Executive summary

There is unequivocal evidence that Earth is warming at an unprecedented rate (NASA, 2018). Climate change is a change in the statistical properties of the climate system that persists for several decades or longer—usually at least 30 years (Australian Academy of Science, 2021). This report aims to analyse temperature data from 1950 to 2021 recorded at Laverton RAAF to find evidence and investigate global warming patterns. It found that temperature shows an increasing sign since 1950, though a short period of decrease was also found in around 1980. A set of various forecast tools for future temperatures are also provided. Yet, the time horizon of data is not long enough to capture a relatively long-term pattern of temperature that seems to occur only every thirty years.

Data features and issues

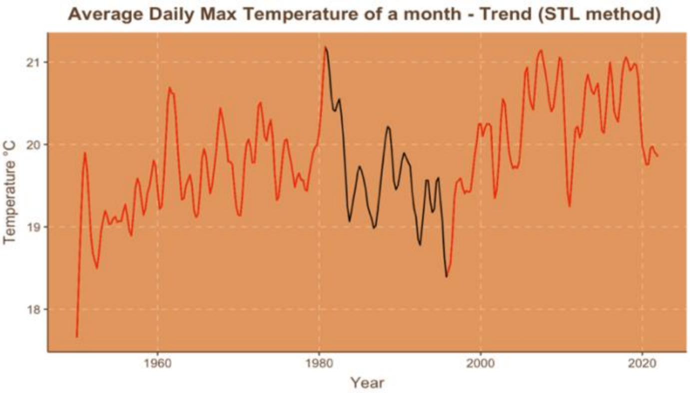
Temperature data has strong seasonality due to its nature. Moreover, percentage change in temperature is relatively small every year. Visualizations of plots for this kind of data are inconvenient and show little insight **(Appendix 1.1)**. It is necessary to break the data into smaller pieces to unveil the mystery behind them. The data used in this report comprises of average daily maximum and minimum, and highest & lowest temperatures of months. The aim is to investigate global warming, the average data aggregating more data samples is more representative for this purpose and thus are mainly used to discover a preserved temperature pattern.

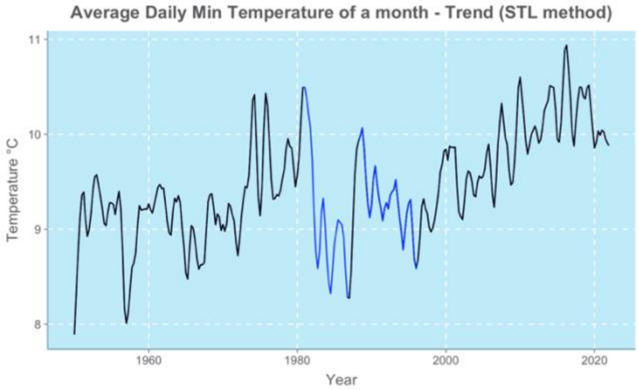
Technical analysis

The report started with STL decomposition which breaks down the data exploration process into the trend, seasonally adjusted, and seasonality components **(Appendix 1.2)**. It then uses findings from these components to construct a regression model to fit the temperature time series data. Model evaluation metrics such as adjusted R squared, CV and BIC are used for selection. Finally, the report also uses the proposed best-performing fit model for forecasting and compares its performance with some other common methods.

STL decomposition – trend

The first component is trend. Below are temperature trend plots for the average daily maximum and minimum temperatures of a month, stripped from seasonal and residual effects. The average

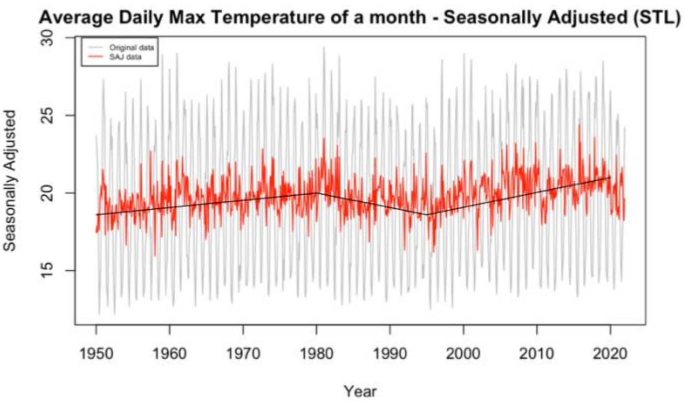


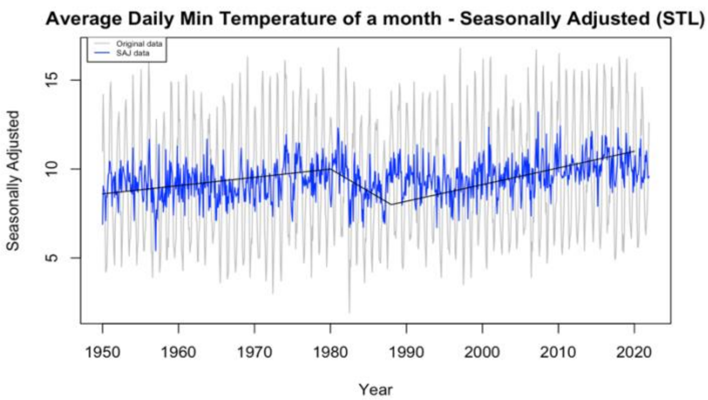


max temperature shows increasing trends, from 1950 to 1980 and 1995 to 2020 denoted by the red lines. However, there was a noticeable decreasing trend from around 1980 to 1995 denoted by the black line. The trend plot for the average minimum temperature shows a similar trend, increasing signs from 1950 to 1980 and 1995 to 2020, but also a changed slope from 1980 to 1995.

STL decomposition – trend with residuals

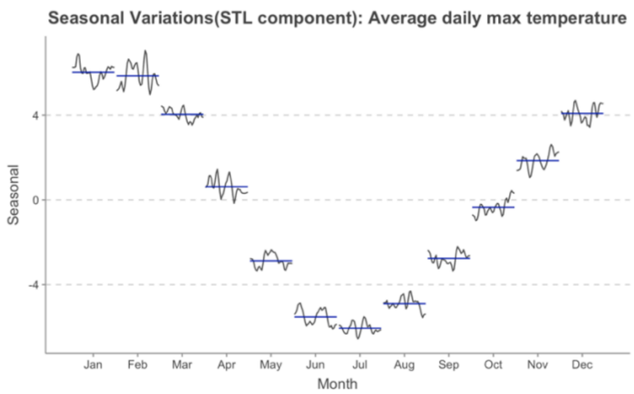
We also look at the temperature trend plots with residuals included (aka. seasonally adjusted). The purpose of investigating a SAJ plot is to understand if these trends persist with consideration of unusual patterns left in residuals. With original data in the background(grey lines) as a reference, we can also detect if the increase is indeed noticeable under the whole range. In fact, both plots show similar patterns, increasing from 1950 to 1980, then decreasing till late 1990, and finally increasing again.

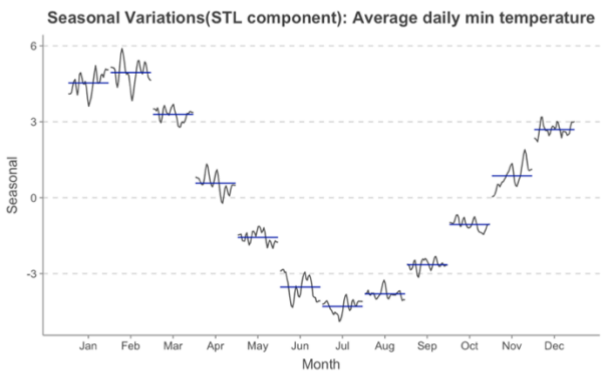




STL decomposition – seasonality

The interest of this report relates to global warming that spans over a larger time horizon. The seasonality change of temperatures year to year is not of particular interest. Nevertheless, below shows the seasonal plots for the average daily max and minimum temperature in the past 70 years.





Consistent with our findings previously, temperatures had a short period of sudden decrease around 1985, shown in the sudden change of slopes in the middle time horizon. On top of that, there appears increasing temperature trends for average daily max temperature for October, November, December, and January (summers) in recent years and a noticeable strong increasing trend in November for average daily minimum temperature. Potential interaction effects of these months with trend dates may be found.

Model Construction – average maximum temperature

Regression of expected mean max temperature has been fitted into three different models: Linear**(Appendix 2.1)**, Cubic **(Appendix 2.2)**, and piecewise **(Appendix 2.3)** respectively. The seasonal effects are included in the table results for concision. **(Appendix 2.4)**

Both linear and piece-wise models suggest that the expected mean temperature/starting temperature was around 25.3 degrees on 1950 January (the reference date). The cubic model has a lower estimation which was about 25 degrees. All models show a statistically positive trend in temperature over time at the lowest 1% significance level, indicating global warming. The cubic model has a steeper slope for the trend in this level coefficient, though this effect is counteracted by the negative quadratic term. For the piece-wise model, the trend effect is slightly higher than in the linear model. Seasonality effects for all models are negative and significant with reference to the January group, which has the highest mean temperature compared to all other months.

Inspired by previous STL decompositions that the "knot" patterns in the data were detected around 1985 and 1995, two piecewise terms are added accordingly to capture the patterns. The data started in 1950, and the coefficient max (trend - 35 * 12, 0) means that the variable effect has been effective since 1985 (35 years * 12 months). Therefore, after 1985, a negative change of the trend slope of -0.007 is expected. After 1995, a positive change in the trend slope of 0.009 was estimated. Not surprisingly, both piecewise variable effects are consistent with the visual representations in the figures before. As suggested also by seasonal plots that the increase of trend shows acceleration for summers in recent years (figure), an interaction term between summer and the trend has been fitted and the coefficient effect is significant at the 10% level.

Lastly, for comparisons of the above models, all measures such as adjusted R square, BIC & CV suggest that the piecewise model fits the best (highest adjusted R square and lowest in the other two), cubic model is the second, and the linear model is the worst in these three. **(Appendix 2.4)**

Model Construction – average minimum temperature

Likewise, regression of expected mean minimum temperature was fitted into the linear **(Appendix 3.1)**, cubic **(Appendix 3.2)**, and piece-wise models **(Appendix 3.3)**. While the baseline expected mean minimum temperature is similar at around 13.45 degrees for all three models, there also appears a positive temperature trend over time, and all significant at 1% level. Again, regressions of minimum temperatures also show signs of global warming. For the piece-wise model, "knots" for temperature plots are captured in the years around 1980 and 1990. A negative change in trend slope was estimated at around 1980 and a positive change was captured at around 1990, consistent with visual representation and both piece-wise variables are significant at 1% level. For the comparison of fit performance, all the statistical measures suggest that the piecewise model is consistently the best, cubic model is the second, and linear is the worst. **(Appendix 3.4)**

Forecast Evaluations - average maximum temperature

A model that fits best with the data does not necessarily have adequate forecasting performance. In the spirit of model training and testing, this section divides the data into two parts, with training data from 1950 to 2009 and testing data from 2010 to 2021. A set of various models has been trained and their forecast performance has been tested accordingly. The results below recorded the models' performance from the worst to the best. The model coefficients are recorded in appendix.

1. Forecast performance for the **cubic model** **(Appendix 4.1)** was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0	1.215	0.947	-0.355	4.784	0.719	0.163	
Test set	-2.000	2.499	2.132	-10.817	11.345	1.618	0.557	0.985

The first model trained was the cubic model, while the fitting performance of a cubic model is adequately great, the higher order on its coefficient makes the forecasting performance inferior.

2. Forecast performance for the **linear model** (Appendix 4.2) was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0	1.253	0.976	-0.379	4.923	0.741	0.212	
Test set	0.175	1.203	0.914	0.411	4.416	0.694	0.314	0.445

The next is the linear model, the forecast performance is much better than the cubic model, evidenced by significant improvements in RMSE, MAE, MPE, MAPE & MASE measures.

3. Forecast performance for the **piece-wise model** (Appendix 4.3) was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0	1.251	0.973	-0.379	4.910	0.739	0.211	
Test set	0.051	1.174	0.902	-0.186	4.395	0.685	0.299	0.435

The last model was the piece-wise model, using a slight parameter tuning process, deleting one of piece-wise variables, forecast performance for this model is even better than the linear model. In fact, forecast performance of this piece-wise model has also been compared to other methods using STL naïve, rwdrift, ets, and arima with the accuracy results included in the appendix **(Appendix 5.1-5.4).** Piece-wise model stands out with its best forecast performance among these all.

Forecast Evaluations- average minimum temperature

1. Forecast performance for **cubic model** (Appendix 4.4) was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-0	1.084	0.857	-1.770	10.298	0.718	0.294	
Test set	10.105	10.272	10.105	50.359	50.359	8.465	0.613	3.854

2. Forecast performance **linear model** (Appendix 4.5) was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0	1.087	0.856	-1.777	10.287	0.717	0.297	
Test set	0.568	1.068	0.848	4.746	8.454	0.711	0.288	0.537

Again, forecast performance of the linear model was much better than the cubic model

3. Forecast performance **piece-wise model** **(Appendix 4.6)** was:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-0	1.070	0.850	-1.751	10.253	0.712	0.290	
Test set	0.139	0.930	0.745	0.164	7.802	0.624	0.291	0.474

The last model was the piece-wise model, using the exact same fit model, forecast performance for the piece-wise model is also better than the linear model. Performance benchmarks for example using STL naïve, rwdrift, ets & Arima methods are included in **(Appendix 6.1-6.4).** The proposed Piece-wise model consistently stands out with its best forecast performance among these all.

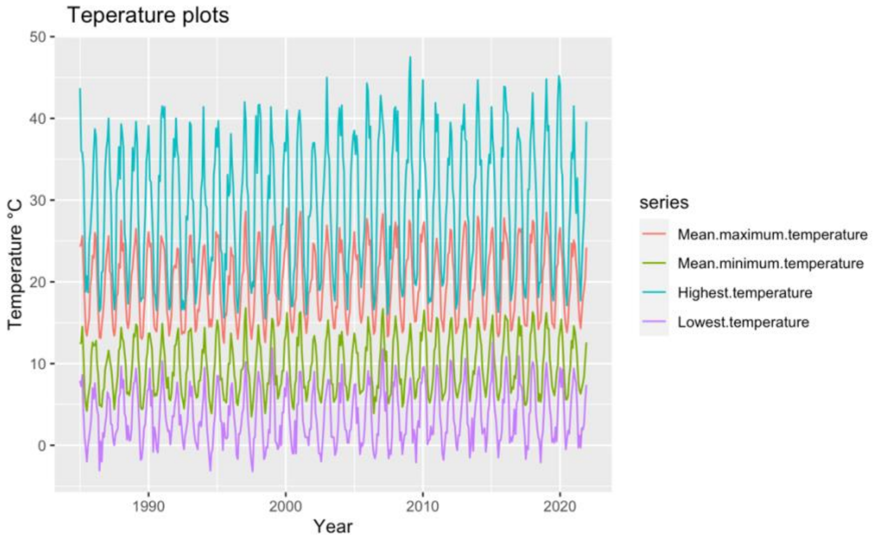
Conclusion and Limitation:

Evidence of global warming has been found under several circumstances in this report. For example, both trend and seasonally adjusted plots under STL decomposition suggest a general increasing trend of temperatures in the past 70 years, despite a short period of decrease in the middle. Alternatively, regression using linear, cubic, and piece-wise models all suggests an estimated positive temperature trend, and all are under the lowest significant level. Piece-wise model that performs the best in both model fitting and forecasting has captured knot findings in line with the STL plot decomposition.

Nevertheless, there are some limitations. Consulting the STL plots and piece-wise variable effect, a decreasing pattern in temperatures seems to occur every 30 years. 2020 seems to be the start of the next decrease and it was the case in the data. However, there has yet been enough evidence for the model proposed in this report to support the sign of such a decrease. Even though temperature in 2020 indeed has started to decrease compared to those previous years, the circumstance may be attributed to exogenous factors such as Covid-19, from which CO_2 emissions decreased by 6.4% in 2020. Secondly, the model construction process in this report uses the approach of forward stepwise, in practice, one may choose backward stepwise which might produce better results in consideration of more predictors, or even the best subsets method, yielding the best model performance, given that the number of predictors other than seasonality is not that large. Thirdly, the Breusch-Godfrey test indicates autocorrelation remains in residuals and has not been captured by the model. Fourthly, the report divides the data into 1950 to 2010 and 2010 to 2020 in the training and testing process, the testing size is a bit smaller than the rule of thumb of 20%, nevertheless, the motivation was to increase the training size to construct a more accurate model for predictions. Lastly, the data used in this report has been collected in only one location at Laverton RAAF. The sample location might not be the temperature circumstances globally.

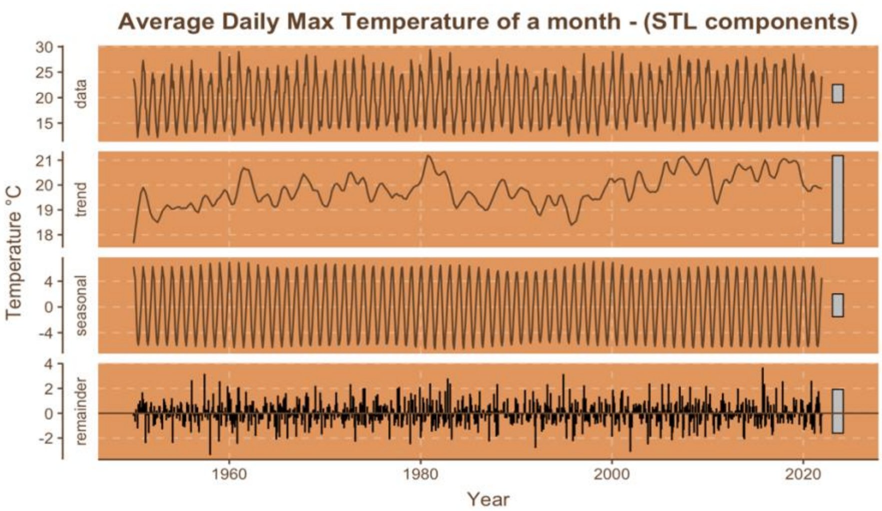
Appendix

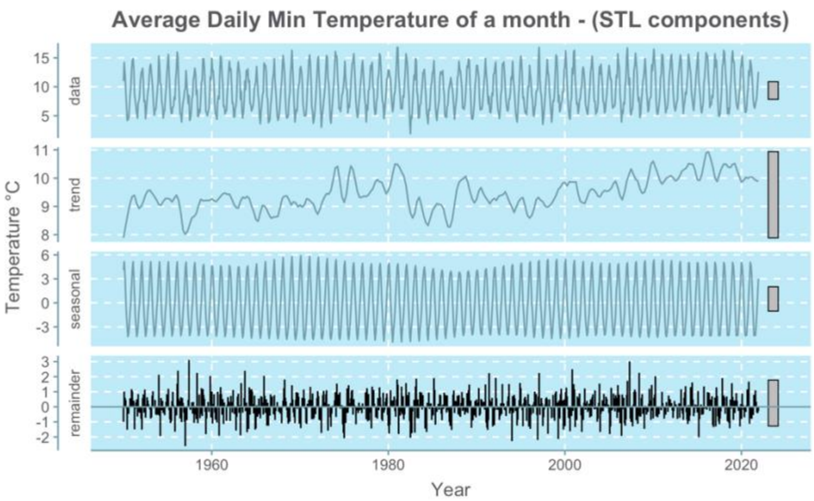
1.1



1.2

STL decompositions full plots – average maximum and minimum





Fitted models – average maximum

2. 1

Linear: $Mean\ max\ temperature_t = 25.295 + 0.001 * trend_t + X_t * \beta_{Seasonlity_t} + e_t$

2. 2

$Cubic: Mean\ max\ tempature_t = 25.984 + 0.006 * trend_t - 0.00002 * trend_t^{\ 2} + 1.26 * 10^{-8} * trend_t + \beta_{Seasonlity_t} + e_t$

2. 3

$Piecewise: Mean\ max\ temperature_t = 25.347 + 0.002 * trend_t - 0.007 * max(trend - 35 * 12, 0) + 0.009 * max(trend - 45 * 12, 0) + 0.001 * (season = summer) * trend + \beta_{Seasonlity_t} + e_t$

2. 4

Regression output results - average maximum

Temperature	**Average max**	**Average max**	**Average max**
Predictors	*Estimates*	*Estimates*	*Estimates*
(Intercept)	25.295 ***	24.984 ***	25.347 ***
trend	0.001 ***	0.006 ***	0.002 ***
season [2]	-0.201	-0.202	-0.202
season [3]	-1.990 ***	-1.992 ***	-1.991 ***
season [4]	-5.418 ***	-5.421 ***	-5.419 ***
season [5]	-8.897 ***	-8.901 ***	-8.898 ***
season [6]	-11.552 ***	-11.557 ***	-11.554 ***
season [7]	-12.068 ***	-12.073 ***	-12.070 ***
season [8]	-10.913 ***	-10.920 ***	-10.916 ***
season [9]	-8.759 ***	-8.767 ***	-8.762 ***
season [10]	-6.363 ***	-6.372 ***	-6.739 ***
season [11]	-4.160 ***	-4.170 ***	-4.537 ***
season [12]	-1.916 ***	-1.926 ***	-2.294 ***
trend^2		-0.000 ***	
trend^3		0.000 ***	
(season == summer) * trend			0.001 *
pmax(trend - 35 * 12, 0)			-0.007 ***
pmax(trend - 45 * 12, 0)			0.009 ***
R^2 / R^2 adjusted	0.921 / 0.920	0.922 / 0.921	0.924 / 0.922
BIC	469.71	468.9	458.8
CV	1.591	1.572	1.547

** p<0.05   ** p<0.01   *** p<0.001*

Fitted models – average minimum

3.1

Linear: $Mean\ min\ temperature_t = 13.40 + 0.0013 * trend_t + X_t * \beta_{Seasonlity_t} + e_t$

3.2

$Cubic: Mean\ min\ tempature_t = 13.46 + 0.0025 * trend_t - 6.7 * 10^{-6} * trend_t^{\ 2} + 6.93 * 10^{-9} * trend_t + \beta_{Seasonlity_t} + e_t$

3.3

$Piecewise: Mean\ min\ temperature_t = 13.45 + 0.0017 * trend_t - 0.006 * max(trend - 30 * 12, 0) + 0.007 * max(trend - 40 * 12, 0) + \beta_{Seasonlity_t} + 0.0015 * (season = 11) * trend + e_t$

3.4

Regression output results - average minimum

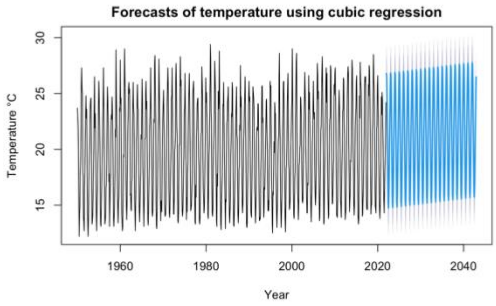
Temperature	**Average min**	**Average min**	**Average min**
Predictors	*Estimates*	*Estimates*	*Estimates*
(Intercept)	13.401 ***	13.459 ***	13.454 ***
trend	0.001 ***	0.003	0.002 ***
season [2]	0.404 *	0.404 *	0.404 *
season [3]	-1.236 ***	-1.237 ***	-1.237 ***
season [4]	-3.950 ***	-3.951 ***	-3.951 ***
season [5]	-6.093 ***	-6.095 ***	-6.094 ***
season [6]	-8.052 ***	-8.055 ***	-8.054 ***
season [7]	-8.825 ***	-8.828 ***	-8.827 ***
season [8]	-8.326 ***	-8.330 ***	-8.328 ***
season [9]	-7.158 ***	-7.162 ***	-7.161 ***
season [10]	-5.573 ***	-5.578 ***	-5.576 ***
season [11]	-3.661 ***	-3.666 ***	-4.334 ***
season [12]	-1.822 ***	-1.827 ***	-1.825 ***
trend^2		-0.000	
trend^3		0.000 *	
(season == 11) * trend			0.002 **
pmin(trend - 30 * 12, 0)			-0.006 ***
pmin(trend - 40 * 12, 0)			0.008 ***
R^2 / R^2 adjusted	0.898 / 0.897	0.900 / 0.898	0.903 / 0.901
BIC	210.995	207.169	190.466
CV	1.179	1.161	1.133

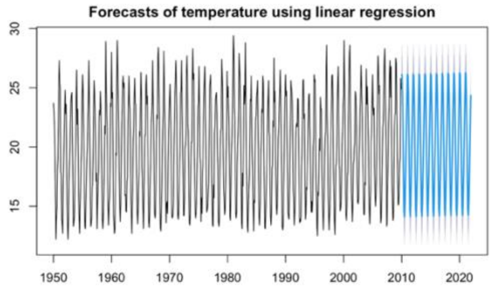
* $p<0.05$ ** $p<0.01$ *** $p<0.001$

Forecasting models for both average maximum and minimum

4. 1

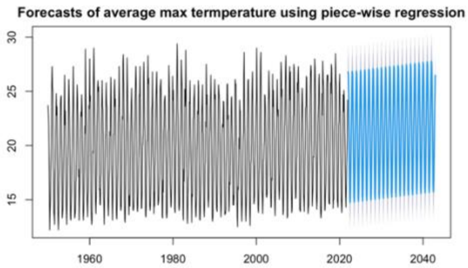
$$Cubic: Mean\ max\ tempature_t = 24.51 + 0.014 * trend_t - 0.000047 * trend_t^{\ 2} + 4.32 * 10^{-8} * trend_t + \beta_{Seasonlity_t} + e_t$$





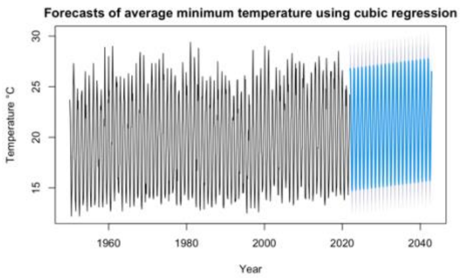
4. 2

$$\text{Linear: } Mean\ max\ temperature_t = 25.29 + 0.00129 * trend_t + X_t * \beta_{Seasonlity_t} + e_t$$



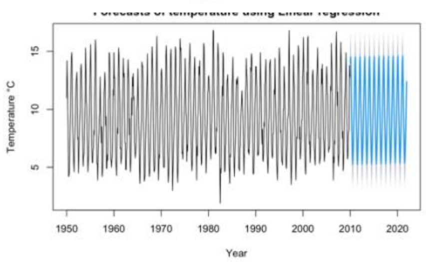
4. 3

$$Piece - wise: Mean\ max\ temperature_t = 25.4 + 0.0068 * trend_t - 0.000839 * max(trend - 35{,}0) + \beta_{Seasonlity_t} + e_t$$



4. 4

$$Cubic: Mean\ min\ tempature_t = 13.7 + 0.004 * trend_t - 0.000013 * trend_t^{\ 2} + 1.16 * 10^{-8} * trend_t + \beta_{Seasonlity_t} + e_t$$

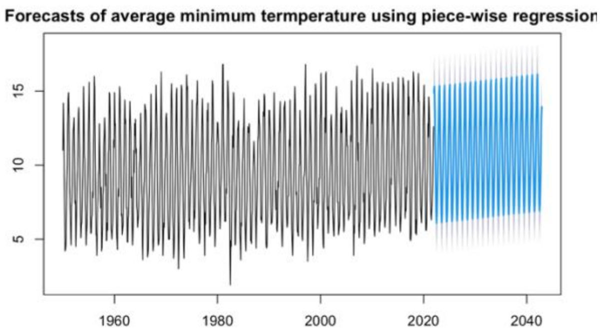


4. 5

$$\text{F5: Linear: } Mean\ min\ temperature_t = 13.44 + 0.00078 * trend_t + X_t * \beta_{Seasonlity_t} + e_t$$

4. 6

$$Piece - wise: Mean\ min\ temperature_t = 13.36 + 0.00168 * trend_t - 0.0059 * max(trend_t - 30{,}0) + 0.0073 * max(trend_t - 40{,}0) + 0.002 * (season_t = 11) * trend + \beta_{Seasonlity_t} + e_t$$



Forecasting evaluation average max, other methods as benchmark

5.1

NAIVE

```
===================================================================
          ME   RMSE   MAE   MPE   MAPE  MASE   ACF1  Theil's U
-------------------------------------------------------------------
Training set 0.004  1.431 1.099 -0.223 5.515 0.835 -0.460
Test set     -0.144 1.188 0.930 -1.203 4.602 0.706 0.287    0.450
-------------------------------------------------------------------
```

5.2

STL+RANDOM WALK WITH DRIFT

```
===================================================================
          ME   RMSE   MAE   MPE   MAPE  MASE   ACF1  Theil's U
-------------------------------------------------------------------
Training set   -0   1.431 1.099 -0.246 5.515 0.835 -0.460
Test set     -0.455 1.273 1.017 -2.810 5.163 0.772 0.297    0.491
-------------------------------------------------------------------
```

5.3

STL+ ETS

```
==================================================================
          ME   RMSE   MAE   MPE   MAPE  MASE  ACF1  Theil's U
------------------------------------------------------------------
Training set 0.026  1.121 0.881 -0.164 4.472 0.668 0.095
Test set     -0.454 1.264 1.011 -2.802 5.138 0.768 0.287   0.491
------------------------------------------------------------------
```

5.4

STL+ ARIMA

```
===================================================================
          ME   RMSE   MAE   MPE   MAPE  MASE   ACF1  Theil's U
-------------------------------------------------------------------
Training set 0.059  1.106 0.863 0.017  4.353 0.655 -0.002
Test set     -0.243 1.202 0.951 -1.713 4.748 0.722 0.285    0.459
-------------------------------------------------------------------
```

Forecasting evaluation average min, other methods as benchmark

6.1
Naive

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.005	1.172	0.926	-1.025	11.176	0.776	-0.459	
Test set	-0.070	0.923	0.759	-2.354	8.386	0.636	0.256	0.494

6.2
STL+RANDOM WALK WITH DRIFT

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0	1.172	0.926	-1.083	11.179	0.776	-0.459	
Test set	-0.412	1.029	0.840	-6.137	9.731	0.704	0.290	0.581

6.3
STL+ ETS

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.013	0.983	0.777	-1.204	9.372	0.651	0.123	
Test set	-0.193	0.941	0.776	-3.713	8.776	0.650	0.256	0.515

6.4
STL+ ARIMA

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.045	0.942	0.741	-0.803	8.865	0.620	0.002	
Test set	0.508	1.049	0.824	4.071	8.227	0.690	0.246	0.522

Reference

NASA. (2018, September 21). Climate Change Evidence: How Do We Know? Climate Change:Vital Signs of the Planet; NASA. https://climate.nasa.gov/evidence/

Australian Academy of Science. (2021). What is climate change? Australian Academy of Science; Australian Academy of Science. https://www.science.org.au/learning/general-audience/science-climate-change/1-what-is-climate-change