

Assignment -1 in L^AT_EX

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Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

- 1) 6
- 2) 12
- 3) 7

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

Z = XY

$$\Pr(X) = \begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \leq x \leq 6 \\ 0 & x > 6 \end{cases} \quad (1)$$

The pmf of Y is also same as above.

$$E(X) = \sum_{X_i=1}^{X_6} X_i \Pr(X_i) = \frac{1}{6} \times \left(\sum_{X=1}^6 X \right) \quad (2)$$

$$= \frac{1}{6} \times \frac{6 \times 7}{2} = \frac{7}{2} \quad (3)$$

Similarly

$$E(Y) = \frac{7}{2} \quad (4)$$

Since X,Y are independent random variables

$$E(XY) = E(X) \times E(Y) \quad (5)$$

$$= \frac{7}{2} \times \frac{7}{2} \quad (6)$$

$$= \frac{49}{4} \quad (7)$$

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ \frac{y}{6} & 1 \leq y \leq 6 \\ 1 & y > 6 \end{cases} \quad (8)$$

$$F_Z(n) = \sum_{k=1}^6 \Pr(X = k) F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (9)$$

$$= \sum_{k=1}^6 \frac{1}{6} \times F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (10)$$

$$= \frac{1}{6} \times \sum_{k=1}^6 F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (11)$$

where $\Pr(X)$ denotes pmf of random variable X
 $F_Y(y)$ denotes cdf of random variable Y

$$F\left(\frac{n}{k}\right) = \begin{cases} 1 & k < \frac{n}{6} \\ \frac{[\frac{n}{k}]}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \notin I \\ \left(\frac{n}{k}\right) & k \geq \frac{n}{6} \cap \frac{n}{k} \in I \end{cases} \quad (12)$$

where $[x]$ denotes the greatest integer less than or equal to x

$$F_Z(n) = \frac{1}{6} \times \sum_{k=1}^6 F_Y\left(\frac{n}{k}\right) \quad (13)$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{[\frac{n}{6}]} F_Y\left(\frac{n}{k}\right) + \sum_{k=[\frac{n}{6}]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (14)$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{[\frac{n}{6}]} 1 + \sum_{k=[\frac{n}{6}]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (15)$$

$$= \frac{1}{6} \times \left\{ \left[\frac{n}{6}\right] + \sum_{k=[\frac{n}{6}]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (16)$$

$$\quad (17)$$

See Fig. 3

$$\Pr(Z = n) = F_Z(n) - F_Z(n-1) \quad (18)$$

See Fig. 3

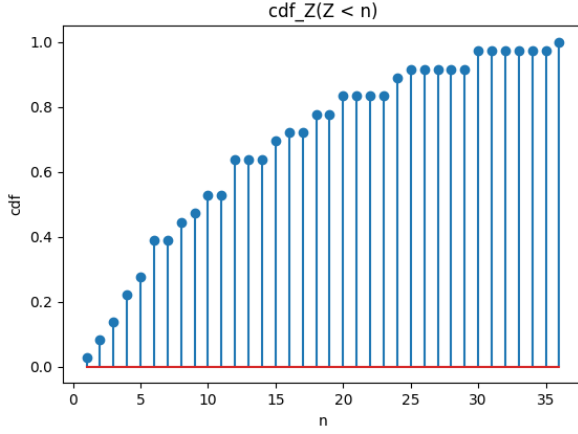


Fig. 3: Plot of cummulative Distribution function

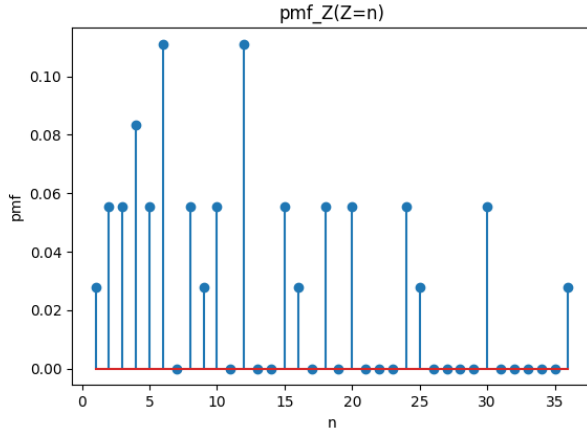


Fig. 3: Plot of Probability Mass Function

Since random variables X, Y are independent

$$\Pr(X, Y) = \Pr(X) \times \Pr(Y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad (19)$$

1) Product = 6

X	1	2	3	6
Y	6	3	2	1
$\Pr(Z=6)$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

TABLE 1: Cases for which product of outcome = 6

No. of events for product to be 6 = 4

$$\Pr(Z = 6) = F_z(6) - F_z(5) \quad (20)$$

$$= 4 \times \frac{1}{36} = \frac{1}{9} \quad (21)$$

2) Product = 12

X	2	3	4	6
Y	6	4	3	2
$\Pr(Z=12)$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

TABLE 2: Cases for which product of outcome = 12

No. of events for product to be 12 = 4

$$\Pr(Z = 12) = F_z(12) - F_z(11) \quad (22)$$

$$= 4 \times \frac{1}{36} = \frac{1}{9} \quad (23)$$

3) Product = 7

$Z = 7$ for $(X, Y) = \{\}$

There are no events for which product of outcome = 7

$$\Pr(Z = 7) = 0 \quad (24)$$