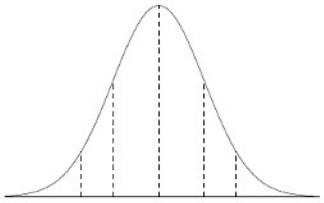
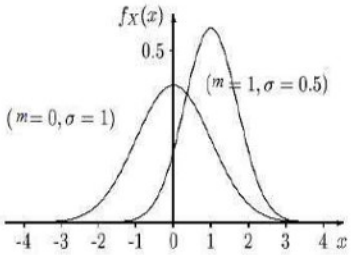


4.7) Distribuição Gaussiana (ou Normal)

Domínio: $S_X = (-\infty, \infty)$ $\quad x \in \mathbb{R}$

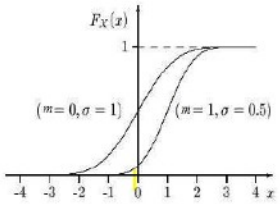
Função densidade de probabilidade

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$



Função distribuição cumulativa

$$F_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{-\frac{(y-m)^2}{2\sigma^2}} dy$$



Aplicações:

- Modelagem de canais (desvanecimento).
- Modelagem de sistemas de comunicações digitais e comunicações móveis.
- Modelagem de processos de ruídos em sistemas de comunicação.

- **Características da distribuição gaussiana**

a) Média

$$E[X] = m$$

b) Variância

$$Var(X) = \sigma_x^2$$

c) Desvio Padrão

$$\sigma_x$$

d) Função Característica

$$\psi(j.w) = e^{j.m.w - \left(\frac{\sigma^2 w^2}{2}\right)}$$

- Tomando uma va Z padronizada:

$$Z = \frac{X - m_x}{\sigma_X} \quad \text{em que} \quad E[Z] = 0, \ \sigma_Z = 1$$

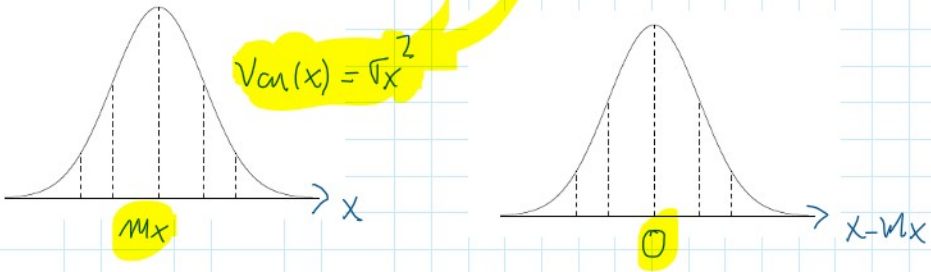
$E[cx] = cE[X]$ $\quad Var(cx) = c^2 . Var(x)$

$$E[z] = E\left[\frac{X-m_x}{\sigma_x}\right] = \frac{1}{\sigma_x} . E[X-m_x] = \frac{1}{\sigma_x} . \{E[X] - E[m_x]\} = 0$$

$m_x \quad m_x$

$$Var(z) = Var\left(\frac{X-m_x}{\sigma_x}\right) = \frac{1}{\sigma_x^2} . Var(X-m_x) = \frac{1}{\sigma_x^2} . \sigma_x^2 = 1 \quad \sigma_z = 1$$

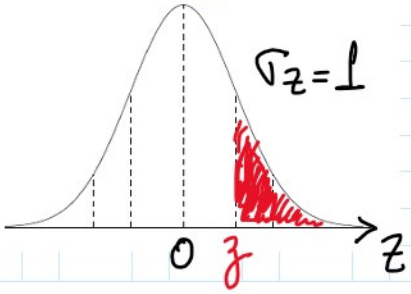
$Var(x) = \sigma_x^2$



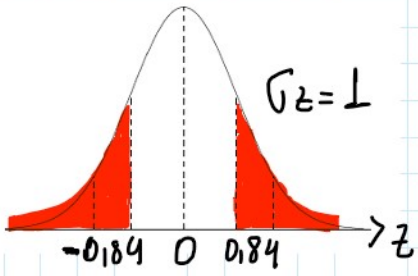
- A função densidade de probabilidade de Z é dada por

$$f_Z(z) = \frac{1}{\sqrt{2.\pi}} . e^{\frac{-Z^2}{2}}$$

$$f_{x(x)} = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{\frac{-(x-m)^2}{2\sigma^2}}$$



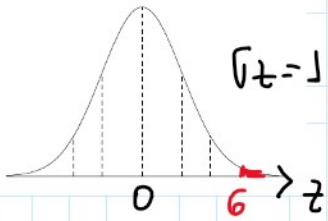
$$P(Z > z) = \int_z^{\infty} \frac{1}{\sqrt{2.\pi}} . e^{\frac{-Z^2}{2}} dz$$



$P(Z > 0{,}84) = 20{,}04\%$

$Q(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000e-1	4.960e-1	4.920e-1	4.880e-1	4.840e-1	4.800e-1	4.760e-1	4.720e-1	4.681e-1	4.641e-1
0.1	4.601e-1	4.562e-1	4.522e-1	4.482e-1	4.443e-1	4.403e-1	4.364e-1	4.325e-1	4.285e-1	4.246e-1
0.2	4.207e-1	4.168e-1	4.129e-1	4.090e-1	4.051e-1	4.012e-1	3.974e-1	3.935e-1	3.897e-1	3.859e-1
0.3	3.820e-1	3.782e-1	3.744e-1	3.707e-1	3.669e-1	3.631e-1	3.594e-1	3.556e-1	3.519e-1	3.482e-1
0.4	3.445e-1	3.409e-1	3.372e-1	3.335e-1	3.299e-1	3.263e-1	3.227e-1	3.191e-1	3.156e-1	3.120e-1
0.5	3.085e-1	3.050e-1	3.015e-1	2.980e-1	2.945e-1	2.911e-1	2.877e-1	2.843e-1	2.809e-1	2.775e-1
0.6	2.742e-1	2.709e-1	2.676e-1	2.643e-1	2.610e-1	2.578e-1	2.546e-1	2.514e-1	2.482e-1	2.450e-1
0.7	2.419e-1	2.388e-1	2.357e-1	2.326e-1	2.296e-1	2.266e-1	2.236e-1	2.206e-1	2.176e-1	2.147e-1
0.8	2.118e-1	2.089e-1	2.061e-1	2.032e-1	2.004e-1	1.976e-1	1.948e-1	1.921e-1	1.894e-1	1.867e-1
0.9	1.840e-1	1.814e-1	1.787e-1	1.761e-1	1.736e-1	1.710e-1	1.685e-1	1.660e-1	1.635e-1	1.610e-1
1.0	1.586e-1	1.562e-1	1.538e-1	1.515e-1	1.491e-1	1.468e-1	1.445e-1	1.423e-1	1.400e-1	1.378e-1
1.1	1.356e-1	1.334e-1	1.313e-1	1.292e-1	1.271e-1	1.250e-1	1.230e-1	1.210e-1	1.190e-1	1.170e-1
1.2	1.150e-1	1.131e-1	1.112e-1	1.093e-1	1.074e-1	1.056e-1	1.038e-1	1.020e-1	1.002e-1	9.852e-2
1.3	9.680e-2	9.509e-2	9.341e-2	9.175e-2	9.012e-2	8.850e-2	8.691e-2	8.534e-2	8.379e-2	8.226e-2



4.8	7.933e-7	7.546e-7	7.177e-7	6.826e-7	6.491e-7	6.173e-7	5.869e-7	5.579e-7	5.304e-7	5.041e-7
4.9	4.791e-7	4.553e-7	4.327e-7	4.111e-7	3.906e-7	3.710e-7	3.524e-7	3.347e-7	3.179e-7	3.018e-7

$P(Z > 4{,}99) \simeq 3 \times 10^{-7} = 0{,}00003\%$

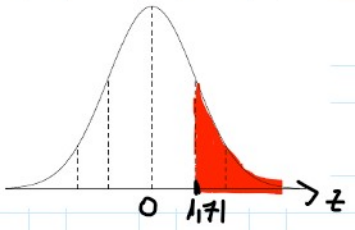
$P(Z > 6) = 0$

- **Exemplo 14:** Para uma variável aleatória gaussiana padronizada, determine:

$E[Z] = 0 \quad \sigma_Z = 1$

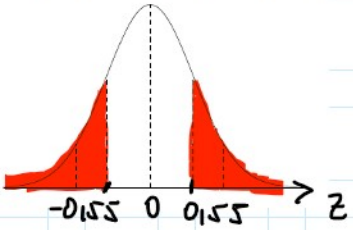
a) $P[Z > 1{,}71]$

$P(Z > 1{,}71) = Q(1{,}71) = 4{,}363\%$



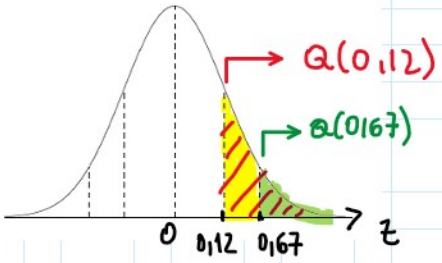
1.6	5.479e-2	5.369e-2	5.261e-2	5.1
1.7	4.456e-2	4.363e-2	4.271e-2	4.1
1.8	3.593e-2	3.514e-2	3.437e-2	3.3
1.9	2.871e-2	2.806e-2	2.742e-2	2.6

b) $P[Z < -0{,}55] = Q(0{,}55) = 29{,}11\%$



0.4	3.445e-1	3.409e-1	3.372e-1	3.335e-1	3.299e-1	3.263e-1	3.227e-1
0.5	3.085e-1	3.050e-1	3.015e-1	2.980e-1	2.945e-1	2.911e-1	2.877e-1
0.6	2.742e-1	2.709e-1	2.676e-1	2.643e-1	2.610e-1	2.578e-1	2.546e-1
0.7	2.419e-1	2.388e-1	2.357e-1	2.326e-1	2.296e-1	2.266e-1	2.236e-1

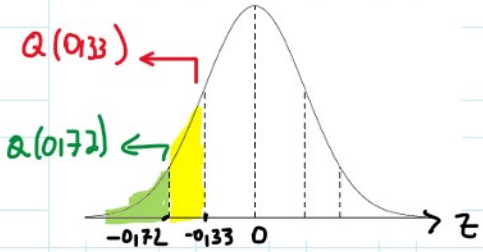
c) $P[0{,}12 < Z < 0{,}67]$ = Q(0,12) - Q(0,67) = 45,22% - 25,14% = 20,08%



z	0.00	0.01	0.02	0.03
0.0	5.000e-1	4.960e-1	4.920e-1	4.880e-1
0.1	4.601e-1	4.562e-1	4.522e-1	4.482e-1

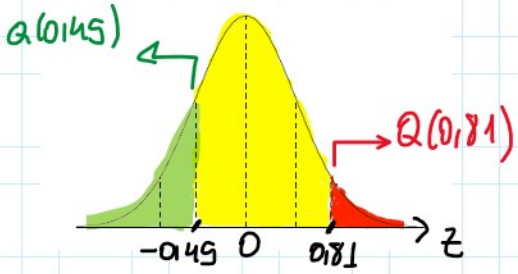
0.5	3.085e-1	3.050e-1	3.015e-1	2.980e-1	2.945e-1	2.911e-1	2.877e-1	2.843e-1	2.809e-1	2.775e-1
0.6	2.742e-1	2.709e-1	2.676e-1	2.643e-1	2.610e-1	2.578e-1	2.546e-1	2.514e-1	2.482e-1	2.450e-1
0.7	2.419e-1	2.388e-1	2.357e-1	2.326e-1	2.296e-1	2.266e-1	2.236e-1	2.206e-1	2.176e-1	2.147e-1

d) $P[-0{,}72 < Z < -0{,}33]$ = Q(0,33) - Q(0,72) = 37,07% - 23,57% = 13,5%



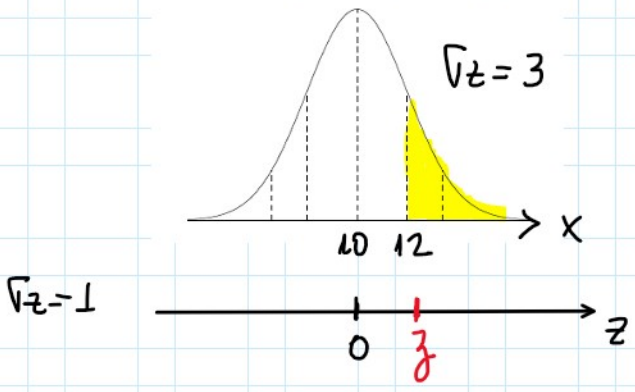
0.2	4.207e-1	4.168e-1	4.129e-1	4.090e-1
0.3	3.820e-1	3.782e-1	3.744e-1	3.707e-1
0.4	3.445e-1	3.409e-1	3.372e-1	3.335e-1
0.5	3.085e-1	3.050e-1	3.015e-1	2.980e-1
0.6	2.742e-1	2.709e-1	2.676e-1	2.643e-1
0.7	2.419e-1	2.388e-1	2.357e-1	2.326e-1
0.8	2.118e-1	2.089e-1	2.061e-1	2.032e-1

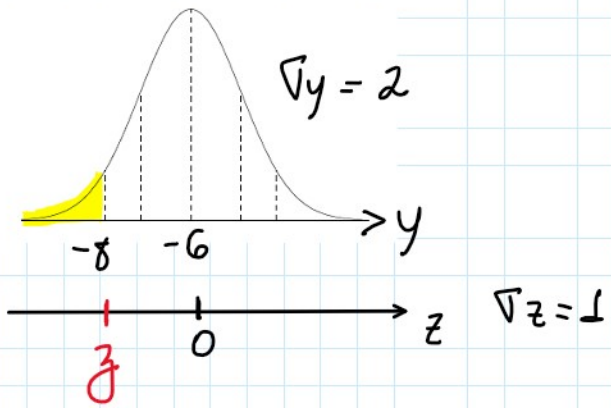
e) $P[-0{,}49 < Z < 0{,}81]$ = 1 - Q(0,49) - Q(0,81) = 100% - 31,2% - 20,89% = 47,91%



0.4	3.445e-1	3.409e-1	3.372e-1	3.335e-1	3.299e-1	3.263e-1	3.227e-1	3.191e-1	3.156e-1	3.120e-1
0.5	3.085e-1	3.050e-1	3.015e-1	2.980e-1	2.945e-1	2.911e-1	2.877e-1	2.843e-1	2.809e-1	2.775e-1
0.6	2.742e-1	2.709e-1	2.676e-1	2.643e-1	2.610e-1	2.578e-1	2.546e-1	2.514e-1	2.482e-1	2.450e-1
0.7	2.419e-1	2.388e-1	2.357e-1	2.326e-1	2.296e-1	2.266e-1	2.236e-1	2.206e-1	2.176e-1	2.147e-1
0.8	2.118e-1	2.089e-1	2.061e-1	2.032e-1	2.004e-1	1.976e-1	1.948e-1	1.921e-1	1.894e-1	1.867e-1

- Para uma variável aleatória Gaussiana de média diferente de zero e desvio padrão diferente de 1, é necessário fazer a padronização para a obtenção de uma probabilidade.





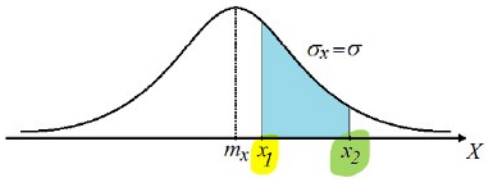
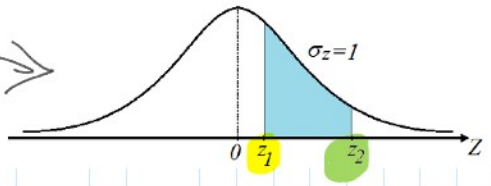
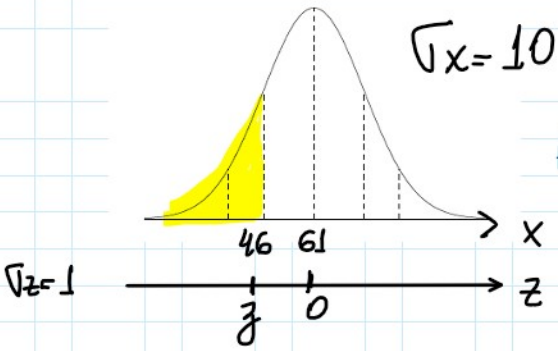


Tabela →



$$Z = \frac{X - m_x}{\sigma_X}$$

- **Exemplo 15:** Se X é uma va Gaussiana com média 61 e variância 100, determine $P[X \leq 46]$.



$z = \frac{46 - 61}{10} = -1{,}5$

1.4	8.075e-2	7.926e-2	7.
1.5	6.680e-2	6.552e-2	6.
1.6	5.479e-2	5.369e-2	5.

$P(X \leq 46) = P(Z \leq -1{,}5) = Q(1{,}5) = 6{,}68\ \%$

- **Exemplo 16:** Uma variável aleatória gaussiana possui média 10 e variância 16. Determine:

a) $P[X > 12]$
b) $P[11 < X < 13]$
c) $P[X < 6]$
d) $P[4 < X < 14]$

$m_X = 10$

$\sigma_X = 4$

$$Z = \frac{X - m_x}{\sigma_X}$$

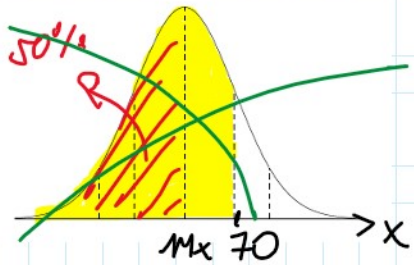
CASA

- **Exemplo 17:** Sabe-se que as notas resultantes de uma avaliação de uma determinada turma seguem uma distribuição gaussiana de variância 16. Se aproximadamente 10,56% da turma tirou nota abaixo de 70, qual foi a média da turma nesta avaliação?

$m_X = ?$

$\sigma_X = 4$

$P(X < 70) = 10{,}56\ \%$



$Z = \frac{X - m_X}{\sigma_X}$

$z = -1{,}25$ $\qquad z = \frac{X - m_X}{\sigma_X}$

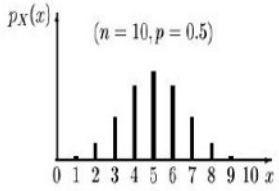
$-1{,}25 = \frac{70 - m_X}{4}$ $\qquad 70 - m_X = -5$ $\qquad m_X = 75$

4.8) Relação entre as distribuições binomial e normal

Função massa de probabilidade

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$x = 0, 1, \ldots, n$$



$E[X] = n \cdot p$ $\qquad \sigma_X = \sqrt{npq}$

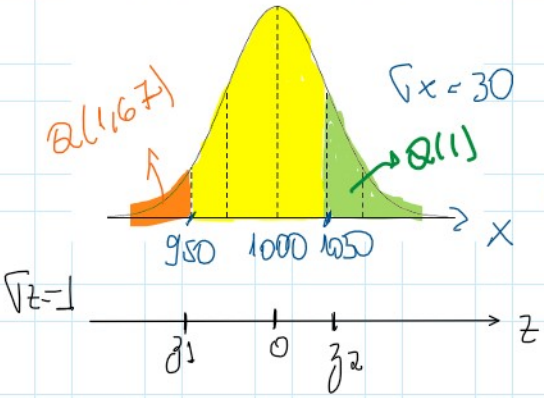
- **Exemplo 18:** Uma grande empresa industrial permite um desconto em qualquer fatura paga dentro de 30 dias. A cada 20 faturas selecionadas aleatoriamente, em média 2 faturas recebem o desconto. Se 10 mil faturas são selecionadas aleatoriamente, determine a probabilidade do número de faturas com desconto estar entre 950 e 1030 faturas.

$X \rightarrow$ nº de faturas com desconto. $\qquad P(950 \leq X \leq 1030) = ?$

$p = 0{,}1$ $\qquad q = 0{,}9$ $\qquad n = 10.000$

$E[X] = np = 10.000 \times 0{,}1 = 1000$

$\sigma_X = \sqrt{npq} = \sqrt{10.000 \times 0{,}1 \times 0{,}9} = 30$



$z_1 = \frac{950 - 1000}{30} = -\frac{5}{3} = -1{,}67$

$z_2 = \frac{1030 - 1000}{30} = 1$

$P(950 \leq X \leq 1030) = P(-1{,}67 \leq Z \leq 1) = 1 - Q(1{,}67) - Q(1)$

= 100 % − 4,745 % − 15,86 % = 79,395 %.

1.0	1.586e-1	1.562e-1	1.538e-1	1.515e-1	1.491e-1	1.468e-1	1.445e-1	1.423e-1	1.400e-1	1.378e-1
1.1	1.356e-1	1.334e-1	1.313e-1	1.292e-1	1.271e-1	1.250e-1	1.230e-1	1.210e-1	1.190e-1	1.170e-1
1.2	1.150e-1	1.131e-1	1.112e-1	1.093e-1	1.074e-1	1.056e-1	1.038e-1	1.020e-1	1.002e-1	9.852e-2
1.3	9.680e-2	9.509e-2	9.341e-2	9.175e-2	9.012e-2	8.850e-2	8.691e-2	8.534e-2	8.379e-2	8.226e-2
1.4	8.075e-2	7.926e-2	7.780e-2	7.635e-2	7.493e-2	7.352e-2	7.214e-2	7.078e-2	6.943e-2	6.811e-2
1.5	6.680e-2	6.552e-2	6.425e-2	6.300e-2	6.178e-2	6.057e-2	5.937e-2	5.820e-2	5.705e-2	5.591e-2
1.6	5.479e-2	5.369e-2	5.261e-2	5.155e-2	5.050e-2	4.947e-2	4.845e-2	4.745e-2	4.647e-2	4.551e-2