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tarefa BASICA: matriz inversa

(1) $A = B^{-1}$, $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$, $x+y = ?$

$*$ $A \cdot A^{-1} = I_n$

↳ $B \cdot A = I_n$

$\begin{cases} 3x - 5 = 1 & \text{(I)} \\ xy + 10 = 0 & \text{(II)} \end{cases}$

$$\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(I)
$3x - 5 = 1$
$3x = 1 + 5$
$x = \frac{6}{3} = 2$

(II)
$xy + 10 = 0$
$2y = -10$
$y = \frac{-10}{2} = -5$

$x + y = 2 + (-5)$
$= 2 - 5 = -3$

↳ NE (C)

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(2) $k = ?$, $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$ — não tem inversa (det = 0)

$1 + 3k + 0 = 3k + 1$

$\det A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix}$

$3 + 0 + k^2 = k^2 + 3$

$= k^2 + 3 - (3k + 1)$

$= k^2 + 3 - 3k - 1$

$= k^2 - 3k + 2 = 0$

$\Delta = 9 - 4 \cdot 1 \cdot 2$

$\Delta = 9 - 8 = 1$

$k_1 = \frac{3 + \sqrt{1}}{2 \cdot 1} = \frac{3+1}{2} = \frac{4}{2} = 2$

$k_{11} = \frac{3 - \sqrt{1}}{2 \cdot 1}$

$k_{k} = \frac{3-1}{2} = \frac{2}{2} = 1$

Alt (c)

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(3) $B = A^{-1}$, $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ ∴ $B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2$

* Regra matriz de ordem 2 $\rightarrow$ $B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$ Alt (C)

$\det A = 12 - 10 = 2$

(4) * Matriz é inversível ($\det \neq 0$)

$20 + 2x + 3x = 20 + 5x$

$$\left[\begin{array}{ccc|cc} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{array}\right]$$

$x^2 + 20 + 6 = x^2 + 26$

$\det = x^2 + 26 - (20 + 5x)$

$x^2 + 26 - 20 - 5x \neq 0$

$x^2 - 5x + 6 \neq 0$

$\Delta = 25 - 4 \cdot 1 \cdot 6$

$\Delta = 25 - 24 = 1$

$x_I \neq \dfrac{5 + \sqrt{1}}{2 \cdot 1} \neq \dfrac{5+1}{2} \neq \dfrac{6}{2} = 3$

$x_{II} \neq \dfrac{5 - \sqrt{1}}{2 \cdot 1} \neq \dfrac{5-1}{2} \neq \dfrac{4}{2} = 2$

$\rightarrow \{x \neq 3 \text{ e } x \neq 2\}$

Alt (A)

(5) $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$, $A + A^{-1} = ?$

$\det A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{matrix} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{matrix} = 7 - 6 = 1$

$2 + 2 + 2 = 6$ $\qquad$ $1 + 2 + 4 = 7$

$A = \begin{bmatrix} -1_{11} & -1_{12} & 2_{13} \\ 2_{21} & 1_{22} & -2_{23} \\ 1_{31} & 1_{32} & -1_{33} \end{bmatrix}$ $\qquad$ $A' = \begin{bmatrix} (-1-(-2)) & (-2-(-2)) & (2-1) \\ (1-2) & (1-2) & (-1-(-1)) \\ (2-2) & (2-4) & (-1-(-2)) \end{bmatrix}$

$i + j =$ ímpar → mudar sinal.

$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ $\qquad$ $\bar{A} = (A')^t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

$A^{-1} = \frac{\bar{A}}{1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Alt (B)

$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

⑥ $(X.A)^t = B$

$((X.A)^t)^t = B^t$ → Matriz transposta de uma transposta é uma matriz normal

$X.A = B^t$

$X.\underbrace{A.A^{-1}}_{I} = B^t.A^{-1}$ → $\boxed{X = B^t.A^{-1}}$. Alt (B)

⑦ $B = \begin{bmatrix} x \\ y \end{bmatrix}$, $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$, A^{-1}?, $AB = C$

$A.\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$

det A = 24-25

det A = -1

* Regra matriz de ordem 2:

$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \Rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$ → Alt (d)

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8) $A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$, soma valores k=? , det A = det A^{-1}

$\det A = 2 - (-2k)$
$\det A = 2 + 2k$

$\det A \cdot \det A^{-1} = 1$
$(2+2k) \cdot (2+2k) = 1$
$4 + 4k + 4k + 4k^2 = 1$

$4k^2 + 8k + 4 - 1 = 0$
$4k^2 + 8k + 3 = 0$

$\Delta = 64 - 4 \cdot 4 \cdot 3$
$\Delta = 64 - 48 = 16$

$k_1 = \frac{-8 + \sqrt{16}}{2 \cdot 4} = \frac{-8+4}{8} = \frac{-4 : 4}{8 : 4} = \frac{-1}{2}$

$k_{11} = \frac{-8 - \sqrt{16}}{2 \cdot 4} = \frac{-8-4}{8} = \frac{-12 : 4}{8 : 4}$

$k_{11} = \frac{-3}{2}$

Soma de valores $k = \frac{-1}{2} - \frac{3}{2} = \frac{-4}{2} = -2$ Alt B

9) $A_{2\times2}$, $B_{2\times2}$, $\det A \neq 0$ e $\det B \neq 0$

a) $(A+B) \cdot (A-B) = \boxed{A^2 - AB + BA - B^2}$. $(AB \neq BA)$

b) $(A+B)^2 = A^2 + 2 \cdot A \cdot B + B^2$ $\boxed{AB = BA}$

c) $\frac{\det(A)}{\det(-A)} \rightarrow \det(-A) = (-1)^2 \cdot \det A = \det A$ $\frac{\det A}{\det(-A)} = \boxed{1}$

d) $B = A^{-1} \rightarrow \det A \cdot \det B = 1$ $\boxed{\det B = \frac{1}{\det A}}$