SIMHEURISTIC APPROACH TO THE SINGLE-MACHINE SCHEDULING PROBLEM WITH STOCHASTIC PROCESSING TIMES AND ENERGY-EFFICIENT

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Abstract

This article addresses the stochastic version of the single-machine scheduling problem with environmental considerations. In this context, jobs processing times behave as random variables, and the total energy consumption depends on which jobs are processed in which periods, since each job has its own energy consumption and each period has its energy tariff due to the Time-Of-Use policy. The objective of this paper is to propose a simheuristic algorithm to minimize total energy consumption under different uncertainty scenarios. The proposed algorithm, called SimSA, combines the metaheuristics Simulated Annealing and Greedy Randomized Adaptive Search Procedure to perform the search in the solution space, along with Monte Carlo Simulation to generate random values. The SimSA results were compared with the simulation of the best solution using deterministic parameters, and SimSA achieved better results in the stochastic metrics Average, Value at Risk (VaR), and Conditional Value at Risk (CVaR). These results highlight the importance of incorporating uncertainties present in processes and emphasize the relevance of the proposed simheuristic.

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1. Literature review

This document presents all the articles used in the three literature reviews conducted in the paper "Simheuristic Approach to the Single-Machine Scheduling Problem with Stochastic Processing Times and Energy-Efficient". This section aims to establish the review protocols, which specify the objectives, the questions to be answered, the databases to be consulted, the types of studies to be analyzed, the keywords to be searched, the languages, the years of publication, and the inclusion and exclusion criteria for the studies obtained. Thus, these protocols seek to provide a clear and pre-defined framework that outlines and guides the entire review process, describing in a transparent manner all the methodological aspects for the search and selection of articles, thereby ensuring the reproducibility of the study.

Therefore, we have three objectives, one for each review. For SSMSP, we aim to identify gaps and trends regarding this model. For SMGSP, to evaluate the state of the art. And for SGSP, to analyze the current landscape of research on this topic. The questions to be answered for each objective are presented in Table 1.

Table 1: Questions to be answered for each review

Review	Questions
SSMSP	(1) Which methods are used to solve this type of problem;
SOMOI	(2) What are the most addressed stochastic parameters in the
	literature.
	(1) Classification of models according to the number of objective
SMGSP	functions;
SMGSI	(2) Which objective function(s) is/are considered;
	(3) What are the main environmental considerations;
	(4) What methods are used to solve this type of problem.
	(1) Which methods are used to solve this type of problem;
SGSP	(2) What are the most addressed stochastic parameters in the
	literature;
	(3) What are the main environmental considerations.

The review was conducted exclusively using the Web of Science database as the source for the literature review. This choice was made because this database is recognized for its multidisciplinary scope and its extensive coverage of renowned international scientific journals. Additionally, the search was limited to only one type of document, which is articles, as they represent a primary and consolidated source of academic knowledge, typically subjected to a rigorous peer-review process, which ensures the credibility and reliability of the information contained in them.

Preliminarily, a search was conducted on Google Scholar to identify keywords present in existing works in the literature. This enabled the formulation of a search string that ensured both the breadth and accuracy of the review, aiming to avoid the exclusion of relevant studies and the inclusion of irrelevant ones. Furthermore, all searches for keywords were performed only in the titles (TI) and the abstract (AB) of the works, and the asterisk ("*") was used as a truncation operator to find all research with the same root. Thus, the search strings adopted can be observed in Table 2.

Table 2: Search string adopted for each review

Review	Search string
SSMSP	$TI = (single machine) AND TI = (sched^*) AND$
	$ (TI=(simheuristic) OR TI=(stoch^*) OR TI=(uncert^*)) AND $
	$(AB=(heuristi^*) OR TI=(simheuris^*) OR TI=(optimiz^*))$
SMGSP	TI=(single machine) AND TI=(sched*) AND (TI=(green)
	OR TI=(sustainab*) OR TI=(energ*) OR TI=(carbon*) OR
	TI=(nois*) OR TI=(pollut*) OR TI=(tariff*) OR TI=(Time
	Of Use) OR TI=(Energy-effici*)OR TI=(Electricity prices*))
SGSP	$TI=(sched^*)$ AND $(TI=(green)$ OR $TI=(sustainab^*)$
	OR $TI=(energ^*)$ OR $TI=(carbon^*)$ OR $TI=(nois^*)$ OR
	$TI=(pollut^*)$ AND $TI=(simheuristic)$ OR $TI=(stoch^*)$ OR
	TI=(uncert*)) AND (TI=(single machine) OR TI=(parallel
	$machines)$ OR $TI=(flow\ shop)$ OR $TI=(flow\ shop)$ OR
	TI = (flowshop) OR $TI = (jop shop)$ OR $TI = (jop shop)$ OR
	TI=(jopshop) OR TI=(open shop) OR TI=(open-shop) OR
	TI = (openshop))

Therefore, as presented in Table 2, for the SSMSP review, the terms "simheuristic", "stoch*", "uncert*" were used to search for works that deal with random variables, and the terms "heuristi*", "simheuris*", and "op-

timiz*" were used to filter only approaches that use optimization methods. This search returned 19 results.

Similarly, for the SMGSP review, the terms "green", "sustainab*", "energ*", "carbon*", "nois*", "pollut*", "tariff*", "Time of use", and "Energy-effici*" were adopted as filters to search for only works that have environmental considerations in their scope. The term "single machine*" was used to delimit the manufacturing environment, and the term "sched*" was adopted to specify the addressed problem. By using this search string, 40 works were retrieved.

Finally, for the SGSP review, where the terms "single machine", "parallel machine", "flow shop", "job shop", "open shop", and their variants are adopted to represent all the manufacturing environments existing in the sequencing literature. This search returned 8 results. All searches and article counting for each review were conducted on 05/03/2025.

The search was conducted without any time limitations or document publication type restrictions (i.e., conferences, symposia, and journals). All works were read in full, with only those that did not address the scheduling topic or were not fully available being excluded.

The tables provide a structured summary of the key aspects analyzed in the reviewed articles, offering a comprehensive overview of the state of the art in these areas. This mapping enables the identification of emerging trends and research gaps, providing insights for the development of new approaches and contributing to advancements in the field.

Table 3: Articles used for the SSMSP review

Ref.	Year	Objective Function	Stochastic parameter	Solution approach
	1988	Inventory costs and setup cost	Demands	Heuristics with Simulation
[2]	1988	Inventory costs and setup cost	Demands	Heuristics with Simulation
[3]	1992	Reward	Release date	Heuristics with Simulation
4	1994	Inventory costs and setup cost	Demands	Heuristics with Simulation
ဩ	1994	Earliness and tardiness costs	Processing times	Heuristics with Simulation
[9]	1997	Total tardiness	Processing times	Robust Programming
[_		Total tardiness	Processing times and due dates	Heuristics with Simulation
∞		Weighted total tardiness	Processing times and due dates	Heuristics with Simulation
[6]		Net present value	Reward value	Branch-and-Bound
[10]	2009	Weighted total tardiness	Machine breakdown	Heuristics with Simulation
[11]		Weighted total tardiness	Processing times	Heuristics with Simulation
[12]		Makespan and Total completion time	Processing times	Branch-and-Bound
[13]		Sum of completion times	Processing times	Robust Programming
[14]		Total tardiness	Setups	Stochastic Programming
[15]	2018	Maximum tardiness	Release date	Robust Programming
[16]	2020	Maximum waiting time	Processing times	Robust Programming
[17]	2020	Makespan	Machine breakdown	Simheuristics
[18]	2022	Makespan	Processing times	Robust Programming
[19]	2024	Earliness and tardiness costs	Machine breakdown	Machine Learning

Table 4: Articles used for the SMGSP review

Table 5: Articles used for the SGSP review

Ref.	Year	Ref. Year Objective Function		Environmental Characteristic Manufacturing Environment Solution approach	Manufacturing Environment	Solution approach
[09]	2016	CO_2 emission and Makespan	Amplitude and magnitude of energy	Sustainable energy storage	Single machine	Interval number theory
[61]	2019	Makespan and TEC	Processing time	Speed adjustment	Flowshop	Heuristic
[62]	2020	Makespan and TEC	Processing time	Speed adjustment	Flowshop	Metaheuristics
[63]		Makespan and TEC	Processing time	Sustainable energy storage	Flowshop	Heuristic
[64]		TEC	Arrival and cancellation of orders	TOU	Flowshop	Metaheuristics
[65]	2023	Makespan and TEC	Processing time	Speed adjustment	Parallel machines	Heuristic
[99]		Profit	Processing time	CO_2 emission	Single machine	Machine learning
[29]	2024	TEC	Amplitude and magnitude of renewable energy	TOU and Renewable energy	Flowshop	Stochastic Programming
[XS]		2024 Minimize total delay and TEC	Processing time	Machine states	Onen shon	Henristic

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