Assignment -1 in$\LaTeX$

Mathew M Philip
EE22BTECH11211

Problem 10.13.3.21:

Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is

1) 6
2) 12
3) 7

Solution:

x = Outcome of the first dice
y = Outcome of the second dice

$$\Pr(xy \le n) = \sum_{k=1}^{6} \Pr(x = k) F_y\left(\frac{n}{k}\right) \tag{1}$$
$$= \sum_{k=1}^{6} \frac{1}{6} \times F_y\left(\frac{n}{k}\right) \tag{2}$$
$$= \frac{1}{6} \times \sum_{k=1}^{6} F_y\left(\frac{n}{k}\right) \tag{3}$$

where $\Pr(X)$ denotes pmf of random variable x
$F_Y(y)$ denotes cdf of random variable y

$$F_y(y) = \begin{cases} 1 & y > 6 \\ \frac{\lfloor y \rfloor}{6} & y \le 6 \end{cases} \tag{4}$$

where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x

$$\Pr(xy \le n) = \frac{1}{6} \times \sum_{k=1}^{6} F_y\left(\frac{n}{k}\right) \tag{5}$$
$$= \frac{1}{6} \times \left[\sum_{k=1}^{\lfloor \frac{n}{6} \rfloor} F_y\left(\frac{n}{k}\right) + \sum_{k=\lfloor \frac{n}{6} \rfloor+1}^{6} F_y\left(\frac{n}{k}\right)\right] \tag{6}$$
$$= \frac{1}{6} \times \left[\sum_{k=1}^{\lfloor \frac{n}{6} \rfloor} 1 + \sum_{k=\lfloor \frac{n}{6} \rfloor+1}^{6} F_y\left(\frac{n}{k}\right)\right] \tag{7}$$
$$= \frac{1}{6} \times \left[\lfloor \frac{n}{6} \rfloor + \sum_{k=\lfloor \frac{n}{6} \rfloor+1}^{6} F_y\left(\frac{n}{k}\right)\right] \tag{8}$$

Since random variables x,y are independent

$$\Pr(x, y) = \Pr(x) \times \Pr(y) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \tag{9}$$

1) Product = 6
   $xy = 6$ for $(x, y) = \{(1, 6), (2, 3), (3, 2), (6, 1)\}$
   No. of events for product to be 6 = 4

$$\Pr(xy = 6) = 4 \times \frac{1}{36} = \frac{1}{9} \tag{10}$$

2) Product = 12
   $xy = 12$ for $(x, y) = \{(2, 6), (3, 4), (4, 3), (6, 2)\}$
   No. of events for product to be 12 = 4

$$\Pr(xy = 12) = 4 \times \frac{1}{36} = \frac{1}{9} \tag{11}$$

3) Product = 7
   $xy = 7$ for $(x, y) = \{\}$

$$\Pr(xy = 7) = 0 \tag{12}$$