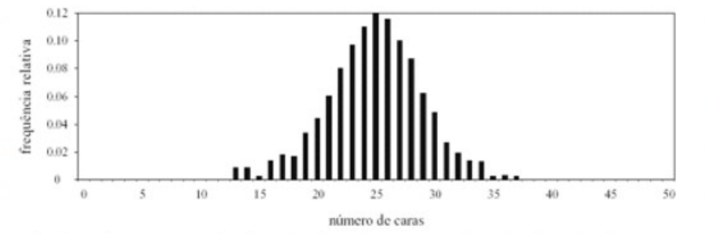
30/09/2021

## 4.9) Teorema do Limite Central




$y = X_1 + X_2 + X_3 + X_4 + \dots + X_n$

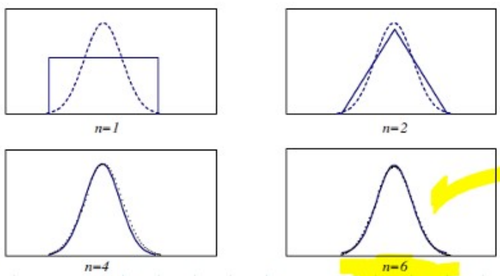
↳ Gaussiana

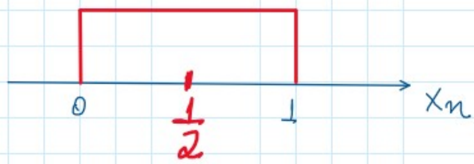
$Z = \dfrac{Y - m_Y}{\sigma_Y}$

$Y = X_1 + X_2 \quad f_Y = f_{X_1} * f_{X_2}$

• **Exemplo 19:** Considere a soma de *n* variáveis aleatórias uniformes e observe o comportamento da fdp desta soma.

$$Y_n = X_1 + X_2 + \dots + X_n$$




$Y = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

↳ Gaussiana

$E[Y] = 3$

• **Exemplo 20:** Suponha que os pedidos da cantina são variáveis aleatórias com média e desvio padrão dados abaixo. Estime a probabilidade de que os primeiros 100 clientes gastem um total superior a R$ 840,00. Estime a probabilidade de que os primeiros 100 clientes gastem um total entre R$ 780,00 e R$ 830,00.

$\mu = R\$8{,}00 \qquad \sigma = R\$2{,}00$

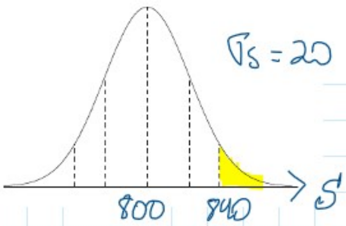
$Z = \dfrac{S - m_S}{\sigma_S}$

$X_i \to$ valor gasto pelo i-ésimo cliente.

$S = X_1 + X_2 + X_3 + \dots + X_{100}$ $\quad E[X_i] = 8$ $\quad$ a) $P(S > 840)$

↳ Gaussiana $\quad \sigma_{X_i} = 2$ $\quad$ b) $P(780 < S < 830)$

$E[S] = 800 \quad \sigma_{X_i}^2 = 4 \quad \sigma_S^2 = 400 \quad \sigma_S = 20$

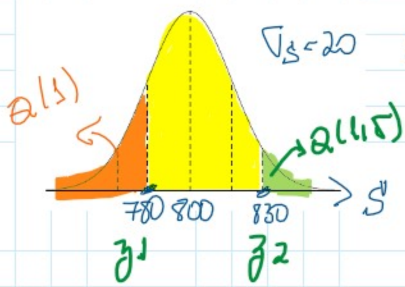



$$z = \frac{840 - 800}{20} = 2$$

$$P(S > 840) = P(Z > 2) = Q(2) = 2{,}275\%$$

| | | | | |
|---|---|---|---|---|
| 1.8 | 3.593e-2 | 3.514e-2 | 3.437 |
| 1.9 | 2.871e-2 | 2.806e-2 | 2.742 |
| 2.0 | 2.275e-2 | 2.221e-2 | 2.169 |
| 2.1 | 1.786e-2 | 1.742e-2 | 1.700 |
| 2.2 | 1.390e-2 | 1.355e-2 | 1.320 |

$$z_1 = \frac{780 - 800}{20} = -1 \qquad z_2 = \frac{830 - 800}{20} = 1{,}5$$




$$P(780 < S < 830) = 1 - Q(1) - Q(1{,}5)$$
$$= 100\% - 15{,}86\% - 6{,}68\% = 77{,}46\%$$

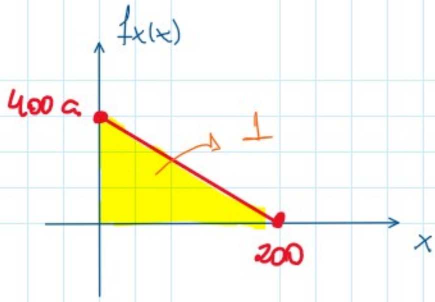
| | | | | |
|---|---|---|---|---|
| 1.0 | 1.586e-1 | 1.562e-1 | 1.538e-1 | 1. |
| 1.1 | 1.356e-1 | 1.334e-1 | 1.313e-1 | 1. |
| 1.2 | 1.150e-1 | 1.131e-1 | 1.112e-1 | 1. |
| 1.3 | 9.680e-2 | 9.509e-2 | 9.341e-2 | 9. |
| 1.4 | 8.075e-2 | 7.926e-2 | 7.780e-2 | 7. |
| 1.5 | 6.680e-2 | 6.552e-2 | 6.425e-2 | 6. |

- **Exemplo 21:** A variável aleatória X representa a demanda diária de arroz em um pequeno supermercado, em quilos, e tem fdp dada por

$$f_X(x) = \begin{cases} -2ax + 400a & 0 < x < 200 \\ 0 & \textit{caso contrário} \end{cases}$$

Pede-se:

a) A demanda diária média de arroz e a variância da demanda diária de arroz.




$$\int_0^{200} (-2ax + 400a)\,dx = 1 \qquad \frac{200 \times 400a}{2} = 1 \qquad a = \frac{1}{40.000}$$

$$f_X(x) = \begin{cases} \dfrac{-x}{20000} + \dfrac{1}{100} & 0 < x < 200 \\ 0 & c.c. \end{cases}$$

$$E[X] = \int_0^{200} x \cdot \left(\frac{-x}{20.000} + \frac{1}{100}\right) dx = \int_0^{200} \left(\frac{-x^2}{20.000} + \frac{x}{100}\right) dx = \left.\frac{-x^3}{60.000} + \frac{x^2}{200}\right|_{x=0}^{200}$$

$$E[X] = -\frac{400}{3} + 200 = \frac{-400 + 600}{3} = \frac{200}{3} \cong 67 \text{ Kg}$$

$$E[X^2] = \int^{200} x^2 \cdot \left(\frac{-x}{20.000} + \frac{1}{100}\right) dx = \int^{200} \left(\frac{-x^3}{20.000} + \frac{x^2}{100}\right) dx = \frac{-x^4}{80.000} + \frac{x^3}{300} \Big|^{200}$$

$$E[X^2] = \int_0^{200} x^2 \cdot \left(\frac{-x}{20.000} + \frac{1}{100}\right) dx = \int_0^{200} \left(\frac{-x^3}{20.000} + \frac{x^2}{100}\right) dx = \frac{-x^4}{80.000} + \frac{x^3}{300} \Bigg|_{x=0}^{200}$$

$$E[X^2] = -20.000 + \frac{80.000}{3} = \frac{-60.000 + 80.000}{3} = \frac{20.000}{3} \ Kg^2$$

$$\sigma_X^2 = E[X^2] - (E[X])^2 = \frac{20.000}{3} - \frac{40.000}{9} = \frac{60.000 - 40.000}{9} = \frac{20.000}{9} \ Kg^2$$

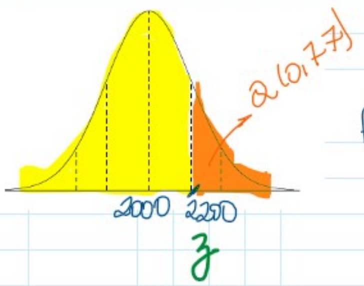
b) A probabilidade da demanda mensal de arroz ser menor que 2200 Kg.

$$Z = \frac{M - m_M}{\sigma_M}$$

$$M = X_1 + X_2 + X_3 + \ldots + X_{30} \qquad E[X_i] = \frac{200}{3} \ Kg \qquad \sigma_{X_i}^2 = \frac{20.000}{9} \ Kg^2$$

↳ Gaussiana $\qquad P(M < 2200) = ?$

$$E[M] = 30 \cdot \frac{200}{3} = 2000 \ Kg \qquad \sigma_M^2 = \cancel{30}^{10} \times \frac{20000}{\cancel{9}_3} = \frac{200.000}{3} \qquad \sigma_M = 258,2 \ Kg$$




$$z = \frac{2200 - 2000}{258,2} = 0,77$$

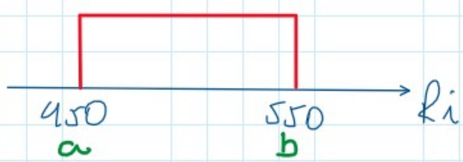
$$P(M < 2200) = 1 - Q(0,77) = 100\% - 22,06\%$$

$$= 77,94\%$$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 3.085e-1 | 3.050e-1 | 3.015e-1 | 2.980e-1 | 2.945e-1 | 2.911e-1 | 2.877e-1 | 2.843e-1 | 2.809e-1 | 2.775e-1 |
| 0.6 | 2.742e-1 | 2.709e-1 | 2.676e-1 | 2.643e-1 | 2.610e-1 | 2.578e-1 | 2.546e-1 | 2.514e-1 | 2.482e-1 | 2.450e-1 |
| 0.7 | 2.419e-1 | 2.388e-1 | 2.357e-1 | 2.326e-1 | 2.296e-1 | 2.266e-1 | 2.236e-1 | 2.206e-1 | 2.176e-1 | 2.147e-1 |
| 0.8 | 2.118e-1 | 2.089e-1 | 2.061e-1 | 2.032e-1 | 2.004e-1 | 1.976e-1 | 1.948e-1 | 1.921e-1 | 1.894e-1 | 1.867e-1 |

**18)** As resistências dos resistores $R_1$, $R_2$, $R_3$ e $R_4$ são variáveis aleatórias independentes, cada uma delas uniformemente distribuída no intervalo $[450,550]$ ohms. Usando o Teorema do Limite Central, calcule a probabilidade da resistência equivalente da associação em série dos quatro resistores estar dentro do intervalo $[1900,2050]$ ohms.

→ 4ª Lista




$$E[R_i] = 500$$

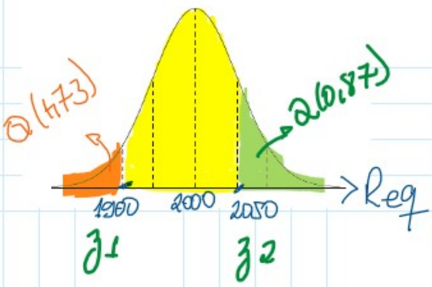
$$R_{eq} = R_1 + R_2 + R_3 + R_4 \qquad P(1900 \leq R_{eq} \leq 2050) = ?$$

↳ Gaussiana

$$\sigma_{R_i}^2 = \frac{(b-a)^2}{12} = \frac{100^2}{12}$$

$$E[R_{eq}] = 2000$$

$$\sigma_{R_{eq}}^2 = 4 \times \frac{100^2}{12} \qquad \sigma_{R_{eq}}^2 = \frac{100^2}{3} \qquad \sigma_{R_{eq}} = \frac{100}{\sqrt{3}}$$




$$z_1 = \frac{1900 - 2000}{\frac{100}{\sqrt{3}}} = -\cancel{100} \cdot \frac{\sqrt{3}}{\cancel{100}} = -\sqrt{3} = -1{,}73$$

$$z_2 = \frac{2050 - 2000}{\frac{100}{\sqrt{3}}} = 50 \cdot \frac{\sqrt{3}}{100} = \frac{\sqrt{3}}{2} = 0{,}87$$

$$P(1900 \leq Reg \leq 2050) = 1 - Q(1{,}73) - Q(0{,}87)$$

$$= 100\% - 4{,}181\% - 19{,}21\% = 76{,}609\%$$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.8 | 2.118e-1 | 2.089e-1 | 2.061e-1 | 2.032e-1 | 2.004e-1 | 1.976e-1 | 1.948e-1 | 1.921e-1 | 1.894e-1 | 1.867e-1 |
| 0.9 | 1.840e-1 | 1.814e-1 | 1.787e-1 | 1.761e-1 | 1.736e-1 | 1.710e-1 | 1.685e-1 | 1.660e-1 | 1.635e-1 | 1.610e-1 |
| 1.0 | 1.586e-1 | 1.562e-1 | 1.538e-1 | 1.515e-1 | 1.491e-1 | 1.468e-1 | 1.445e-1 | 1.423e-1 | 1.400e-1 | 1.378e-1 |
| 1.1 | 1.356e-1 | 1.334e-1 | 1.313e-1 | 1.292e-1 | 1.271e-1 | 1.250e-1 | 1.230e-1 | 1.210e-1 | 1.190e-1 | 1.170e-1 |
| 1.2 | 1.150e-1 | 1.131e-1 | 1.112e-1 | 1.093e-1 | 1.074e-1 | 1.056e-1 | 1.038e-1 | 1.020e-1 | 1.002e-1 | 9.852e-2 |
| 1.3 | 9.680e-2 | 9.509e-2 | 9.341e-2 | 9.175e-2 | 9.012e-2 | 8.850e-2 | 8.691e-2 | 8.534e-2 | 8.379e-2 | 8.226e-2 |
| 1.4 | 8.075e-2 | 7.926e-2 | 7.780e-2 | 7.635e-2 | 7.493e-2 | 7.352e-2 | 7.214e-2 | 7.078e-2 | 6.943e-2 | 6.811e-2 |
| 1.5 | 6.680e-2 | 6.552e-2 | 6.425e-2 | 6.300e-2 | 6.178e-2 | 6.057e-2 | 5.937e-2 | 5.820e-2 | 5.705e-2 | 5.591e-2 |
| 1.6 | 5.479e-2 | 5.369e-2 | 5.261e-2 | 5.155e-2 | 5.050e-2 | 4.947e-2 | 4.845e-2 | 4.745e-2 | 4.647e-2 | 4.551e-2 |
| 1.7 | 4.456e-2 | 4.363e-2 | 4.271e-2 | 4.181e-2 | 4.092e-2 | 4.005e-2 | 3.920e-2 | 3.836e-2 | 3.753e-2 | 3.672e-2 |