Assignment -1 inLATEX

Mathew M Philip
EE22BTECH11211

Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

1) 6
2) 12
3) 7

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

$$
F_y(y) = \begin{cases} 1 & y > 6 \\ \frac{[y]}{6} & y \le 6 \cap y \in I \\ \frac{y}{6} & y \le 6 \cap y \notin I \end{cases} \tag{1}
$$

$$
\Pr(XY \le n) = \sum_{k=1}^{6} \Pr(X = k) F_Y\left(\left(\frac{n}{k}\right) | k\right) \tag{2}
$$

$$
= \sum_{k=1}^{6} \frac{1}{6} \times F_Y\left(\left(\frac{n}{k}\right) | k\right) \tag{3}
$$

$$
= \frac{1}{6} \times \sum_{k=1}^{6} F_Y\left(\left(\frac{n}{k}\right) | k\right) \tag{4}
$$

where $\Pr(X)$ denotes pmf of random variable X

$F_Y(y)$ denotes cdf of random variable Y

$$
F\left(\frac{n}{k}\right) = \begin{cases} 1 & k < \frac{n}{6} \\ \frac{\left[\frac{n}{k}\right]}{6} & k \ge \frac{n}{6} \cap \frac{n}{k} \notin I \\ \frac{\left(\frac{n}{k}\right)}{6} & k \ge \frac{n}{6} \cap \frac{n}{k} \in I \end{cases} \tag{5}
$$

where $[x]$ denotes the greatest integer less than or equal to x

$$
\Pr(XY \le n) = \frac{1}{6} \times \sum_{k=1}^{6} F_Y\left(\frac{n}{k}\right) \tag{6}
$$

$$
= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} F_Y\left(\frac{n}{k}\right) + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{7}
$$

$$
= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} 1 + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{8}
$$

$$
= \frac{1}{6} \times \left\{ \left[\frac{n}{6}\right] + \sum_{k=\left[\frac{n}{6}\right]+1}^{6} F_Y\left(\frac{n}{k}\right) \right\} \tag{9}
$$

Since random variables X,Y are independent

$$
\Pr(X, Y) = \Pr(X) \times \Pr(Y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \tag{10}
$$

1) Product = 6
$XY = 6$ for $(X, Y) = \{(1, 6), (2, 3), (3, 2), (6, 1)\}$
No. of events for product to be 6 = 4

$$
\Pr(XY = 6) = 4 \times \frac{1}{36} = \frac{1}{9} \tag{11}
$$

2) Product = 12
$XY = 12$ for $(X, Y) = \{(2, 6), (3, 4), (4, 3), (6, 2)\}$
No. of events for product to be 12 = 4

$$
\Pr(XY = 12) = 4 \times \frac{1}{36} = \frac{1}{9} \tag{12}
$$

3) Product = 7
$XY = 7$ for $(X, Y) = \{\}$

$$
\Pr(XY = 7) = 0 \tag{13}
$$