MATH 554 - Homework #7 - Mathew Houser \$3:2,5,7,13,15,17

2. Define $f: [-4,0] \rightarrow \mathbb{R}$ by $f(x) = \frac{2 \times ^2 - 18}{x + 3}$ for $x \neq -3$ and f(-3) = -12. Show that f: s continuous at -3.

Then $\forall x \in [-4, 0]$, $x \neq -3$, $\lim_{x \to -3} f(x) = \lim_{x \to -3} \frac{2x^2 - 18}{x + 3} = \lim_{x \to -3} \frac{2(x + 3)(x - 3)}{x + 3} = \lim_{x \to -3} \frac{2x - 6}{x + 3}$

=2(-3)-6=-6-6=-12=f(-3).

Thus by Theorem 3.1, f is continuous at -3.

5. Define $f:(0,1) \to \mathbb{R}$ by $f(x) = \sqrt{x} - \sqrt{\frac{x+1}{x}}$. Can onedefine f(0) tomake f continuous at 0? Explain.

Assume x = 0. Then \x \(\epsilon(0,1) \),

lim f(x) = lim (1/x) - (1im (1/x) . [lim (x+1))

1im (X+1)=1. J=1. lim (大)·1= lim (元).

Then x+0 f(x) = x+0 (\frac{1}{\sigma}) - \frac{1 im}{x+0} (\frac{1}{\sigma}) = 0.

Now suppose f(0) =0. Then x =0 f(x) = 0 = f(0)

Thus by Theorem 3.1, f is continuous at O.

7. Suppose $f: \mathbb{R} \to \mathbb{R}$ is continuous and $f(r) = r^2$ for each rational number r. Determine $f(\sqrt{2})$ and Justify. Consider the sequence of rational numbers $\frac{2}{7}r^3 \frac{2}{7}r^2 = \frac{1}{7}$ where $r_n \in (\sqrt{2} - \frac{1}{7}, \sqrt{2} + \frac{1}{7})$, Clearly $|r_n - \sqrt{2}| < \frac{1}{7}$, hence $\frac{2}{7}r^3 = \frac{2}{7}r^2 = \frac{1}{7}r^2 = \frac{1$

Thus f(12) = 2.



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13. Let $f: D \ni \mathbb{R}$ be continuous at $x \in D$. Prove that there is M > 0 and a neighborhood Q of x_0 such that $|f(x)| \leq M$ $\forall x \in Q \cap D$.

Choose any E > 0, and let $Q = (x_0 - \delta, x_0 + \delta)$ be a neigh borhood of x_0 . Then $\exists \delta > 0$ such that $\forall x \in Q \cap D$, $|f(x) - f(x_0)| < E$. (Since f is continuous) Let $M = |f(x_0)| + E$. Clearly M > 0. Then it follows that $|f(x) - f(x_0)| < E \implies |f(x)| - |f(x_0)| \leq E \implies |f(x_0)| + E = M$. Therefore $\exists M > 0$ and a neighborhood Q of x_0 such that $|f(x)| \leq M$ $\forall x \in Q \cap D$.



15.

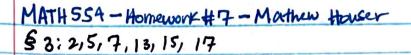
Suppose $f, g: D \rightarrow \mathbb{R}$ are both continuous on D.

Define $h: D \rightarrow \mathbb{R}$ by $h(x) = \max \{f(x), g(x)\}\}$.

Show that h is continuous on D.

Choose E > 0 arbitrarily. Let $E^* = 1$ Then f(x) = 1 and f(x) = 1 and





17. Suppose $f: D \rightarrow \mathbb{R}$ with $f(x) \ge 0 \ \forall x \in D$. Show that if fis continuous at x_0 , then Jf is continuous at x_0 .

Assume f is continuous at x_0 . $\lim_{x \to x_0} f(x) = f(x_0)$.

Then f has a limit at $f(x) = \int_{x \to x_0}^{x \to x_0} f(x) = f(x_0)$.

Then $\lim_{x \to x_0} \int_{x \to x_0}^{x \to x_0} f(x) = \int_{x \to x_0}^{x \to x_0} f(x)$