

# Assignment -1 in L<sup>A</sup>T<sub>E</sub>X

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Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

- 1) 6
- 2) 12
- 3) 7

Solution:

x = Outcome of the first dice  
y = Outcome of the second dice

$$\Pr(xy \leq n) = \sum_{k=1}^6 \Pr(x = k) F_y\left(\frac{n}{k}\right) \quad (1)$$

$$= \sum_{k=1}^6 \frac{1}{6} \times F_y\left(\frac{n}{k}\right) \quad (2)$$

$$= \frac{1}{6} \times \sum_{k=1}^6 F_y\left(\frac{n}{k}\right) \quad (3)$$

where  $\Pr(X)$  denotes pmf of random variable x  
 $F_Y(y)$  denotes cdf of random variable y

$$F\left(\frac{n}{k}\right) = \begin{cases} 1 & k < \frac{n}{6} \\ \frac{\lfloor \frac{n}{k} \rfloor}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \notin I \\ \frac{(\frac{n}{k})}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \in I \end{cases} \quad (4)$$

where  $\lfloor x \rfloor$  denotes the greatest integer less than or equal to x

$$\Pr(xy \leq n) = \frac{1}{6} \times \sum_{k=1}^6 F_y\left(\frac{n}{k}\right) \quad (5)$$

$$= \frac{1}{6} \times \left[ \sum_{k=1}^{\lfloor \frac{n}{6} \rfloor} F_y\left(\frac{n}{k}\right) + \sum_{k=\lfloor \frac{n}{6} \rfloor + 1}^6 F_y\left(\frac{n}{k}\right) \right] \quad (6)$$

$$= \frac{1}{6} \times \left[ \sum_{k=1}^{\lfloor \frac{n}{6} \rfloor} 1 + \sum_{k=\lfloor \frac{n}{6} \rfloor + 1}^6 F_y\left(\frac{n}{k}\right) \right] \quad (7)$$

$$= \frac{1}{6} \times \left[ \left\lfloor \frac{n}{6} \right\rfloor + \sum_{k=\lfloor \frac{n}{6} \rfloor + 1}^6 F_y\left(\frac{n}{k}\right) \right] \quad (8)$$

Since random variables x,y are independent

$$\Pr(x, y) = \Pr(x) \times \Pr(y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad (9)$$

1) Product = 6

$xy = 6$  for  $(x, y) = \{(1, 6), (2, 3), (3, 2), (6, 1)\}$

No. of events for product to be 6 = 4

$$\Pr(xy = 6) = 4 \times \frac{1}{36} = \frac{1}{9} \quad (10)$$

2) Product = 12

$xy = 12$  for  $(x, y) = \{(2, 6), (3, 4), (4, 3), (6, 2)\}$

No. of events for product to be 12 = 4

$$\Pr(xy = 12) = 4 \times \frac{1}{36} = \frac{1}{9} \quad (11)$$

3) Product = 7

$xy = 7$  for  $(x, y) = \{\}$

$$\Pr(xy = 7) = 0 \quad (12)$$