

Assignment -1 in L^AT_EX

Mathew M Philip
EE22BTECH11211

Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

- 1) 6
- 2) 12
- 3) 7

Solution:

X = Outcome of the first dice

Y = Outcome of the second dice

Z = XY

X	1	2	3	4	5	6
Pr(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

The pmf of Y is also similar to the table above

$$E(X) = \sum_{X_i=1}^{X_6} X_i \Pr(X_i) = \frac{1}{6} \times \left(\sum_{X=1}^6 X \right) \quad (1)$$

$$= \frac{1}{6} \times \frac{6 \times 7}{2} = \frac{7}{2} \quad (2)$$

Similarly

$$E(Y) = \frac{7}{2} \quad (3)$$

Since X,Y are independent random variables

$$E(XY) = E(X) \times E(Y) \quad (4)$$

$$= \frac{7}{2} \times \frac{7}{2} \quad (5)$$

$$= \frac{49}{4} \quad (6)$$

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ \frac{y}{6} & 1 \leq y \leq 6 \\ 1 & y > 6 \end{cases} \quad (7)$$

Y	< 1	1	2	3	4	5	6	> 6
F _Y (y)	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1	1

$$\Pr(Z \leq n) = \sum_{k=1}^6 \Pr(X = k) F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (8)$$

$$= \sum_{k=1}^6 \frac{1}{6} \times F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (9)$$

$$= \frac{1}{6} \times \sum_{k=1}^6 F_Y\left(\left(\frac{n}{k}\right) | k\right) \quad (10)$$

where Pr(X) denotes pmf of random variable X
F_Y(y) denotes cdf of random variable Y

$$F\left(\frac{n}{k}\right) = \begin{cases} 1 & k < \frac{n}{6} \\ \frac{[\frac{n}{k}]}{6} & k \geq \frac{n}{6} \cap \frac{n}{k} \notin I \\ \left(\frac{n}{k}\right) & k \geq \frac{n}{6} \cap \frac{n}{k} \in I \end{cases} \quad (11)$$

where [x] denotes the greatest integer less than or equal to x

$$\Pr(Z \leq n) = \frac{1}{6} \times \sum_{k=1}^6 F_Y\left(\frac{n}{k}\right) \quad (12)$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} F_Y\left(\frac{n}{k}\right) + \sum_{k=\left[\frac{n}{6}\right]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (13)$$

$$= \frac{1}{6} \times \left\{ \sum_{k=1}^{\left[\frac{n}{6}\right]} 1 + \sum_{k=\left[\frac{n}{6}\right]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (14)$$

$$= \frac{1}{6} \times \left\{ \left[\frac{n}{6}\right] + \sum_{k=\left[\frac{n}{6}\right]+1}^6 F_Y\left(\frac{n}{k}\right) \right\} \quad (15)$$

Since random variables X,Y are independent

$$\Pr(X, Y) = \Pr(X) \times \Pr(Y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad (16)$$

1) Product = 6

Z = 6 for (X, Y) = {(1, 6), (2, 3), (3, 2), (6, 1)}

No. of events for product to be 6 = 4

$$\Pr(Z = 6) = F_z(6) - F_z(5) \quad (17)$$

$$= 4 \times \frac{1}{36} = \frac{1}{9} \quad (18)$$

2) Product = 12

$Z = 12$ for $(X, Y) = \{(2, 6), (3, 4), (4, 3), (6, 2)\}$

No. of events for product to be 12 = 4

$$\Pr(Z = 12) = F_z(12) - F_z(11) \quad (19)$$

$$= 4 \times \frac{1}{36} = \frac{1}{9} \quad (20)$$

3) Product = 7

$Z = 7$ for $(X, Y) = \{\}$

$$\Pr(Z = 7) = 0 \quad (21)$$