B[i]);

multiply(a, b, n);

vector<int> r;
for(int i = 0; i < n - 1; i++)
    r.push_back(a[i].x + 0.5);

reverse(r.begin(), r.end());

r = normalize(r);

while(r.back() == 0)
    r.pop_back();

reverse(r.begin(), r.end());

string ans;

for(int &c: r)
    ans.push_back(c + '0');

return ans;
}

int main()
{

    return 0;
}

```

## 5.7 Gaussian Elimination For Max Subset Xor

```

#include <bits/stdc++.h>
using namespace std;

#define ull unsigned long long

int MSB(ull n)
{
    int cnt = 0;
    while(n)
    {
        cnt++;
        n >>= 1;
    }
    return cnt;
}

int main()
{
    int n;
    cin >> n;
    ull a[n];

```

```

for(int i = 0; i < n; i++)
    cin >> a[i];
int lengths[n];
for(int i = 0; i < n; i++)
    lengths[i] = MSB(a[i]);
//eh um array que armazena os coeficientes
//das equacoes
vector<ull> buckets[65];
//para a Gaussian Elimination, semelhante
//a linha da matriz em algebra linear
for(int i = 0; i < n; i++)
    buckets[lengths[i]].push_back(a[i]);
ull modified_array[100], m_index = 0;

// Gaussian Elimination
for(int i = 64; i > 0; i--)
    if(buckets[i].size())
    {
        modified_array[m_index++] = buckets[i][0];
        for(int j = 1; j < buckets[i].size(); j++)
        {
            ull temp = buckets[i][0] ^ buckets[i][j];
            int len = MSB(temp);
            buckets[len].push_back(temp);
        }
    }
ull ans = 0;
for(int i = 0; i < m_index; i++)
    if(ans < (ans ^ modified_array[i]))
        ans = (ans ^ modified_array[i]);
cout << ans << '\n';

return 0;
}

```

## 5.8 Karatsuba

```

#include <bits/stdc++.h>
using namespace std;

typedef vector<long long> vll;

vll karatsubaMultiply(const vll &a, const vll &b) {
    int n = a.size();
    vll res(n + n);
    if (n <= 32) {
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                res[i + j] += a[i] * b[j];
        return res;
    }

    int k = n >> 1;
    vll a1(a.begin(), a.begin() + k);
    vll a2(a.begin() + k, a.end());
    vll b1(b.begin(), b.begin() + k);
    vll b2(b.begin() + k, b.end());

    vll a1b1 = karatsubaMultiply(a1, b1);
    vll a2b2 = karatsubaMultiply(a2, b2);

```



```

for(int i = 0; i < k; i++)
    a2[i] += a1[i];
for(int i = 0; i < k; i++)
    b2[i] += b1[i];

vll r = karatsubaMultiply(a2, b2);

for(int i = 0; i < (int) albl.size(); i++)
    r[i] -= albl[i];
for(int i = 0; i < (int) a2b2.size(); i++)
    r[i] -= a2b2[i];

for(int i = 0; i < (int) r.size(); i++)
    res[i + k] += r[i];
for(int i = 0; i < (int) albl.size(); i++)
    res[i] += albl[i];
for(int i = 0; i < (int) a2b2.size(); i++)
    res[i + n] += a2b2[i];
return res;
}

int main()
{
    vll a = {8, 7, 5};
    vll b = {12};

    vll c = karatsubaMultiply(a, b);

    for(auto it : c) cout << it << ' '; puts("");

    return 0;
}

```

## 5.9 Matrix Exponentiation

```

#include <bits/stdc++.h>
using namespace std;
#define matrix vector<vector<int>>

matrix init(int n, int m, int value = 0)
{
    return vector<vector<int>>(n, vector<int>(m, value));
}

void printttt(const matrix &M)
{
    for(int i = 0; i < M.size(); i++)
    {
        for(int j = 0; j < M[0].size(); j++)
            cout << M[i][j] << ' ';
        puts("");
    }
}

matrix multiply(const matrix &A, const matrix &B)
{
    matrix C = init(A.size(), B[0].size());
    for(int i = 0; i < A.size(); i++)
        for(int j = 0; j < B[i].size(); j++)

```

```

            for(int k = 0; k < B.size(); k++)
                C[i][j] += A[i][k] * B[k][j];
    return C;
}

matrix exp(matrix M, int k)
{
    matrix I = init(M.size(), M[0].size());
    for(int i = 0; i < M.size(); i++) I[i][i] = 1;
    while(k)
        if(k & 1) I = multiply(I, M), k--;
        else M = multiply(M, M), k /= 2;
    return I;
}

int determinantOfMatrix(matrix mat)
{
    int n = mat.size();
    int num1, num2, det = 1, index, total = 1;
    int temp[n + 1];
    for(int i = 0; i < n; i++)
    {
        index = i;
        while(mat[index][i] == 0 and index < n)
            index++;
        if(index == n)
            continue;
        if(index != i)
        {
            for(int j = 0; j < n; j++)
                swap(mat[index][j], mat[i][j]);
            det = det * pow(-1, index - i);
        }
        for(int j = 0; j < n; j++)
            temp[j] = mat[i][j];
        for(int j = i + 1; j < n; j++)
        {
            num1 = temp[i];
            num2 = mat[j][i];
            for(int k = 0; k < n; k++)
                mat[j][k] = (num1 * mat[j][k]) - (num2 * temp[k]);
            total = total * num1;
        }
        for(int i = 0; i < n; i++)
            det = det * mat[i][i];
        return (det / total);
    }
}

int32_t main()
{
    int n, m;

    cin >> n >> m;
    matrix A = init(n, m);
    for(int i = 0; i < n; i++)
        for(int j = 0; j < m; j++)
            cin >> A[i][j];
    matrix C = exp(A, 7);

```

```

    printtt(C);

    return 0;
}

```

## 5.10 Miller Rabin

```

#include <bits/stdc++.h>
using namespace std;
#define ll long long

ll add(ll a, ll b, ll c)
{
    ll ans = (a + b) % c;
    if(ans < 0) ans += c;
    return ans;
}

ll mulmod(ll a, ll b, ll c)
{
    ll ans = 0;
    while(b)
    {
        if(b & 1) ans = add(ans, a, c);
        a = add(a, a, c);
        b /= 2;
    }
    return ans;
}

ll fexp(ll a, ll b, ll c)
{
    ll ans = 1;
    while(b)
    {
        if(b & 1) ans = mulmod(ans, a, c);
        a = mulmod(a, a, c);
        b /= 2;
    }
    return ans;
}

bool miller(ll a, ll n)
{
    if (a >= n) return true;
    ll s = 0, d = n - 1;
    while(d%2 == 0 and d) d >>= 1, s++;
    ll x = fexp(a, d, n);
    if(x == 1 or x == n - 1) return true;
    for(int r = 0; r < s; r++, x = mulmod(x, x, n))
    {
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}

bool isprime(ll n)
{
    int base[] = {2, 3, 5, 7, 11,

```

```

    13, 17, 19, 23, 29, 31, 37};
    for(int i = 0; i < 12; i++)
        if(!miller(base[i], n))
            return false;
    return true;
}

int32_t main()
{
    ll n;
    cin >> n;
    cout << (isprime(n) ? "PRIME\n"
        : "NOT PRIME\n");

    return 0;
}

```

## 5.11 Mobius

```

/*
-  $\mu(n) = 0$  se  $n$  tem como divisor um outro numero
natural ao quadrado
-  $\mu(n) = 1$  se  $n$  nao tem como divisor um outro
numero natural ao quadrado
e eh decomposto em uma quantidade par de
numeros primos
-  $\mu(n) = -1$  se  $n$  nao tem como divisor um outro
numero natural ao quadrado
e eh decomposto em uma quantidade impar de
numeros primos
*/

#include "bits/stdc++.h"
using namespace std;
const int MAX = 1e6;

bool np[MAX];
int mob[MAX];

void mobius()
{
    for(int i = 1; i < MAX; i++)
        mob[i] = 1;
    for(int i = 2; i < MAX; i++)
    {
        if(np[i]) continue;
        for(int j = i; j < MAX; j += i)
        {
            np[j] = true;
            mob[j] *= -1;
            if((j / i) % i == 0)
                mob[j] = 0;
        }
    }
}

int main()
{
    mobius();
}

```

```

    for(int i = 2; i <= 10; i++)
        cout << i << ' ' << mob[i] << '\n';
    puts("");

    return 0;
}

```

## 5.12 Mod Gaussian Elimination

```

#include<bits/stdc++.h>
using namespace std;
// #define int long long
#define pb push_back
#define inf 0x3f3f3f3f

int MOD = 1000000007LL;

inline int prod(int a, int b)
{
    return (((a % MOD) * 1LL * (b % MOD)) % MOD) + MOD) % MOD;
}

inline int sub(int a, int b)
{
    return (((a % MOD) - (b % MOD)) % MOD) + MOD) % MOD;
}

inline int expMod(int x, int e)
{
    int ans = 1;
    while(e > 0)
    {
        if(e & 1LL) ans = prod(ans, x), e--;
        else x = prod(x, x), e /= 2;
    }
    return ans;
}

inline int inv(int x)
{
    return expMod(x, MOD - 2);
}

inline int gauss (vector<vector<int>> a, int mod)
{
    MOD = mod;

    int n = (int) a.size();
    int m = (int) a[0].size();

    vector<int> where (m, -1);
    for(int col = 0, row = 0; col < m and row < n; ++col)
    {
        int sel = row;
        for(int i = row; i < n; ++i)
            if(abs(a[i][col]) > abs(a[sel][col]))
                sel = i;
        if(a[sel][col] == 0)
            continue;
        for(int i = col; i < m; ++i)

```

```

            swap(a[sel][i], a[row][i]);
            where[col] = row;
            for(int i = row + 1; i < n; ++i)
            {
                int c = prod(a[i][col], inv(a[row][col]));
                for(int j = col; j < m; ++j)
                    a[i][j] = sub(a[i][j], prod(a[row][j], c));
            }
            ++row;
        }
        int ans = 0;
        for(int i = 0; i < m; ++i)
            if(where[i] != -1)
                ans++;
        return n - ans;
    }

int32_t main()
{
    int n, m, a, k, t, caso = 1;

    cin >> t;
    while(t--)
    {
        scanf(" %d %d %d", &n, &m, &k);
        vector<vector<int>> A(n, vector<int>(m));
        while(m--)
        {
            int u, v;
            scanf(" %d %d", &u, &v), u--; v--;
            A[u][v] = A[v][u] = 1;
            if(u != v) A[u][v] = A[v][u] = k - 1;
        }
        for(int i = 0; i < n; i++) A[i][i] = 1;
        int ans = gauss(A, k);
        MOD = 1000000007LL;
        printf("Case %d: %d\n", caso++, expMod(k, ans));
    }

    return 0;
}

```

## 5.13 Mod Integer

```

#include <bits/stdc++.h>
using namespace std;

template <typename T>
struct ModInt {
    const static T MOD = 1000000007; //1e9 + 7

    T v; explicit operator T() const { return v; }

    ModInt() { v = 0; }
    ModInt(T _v) {
        v = (-MOD < _v && _v < MOD) ? _v : _v % MOD;
        if (v < 0) v += MOD;
    }
    friend bool operator==(const ModInt& a, const ModInt& b) {

```

```

    return a.v == b.v; }
friend bool operator!=(const ModInt& a, const ModInt& b) {
    return !(a == b); }
friend bool operator<(const ModInt& a, const ModInt& b) {
    return a.v < b.v; }

ModInt& operator+=(const ModInt& m) {
    if ((v += m.v) >= MOD) v -= MOD;
    return *this; }
ModInt& operator-=(const ModInt& m) {
    if ((v -= m.v) < 0) v += MOD;
    return *this; }
ModInt& operator*=(const ModInt& m) {
    v = v*m.v%MOD; return *this; }
ModInt& operator/=(const ModInt& m) { return (*this) *= inv(m); }
friend ModInt pow(ModInt a, T p) {
    assert(p >= 0);
    ModInt ans = 1;
    while(p) {
        if(p & 1) ans *= a;
        a *= a;
        p /= 2;
    }
    return ans;
}
friend ModInt inv(const ModInt& a) { assert(a.v != 0);
    return pow(a, MOD-2); }

ModInt operator-() const { return ModInt(-v); }
ModInt& operator++() { return *this += 1; }
ModInt& operator--() { return *this -= 1; }
ModInt operator++(int) { ModInt temp; temp.v = v++; return temp; }
ModInt operator--(int) { ModInt temp; temp.v = v--; return temp; }
friend ModInt operator+(ModInt a, const ModInt& b) { return a += b;
}
friend ModInt operator-(ModInt a, const ModInt& b) { return a -= b;
}
friend ModInt operator*(ModInt a, const ModInt& b) { return a *= b;
}
friend ModInt operator/(ModInt a, const ModInt& b) { return a /= b;
}
friend ostream& operator<<(ostream& os, const ModInt& m) {
    os << m.v; return os;
}
friend istream& operator>>(istream& is, ModInt& m) {
    T x; is >> x;
    m.v = x;
    return is;
}
}

int main(){
    ModInt<long long> x = 5;
    cout << x << ' ' << inv(x) << '\n';
    cout << x.MOD << '\n';

    return 0;
}

```

## 5.14 Modular Arithmetic

```

#include <bits/stdc++.h>
using namespace std;
#define int long long
const int MOD = 1000000007LL;

int normalize(int x)
{
    x = x % MOD;
    if(x < 0) x += MOD;
    return x;
}

int add(int a, int b)
{
    return normalize(normalize(a) + normalize(b));
}

int prod(int a, int b)
{
    return normalize(normalize(a) * normalize(b));
}

int sub(int a, int b)
{
    return normalize(normalize(a) - normalize(b));
}

int expMod(int x, int e)
{
    int ans = 1;
    while(e > 0)
    {
        if(e & 1LL) ans = prod(ans, x), e--;
        else x = prod(x, x), e /= 2;
    }
    return normalize(ans);
}

int inv(int x)
{
    return expMod(x, MOD - 2);
}

int extended_euclidean(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = extended_euclidean(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

int inv(int a, int m) {

```

```

int x, y;
int g = extended_euclidean(a, m, x, y);
if (g != 1) return -1; // nao tem inverso
return ((x % m) + m) % m;
}

```

## 5.15 Mulmod Trick

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

ll mulmod(ll a, ll b, ll m)
{
    ll q = ll((long double)a*b/m);
    ll r = a * b - m * q;
    while(r < 0) r += m;
    while(r >= m) r -= m;
    return r;
}

int main()
{
    ll a, b, c;

    cin >> a >> b >> c;
    cout << mulmod(a, b, c) << '\n';

    return 0;
}

```

## 5.16 Pollard Rho

```

#include <bits/stdc++.h>
using namespace std;
#define ll long long

ll llrand()
{
    ll tmp = rand();
    return (tmp << 31) | rand();
}

ll add(ll a, ll b, ll c)
{
    ll ans = (a + b) % c;
    if(ans < 0) ans += c;
    return ans;
}

ll mulmod(ll a, ll b, ll c)
{
    ll ans = 0;
    while(b)
    {
        if(b & 1) ans = add(ans, a, c);
        a = add(a, a, c);
        b /= 2;
    }
}

```

```

    }
    return ans;
}

ll rho(ll n)
{
    if(n % 2 == 0) return 2;
    ll d = n;
    while(d == n)
    {
        ll c = llrand() % n, x = llrand() % n, y = x;
        do
        {
            x = add(mulmod(x, x, n), c, n);
            y = add(mulmod(y, y, n), c, n);
            y = add(mulmod(y, y, n), c, n);
            d = __gcd(abs(x - y), n);
        }while(d == 1);
    }
    return d;
}

```

*// Miller-Rabin AQUI*

```

vector<ll> fac;

void factors(ll n) // encontrar os fatores primos de N
{ // Usar Miller-Rabin para testar se N eh primo
    if(n == 1) return;
    if(isprime(n)) { fac.push_back(n); return; }
    ll d = rho(n);
    factors(d);
    factors(n / d);
}

int32_t main()
{
    srand(time(NULL));
    ll n;
    cin >> n;
    cout << rho(n) << '\n';

    return 0;
}

```

## 6 Geometry

### 6.1 Andrew Algorithm Convex Hull

```

#include <bits/stdc++.h>
using namespace std;
#define X first
#define Y second
typedef pair<int, int> ii;

int cross(ii O, ii A, ii B)
{
    return ((A.X - O.X) * (B.Y - O.Y)) - ((A.Y - O.Y) * (B.X - O.X));
}

```

```

}

vector<ii> ConvexHull(vector<ii> P)
{
    if(P.size() <= 1) return P;
    vector<ii> H(2*P.size());
    int k = 0;
    sort(P.begin(), P.end());
    //lower hull
    for(int i = 0; i < P.size(); i++)
    {
        while(k >= 2 and cross(H[k-2], H[k-1], P[i]) < 0) k--;
        H[k++] = P[i];
    }
    //upper hull
    for(int i = P.size()-2, l = k + 1; i >= 0; i--)
    {
        while(k >= 1 and cross(H[k-2], H[k-1], P[i]) < 0) k--;
        H[k++] = P[i];
    }
    H.resize(k-1);
    return H;
}

int main()
{
    int n, x, y;
    vector<ii> P;

    cin >> n;
    while(n--)
    {
        cin >> x >> y;
        P.push_back({x, y});
    }

    vector<ii> H = ConvexHull(P);

    for(int i = 0; i < H.size(); i++)
        cout << H[i].X << ' ' << H[i].Y << '\n';

    return 0;
}

```

## 6.2 Build Two Lines That Go Through All Points Of A Set

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e6 + 10;
typedef long long ll;

struct Point
{
    ll x, y;
    Point(ll _x, ll _y) : x(_x), y(_y) {}
    Point() {}
    Point operator-(const Point& p)
    {
        return Point(x - p.x, y - p.y);
    }
}

```

```

}
ll operator*(const Point& p)
{
    return (x * p.y) - (y * p.x);
}

int n;
int visit[MAX];
Point point[MAX];

bool inLine(int a, int b, int c)
{
    return ((point[a] - point[c]) * (point[b] - point[c])) == 0LL;
}

bool check(int a, int b) // traca a reta AB e verifica se todos os
//pontos que nao estao em AB estao contidos em uma mesma reta
{
    memset(visit, 0, sizeof(visit));
    visit[a] = visit[b] = 1;
    for(int i = 0; i < n; i++)
        if(!visit[i] and inLine(a, b, i))
            visit[i] = 1; // marco todos os pontos que estao na reta AB
    vector<int> c;
    for(int i = 0; i < n and c.size() < 2; i++)
        if(!visit[i])
            c.push_back(i); // procuro dois pontos que nao estao na
            //reta AB
    if(c.size() < 2) return true;
    visit[c[0]] = visit[c[1]] = 1;
    for(int i = 0; i < n; i++)
        if(!visit[i])
        {
            // checo se o ponto que nao esta na reta AB esta na reta
            //COC1
            if(inLine(c[0], c[1], i))
                visit[i] = 1;
            else
                return false;
        }
    return true;
}

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> point[i].x >> point[i].y;
    if(n <= 2) return cout << "YES\n", 0;
    int k = 2;
    while(k < n and inLine(0, 1, k)) k++;
    if(k == n) return cout << "YES\n", 0;
    cout << ((check(0, 1) or check(0, k)
        or check(1, k)) ? "YES\n" : "NO\n");

    return 0;
}

```

## 6.3 Check If A Point Is Inside A Convex Polygon

```

#include <bits/stdc++.h>
using namespace std;
#define int long long
#define ii pair<int, int>
#define fi first
#define se second

int n;
vector<ii> P;

ii operator-(ii a, ii b)
{
    return {a.fi - b.fi, a.se - b.se};
}

int operator*(ii a, ii b)
{
    return a.fi * b.se - a.se * b.fi;
}

void setFirstPoint()
{
    int pos = 0;
    for(int i = 0; i < n; i++)
        if(P[i].fi < P[pos].fi or P[i].fi == P[pos].fi and P[i].se < P[pos].se)
            pos = i;
    rotate(P.begin(), P.begin() + pos, P.end());
}

bool pointInTriangle(ii a, ii b, ii c, ii p)
{
    int s1 = abs((a - c) * (b - c));
    int s2 = abs((a - p) * (b - p)) + abs((b - p) * (c - p)) +
        abs((c - p) * (a - p));
    return s1 == s2; //mesma area
}

int dist(ii a, ii b)
{
    return (a.fi - b.fi) * (a.fi - b.fi) + (a.se - b.se) * (a.se - b.se);
}

// O(logN) per query
bool pointInConvexPolygon(ii p)
{
    //adicionar = desconsidera pontos na borda
    if((P[1] - P[0]) * (p - P[0]) < 0)
        return false;
    //adicionar = desconsidera pontos na borda
    if((p - P[0]) * (P[n - 1] - P[0]) < 0)
        return false;
    //o ponto esta em cima do segmento P[0], P[n-1]
    if((p - P[0]) * (P[n - 1] - P[0]) == 0)
        return dist(P[0], p) <= dist(P[0], P[n - 1]) and dist(P[n - 1], p) <= dist(P[0], P[n - 1]);
    //o ponto esta em cima do segmento P[0], P[1]
    if((P[1] - P[0]) * (p - P[0]) == 0)
        return dist(P[0], p) <= dist(P[0], P[1]) and dist(P[1], p) <= dist(P[0], P[1]);
}

```

```

// se o ponto esta entre os segmentos P[0], P[n]
int l = 0, e = n - 1, ans = 0;
while(l <= e)
{
    int m = l + (e - l) / 2;
    if((P[m] - P[0]) * (p - P[0]) >= 0) l = m + 1, ans = m;
    else e = m - 1;
}
return pointInTriangle(P[ans], P[ans + 1], P[0], p);
}

int32_t main()
{
    int q, x, y;
    cin >> n >> q;
    for(int i = 0; i < n; i++)
        //poligono no sentido anti-horario
        cin >> x >> y;
        P.push_back({x, y});
}
setFirstPoint();
while(q--)
{
    cin >> x >> y;
    cout << (pointInConvexPolygon({x, y}) ? "Dentro" : "Fora") <<
        '\n';
}

return 0;
}

```

## 6.4 Convex Hull Trick

```

#include <bits/stdc++.h>
using namespace std;
#define type int
const int MAX = 1e5;
const int OO = 0x3f3f3f3f;

struct line
{
    type m, b;
    line(type _m, type _b){ m = _m, b = _b; }
};

int pointer; //Keeps track of the best line from previous query
vector<line> hull; //store hull

//Returns true if line l3 is always better than line l2
bool bad(int l1, int l2, int l3)
{
    /*
    intersection(l1,l2) has x-coordinate (b1-b2)/(m2-m1)
    intersection(l1,l3) has x-coordinate (b1-b3)/(m3-m1)
    set the former greater than the latter, and cross-multiply to
    eliminate division
    */
    line L1 = hull[l1], L2 = hull[l2], L3 = hull[l3];
    return (L3.b-L1.b)*(L1.m-L2.m) < (L2.b-L1.b)*(L1.m-L3.m);
}

```

```
//Adds a new line
void add(type m, type b)
{
    if(hull.size() > 0 and hull.back().m == m) return;
    //First, let's add it to the end
    hull.emplace_back(m, b);
    //If the penultimate is now made irrelevant between the
    antepenultimate
    //and the ultimate, remove it. Repeat as many times as necessary
    while(hull.size() >= 3 and bad(hull.size()-3, hull.size()-2, hull.size()
    (-1))
        hull.erase(hull.end()-2);
}

//Returns y value of a function i
type eval(int i, type x)
{
    return hull[i].m * x + hull[i].b;
}

//Returns the minimum y-coordinate of any intersection
//between a given vertical line and the lower envelope
//O(N) for all queries (queries are in ascending order of x)
type query(type x)
{
    if(pointer >= hull.size())
        pointer = hull.size() - 1;
    while(pointer < hull.size()-1 and eval(pointer+1, x) < eval(
        pointer, x))
        pointer++;
    return eval(pointer, x);
}

//Returns the minimum y-coordinate of any intersection
//between a given vertical line and the lower envelope
//O(LogN) time (queries are in any order of x)
type binarySearch(type x)
{
    int b = 0, e = hull.size() - 1;
    while(b < e)
    {
        int mid = (b + e) / 2;
        if(eval(mid+1, x) < eval(mid, x)) b = mid + 1;
        else e = mid;
    }
    return eval(b, x);
}

/*
Maximum Y coordinate query, we have two options:
1) Maximum Y-coordinate query: multiply m and b by -1 and
make minimum Y-coordinate query...

2) Order lines by increasing m if m is not equal, otherwise by
decreasing b
in the function query and binary Search change < to >
eval(pointer+1, x) < eval(pointer, x)
to,
eval(pointer+1, x) > eval(pointer, x)
*/
```

```
int main()
{
    int n;
    cin >> n;
    //Order lines by decreasing m if m is not equal, otherwise by
    increasing b
    for(int i = 0; i < n; i++)
    {
        int m, b;
        cin >> m >> b;
        add(m, b);
    }
    int q;
    cin >> q;
    vector<int> queries(q); //queries are in ascending order of x - run
    in O(N)
    for(int &w : queries)
        cin >> w;
    //processing queries in ascending order of x
    for(int &w : queries)
        cout << query(w) << '\n';

    int x;
    while(cin >> x) //queries are in any order of x - run in O(logN)
        cout << binarySearch(x) << '\n';

    return 0;
}
```

## 6.5 Distance Between Nearest Pair Of Points

```
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e4;
const double EPS = 1e7;
typedef pair<double, double> ii;

vector<ii> v;
set<ii> sy, sx;
int n;

int main()
{
    double x, y;
    cin >> n;
    for(int i = 0; i < n; i++)
    {
        cin >> x >> y;
        v.push_back({x, y});
    }
    sort(v.begin(), v.end());
    double d = 0x3f3f3f3f;
    for(int i = 0; i < n; i++)
    {
        x = v[i].first, y = v[i].second;
        while(!sx.empty())
        {
            ii p = *(sx.begin());
            if(p.first + d < x)
```



```

    {
        sy.erase({p.second, p.first});
        sx.erase(p);
    }
    else
        break;
}
auto it = sy.lower_bound({int(floor(y-d))-1, 0});
while(it != sy.end() and it->first < y + d + 1)
{
    d = min(d, hypot(x - it->second, y - it->first));
    it++;
}
sy.insert({y, x});
sx.insert({x, y});
}
cout << d << '\n';

return 0;
}

```

## 6.6 Dynamic Convex Hull Trick

```

#include <bits/stdc++.h>
using namespace std;
#define type __int128
#define int __int128
#define gc getchar
#define pc putchar
#define Min(a, b) (a > b ? b : a)

inline void scanint(int &k)
{
    bool sinal = true;
    register char c;
    k = 0;
    for(c = gc(); sinal and (c < '0' or c > '9'); c = gc())
        if(c == '-')
            sinal = false;
    for(; c >= '0' and c <= '9'; c = gc())
        k = (k << 3) + (k << 1) + c - '0';
    if(!sinal) k = -k;
}

inline void printint(int n)
{
    if(n < 0) pc('-');
    n = abs(n);
    int rev = n, cnt = 0;
    if(!n)
    {
        pc('0');
        pc('\n');
        return;
    }
    while(!(rev % 10))
        cnt++, rev /= 10;
    rev = 0;
    while(n)
        rev = (rev << 3) + (rev << 1) + n % 10, n /= 10;
}

```

```

while(rev)
    pc(rev % 10 + '0'), rev /= 10;
while(cnt-->0)
    pc('0');
pc('\n');
}

struct line
{
    type m, b;
    line(type _m, type _b){ m = _m, b = _b; }
    line(){ m = 0, b = 0; }
};

bool bad(int l1, int l2, int l3, vector<line> &hull)
{
    line L1 = hull[l1], L2 = hull[l2], L3 = hull[l3];
    return (L3.b-L1.b)*(L1.m-L2.m) < (L2.b-L1.b)*(L1.m-L3.m);
}

void add(type m, type b, vector<line> &hull)
{
    if(hull.size() > 0 and hull.back().m == m) return;
    hull.emplace_back(m, b);
    while(hull.size() >= 3 and bad(hull.size()-3, hull.size()-2, hull.size()-1, hull))
        hull.erase(hull.end()-2);
}

type eval(int i, type x, vector<line> &hull)
{
    return hull[i].m * x + hull[i].b;
}

type binarySearch(type x, vector<line> &hull)
{
    int b = 0, e = hull.size() - 1;
    while(b < e)
    {
        int mid = (b + e) / 2;
        if(eval(mid+1, x, hull) < eval(mid, x, hull)) b = mid + 1;
        else e = mid;
    }
    return eval(b, x, hull);
}

//#####DAQUI PRA BAIXO EH O SUCESSO#####

vector<line> merge(vector<line> a, vector<line> b)
{
    if(a.size() < b.size()) swap(a, b);
    for(int i = 0; i < b.size(); i++)
        a.push_back(b[i]);
    sort(a.begin(), a.end(), [](line c, line d)
        { return c.m == d.m ? c.b < d.b : c.m > d.m; });
    b.clear();
    for(int i = 0; i < a.size(); i++)
        add(a[i].m, a[i].b, b);
    return b;
}

```

```

vector<vector<line>> groups;

void add(line l)
{
    vector<line> g = {l};
    while(!groups.empty() and groups.back().size() <= g.size())
    {
        g = merge(g, groups.back());
        groups.pop_back();
    }
    groups.push_back(g);
}

type query(int x)
{
    int ans = 0;
    for(int i = 0; i < groups.size(); i++)
        ans = Min(ans, binarySearch(x, groups[i]));
    return -ans;
}

int32_t main()
{
    int n, q;
    scanint(n);
    scanint(q);

    vector<line> cyc(n + 1);

    while(q--)
    {
        int t, T;
        scanint(t);
        scanint(T);
        if(t % 2 == 1)
        {
            int N, C;
            scanint(C);
            scanint(N);
            int b = - N * T + cyc[C].m * T + cyc[C].b;
            //cout << N << ' ' << b << '\n';
            add(line(-N, -b));
            cyc[C] = line(N, b);
        }
        else
            printint(query(T));
    }

    return 0;
}

```

## 6.7 Enclosing Circle R2

```

#include <cstdio>
#include <cmath>
int n;
double x[1005], y[1005], X, Y, d, e;
double dist(double a, double b) {
    return a*a + b*b;
}

```

```

}
int main() {
    scanf("%d", &n);
    for (int i = 0; i < n; i++) {
        scanf("%lf%lf", &x[i], &y[i]);
        X += x[i]; Y += y[i];
    }
    X /= n; Y /= n;
    double P = 0.1;
    for (int i = 0; i < 30000; i++) {
        int f = 0;
        d = dist(X - x[0], Y - y[0]);
        for (int j = 1; j < n; j++) {
            e = dist(X - x[j], Y - y[j]);
            if (d < e) { d = e; f = j; }
        }
        X += (x[f] - X)*P;
        Y += (y[f] - Y)*P;
        P *= 0.999;
    }
    printf("%.3lf %.3lf\n%.3lf", X, Y, sqrt(d));
}

```

## 6.8 Enclosing Circle R3

```

#include <stdio>
#include <cmath>
int n;
double x[105], y[105], z[105], X, Y, Z, d, e;
double dist(double a, double b, double c) {
    return a*a + b*b + c*c;
}
int main() {
    scanf("%d", &n);
    for (int i = 0; i < n; i++) {
        scanf("%lf%lf%lf", &x[i], &y[i], &z[i]);
        X += x[i];
        Y += y[i];
        Z += z[i];
    }
    X /= n; Y /= n; Z /= n;
    double P = 0.1;
    for (int i = 0; i < 70000; i++) {
        int f = 0;
        d = dist(X - x[0], Y - y[0], Z - z[0]);
        for (int j = 1; j < n; j++) {
            e = dist(X - x[j], Y - y[j], Z - z[j]);
            if (d < e) {
                d = e;
                f = j;
            }
        }
        X += (x[f] - X)*P;
        Y += (y[f] - Y)*P;
        Z += (z[f] - Z)*P;
        P *= 0.998;
    }
    printf("%.10lf %.10lf %.10lf", X, Y, Z);
}

```

## 6.9 Geometry Stan

```

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
    bool operator == (PT p) const {
        return (fabs(x-p.x) < EPS && (fabs(y-p.y) < EPS)); }
    bool operator < (PT p) const {
        if(fabs(x-p.x) > EPS) return x<p.x; return y<p.y; }
};

// dot(p,q) = length(p)*length(q)*cos(angle between p and q)
double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double dist(PT p, PT q) { return sqrt(dist2(p,q)); }
double mdist(PT p, PT q) { return fabs(p.x-q.x)+fabs(p.y-q.y); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {return os<<"("<<p.x<<"
    "<<p.y<<")";}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// returns angle aob in rad
double angle(PT a, PT o, PT b){
    return acos(dot(a-o,b-o)/sqrt(dot(a-o,a-o)*dot(b-o,b-o)));
}

// returns true if point r is on the left side of line pq
bool ccw(PT p, PT q, PT r) {
    return cross(p,q)+cross(q,r)+cross(r,p) > 0;
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
double a, double b, double c, double d){
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

```

```

}
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }
    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
    return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=d-c; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90
        (a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[
                i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {

```

```

    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) <
            EPS)
            return true;
    return false;
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}
// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}
// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}
double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}
// gravity center
PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

```

```

// tests whether or not a given polygon (in CW or CCW order) is simple
// segments do not intersect
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}
// compute two center's of circle given two points a and b
// and a radius r
pair<PT, PT> ComputeTwoCircleCenter(PT a, PT b, double R)
{
    if(dist(a, b) < EPS)
        return {a, b};

    PT middle = (a + b) / 2;
    PT v = RotateCCW90(middle - a);
    v = v / sqrt(dot(v, v));

    double l = 0, r = 100, escalar;

    for(int i = 0; i < 100; i++)
    {
        double mid = (l + r) / 2;
        PT u = v * mid + middle;
        if(dist(a, u) <= R + EPS)
            l = mid, escalar = mid;
        else
            r = mid;
    }

    PT c1 = v * escalar + middle;
    PT c2 = v * (-escalar) + middle;

    return {c1, c2};
}

```

## 6.10 Graham Scan

```

#include <bits/stdc++.h>
using namespace std;
#define ii pair<int, int>
#define fi first
#define se second

vector<ii> P;

ii operator-(ii a, ii b)
{
    return ii(a.fi - b.fi, a.se - b.se);
}

ii operator+(ii a, ii b)
{

```

```

    return ii(a.fi + b.fi, a.se + b.se);
}

int operator*(ii a, ii b)
{
    return a.fi * b.se - a.se * b.fi;
}

int dist(ii a, ii b)
{
    return (a.fi - b.fi) * (a.fi - b.fi) + (a.se - b.se) * (a.se - b.se);
}

bool cmp(ii a, ii b)
{
    int cross = (a - P[0]) * (b - P[0]);
    if(!cross) return dist(P[0], a) > dist(P[0], b);
    return cross > 0;
}

void setFirstPoint()
{
    for(int i = 1; i < P.size(); i++)
        if(P[i].fi < P[0].fi or P[i].fi == P[0].fi and P[i].se < P[0].se)
            swap(P[0], P[i]);
}

vector<ii> GrahamScan()
{
    setFirstPoint();
    sort(P.begin() + 1, P.end(), cmp);
    vector<ii> H(P.size() * 2);
    int k = 0;
    for(int i = 0; i < P.size(); i++)
    {
        //crsso <= 0 para remover os pontos colineares
        while(k > 2 and (H[k - 1] - H[k - 2]) * (P[i] - H[k - 1]) < 0)
            k--;
        H[k++] = P[i];
    }
    H.resize(k);
    return H;
}

int main()
{
    int n;
    cin >> n;
    for(int i = 0; i < n; i++)
    {
        ii p;
        cin >> p.fi >> p.se;
        P.push_back(p);
    }
    vector<ii> H = GrahamScan();
    for(int i = 0; i < H.size(); i++)
        cout << H[i].fi << ' ' << H[i].se << '\n';
    return 0;
}

```

## 6.11 Maximum Dot Product

```

#include <bits/stdc++.h>
using namespace std;
#define int long long
#define X first
#define Y second
const int OO = 0x3f3f3f3f3f3f3f3f;
typedef pair<int, int> ii;

int cross(ii O, ii A, ii B)
{
    return ((A.X - O.X) * (B.Y - O.Y)) - ((A.Y - O.Y) * (B.X - O.X));
}

int dot(ii a, ii b)
{
    return a.X * b.X + a.Y * b.Y;
}

vector<ii> ConvexHull(vector<ii> P)
{
    if(P.size() <= 1) return P;
    vector<ii> H(2*P.size());
    int k = 0;
    sort(P.begin(), P.end());
    for(int i = 0; i < P.size(); i++)
    {
        while(k >= 2 and cross(H[k-2], H[k-1], P[i]) <= 0) k--;
        H[k++] = P[i];
    }
    for(int i = P.size()-2, l = k + 1; i >= 0; i--)
    {
        while(k >= 1 and cross(H[k-2], H[k-1], P[i]) <= 0) k--;
        H[k++] = P[i];
    }
    H.resize(k-1);
    return H;
}

vector<ii> merge(vector<ii> H1, vector<ii> H2)
{
    for(auto &it : H2) H1.push_back(it);
    return ConvexHull(H1);
}

int maxConcavityUp(int b, int e, vector<ii> &H, ii p)
{
    if(b > e) return -OO;
    return max(dot(H[b], p), dot(H[e], p));
}

int maxConcavityDown(int b, int e, vector<ii> &H, ii p)
{
    if(b > e) return -OO;
    b--;
    while(e - b > 1)
    {
        int m = b + (e - b) / 2;

```

```

    if(dot(H[m], p) > dot(H[m + 1], p))
        e = m;
    else
        b = m;
}
return dot(H[e], p);
}

int maximumDot(vector<ii> &H, ii p)
{
    bool growing = dot(H[0], p) <= dot(H[1], p);
    if(growing)
    {
        int b = 0, e = H.size() - 1, w = -1;
        while(b <= e)
        {
            int m = (b + e) / 2;
            if(dot(H[0], p) <= dot(H[m], p))
                b = m + 1, w = m;
            else
                e = m - 1;
        }
        return max(maxConcavityUp(w, H.size() - 1, H, p),
            maxConcavityDown(0, w, H, p));
        //cout << "caso #1\n0 " << w << " concavidade para baixo\n"
        //      << w + 1 << ' ' << H.size() - 1 << " concavidade para
        //      cima\n";
    }
    else
    {
        int b = 0, e = H.size() - 1, w = -1;
        while(b <= e)
        {
            int m = (b + e) / 2;
            if(dot(H[0], p) >= dot(H[m], p))
                b = m + 1, w = m;
            else
                e = m - 1;
        }
        //cout << "caso #2\n0 " << w << " concavidade para cima\n"
        //      << w + 1 << ' ' << H.size() - 1 << " cocavidade para
        //      baixo\n";
        return max(maxConcavityUp(0, w, H, p),
            maxConcavityDown(w, H.size() - 1, H, p));
    }
}

vector<vector<ii>> st;

void add(ii p)
{
    vector<ii> g = {p};
    while(!st.empty() and st.back().size() <= g.size())
    {
        g = merge(g, st.back());
        st.pop_back();
    }
    st.push_back(g);
}

int query(ii p)

```

```

{
    int ans = -OO;
    for(int i = 0; i < st.size(); i++)
        ans = max(ans, maximumDot(st[i], p));
    return ans;
}

int32_t main()
{
    int n, q;
    scanf("%lld", &n);
    for(int i = 0; i < n; i++)
    {
        int x, y;
        scanf("%lld %lld", &x, &y);
        add({x, y});
    }
    scanf("%lld", &q);
    while(q--)
    {
        char s[10];
        int x, y;
        scanf("%s %lld %lld", s, &x, &y);
        if(s[0] == 'a') add({x, y});
        else printf("%lld\n", query({x, y}));
    }

    return 0;
}

```

## 6.12 Radial Sort

```

#include <bits/stdc++.h>
using namespace std;
#define type int
#define point pair<type, type>
#define X first
#define Y second

point operator-(point a, point b)
{
    return {a.X - b.X, a.Y - b.Y};
}

type operator*(point a, point b)
{
    return a.X * b.Y - a.Y * b.X;
}

int n;
vector<point> P;
point R;

int dist(point a, point b)
{
    return (a.X - b.X) * (a.X - b.X) + (a.Y - b.Y) * (a.Y - b.Y);
}

bool cmp(point a, point b)
{

```

```

    if((a - R).Y * (b - R).Y <= 0) return a.Y > R.Y;
    int c = (a - R) * (b - R);
    if(c == 0) return dist(R, a) <= dist(R, b);
    return c > 0;
}

int main()
{
    cin >> n >> R.X >> R.Y;
    for(int i = 0; i < n; i++)
    {
        type x, y;
        cin >> x >> y;
        P.push_back({x, y});
    }
    sort(P.begin(), P.end(), cmp);
    for(point p : P) cout << p.X << ' ' << p.Y << '\n';

    return 0;
}

```

## 6.13 Segment Intersection

```

#define ll long long
#define Pll complex<ll>
#define Y imag()
#define X real()

bool intersec(Pll a, Pll b, Pll c, Pll d)
{
    if(cross(c - a, c - b) * cross(d - a, d - b) <= 0 and
       cross(a - c, a - d) * cross(b - c, b - d) <= 0)
        return true;
    return false;
}

```

# 7 Miscellaneous

## 7.1 Big Num Product

```

string mul(string a, string b)
{
    while(a.size() > b.size()) b = "0" + b;
    while(a.size() < b.size()) a = "0" + a;
    a = "00" + a;
    b = "00" + b;
    int ans = 0, n = a.size(), carry = 0;
    vector<int> num(2 * n, 0);
    for(int i = n - 1; i >= 0; i--)
        for(int j = n - 1; j >= 0; j--)
        {
            int di = a[i] - '0';
            int dj = b[j] - '0';
            num[i + j + 1] += (di * dj) + carry;
            carry = (num[i + j + 1] / 10);
            num[i + j + 1] %= 10;
        }
}

```

```

string r;
for(int i = 0, fl = 0; i < 2 * n; i++)
{
    if(num[i]) fl = 1;
    if(fl) r.push_back(num[i] + '0');
}
return r;

```

## 7.2 Count Divisors

```

#include <bits/stdc++.h>
using namespace std;

long long add(long long a, long long b, long long c)
{
    long long ans = (a + b) % c;
    if(ans < 0) ans += c;
    return ans;
}

long long mulmod(long long a, long long b, long long c)
{
    long long ans = 0;
    while(b)
    {
        if(b & 1) ans = add(ans, a, c);
        a = add(a, a, c);
        b /= 2;
    }
    return ans;
}

long long fexp(long long a, long long b, long long c)
{
    long long ans = 1;
    while(b)
    {
        if(b & 1) ans = mulmod(ans, a, c);
        a = mulmod(a, a, c);
        b /= 2;
    }
    return ans;
}

bool miller(long long a, long long n)
{
    if (a >= n) return true;
    long long s = 0, d = n - 1;
    while(d%2 == 0 and d >>= 1, s++);
    long long x = fexp(a, d, n);
    if(x == 1 or x == n - 1) return true;
    for(int r = 0; r < s; r++, x = mulmod(x, x, n))
    {
        if (x == 1) return false;
        if (x == n-1) return true;
    }
    return false;
}

bool isprime(long long n)

```

```

{
    if(n < 2) return false;
    int base[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
    for(int i = 0; i < 12; i++)
        if(!miller(base[i], n))
            return false;
    return true;
}

vector<int> prime;
bitset<10000000> composite;

void sieve()
{
    for(int i = 2; i < 10000000; i++)
        if(!composite[i])
        {
            prime.push_back(i);
            for(int j = 2; i * j < 10000000; j++)
                composite[i * j] = 1;
        }
}

long long countDivisors(long long n)
{
    int idx = 1;
    long long ans = 1, p = prime[0];
    while(p * p * p <= n)
    {
        int cnt = 1;
        while(n % p == 0)
            n /= p, cnt++;
        ans *= cnt;
        p = prime[idx++];
    }
    if(n == 1) return ans;
    if(isprime(n)) ans *= 2;
    else
    {
        long long sq = sqrt(n);
        if(sq * sq == n)
            ans *= 3;
        else if(n != 1)
            ans *= 4;
    }
    return ans;
}

int main()
{
    long long n;
    cin >> n;
    sieve();
    cout << countDivisors(n) << '\n';

    return 0;
}

```

## 7.3 Count Sort

```

#include <bits/stdc++.h>
using namespace std;

int n, m;
int arr[100];
int cnt[10000];
int aux[100];

void count_sort()
{
    for(int i = 0; i < n; i++)
        cnt[arr[i]]++;
    for(int i = 1; i <= m; i++)
        cnt[i] += cnt[i-1];
    for(int i = 0; i < n; i++)
        aux[--cnt[arr[i]]] = arr[i];
    memcpy(arr, aux, n*sizeof(int));
}

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> arr[i], m = max(arr[i], m);

    count_sort();

    for(int i = 0; i < n; i++)
        cout << arr[i] << ' '; cout << '\n';

    return 0;
}

```

## 7.4 Counting Different Elements In A Path With Mo

```

//COT - Count on a tree (SPOJ)
//Em cada vertice existe um valor
//A resposta para uma query eh quantos valores
//distintos existem no caminho de u a v
#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e6;

typedef long long ll;

struct Query
{
    int x, y, l, r, lc, res;
};

int n, q, max_log, tempo, blk, ans;
vector<int> G[MAX];
int value[MAX], pos[MAX], anc[MAX][25], depth[MAX];
int tl[MAX], ST[MAX], EN[MAX], freq[MAX], node[MAX];
Query Q[MAX];
ll arr[MAX];

void dfs(int v, int p, int d)
{
    anc[v][0] = p;

```



```

depth[v] = d;
tl[tempo] = v;
ST[v] = tempo++;
if(d) max_log = max(max_log, (int)log2(d));
for(const int &u : G[v])
    if(u != p)
        dfs(u, v, d + 1);
tl[tempo] = v;
EN[v] = tempo++;
}

int walk(int v, int k)
{
    while(k) v = anc[v][(int)log2(k&-k)], k -= k&-k;
    return v;
}

int lca(int u, int v)
{
    if(depth[u] > depth[v]) u = walk(u, depth[u] - depth[v]);
    if(depth[u] < depth[v]) v = walk(v, depth[v] - depth[u]);
    if(u == v) return u;
    for(int i = max_log; i >= 0; i--)
        if(anc[u][i] != anc[v][i])
        {
            u = anc[u][i];
            v = anc[v][i];
        }
    return anc[u][0];
}

void build()
{
    memset(anc, -1, sizeof(anc));
    dfs(0, -1, 0);
    for(int j = 1; j <= max_log; j++)
        for(int i = 0; i < n; i++)
            if(anc[i][j-1] != -1)
                anc[i][j] = anc[anc[i][j-1]][j-1];
}

inline void mo(int i)
{
    int u = tl[i];
    if(node[u] and --freq[value[u]] == 0) ans--;
    else if(!node[u] and ++freq[value[u]] == 1) ans++;
    node[u] ^= 1;
}

bool compare(int a, int b)
{
    if(Q[a].l/blk != Q[b].l/blk)
        return Q[a].l < Q[b].l;
    return Q[a].r > Q[b].r;
}

int main()
{
    scanf("%d %d", &n, &q);
    for(int i = 0; i < n; i++) //values
        scanf("%d", &arr[i]), pos[i] = i;

```

```

sort(pos, pos + n, [](ll a, ll b){return arr[a] < arr[b]});
for(int i = 0, j = 1; i < n; i++)
    if(!i)
        value[pos[i]] = j++;
    else if(arr[pos[i]] != arr[pos[i-1]])
        value[pos[i]] = j++;
    else value[pos[i]] = value[pos[i-1]];

for(int i = 0; i < n-1; i++)
{
    int u, v;
    scanf("%d %d", &u, &v); u--; v--;
    G[u].push_back(v);
    G[v].push_back(u);
}
build();

for(int i = 0; i < q; i++)
{
    int u, v;
    scanf("%d %d", &u, &v); u--; v--;
    if(ST[u] > ST[v]) swap(u, v);
    Q[i].lc = lca(u, v);
    Q[i].x = u, Q[i].y = v;
    if(u == Q[i].lc)
        Q[i].l = ST[u], Q[i].r = ST[v];
    else
        Q[i].l = EN[u], Q[i].r = ST[v];
    pos[i] = i;
}

blk = sqrt(tempo);
sort(pos, pos + q, compare);

int curL = 0, curR = 0;
for(int i = 0; i < q; i++)
{
    int L = Q[pos[i]].l, R = Q[pos[i]].r;
    while(curL < L)
        mo(curL++);
    while(curL > L)
        mo(--curL);
    while(curR < R + 1)
        mo(curR++);
    while(curR > R + 1)
        mo(--curR);

    if(Q[pos[i]].x != Q[pos[i]].lc)
        mo(ST[Q[pos[i]].lc]);
    Q[pos[i]].res = ans;
    if(Q[pos[i]].x != Q[pos[i]].lc)
        mo(ST[Q[pos[i]].lc]);
}
for(int i = 0; i < q; i++)
    printf("%d\n", Q[i].res);

return 0;
}

```

## 7.5 Custom Hash Function Unordered Map Or Set

```
#include <bits/stdc++.h>
using namespace std;

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now()
            .time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};

//teste
const int N = 2e5;

void insert_numbers(long long x)
{
    clock_t begin = clock();
    unordered_map<long long, int, custom_hash> numbers;

    for (int i = 1; i <= N; i++)
        numbers[i * x] = i;

    long long sum = 0;

    for (auto &entry : numbers)
        sum += (entry.first / x) * entry.second;

    printf("x = %lld: %.3lf seconds, sum = %lld\n", x, (double) (clock()
        - begin) / CLOCKS_PER_SEC, sum);
}

int main()
{
    insert_numbers(107897);
    insert_numbers(126271);

    return 0;
}
```

## 7.6 Divide Conquer Optimization

```
#include <bits/stdc++.h>
using namespace std;

#define maxn 20005
#define maxnlog 22
const long long OO = 0x3f3f3f3f3f3f3f3f;
```

```
struct SparseTableDS
{
    int Sparse_Table[maxnlog][maxn];
    bool maxi;
    int n;

    void build()
    {
        for(int i = 1; (1 << i) <= n; i++)
            for(int j = 0; j + (1 << i) <= n; j++)
            {
                if(maxi)
                    Sparse_Table[i][j] = max(Sparse_Table[i-1][j],
                        Sparse_Table[i-1][j+(1 << (i-1))]);
                else
                    Sparse_Table[i][j] = min(Sparse_Table[i-1][j],
                        Sparse_Table[i-1][j+(1 << (i-1))]);
            }
    }

    int query(int i, int j)
    {
        int sz = log2(j-i+1);
        if(maxi)
            return max(Sparse_Table[sz][i], Sparse_Table[sz][j+1-(1 << sz)]);
        return min(Sparse_Table[sz][i], Sparse_Table[sz][j+1-(1 << sz)]);
    }

    void init(bool fl, vector<int> &arr)
    {
        maxi = fl;
        if(!maxi) memset(Sparse_Table, 63, sizeof(Sparse_Table));
        n = arr.size();
        for(int i = 0; i < n; i++)
            Sparse_Table[0][i] = arr[i];
        build();
    }
};

int n, k;
SparseTableDS maxi, mini;

long long dp_before[maxn];
long long dp_cur[maxn];

int get(int l, int r)
{
    int a = maxi.query(l, r);
    int b = mini.query(l, r);
    return abs(a - b);
}

void compute(int l, int r, int optl, int optr)
{
    if(l > r) return;

    int mid = (l + r) >> 1;
    int best = 0;
    int opt = optl;
```

```

for(int k = optl; k < min(mid, optr + 1); k++)
    if(best < dp_before[k] + get(k + 1, mid))
    {
        best = dp_before[k] + get(k + 1, mid);
        opt = k;
    }
dp_cur[mid] = best;

compute(l, mid - 1, optl, opt);
compute(mid + 1, r, opt, optr);
}

int32_t main()
{
    cin >> n >> k;
    vector<int> arr(n);
    for(int &w : arr) scanf("%d", &w);

    maxi.init(true, arr);
    mini.init(false, arr);

    for(int i = 0; i < n; i++)
        dp_cur[i] = get(0, i);

    for(int i = 2; i <= k; i++)
    {
        for(int j = 0; j < n; j++)
        {
            dp_before[j] = dp_cur[j];
            dp_cur[j] = 0;
        }
        compute(i - 2, n - 1, i - 2, n - 1);
    }

    cout << dp_cur[n - 1] << endl;

    return 0;
}

```

## 7.7 FastIO

```

#include <bits/stdc++.h>
using namespace std;

#define gc getchar_unlocked
#define pc putchar_unlocked

inline void scanint(int &k)
{
    bool sinal = true;
    register char c;
    k = 0;
    for(c = gc(); sinal and (c < '0' or c > '9'); c = gc())
        if(c == '-')
            sinal = false;
    for(; c >= '0' and c <= '9'; c = gc())
        k = (k << 3) + (k << 1) + c - '0';
    if(!sinal) k = -k;
}

```

```

inline void printint(int n)
{
    if(n < 0) pc('-');
    n = abs(n);
    int rev = n, cnt = 0;
    if(!n)
    {
        pc('0');
        pc('\n');
        return;
    }
    while(!(rev % 10))
        cnt++, rev /= 10;
    rev = 0;
    while(n)
        rev = (rev << 3) + (rev << 1) + n % 10, n /= 10;
    while(rev)
        pc(rev % 10 + '0'), rev /= 10;
    while(cnt--)
        pc('0');
    pc('\n');
}

inline void scanstr(string &k)
{
    register char c;
    k = "";
    for(c = gc(); c < 'a' or c > 'z'; c = gc());
    for(; c >= 'a' and c <= 'z'; c = gc()) k.push_back(c);
}

inline void printstr(string &k)
{
    for(char &c : k) putchar(c);
    putchar('\n');
}

int main()
{
    int k;
    scanint(k);
    printint(k);

    return 0;
}

```

## 7.8 Fence Problem With Max Flow

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e4;
const int OO = 0x3f3f3f3f;
int SOURCE, SINK;

struct edge
{
    int v, f, c;
    edge() {}
    edge(int _v, int _f, int _c)

```

```

    {
        v = _v, f = _f, c = _c;
    }
};

vector<edge> edges;
vector<int> G[MAX];
int dist[MAX], work[MAX];

void add_edge(int u, int v, int cp, int rc){
    edges.push_back(edge(v, 0, cp));
    G[u].push_back(edges.size()-1);
    edges.push_back(edge(u, 0, rc));
    G[v].push_back(edges.size()-1);
}

bool bfs(int s, int t)
{
    memset(dist, -1, sizeof(dist));
    dist[s] = 0;
    queue<int> q;
    q.push(s);
    while(!q.empty())
    {
        int u = q.front();
        q.pop();
        for(int e : G[u])
            if(dist[edges[e].v] == -1 and edges[e].c-edges[e].f > 0)
            {
                q.push(edges[e].v);
                dist[edges[e].v] = dist[u] + 1;
            }
        return dist[t] != -1;
    }
}

int dfs(int s, int t, int f)
{
    if(s == t) return f;
    for(int &i = work[s]; i < G[s].size(); i++)
    {
        int e = G[s][i];
        if(dist[edges[e].v] == dist[s] + 1 and edges[e].c-edges[e].f > 0)
            if(int a = dfs(edges[e].v, t, min(f, edges[e].c-edges[e].f)))
            {
                edges[e].f += a;
                edges[e^1].f -= a;
                return a;
            }
    }
    return 0;
}

int MaxFlow(int s, int t)
{
    int mf = 0;
    while(bfs(s, t))
    {
        memset(work, 0, sizeof(work));

```

```

        while(int a = dfs(s, t, 00))
            mf += a;
    }
    return mf;
}

int n, m, a, b;
int dx[] = {1, 0, -1, 0};
int dy[] = {0, -1, 0, 1};
char ANS[60][60];
bool cor[MAX];

bool check(int x, int y)
{
    return x >= 0 and x < n and y >= 0 and y < m;
}

int vertexIn(int i, int j)
{
    return i * m + j;
}

int vertexOut(int i, int j)
{
    return i * m + j + n * m + 1;
}

void mountANS(int v)
{
    cor[v] = true;
    for(int &e : G[v])
    {
        if(cor[edges[e].v]) continue;
        if(edges[e].c - edges[e].f > 0)
            mountANS(edges[e].v);
    }
}

int main()
{
    memset(ANS, '.', sizeof(ANS));
    cin >> n >> m >> a >> b; a--; b--;
    SOURCE = 2 * n * m + 2;
    SINK = 2 * n * m + 3;
    for(int i = 0; i < n; i++)
        for(int j = 0; j < m; j++)
        {
            int cost;
            cin >> cost;
            if(a == i and b == j) cost = 00;
            add_edge(vertexIn(i, j), vertexOut(i, j), cost, 0);
            if(cost == 00) add_edge(vertexOut(i, j), SINK, 00, 0);
            for(int k = 0; k < 4; k++)
            {
                int x = i + dx[k], y = j + dy[k];
                if(check(x, y))
                    add_edge(vertexOut(i, j),
                        vertexIn(x, y), 00, 0);
            }
            if(!i or !j or i == n - 1 or j == m - 1)
                add_edge(SOURCE, vertexIn(i, j), 00, 0);

```

```

    }
    cout << MaxFlow(SOURCE, SINK) << '\n';
    mountANS(SOURCE);
    for(int i = 0; i < n * m; i++)
        for(int &e : G[i])
            if(!(e & 1) and cor[i] and !cor[edges[e].v])
                ANS[i / m][i % m] = 'X';
    for(int i = 0; i < n; i++)
    {
        for(int j = 0; j < m; j++)
            cout << ANS[i][j];
        puts("");
    }
    return 0;
}

```

## 7.9 Gen Random Tree

```

#include <bits/stdc++.h>
using namespace std;
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

int rand(int a, int b) {
    return a + rng() % (b - a + 1);
}

int main() {
    int n = rand(4, 20);
    cout << n << endl;
    vector<pair<int,int>> edges;
    for(int i = 2; i <= n; i++)
        edges.emplace_back(rand(1, i - 1), i);

    // re-naming vertices
    vector<int> perm(n + 1); // re-naming vertices
    for(int i = 1; i <= n; ++i)
        perm[i] = i;

    // random order of labels
    shuffle(perm.begin() + 1, perm.end(), rng);
    // random order of edges
    shuffle(edges.begin(), edges.end(), rng);

    for(auto [u, v] : edges) {
        // random order of two vertices
        if(rng() % 2) swap(u, v);
        cout << perm[u] << ' ' << perm[v] << endl;
    }

    return 0;
}

```

## 7.10 Histogram

```

#include <bits/stdc++.h>
using namespace std;

typedef long long ll;

```

```

int n, vet[1000000];

ll histogram()
{
    stack<ll> s;
    ll ans = 0, tp, cur;
    int i = 0;
    while(i < n or !s.empty())
    {
        if(i < n and (s.empty() or vet[s.top()] <= vet[i]))
            s.push(i++);
        else
        {
            tp = s.top();
            s.pop();
            cur = vet[tp] * (s.empty() ? i : i - s.top() - 1);
            if(ans < cur)
                ans = cur;
        }
    }
    return ans;
}

int main()
{
    while(cin >> n and n)
    {
        for(int i = 0; i < n; i++)
            cin >> vet[i];
        cout << histogram() << '\n';
    }
    return 0;
}

```

## 7.11 Inclusion Exclusion

```

/*
    contar a quantidade de numeros na range [1, b]
    que sao multiplos de pelo menos um numero na range [1, a]
*/

#include <bits/stdc++.h>
using namespace std;
#define bug(x) cout << #x << " >>>>>> " << x << '\n'
#define _ << " , " <<
#define int long long
#define Max(a, b) (a > b ? a : b)
#define Min(a, b) (a < b ? a : b)
#define ii pair<int, int>
#define fi first
#define se second
#define SZ(v) (int)v.size()
#define UNTIL(t) while (clock() < (t) * CLOCKS_PER_SEC)
const long long MAX = (long long)1e15; //2 * 10^5
const int MOD = 1000000007; //10^9 + 7
const int OO = 0x3f3f3f3f; //3f3f3f3f;
const double EPS = 1e-9; //10^-9
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

```

```

vector<int> prime = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43,
    47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109,
    113, 127};
vector<vector<ii>> lista;

void add(int id)
{
    vector<ii> aux;
    for(int k = 0; k < id; k++)
    {
        int i = SZ(lista[k]) - 1;
        while(i >= 0 and MAX / lista[k][i].fi / prime[id] == 0) i--;
        for(int j = 0; j <= i; j++)
            aux.push_back({lista[k][j].fi * prime[id], !lista[k][j].se});
    }
    aux.push_back({prime[id], 1});
    sort(aux.begin(), aux.end());
    lista.push_back(aux);
}

int32_t main()
{
    for(int i = 0; i < SZ(prime); i++)
        add(i);

    int t;

    scanf("%lld", &t);

    while(t--)
    {
        int a, b;

        scanf("%lld %lld", &a, &b);

        int ans = b / 2;
        int cnt_p = 0;

        for(int &w : prime) cnt_p += (w <= a);

        for(int i = 0; i < cnt_p; i++)
        {
            for(int j = 0; j < SZ(lista[i]); j++)
            {
                if(lista[i][j].fi > b) break;
                if(lista[i][j].se) ans += (b / lista[i][j].fi + 1) / 2;
                else ans -= (b / lista[i][j].fi + 1) / 2;
            }
        }
        printf("%lld\n", ans);
    }

    return 0;
}

```

## 7.12 Index Compression

```

#include <bits/stdc++.h>
const int MAX = 1e6 + 10;
using namespace std;

```

```

int n, arr[MAX], pos[MAX], newArr[MAX], realValue[MAX];

int main()
{
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> arr[i], pos[i] = i;
    sort(pos, pos + n, [](int i, int j){ return arr[i] < arr[j]; });
    int id = 1;
    for(int i = 0; i < n; i++)
    {
        if(!i) newArr[pos[i]] = id, realValue[id] = arr[pos[i]];
        else if(arr[pos[i-1]] == arr[pos[i]]) newArr[pos[i]] = newArr[pos[i-1]];
        else newArr[pos[i]] = ++id, realValue[id] = arr[pos[i]];
    }
    for(int i = 0; i < n; i++)
        cout << arr[i] << ' ' << newArr[i] <<
            ' ' << realValue[newArr[i]] << '\n';

    return 0;
}

```

## 7.13 Karp Rabin

```

/*
Funcao de Hash:

Tome uma string S[0 ... n-1] e deois inteiros A e B

(S[0]*A^n-1 + S[1]*A^n-2 + S[2]*A^n-3 + S[3]*A^n-4
... S[N-1]*A^0) mod B
*/

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

typedef long long ll;

ll A = 911382323, B = 972663749;
ll h1[MAX], h2[MAX], p[MAX];
int arr[MAX];
string s;

ll buildH(ll *H, int k)
{
    if(k == 0)
        return H[0] = s[0];
    return H[k] = (buildH(H, k - 1)*A + s[k]) % B;
}

ll buildP(int k)
{
    if(k == 0)
        return p[0] = 1;
    return p[k] = (buildP(k - 1)*A) % B;
}

```

```

11 vhash(11 *H, int a, int b)
{
    if(a == 0)
        return H[b];
    11 ans = (H[b] - H[a - 1] * p[b - a + 1]) % B;
    if(ans < 0)
        ans += B;
    return ans;
}

bool slidingWindow(int k)
{
    if(k < 0 or k > s.size()) return false;
    for(int i = 0; i + k - 1 < s.size(); i++)
        if(vhash(h1, i, i + k - 1) == vhash(h2, s.size()
            - (i + k - 1) - 1, s.size() - (i + k - 1) - 2 + k))
            return true; // A substring [i, i + k - 1] eh palindromo
    return false;
}

int buscab()
{
    int tam = 0;
    for(int i = 0; i < s.size(); i++)
        arr[i] = 2*i + 1, tam++;
    int b = 0, e = tam, m, ans = 0;
    while(b <= e)
    {
        m = (b + e) / 2;
        slidingWindow(arr[m]) ? b = m + 1, ans = arr[m] : e = m - 1;
    }
    tam = 0;
    for(int i = 0; i < s.size(); i++)
        arr[i] = 2*i, tam++;
    b = 0, e = tam;
    while(b <= e)
    {
        m = (b + e) / 2;
        slidingWindow(arr[m]) ? b = m + 1,
            ans = max(arr[m], ans) : e = m - 1;
    }
    return ans;
}

int main()
{
    cin >> s;
    buildH(h1, s.size()-1);
    reverse(s.begin(), s.end());
    buildH(h2, s.size()-1);
    buildP(s.size()-1);
    reverse(s.begin(), s.end());
    cout << buscab() << '\n';

    return 0;
}

```

## 7.14 Knapsack With Backtraking

```

#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
#define fi first
#define se second
#define pb push_back

int n,c;
vector<pii> v;
int res,aux;
double c2,aux2;

void bt(int i){
    if(i == n) return;

    aux2 = 0; c2 = c;
    for(int j=i; j<n && c2; j++){
        if(v[j].fi <= c2){
            c2 -= v[j].fi; aux2 += v[j].se;
        } else {
            aux2 += (v[j].se*c2)/v[j].fi;
            c2 = 0;
        }
    }
    if(aux2 + aux <= res) return;

    if(v[i].fi <= c){
        c -= v[i].fi;
        aux += v[i].se;
        if(aux > res) res = aux;
        bt(i+1);
        aux -= v[i].se;
        c += v[i].fi;
    }
    bt(i+1);
}

int32_t main(){
    ios::sync_with_stdio(false);cin.tie(0);

    cin>>n>>c;
    for(int i = 0; i < n; i++)
    {
        int wei,value; cin>>wei>>value;
        v.pb({wei,value});
    }

    sort(v.begin(), v.end(), [](pii a, pii b){
        return (a.se+0.0)/a.fi > (b.se+0.0)/b.fi;
    });

    bt(0);
    cout<<res<<endl;

    return 0;
}

```

## 7.15 Knuth Optimization

```

#include <bits/stdc++.h>

```

## 7.16 Longest Substring That Is A Correct Bracket Sequence

```
using namespace std;

// Knuth Optimization

int pf[6000], n;
int dp[6000][6000];

int sum(int l, int r)
{
    return pf[r] - pf[l - 1];
}

int solve(int l, int r)
{
    if(l > r) return 0;
    if(dp[l][r] != -1) return dp[l][r];
    int ans = (l <= 30);
    for(int i = l; i <= r; i++)
        ans = min(ans, sum(l, r) + solve(l, i - 1) + solve(i + 1, r));
    return dp[l][r] = ans;
}

#define ii pair<int, int>
#define fi first
#define se second

ii DP[6000][6000];

//Point(l, r - 1) <= Point(l, r) <= Point(l + 1, r)
ii knuth(int l, int r)
{
    if(l == r) return {sum(l, r), l};
    if(DP[l][r] != ii(-1, -1)) return DP[l][r];
    int lef = knuth(l, r - 1).se;
    int rig = knuth(l + 1, r).se;
    int point = l, ans = (l <= 30);
    for(int i = lef; i <= rig; i++)
    {
        int cur = sum(l, r);
        if(i - 1 >= l) cur += knuth(l, i - 1).fi;
        if(i + 1 <= r) cur += knuth(i + 1, r).fi;
        if(cur < ans) ans = cur, point = i;
    }
    return DP[l][r] = {ans, point};
}

int main()
{
    memset(dp, -1, sizeof(dp));
    cin >> n;
    //for(int i = 1; i <= n; i++)
    //    cin >> pf[i], pf[i] += pf[i - 1];
    //cout << solve(1, n) << endl;

    memset(DP, -1, sizeof(DP));
    cout << knuth(1, n).fi << endl;

    return 0;
}
```

```
#include <bits/stdc++.h>
using namespace std;
#define OO 0x3f3f3f3f
#define gc getchar
#define pc putchar
#define offset 1000000

string str;
int Sparse_Table[22][1000002], n;
vector<int> forest[2000002];
int pf[1000002], cnt[1000002];

inline void build()
{
    for(int i = 1; (1 << i) < n; i++)
        for(int j = 0; j + (1 << i) < n; j++)
            Sparse_Table[i][j] = min(Sparse_Table[i-1][j],
                                     Sparse_Table[i-1][j+(1 << (i-1))]);
}

inline int range_query(int i, int j)
{
    int sz = log2(j-i+1);
    return min(Sparse_Table[sz][i], Sparse_Table[sz][j+1-(1 << sz)]);
}

inline int countNumberOfElementEqualToXinLR(int l, int r, int x)
{
    int p1 = lower_bound(forest[x + offset].begin(), forest[x + offset].
                        end(), l) - forest[x + offset].begin();
    int p2 = upper_bound(forest[x + offset].begin(), forest[x + offset].
                        end(), r) - forest[x + offset].begin() - 1;
    if(p1 > p2) return -1;
    return p2 - p1 + 1;
}

inline int nxt(int i, int x)
{
    int b = i, e = n - 1, ans = -1;
    while(b <= e)
    {
        int m = (b + e) >> 1;
        int v = range_query(i, m);
        if(v >= x) ans = m, b = m + 1;
        else e = m - 1;
    }
    return ans;
}

inline int queryIndex(int l, int r, int x)
{
    int b = l, e = r, ans = -1;
    while(b <= e)
    {
        int m = (b + e) >> 1;
        if(countNumberOfElementEqualToXinLR(m, r, x) > 0) ans = m, b = m +

```



```

        1;
    else e = m - 1;
}
return ans;
}

inline void scanstr(string &k)
{
    register char c;
    k = "";
    for(c = gc(); c != '(' and c != ')'; c = gc());
    for(; c >= '(' and c <= ')'; c = gc()) k.push_back(c);
}

int main()
{
    scanstr(str);
    n = str.size() + 1;
    for(int i = 1; i < n; i++)
    {
        pf[i] = pf[i - 1] + (str[i - 1] == '(' ? 1 : -1);
        Sparse_Table[0][i] = pf[i];
        forest[pf[i] + offset].push_back(i);
    }
    build();
    int ans = 0;
    cnt[0] = 1;
    for(int i = 1; i < n; i++)
    {
        if(str[i - 1] != '(') continue;
        int e = nxt(i, pf[i] - 1);
        if(e < i) continue;
        int p = queryIndex(i, e, pf[i] - 1);
        if(p < i) continue;
        int l = p - i + 1;
        cnt[l]++;
        if(l > ans) ans = l;
    }
    printf("%d %d\n", ans, cnt[ans]);

    return 0;
}

```

## 7.17 Maximum Subarray Xor

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e4 + 10;
const int OO = 0x3f3f3f3f;
const double EPS = 1e-9;

#define bug(x) cout << #x << " = " << x << '\n'
#define FOR(i, a, n) for(int i = a; i < n; i++)
#define REP(i, n) FOR(i, 0, n)
#define fi first
#define se second
#define pb push_back
#define mt make_tuple
#define all(vetor) vetor.begin(), vetor.end()

```

```

#define X real()
#define Y imag()
// #define gc getchar_unlocked

typedef long long ll;
typedef long double ld;
typedef pair<ll, ll> ii;
typedef pair<int, ii> iii;
typedef complex<ll> Pll;
typedef complex<ld> Pld;

struct TrieNode
{
    int value;
    TrieNode *children[2];
    TrieNode()
    {
        value = 0;
        children[0] = children[1] = nullptr;
    }
};

void insert(TrieNode *root, int n)
{
    TrieNode *aux = root;
    for(int i = 31; i >= 0; i--)
    {
        bool b = (n & (1 << i));
        if(aux->children[b] == nullptr)
            aux->children[b] = new TrieNode();
        aux = aux->children[b];
    }
    aux->value = n;
}

int query(TrieNode *root, int n)
{
    TrieNode *aux = root;
    for(int i = 31; i >= 0; i--)
    {
        bool b = (n & (1 << i));
        if(aux->children[1-b] != nullptr)
            aux = aux->children[1-b];
        else
            aux = aux->children[b];
    }
    return n ^ aux->value;
}

void maxSubArrayXor(int *arr, int n)
{
    TrieNode *root = new TrieNode();
    insert(root, 0);
    int px = 0, ans = INT_MIN, r = -1;
    for(int i = 0; i < n; i++)
    {
        px = px ^ arr[i];
        insert(root, px);
        int num = max(ans, query(root, px));
        if(num > ans)
            ans = num, r = i;
    }
}

```

```

    }
    int l = r, xo = 0;
    for(; l >= 0; l--)
    {
        xo ^= arr[l];
        if(xo == ans)
            break;
    }
    cout << "O Xor maximo eh: " << ans << '\n';
    while(l <= r)
        cout << arr[l++] << ' ';
    cout << '\n';
}

int main()
{
    int arr[MAX], n;

    cin >> n;
    REP(i, n) cin >> arr[i];
    maxSubArrayXor(arr, n);

    return 0;
}

```

## 7.18 Mo

```

//com hash no braco

#include <bits/stdc++.h>
using namespace std;
const int MAX = 3 * 1e5;

struct Query
{
    int l, r, qt;
    vector<int> el;
};

Query q[MAX];
int n, m, blk, ans;
int arr[MAX], freq[MAX], qtd[MAX], pos[MAX];
vector<vector<int>> tab;

void add(int i)
{
    if(freq[arr[i]])
        qtd[freq[arr[i]]]--;
    freq[arr[i]]++;
    qtd[freq[arr[i]]]++;
    if(ans <= freq[arr[i]])
    {
        ans = freq[arr[i]];
        tab[ans].push_back(arr[i]);
    }
}

void sub(int i)
{
    qtd[freq[arr[i]]]--;

```

```

    if(!qtd[ans])
    {
        for(int j = 0; j < tab[ans].size(); j++)
            if(tab[ans][j] == arr[i])
            {
                tab[ans].erase(tab[ans].begin() + j);
                break;
            }
        ans--;
        tab[ans].push_back(arr[i]);
    }
    freq[arr[i]]--;
    if(freq[arr[i]])
        qtd[freq[arr[i]]]++;
}

bool compare(int a, int b)
{
    if(q[a].l/blk != q[b].l/blk)
        return q[a].l < q[b].l;
    return q[a].r < q[b].r;
}

int main()
{
    while(cin >> n >> m and n)
    {
        blk = sqrt(n);
        for(int i = 0; i < n; i++)
        {
            cin >> arr[i];
            arr[i] += 100000;
        }
        for(int i = 0; i < m; i++)
            cin >> q[i].l >> q[i].r, pos[i] = i;

        sort(pos, pos + m, compare);

        ans = 0;
        int curL = 0, curR = 0;
        int L, R;

        memset(freq, 0, sizeof(freq));
        memset(qtd, 0, sizeof(qtd));
        tab.clear();
        tab.resize(MAX);

        for(int j = 0; j < m; j++)
        {
            L = q[pos[j]].l - 1;
            R = q[pos[j]].r - 1;
            while(curL < L)
                sub(curL++);
            while(curL > L)
                add(--curL);
            while(curR < R + 1)
                add(curR++);
            while(curR > R + 1)
                sub(--curR);
            q[pos[j]].qt = ans;
            q[pos[j]].el = tab[ans];

```

```

    }
    for(int j = 0; j < m; j++)
        for(int i = 0; i < q[j].el.size(); i++)
            cout << q[j].el[i]-100000 << '\n';
}

return 0;
}

////////////////////
//com unordered_multimap

#include <bits/stdc++.h>
using namespace std;
const int MAX = 3 * 1e5;

struct Query
{
    int l, r, qt, morefrequent;
};

Query q[MAX];
int n, m, blk, ans;
int arr[MAX], freq[MAX], qtd[MAX], pos[MAX];
unordered_multimap<int, int> tab;

void add(int i)
{
    if(freq[arr[i]])
        qtd[freq[arr[i]]]--;
    freq[arr[i]]++;
    qtd[freq[arr[i]]]++;
    if(ans <= freq[arr[i]])
    {
        ans = freq[arr[i]];
        tab.insert({ans, arr[i]});
    }
}

void sub(int i)
{
    qtd[freq[arr[i]]]--;
    if(!qtd[ans])
    {
        int k = 0, sz = tab.bucket(ans);
        auto it = tab.find(ans);
        for(int j = 0; j < sz; j++)
            if(it->second == arr[i])
            {
                tab.erase(it);
                j = sz;
            }
        else it++;

        ans--;
        tab.insert({ans, arr[i]});
    }
    freq[arr[i]]--;
    if(freq[arr[i]])
        qtd[freq[arr[i]]]++;
}

```

```

bool compare(int a, int b)
{
    if(q[a].l/blk != q[b].l/blk)
        return q[a].l < q[b].l;
    return q[a].r < q[b].r;
}

int main()
{
    while(cin >> n >> m and n)
    {
        blk = sqrt(n);
        for(int i = 0; i < n; i++)
        {
            cin >> arr[i];
            arr[i] += 100000;
        }
        for(int i = 0; i < m; i++)
            cin >> q[i].l >> q[i].r, pos[i] = i;

        sort(pos, pos + m, compare);

        ans = 0;
        int curL = 0, curR = 0;
        int L, R;

        memset(freq, 0, sizeof(freq));
        memset(qtd, 0, sizeof(qtd));
        tab.clear();

        for(int j = 0; j < m; j++)
        {
            L = q[pos[j]].l - 1;
            R = q[pos[j]].r - 1;
            while(curL < L)
                sub(curL++);
            while(curL > L)
                add(--curL);
            while(curR < R + 1)
                add(curR++);
            while(curR > R + 1)
                sub(--curR);
            q[pos[j]].qt = ans;
            q[pos[j]].morefrequent = tab.find(ans)->second;
        }
        for(int j = 0; j < m; j++)
            cout << q[j].morefrequent-100000 << '\n';
    }

    return 0;
}

////////////////////
Mo em arvore parte 1

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5 + 10;

struct Query

```

```

{
    int l, r, v;
};

int n, q;
int freq[MAX], inicio[MAX], fim[MAX], pos[MAX], value[MAX];
vector<int> tree_linearization, G[MAX];
Query Q[MAX];
int ans, blk;

void TreeLinearization(int v, int p)
{
    inicio[v] = tree_linearization.size();
    tree_linearization.push_back(v);
    for(const int &u : G[v])
        if(u != p)
            TreeLinearization(u, v);
    fim[v] = tree_linearization.size() - 1;
}

void add(int i)
{
    if(!freq[value[tree_linearization[i]]])
        ans++;
    freq[value[tree_linearization[i]]]++;
}

void sub(int i)
{
    freq[value[tree_linearization[i]]]--;
    if(!freq[value[tree_linearization[i]]])
        ans--;
}

bool compare(int a, int b)
{
    if(Q[a].l/blk != Q[b].l/blk)
        return Q[a].l/blk < Q[b].l/blk;
    return Q[a].r < Q[b].r;
}

int main()
{
    int u, v;
    cin >> n >> q;
    for(int i = 0; i < n; i++)
        cin >> value[i];
    for(int i = 0; i < n-1; i++)
    {
        cin >> u >> v;
        u--; v--;
        G[u].push_back(v);
        G[v].push_back(u);
    }
    TreeLinearization(0, -1);
    for(int i = 0; i < q; i++)
    {
        cin >> u;
        u--;
        Q[i].l = inicio[u];
        Q[i].r = fim[u];
    }

```

```

        pos[i] = i;
    }
    blk = sqrt(n);
    sort(pos, pos+q, compare);
    int curL = 0, curR = 0;
    for(int i = 0; i < q; i++)
    {
        int L = Q[pos[i]].l, R = Q[pos[i]].r;
        while(curL < L)
            sub(curL++);
        while(curL > L)
            add(--curL);
        while(curR < R + 1)
            add(curR++);
        while(curR > R + 1)
            sub(--curR);
        Q[pos[i]].v = ans;
    }
    for(int i = 0; i < q; i++)
        cout << Q[i].v << '\n';

    return 0;
}

```

## 7.19 Odd Rectangles Area

*/\*  
You are given several axis-aligned rectangles. Compute the sum  
of the area of the regions that are covered by an odd number of  
rectangles.  
  
input: The first line of input contains a single integer n (1<=n  
<=10^5), representing  
the number of rectangles. Each of the next n lines contains four space  
-separated  
integers x1, y1, x2, and y2, each between 0 and 109, describing the  
coordinates of  
a rectangle.*

*Output: Print, on one line, the total area covered by an odd number of  
rectangles  
as an exact integer.*

*\*/*

```

#include <bits/stdc++.h>
using namespace std;

struct Event
{
    int x1, x2, y, t;
    Event(int _x1, int _x2, int _y, int _t)
    {
        x1 = _x1, x2 = _x2, y = _y, t = _t;
    }
    Event() {}
};

struct Node
{

```

```

    int l, r, value;
};

int n;
vector<Node> tree;
vector<int> lazy;
vector<Event> arr;

int init()
{
    tree.clear();
    lazy.clear();
    tree.emplace_back();
    lazy.push_back(0);
}

void createL(int node)
{
    tree[node].l = tree.size();
    tree.emplace_back();
    lazy.push_back(0);
}

void createR(int node)
{
    tree[node].r = tree.size();
    tree.emplace_back();
    lazy.push_back(0);
}

void calc(int node)
{
    tree[node].value = 0;
    if(tree[node].l) tree[node].value += tree[tree[node].l].value;
    if(tree[node].r) tree[node].value += tree[tree[node].r].value;
}

void push(int node, int start, int end)
{
    tree[node].value = (end - start + 1) - tree[node].value;
    if(start != end)
    {
        if(tree[node].l == 0) createL(node);
        if(tree[node].r == 0) createR(node);
        lazy[tree[node].l] ^= 1;
        lazy[tree[node].r] ^= 1;
    }
    lazy[node] = 0;
}

void update(int node, int start, int end, int l, int r)
{
    if(lazy[node])
        push(node, start, end);
    if(start > r or l > end) return;
    if(l <= start and end <= r)
    {
        push(node, start, end);
    }
    else
    {

```

```

        int mid = (start + end) / 2;
        if(tree[node].l == 0) createL(node);
        update(tree[node].l, start, mid, l, r);
        if(tree[node].r == 0) createR(node);
        update(tree[node].r, mid + 1, end, l, r);
        calc(node);
    }
}

int query(int node, int start, int end, int l, int r)
{
    if(lazy[node])
        push(node, start, end);

    if(start > r or l > end) return 0;

    if(l <= start and end <= r) return tree[node].value;

    int mid = (start + end) / 2, q1 = 0, q2 = 0;
    if(tree[node].l) q1 = query(tree[node].l, start, mid, l, r);
    if(tree[node].r) q2 = query(tree[node].r, mid + 1, end, l, r);
    return q1 + q2;
}

bool cmp(Event a, Event b)
{
    if(a.y != b.y) return a.y < b.y;
    return a.t < b.t;
}

int32_t main()
{
    scanf("%d", &n);
    for(int i = 0; i < n; i++)
    {
        int x1, y1, x2, y2;
        scanf("%d %d %d %d", &x1, &y1, &x2, &y2);
        int xmi = min(x1, x2);
        int ymi = min(y1, y2);
        int xma = max(x1, x2);
        int yma = max(y1, y2);
        xma--;
        //yma--;
        arr.emplace_back(xmi, xma, ymi, -1);
        arr.emplace_back(xmi, xma, yma, 1);
    }
    sort(arr.begin(), arr.end(), cmp);
    long long Y = 0, ans = 0;
    init();
    for(int i = 0; i < arr.size(); i++)
    {
        //cout << "op " << arr[i].t << '\n';
        long long s = query(0, 0, 1000000008, 0, 1000000007);
        long long aux = s * 1LL * (arr[i].y - Y);
        //cout << "this " << aux << ' ' << arr[i].y1 << ' ' <<
            Y << ' ' << s << '\n';
        update(0, 0, 1000000008, arr[i].x1, arr[i].x2);
        Y = arr[i].y;
        ans += aux;
    }
    cout << ans << '\n';
}

```

```

    return 0;
}

```

## 7.20 Quick Sort And Select

```

#include <bits/stdc++.h>
using namespace std;

int n, arr[10000];

int quickselect(int l, int r, int k)
{
    int j = l - 1;
    for(int i = l; i < r; i++)
        if(arr[i] <= arr[r])
            swap(arr[++j], arr[i]);
    swap(arr[j+1], arr[r]);
    if(j+1 < k) return quickselect(j+2, r, k);
    else if(j+1 > k) return quickselect(l, j, k);
    return arr[j+1];
}

void quicksort(int l, int r)
{
    int j = l - 1;
    for(int i = l; i < r; i++)
        if(arr[i] <= arr[r])
            swap(arr[++j], arr[i]);
    swap(arr[j+1], arr[r]);
    if(l < j)
        quicksort(l, j);
    if(j+2 < r)
        quicksort(j+2, r);
}

int main()
{
    int k;
    cin >> n >> k;
    for(int i = 0; i < n; i++) cin >> arr[i];
    cout << quickselect(0, n-1, k-1) << '\n';

    return 0;
}

```

## 7.21 Rectangles Union Area

```

#include <bits/stdc++.h>
using namespace std;
#define ii pair<int, int>
#define fi first
#define se second

struct Event
{
    int x1, x2, y, t;
    Event(int _x1, int _x2, int _y, int _t)

```

```

    {
        x1 = _x1, x2 = _x2, y = _y, t = _t;
    }
    Event(){}
};

ii tree[500800];
int lazy[500800];

int n;
vector<pair<ii, ii>> segments, rect;
int X1, Y1, X2, Y2, P;

ii calc(ii a, ii b)
{
    if(a.fi > b.fi) return b;
    else if(a.fi < b.fi) return a;
    return {a.fi, a.se + b.se};
}

void build(int node, int start, int end)
{
    if(start == end)
        tree[node] = {0, 1}, lazy[node] = 0;
    else
    {
        int mid = (start + end) / 2;
        build(2 * node, start, mid);
        build(2 * node + 1, mid + 1, end);
        tree[node] = calc(tree[2 * node], tree[2 * node + 1]);
        lazy[node] = 0;
    }
}

void push(int node, int start, int end)
{
    tree[node].fi += lazy[node];
    if(start != end)
    {
        lazy[2 * node] += lazy[node];
        lazy[2 * node + 1] += lazy[node];
    }
    lazy[node] = 0;
}

void update(int node, int start, int end, int l, int r, int v)
{
    if(lazy[node]) push(node, start, end);
    if(start > r or end < l) return;
    if(l <= start and end <= r)
    {
        lazy[node] += v;
        push(node, start, end);
        return;
    }
    int mid = (start + end) / 2;
    update(2 * node, start, mid, l, r, v);
    update(2 * node + 1, mid + 1, end, l, r, v);
    tree[node] = calc(tree[2 * node], tree[2 * node + 1]);
}

```

```

int query(int node, int start, int end)
{
    if(lazy[node]) push(node, start, end);
    return end - start + 1 - tree[node].se;
}

void mount(int r)
{
    rect.clear();
    for(auto &it : segments)
    {
        int x1 = max(min(it.fi.fi, it.se.fi) - r, X1);
        int y1 = max(min(it.fi.se, it.se.se) - r, Y1);
        int x2 = min(max(it.fi.fi, it.se.fi) + r, X2);
        int y2 = min(max(it.fi.se, it.se.se) + r, Y2);
        x2--;
        rect.push_back({x1, y1}, {x2, y2});
    }
}

bool cmp(Event a, Event b)
{
    if(a.y != b.y) return a.y < b.y;
    return a.t > b.t;
}

long long area(int r)
{
    mount(r);
    vector<Event> eve;
    for(auto &it : rect)
    {
        eve.emplace_back(it.fi.fi, it.se.fi, it.fi.se, 1);
        eve.emplace_back(it.fi.fi, it.se.fi, it.se.se, -1);
    }
    sort(eve.begin(), eve.end(), cmp);
    build(1, 0, 100001);
    long long Y = 0, ans = 0;
    for(int i = 0; i < eve.size(); i++)
    {
        long long s = query(1, 0, 100001);
        long long aux = s * 1LL * (eve[i].y - Y);
        update(1, 0, 100001, eve[i].x1, eve[i].x2, eve[i].t);
        Y = eve[i].y;
        ans += aux;
    }
    return ans;
}

int32_t main()
{
    scanf("%d", &n);
    for(int i = 0; i < n; i++)
    {
        int x1, y1, x2, y2;
        scanf("%d %d %d %d", &x1, &y1, &x2, &y2);
        segments.push_back({x1, y1}, {x2, y2});
    }
    scanf("%d %d %d %d %d", &P, &X1, &Y1, &X2, &Y2);

    long long tot = (X2 - X1) * 1LL * (Y2 - Y1);

```

```

int b = 0, e = 100000, ans = 0;
while(b <= e)
{
    int mid = (b + e) / 2;
    long long A = area(mid);
    if(P * 1LL * tot <= 100LL * A) ans = mid, e = mid - 1;
    else b = mid + 1;
}
cout << ans << '\n';

return 0;
}

```

## 7.22 Small To Large

<https://codeforces.com/blog/entry/44351>

*// Small To Large (using map)  
 // Given a tree, every vertex has color. Query is  
 // how many vertices in subtree of vertex v are  
 // colored with color c?  
 // O(N\*logN\*logN), (we are using map)*

```

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e4 + 10;

vector<int> g[MAX];
int sz[MAX], col[MAX];
map<int, int> *cnt[MAX];

void getsz(int v, int p)
{
    sz[v] = 1;
    for(auto u : g[v])
        if(u != p) {
            getsz(u, v); sz[v] += sz[u]; } }

void dfs(int v, int p)
{
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p)
        {
            dfs(u, v);
            if(sz[u] > mx)
                mx = sz[u], bigChild = u;
        }
    if(bigChild != -1)
        cnt[v] = cnt[bigChild];
    else
        cnt[v] = new map<int, int> ();
    (*cnt[v])[col[v]]++;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            for(auto x : *cnt[u])
                (*cnt[v])[x.first] += x.second;
    //now (*cnt[v])[c] is the number of vertices in

```

```

        //subtree of vertex v that has color c. You can
        //answer the queries easily.
    }

int32_t main()
{
    int n, m;

    cin >> n >> m;
    for(int i = 0; i < n; i++)
        cin >> col[i];
    for(int i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    getsz(0, -1);
    dfs(0, -1);

    return 0;
}

////////////////////////////////////

// dsu on tree (using vector)
// Given a tree, every vertex has color. Query is
//how many vertices in subtree of vertex v are
// colored with color c?
// O(N*logN)

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e4 + 10;

vector<int> g[MAX];
int sz[MAX], col[MAX];
vector<int> *vec[MAX];
int cnt[MAX];

void getsz(int v, int p)
{
    sz[v] = 1;
    for(auto u : g[v])
        if(u != p){
            getsz(u, v); sz[v] += sz[u]; } }

void dfs(int v, int p, bool keep)
{
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p && sz[u] > mx)
            mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            dfs(u, v, 0);

```

```

        if(bigChild != -1)
            dfs(bigChild, v, 1), vec[v] = vec[bigChild];
        else
            vec[v] = new vector<int> ();
        vec[v]->push_back(v);
        cnt[ col[v] ]++;
        for(auto u : g[v])
            if(u != p && u != bigChild)
                for(auto x : *vec[u]){
                    cnt[ col[x] ]++;
                    vec[v] -> push_back(x);
                }
        //now (*cnt[v])[c] is the number of vertices in subtree
        //of vertex v that has color c. You can answer the queries
        //easily. note that in this step *vec[v] contains all of
        //the subtree of vertex v.
        if(keep == 0)
            for(auto u : *vec[v])
                cnt[ col[u] ]--;
    }

int32_t main()
{
    int n, m;

    cin >> n >> m;
    for(int i = 0; i < n; i++)
        cin >> col[i];
    for(int i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    getsz(0, -1);
    dfs(0, -1, 0);

    return 0;
}

// Small To Large (heavy-light decomposition style)
// Given a tree, every vertex has color. Query is
//how many vertices in subtree of vertex v are
// colored with color c?
// O(N*logN)

#include <bits/stdc++.h>
using namespace std;

const int MAX = 1e4 + 10;

vector<int> g[MAX];
int sz[MAX], col[MAX];
bool big[MAX];
int cnt[MAX];

void getsz(int v, int p)
{
    sz[v] = 1;
    for(auto u : g[v])

```



```

    if(u != p){
        getsz(u, v); sz[v] += sz[u]; } }

void add(int v, int p, int x)
{
    cnt[ col[v] ] += x;
    for(auto u: g[v])
        if(u != p && !big[u])
            add(u, v, x);
}

void dfs(int v, int p, bool keep)
{
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p && sz[u] > mx)
            mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            // run a dfs on small childs and clear them from cnt
            dfs(u, v, 0);
    if(bigChild != -1)
        // bigChild marked as big and not cleared from cnt
        dfs(bigChild, v, 1), big[bigChild] = 1;
    add(v, p, 1);
    //now cnt[c] is the number of vertices in subtree of
    //vertex v that has color c. You can answer the queries easily.
    if(bigChild != -1)
        big[bigChild] = 0;
    if(keep == 0)
        add(v, p, -1);
}

int32_t main()
{
    int n, m;

    cin >> n >> m;
    for(int i = 0; i < n; i++)
        cin >> col[i];
    for(int i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    getsz(0, -1);
    dfs(0, -1, 0);

    return 0;
}

// Small To Large (using nesting intervals)
// Given a tree, every vertex has color. Query is
//how many vertices in subtree of vertex v are
// colored with color c?
// O(N*logN)

#include <bits/stdc++.h>
using namespace std;

```

```

const int MAX = 1e4 + 10;

vector<int> g[MAX];
int sz[MAX], col[MAX];
int st[MAX], ft[MAX];
int cnt[MAX], ver[MAX];
int tempo = 0;

void getsz(int v, int p)
{
    sz[v] = 1;
    ver[tempo] = v;
    st[v] = tempo++;
    for(auto u : g[v])
        if(u != p){
            getsz(u, v); sz[v] += sz[u]; }
    ft[v] = tempo++;
}

void dfs(int v, int p, bool keep)
{
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
        if(u != p && sz[u] > mx)
            mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            // run a dfs on small childs and
            // clear them from cnt
            dfs(u, v, 0);
    if(bigChild != -1)
        // bigChild marked as big and not cleared from cnt
        dfs(bigChild, v, 1);
    for(auto u : g[v])
        if(u != p && u != bigChild)
            for(int p = st[u]; p < ft[u]; p++)
                cnt[ col[ ver[p] ] ]++;
    cnt[ col[v] ]++;
    //now cnt[c] is the number of vertices in subtree of vertex
    //v that has color c. You can answer the queries easily.
    if(v == 1) cout << cnt[2] << '\n';
    if(keep == 0)
        for(int p = st[v]; p < ft[v]; p++)
            cnt[ col[ ver[p] ] ]--;
}

int32_t main()
{
    int n, m;

    cin >> n >> m;
    for(int i = 0; i < n; i++)
        cin >> col[i];
    for(int i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v; u--; v--;
        g[u].push_back(v);
        g[v].push_back(u);
    }
}

```

```

    getsz(0, -1);
    dfs(0, -1, 0);

    return 0;
}

/*
But why it is ? You know that why dsu has time
(for q queries); the code uses the same method.
Merge smaller to greater.

If you have heard heavy-light decomposition you
will see that function add will go light edges
only, because of this, code works in time.

*/

```

## 7.23 Square Root Decomposition

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5;

int n, blk_sz;
int arr[MAX], block[MAX];

void update(int idx, int val)
{
    int blockNumber = idx/blk_sz;
    block[blockNumber] += val - arr[idx];
    arr[idx] = val;
}

int query(int l, int r)
{
    int sum = 0;
    while(l < r and l%blk_sz != 0 and l != 0)
        sum += arr[l], l++;
    while(l+blk_sz <= r)
        sum += block[l/blk_sz], l += blk_sz;
    while(l <= r)
        sum += arr[l], l++;
    return sum;
}

void build()
{
    int blk_idx = -1;
    blk_sz = sqrt(n);
    for(int i = 0; i < n; i++)
    {
        if(i%blk_sz == 0)
            blk_idx++;
        block[blk_idx] += arr[i];
    }
}

int main()
{
    cin >> n;

```

```

    for(int i = 0; i < n; i++)
        cin >> arr[i];
    build();
    int o, l, r;
    while(cin >> o >> l >> r and o)
        if(o == 1)
            update(l-1, r);
        else
            cout << query(l-1, r-1) << '\n';

    return 0;
}

```

## 7.24 String Matching Hash Sqrtdecomp

```

#include <bits/stdc++.h>
using namespace std;
const int MAX = 1e5 + 100;

typedef long long ll;

#define ii pair<int, long long>
#define fi first
#define se second

ll A = 911382323, B = 972663749;
ll h[MAX], p[MAX];
string s;

ll buildP(int k)
{
    if(k == 0)
        return p[0] = 1;
    return p[k] = (buildP(k - 1)*A) % B;
}

int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);

    cin >> s;
    buildP(s.size() + 10);

    int n = s.size();
    int sz = sqrt(n);

    unordered_set<ll> AAAA;
    vector<ii> BBBB;

    string aux;

    while(cin >> aux)
    {
        long long value = 0, j = 0;
        while(j < aux.size())
        {
            if(j == 0) value = aux[j];
            else value = (value * A + aux[j]) % B;
            j++;

```

```

    }
    if(aux.size() > sz)
        BBBB.push_back({aux.size(), value});
    else
        AAAA.insert(value);
}
sort(BBBB.begin(), BBBB.end());

string ans;

int j = 0, i = 0;

while(i < s.size())
{
    ans.push_back(s[i]);

    if(j == 0) h[j] = s[i];
    else h[j] = (h[j - 1] * A + s[i]) % B;

    int leng = -1;
    long long vh;

    for(int k = j; k >= 0 and k >= j - sz - 1 and leng < 0; k--)
    {
        if(k == 0) vh = h[j];
        else
        {
            vh = (h[j] - h[k - 1] * p[j - k + 1]) % B;
            if(vh < 0) vh += B;
        }
        if(AAAA.count(vh))
            leng = j - k + 1;
        //length j - k + 1
    }

    if(leng == -1)
    {
        for(int k = 0; k < BBBB.size(); k++)
        {
            int a = j - BBBB[k].first + 1;

            if(a < 0) break;

            if(a == 0) vh = h[j];
            else
            {
                vh = (h[j] - h[a - 1] * p[j - a + 1]) % B;
                if(vh < 0) vh += B;
            }

            if(vh == BBBB[k].se)
                leng = BBBB[k].fi;
        }
    }

    if(leng != -1)
    {
        j -= leng;
        while(leng--> 0) ans.pop_back();
    }

    ++j;
}

```

```

        i++;
    }
    cout << ans << '\n';

    return 0;
}

```

## 8 Useful Scripts

### 8.1 Brute

```

#include <bits/stdc++.h>
using namespace std;
// implement the brute force here

int32_t main() {

    return 0;
}

```

### 8.2 Gen Tree2

```

// generating a tree in a not-so-stupid way (from Errichto)
#include <bits/stdc++.h>
using namespace std;

int rand(int a, int b) {
    return a + rand() % (b - a + 1);
}

int main(int argc, char* argv[]) {
    srand(atoi(argv[1]));
    int n = rand(2, 20);
    cout << n << '\n';
    vector<pair<int, int>> edges;
    for(int i = 2; i <= n; ++i)
        edges.emplace_back(rand(1, i - 1), i);

    vector<int> perm(n + 1); // re-naming vertices
    for(int i = 1; i <= n; ++i)
        perm[i] = i;

    random_shuffle(perm.begin() + 1, perm.end());

    random_shuffle(edges.begin(), edges.end()); // random order of
        edges

    for(pair<int, int> edge : edges) {
        int a = edge.first, b = edge.second;
        if(rand() % 2)
            swap(a, b); // random order of two vertices
        cout << perm[a] << ' ' << perm[b] << '\n';
    }

    return 0;
}

```

## 8.3 Gen Tree

```
// generating a tree in a simple way (from Errichto)
#include <bits/stdc++.h>
using namespace std;

int rand(int a, int b) {
    return a + rand() % (b - a + 1);
}

int main(int argc, char* argv[]) {
    srand(atoi(argv[1]));
    int n = rand(2, 20);
    cout << n << '\n';
    for(int i = 2; i <= n; ++i)
        cout << rand(1, i - 1) << ' ' << i << '\n';
    return 0;
}
```

## 8.4 Gen

```
// simple matrix generator

#include <bits/stdc++.h>
using namespace std;

int rand(int a, int b) {
    return a + rand() % (b - a + 1);
}

int main(int argc, char* argv[]) {
    // atoi(s) converts an array of chars to int
    srand(atoi(argv[1]));

    int n = rand(1, 3), m = rand(1, 3);

    cout << n << ' ' << m << endl;
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < m; j++) {
            cout << rand(1, 5) << ' ';
        }
        cout << endl;
    }

    return 0;
}
```

## 8.5 Makefile

```
CXX = g++
CXXFLAGS = -fsanitize=address,undefined -fno-omit-frame-pointer -g -
Wall -Wshadow -std=c++17 -Wno-unused-result -Wno-sign-compare -Wno-
-char-subscripts #-fuse-ld=gold
```

## 8.6 Sol

```
#include <bits/stdc++.h>
using namespace std;
// implement your solution here

int32_t main() {

    return 0;
}
```

## 8.7 Stress Test Animated

```
make sol brute gen

cmatrix &

for ((i = 1; ; i++)) do
    ./gen $i > in
    ./sol < in > out
    ./brute < in > out2
    if (! cmp -s out out2) then
        kill -9 `pgrep -u matheus cmatrix`
        reset
        clear
        echo "--> entrada:"
        cat in
        echo "--> saida sol"
        cat out
        echo "--> saida2 brute"
        cat out2
        break;
    fi
    echo $i

done
```

## 8.8 Stress With Checker

```
#!/bin/bash

make sol brute gen check

for ((i = 1; ; i++)) do
    ./gen $i > in
    ./sol < in > out
    ./brute < in > out2
    ./check > log

    expected=correct
    result=$(  
check.txt)

    if [ "$expected" != "$result" ]; then
        echo "--> entrada:"
```

```
        cat in
        echo "--> saida sol"
        cat out
        echo "--> saida2 brute"
        cat out2
        break;
    fi
    echo $i
done
```

---

## 8.9 Stress

```
make sol brute gen

for ((i = 1; ; i++)) do
```

```
    ./gen $i > in
    ./sol < in > out
    ./brute < in > out2
    if (! cmp -s out out2) then
        echo "--> entrada:"
        cat in
        echo "--> saida sol"
        cat out
        echo "--> saida2 brute"
        cat out2
        break;
    fi
    echo $i
done
```

---