

- 19. Show that $\cos x = x^3 + x^2 + 4x$ has exactly one root in $[0, \mathbb{7}/2]$.

 Define $f(x) = x^3 + x^2 + 4x \cos x \rightarrow \text{continuous } L$ $f(0) = -1 < 0 < \mathbb{F}^3 + \mathbb{T}^2 + 4\mathbb{T} = f(\mathbb{7}/2); \text{ thus by Bolzano's}$ Theorem, $\exists z \in (0, \mathbb{7}/2)$ such that f(z) = 0. Furthermore $f'(x) = 3x^2 + 2x + 4 + \sin x > 0 \quad \forall x \in [0, \mathbb{7}/2], \text{ thus by}$ Theorem $4.9 \quad f$ is increasing and 1-1 on the interval $[0, \mathbb{7}/2]$.

 Therefore $\cos x = x^3 + x^2 + 4x$ has exactly one root in $[0, \mathbb{7}/2]$.
- 21. Let $f:[0,1] \rightarrow \mathbb{R}$ and $g:[0,1] \rightarrow \mathbb{R}$ be differentiable with f(0) = g(0) and $f'(x) > g'(x) \forall x \in [0,1]$.

 Prove that $f(x) > g(x) \forall x \in (0,1]$.

 Fand g are both continuous and differentiable

 By the cauchy Mean Value Theorem, $\exists c \in (0,1)$ such that $f(x) f(0) = f'(c) = f'(x) > g'(x) \forall x \in [0,1]$, g(x) g(0) = g'(c) = g'(c) = g(x) + g(x) = g(x).

 It follows that f(x) f(0) > 1 = f(x) f(0) > g(x) g(0). g(x) g(0) = f(x) g(0) = g(0),

 we conclude that $f(x) > g(x) \forall x \in (0,1]$
- 23. Use the Mean Value Theorem to prove that $\sqrt{1+h} < 1 + \frac{1}{2}h$ $\sqrt{h} > 0$ Let $f(x) = \sqrt{1+k} \Rightarrow 0$ Such that $f'(t) = \frac{f(h) f(0)}{h 0}$ Then, $f(h) f(0) = \sqrt{1+h} \sqrt{1} = \sqrt{1+h} 1 = f'(t) = 2\sqrt{1+t}$. $\leq \frac{1}{2}$ $= \sqrt{1+h} < \frac{1}{2}h + 1$ $\forall h > 0$.

MATH 554 Homework #10-Mathew House \$4#19, 21,23, 28,32, 33, 36.

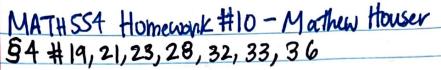
- 28. Prove that $f(x) = 2x^3 + 3x^2 36x + 5$ is 1-1 on the interval [-1,1], is f increasing or decreasing? Is obviously differentiable, Observe that $f'(x) = 6x^2 + 6x 36 = 6(x^2 + x 6) = 6(x + 3)(x 2). \rightarrow f'(x) \neq 0 \ \forall \ x \in [-1,1] \ \text{Thus } f \text{ is } 1-1 \ \text{ and } monotone \ \text{on } [-1,1].$ $f(1) = 42 \ \text{and} \ f(1) = -26 \Rightarrow f \text{ is } decreasing.$
- 32. Use L'Hospital's Rule to find the limits
- (a) $\lim_{x \to 1} \frac{\ln x}{x-1} = 0 \Rightarrow Apply \lim_{x \to 1} \frac{1}{x} = 1$
- (b) $\lim_{x\to 0} \frac{x}{e^x-1} = \frac{O}{O} \rightarrow APPly \quad \lim_{x\to 0} \frac{1}{e^x} = 1$
- (c) $\lim_{x\to 0} \frac{\sin x}{x} = 0 \to Apply \lim_{x\to 0} \frac{\cos x}{x} = 1$
- 33. Use L'Hospital's Rule to find the limit:

 $\lim_{X \to 0} \frac{\chi^2 \sin X}{\sin X - \chi \cos X} = \frac{0}{0}$

 $\lim_{x \to 0} \frac{x^2 \cos x + 2 \times \sin x}{x \sin x} = 0$

 $\lim_{x\to 0} \frac{4x(\cos x + 2\sin x - x^2 \sin x)}{x\cos x + \sin x} = 0$

 $\lim_{x\to 0} \frac{6\cos x - 6x\sin x - x^2\cos x}{2\cos x - x\sin x} = \frac{6}{2} = 3.$



36. Use the Inverse-Function Theorem to derive the formula for the derivative of the inverse of $Sin \times On$ the interval [-7/2, 7/2]. Consider f(x) = Sin (x). It is obviously continuous and differentiable on [-7/2, 7/2] and the derivative $f'(x) = Cos(x) \neq O \ \forall x \in [-7/2, 7/2]$. Thus f is 1-1 and the inverse exists. Now consider g(x) = arc sin(x). Then

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\cos(g(x))}$$
and since $\cos^2 x + \sin^2 x = 1$, it follows that
$$g'(x) = \frac{1}{\sqrt{1 - \sin^2(g(x))}} = \frac{1}{\sqrt{1 - \left(\sin(g(x))\right)^2}}$$

$$= \frac{1}{\sqrt{1-(\sin(\arcsin\cos))^2}} = \frac{1}{\sqrt{1-x^2}}$$