Assignment -1 in$\LaTeX$

Mathew M Philip
EE22BTECH11211

Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

1) 6
2) 12
3) 7

Solution:

X = Outcome of the first dice
Y = Outcome of the second dice
Z = XY

X	1	2	3	4	5	6
Pr(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

TABLE 3: Pmf of outcome when a single die is tossed

The pmf of Y is also similar to the table above

$$E\left(X\right) = \sum_{X_i=1}^{X_6} X_i \Pr\left(X_i\right) = \frac{1}{6} \times \left(\sum_{X=1}^{6} X\right) \tag{1}$$

$$= \frac{1}{6} \times \frac{6 \times 7}{2} = \frac{7}{2} \tag{2}$$

Similarly

$$E\left(Y\right) = \frac{7}{2} \tag{3}$$

Since X,Y are independent random variables

$$E\left(XY\right) = E\left(X\right) \times E\left(Y\right) \tag{4}$$

$$= \frac{7}{2} \times \frac{7}{2} \tag{5}$$

$$= \frac{49}{4} \tag{6}$$

$$F_Y\left(y\right) = \begin{cases} 0 & y < 1 \\ \frac{y}{6} & 1 \leq y \leq 6 \\ 1 & y > 6 \end{cases} \tag{7}$$

Y	<1	1	2	3	4	5	6	>6
$F_Y(Y)$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1	1

TABLE 3: Cdf of outcome when a single die is tossed

$$F_Z(n) = \sum_{k=1}^{6} \Pr\left(X = k\right) F_Y\left(\left(\frac{n}{k}\right)|k\right) \tag{8}$$

$$= \sum_{k=1}^{6} \frac{1}{6} \times F_Y\left(\left(\frac{n}{k}\right)|k\right) \tag{9}$$

$$= \frac{1}{6} \times \sum_{k=1}^{6} F_Y\left(\left(\frac{n}{k}\right)|k\right) \tag{10}$$

where $\Pr\left(X\right)$ denotes pmf of random variable X
$F_Y(y)$ denotes cdf of random variable Y

$$F\left(\frac{n}{k}\right) = \begin{cases} 1 & k < \frac{n}{6} \\ \frac{\left[\frac{n}{k}\right]}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \notin I \\ \frac{\left(\frac{n}{k}\right)}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \in I \end{cases} \tag{11}$$

where $[x]$ denotes the greatest integer less than or equal to x

$$F_Z\left(n\right) = \frac{1}{6} \times \sum_{k=1}^{6} F_Y\left(\frac{n}{k}\right) \tag{12}$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} F_Y\left(\frac{n}{k}\right) + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{13}$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} 1 + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{14}$$

$$= \frac{1}{6} \times \left\{ \left[\frac{n}{6}\right] + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{15}$$

$$\tag{16}$$

See Fig. 3

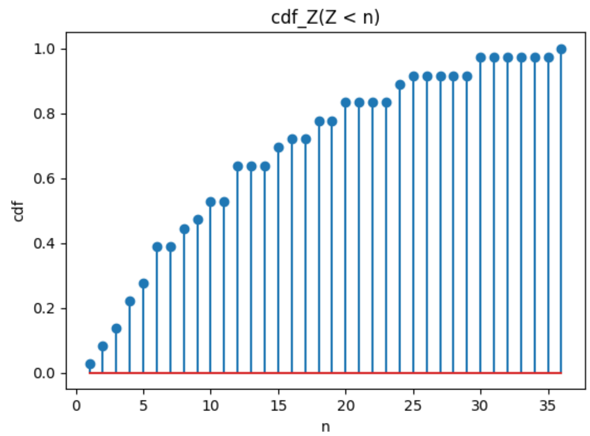

Fig. 3: Plot of cummulative Distribution function

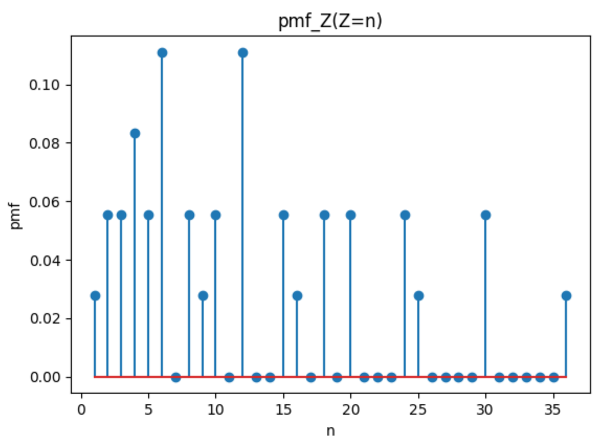

Fig. 3: Plot of Probability Mass Function

$$\Pr(Z = n) = F_z(n) - F_z(n-1) \tag{17}$$

See Fig. 3

Since random variables X,Y are independent

$$\Pr(X, Y) = \Pr(X) \times \Pr(Y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \tag{18}$$

1) Product = 6

X	1	2	3	6
Y	6	3	2	1
Pr(Z=6)	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

TABLE 1: Cases for which product of outcome = 6

No. of events for product to be 6 = 4

$$\Pr(Z = 6) = F_z(6) - F_z(5) \tag{19}$$
$$= 4 \times \frac{1}{36} = \frac{1}{9} \tag{20}$$

2) Product = 12

X	2	3	4	6
Y	6	4	3	2
Pr(Z=12)	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

TABLE 2: Cases for which product of outcome = 12

No. of events for product to be 12 = 4

$$\Pr(Z = 12) = F_z(12) - F_z(11) \tag{21}$$
$$= 4 \times \frac{1}{36} = \frac{1}{9} \tag{22}$$

3) Product = 7
$Z = 7$ for $(X, Y) = \{\}$
There are no events for which product of outcome = 7

$$\Pr(Z = 7) = 0 \tag{23}$$