Teoria de Filas – Formulário

Para todas as filas:

$$\rho = \frac{\lambda}{\mu} \quad E(t_s) = \frac{1}{\mu} \quad E(q) = E(w) + E(S) = \sum_{k=0}^{\infty} k \cdot P_k \quad E(t_q) = E(t_w) + E(t_s) \quad U = 1 - P_0$$

Little para filas sem bloqueio (buffer infinito):

$$E(t_q) = \frac{E(q)}{\lambda} \qquad E(t_w) = \frac{E(w)}{\lambda} \qquad E(t_s) = \frac{E(S)}{\lambda} \qquad E(S) = \rho$$

Little para filas com bloqueio (buffer finito):

$$E(t_q) = \frac{E(q)}{\lambda(1-P_B)} \qquad E(t_w) = \frac{E(w)}{\lambda(1-P_B)} \qquad E(t_s) = \frac{E(S)}{\lambda(1-P_B)} \qquad E(S) = \rho(1-P_B)$$

Fila M/M/1/∞/∞/∞/FIFO:

$$P_0 = (1-\rho) \qquad P_k = \rho^k \cdot P_0 \qquad E(q) = \frac{\rho}{1-\rho} \qquad E(w) = \frac{\rho^2}{1-\rho} \qquad E(t_q) = \frac{1}{\mu-\lambda}$$

$$P(q \geq N) = \rho^N \qquad P(t_q \geq T) = \mathrm{e}^{-(\mu-\lambda)\cdot T}$$

Fila M/M/1/J/J+1/∞/FIFO:

$$\mathrm{P_k} = \rho^{\mathrm{k}} \cdot \mathrm{P_0} \qquad P_0 = \frac{1-\rho}{1-\rho^{J+2}} \qquad P_B = P_{J+1} \qquad E(q) = \frac{\rho}{1-\rho} - \frac{(J+2)\cdot\rho^{J+2}}{1-\rho^{J+2}}$$

Fila M/M/m/0/m/∞/ FIFO:

$$\mathrm{P_k} = \frac{\rho^{\mathrm{k}}}{\mathrm{k}!} \cdot \mathrm{P_0} \qquad P_0 = \frac{1}{\sum_{k=0}^{m} \frac{\rho^k}{k!}} \qquad P_B = P_m = \frac{\rho^m}{m!} \cdot P_0$$

$$E[q] = E[S] = \rho(1-P_B)$$

$$E[t_q] = E[t_S] = \frac{1}{\mu}$$

Fila M/M/m/∞/∞/∞/FIFO:

$$P_k = \frac{\rho^k}{k!} \cdot P_0\ ,\ k \leq m \qquad \mathrm{P_k} = \frac{\rho^{\mathrm{k}}}{\mathrm{m}^{\mathrm{k-m}} \cdot \mathrm{m}!} \cdot \mathrm{P_0}\ ,\ \mathrm{k} \geq \mathrm{m}$$

$$P_0 = \frac{1}{\left(\sum_{k=0}^{m-1} \frac{\rho^k}{k!}\right) + \frac{\rho^m}{m! \cdot \left(1-\frac{\rho}{m}\right)}} \qquad E(w) = \frac{P_0 \cdot \rho^m}{m!} \cdot \left(\frac{\frac{\rho}{m}}{\left(1-\frac{\rho}{m}\right)^2} \right)$$

Fila M/M/m/J/K/∞/FIFO:

$$P_k = \frac{\rho^k}{k!} \cdot P_0 \text{ , } k \le m \qquad P_k = \frac{\rho^k}{m^{k-m} \cdot m!} \cdot P_0 \text{ , } m \le k \le J+m \qquad P_B = P_{J+m}$$

$$P_0 = \frac{1}{\sum_{k=0}^{m} \frac{\rho^k}{k!} + \sum_{k=m+1}^{J+m} \frac{\rho^k}{m! \cdot m^{k-m}}} \qquad E(q) = \sum_{k=1}^{m} \frac{k \cdot \rho^k}{k!} P_0 + \sum_{k=m+1}^{J+m} \frac{k \cdot \rho^k}{m! \cdot m^{k-m}} P_0$$

Fila M/G/1/∞/∞/∞/FIFO:

$$E(w) = \frac{\lambda^2 \cdot E(t_s^2)}{2(1-\rho)} \qquad E(t_w) = \frac{\lambda \cdot E(t_s^2)}{2(1-\rho)}$$

Para atendimento exponencial (M/M/1): $\sigma_{ts}^2 = \frac{1}{\mu^2}$ $\quad \left[E(t_s)\right]^2 = \frac{1}{\mu^2}$ $\quad E(t_s^2) = \frac{2}{\mu^2}$

$$E(w) = \frac{\lambda^2 \cdot E(t_s^2)}{2(1-\rho)} = \frac{\rho^2}{1-\rho} \qquad E(q) = \frac{\rho^2}{1-\rho} + E(s) = \frac{\rho}{1-\rho}$$

Para atendimento constante: $\sigma_{ts}^2 = 0$ $\quad \left[E(t_s)\right]^2 = \frac{1}{\mu^2}$ $\quad E(t_s^2) = \frac{1}{\mu^2}$

$$E(w) = \frac{\lambda^2 \cdot E(t_s^2)}{2(1-\rho)} = \frac{\rho^2}{2(1-\rho)} \qquad E(q) = \frac{\rho^2}{2(1-\rho)} + E(s) = \frac{\rho^2}{2(1-\rho)} + \rho$$

Para qualquer atendimento (incluindo os casos anteriores):

$$E(t_s) = \sum t_s \cdot f_{T_s}(t_s) \qquad E(t_s^2) = \sum t_s^2 \cdot f_{T_s}(t_s)$$

Filas com prioridades:

$$\lambda = \sum_{r=1}^{R} \lambda_r \qquad \rho = \sum_{r=1}^{R} \rho_r \qquad E(t_s) = \sum_{r=1}^{R} \frac{\lambda_r}{\lambda} E(t_{s_r}) \qquad E(t_s^2) = \sum_{r=1}^{R} \frac{\lambda_r}{\lambda} E(t_{s_r}^2)$$

$$E\left\{w_{(p)}\right\} = \frac{\lambda.\lambda_{(p)}.E\left\{t_s^2\right\}}{2 \cdot \left(1-\beta_{(p-1)}\right) \cdot \left(1-\beta_{(p)}\right)} \qquad \begin{cases} \beta_{(i)} = \sum_{k=1}^{i} \rho_{(k)} & \lambda_{(p)} = \lambda_r \qquad \rho_{(p)} = \rho_r \\ \beta_{(0)} = 0 \end{cases}$$

$$E\left\{t_{w_{(p)}}\right\} = \frac{\lambda.E\left\{t_s^2\right\}}{2 \cdot \left(1-\beta_{(p-1)}\right) \cdot \left(1-\beta_{(p)}\right)} \qquad \begin{cases} \beta_{(i)} = \sum_{k=1}^{i} \rho_{(k)} & \lambda_{(p)} = \lambda_r \qquad \rho_{(p)} = \rho_r \\ \beta_{(0)} = 0 \end{cases}$$

$$E\left\{w\right\} = \sum_{p=1}^{P} E\left\{w_{(p)}\right\} \qquad E\left\{t_w\right\} = \sum_{p=1}^{P} E\left\{t_{w_{(p)}}\right\}$$

Equacionamento para o caso sem prioridades: $E(w) = \frac{\lambda^2 \cdot E(t_s^2)}{2(1-\rho)}$ $\qquad E(t_w) = \frac{\lambda \cdot E(t_s^2)}{2(1-\rho)}$

Fila M/M/m/0/m/S/FIFO (População finita):

$$P_K = P_0.\rho^K.\binom{S}{K} = P_0.\rho^K.\frac{S!}{(S-K)!K!} \qquad P_0 = \frac{1}{\sum_{K=0}^{m}\rho^K.\binom{S}{K}}$$

Fila M/M/m/J/K/S/FIFO (População finita):

$$P_K = P_0.\rho^K.\binom{S}{K} = P_0.\rho^K.\frac{S!}{(S-K)!K!} \quad K \le m$$

$$P_K = \frac{P_0.\rho^K}{m^{K-m}}.\binom{S}{m}\cdot\prod_{K=1}^{J}\left(S-m-K+1\right) \quad m < K \le m+J$$

$$P_0 = \frac{1}{\sum_{K=0}^{m}\rho^K.\binom{S}{K} + \sum_{K=m+1}^{m+J}\left[\frac{\rho^K}{m^{K-m}}.\binom{S}{m}\cdot\prod_{i=1}^{J}\left(S-m-i+1\right)\right]}$$

Teorema de Little para filas de população finita:

$$\lambda_e = \sum_{K=0}^{m+J}\left(S-K\right)\cdot\lambda\cdot P_K \qquad E\{t_q\} = \frac{E\{q\}}{\lambda_e\left(1-P_B\right)} \qquad E\{t_w\} = \frac{E\{w\}}{\lambda_e\left(1-P_B\right)} \qquad E\{S\} = \frac{\lambda_e\left(1-P_B\right)}{\mu}$$

Sistemas multidimensionais (filas com múltiplos serviços):

$$P_{a,b,c,\ldots,K} = P_{0,0,0,\ldots,0}.\prod_{L=1}^{K}\binom{S_L}{i_L}\left(\frac{\lambda_L}{\mu_L}\right)^{i_L}, \text{ com } i_1 = a, i_2 = b, \ldots, i_K = K$$

Redes de filas. Teorema de Jackson:

$$\lambda_i = \gamma_i + \sum_{j=1}^{M} r_{ji}\lambda_j \qquad p\left(q_1, q_2, q_3, \ldots, q_M\right) = \prod_{i=1}^{M} p\left(q_i\right)$$