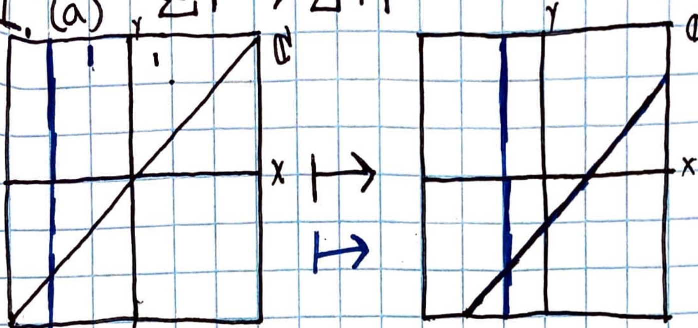
1. (a) $z \longmapsto z+1$



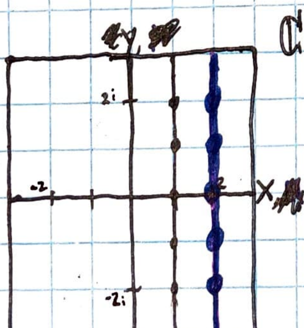
$T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

• = Center of circle.

Line: $x=1$	
$z = 1+iy$	$f(z) = 2+iy$
$1-2i$	$2-2i$
$1-i$	$2-i$
1	2
$1+i$	$2+i$
$1+2i$	$2+2i$

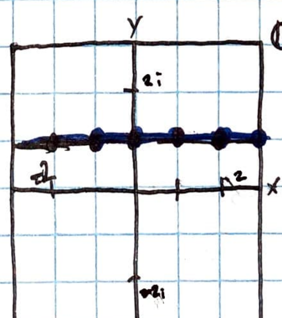
Line: $y=1$	
$z = x+i$	$f(z) = x+1+i$
$-2+i$	$-1+i$
$-1+i$	i
i	$1+i$
$1+i$	$2+i$
$2+i$	$3+i$

Circle: $(x-1)^2+(y-1)^2=1$	
$\lvert z-1-i\rvert^2=1$	$\lvert z-2-i\rvert^2=1$
$2+i$	$3+i$
$1+2i$	$2+2i$
1	2
i	$1+i$
$(1+i)$	$(2+i)$



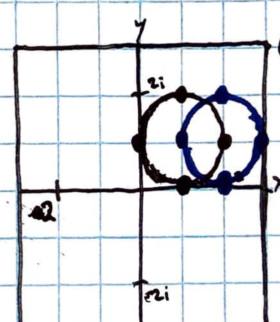
$z = 1+iy$

$\mapsto f(z) = 2+iy$



$z = x+i \mapsto$

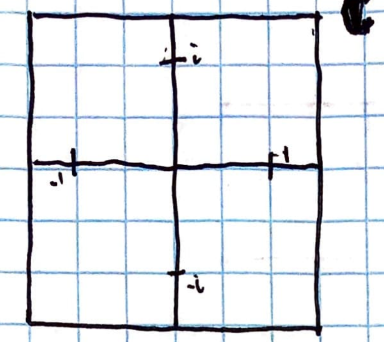
$f(z) = x+1+i$

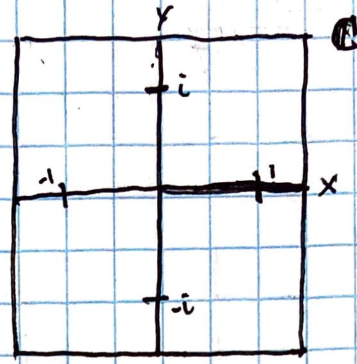


$\lvert z-1-i\rvert^2 = 1 \mapsto$

$\lvert z-2-i\rvert^2 = 1$

1(b). $Z \mapsto -1/Z$





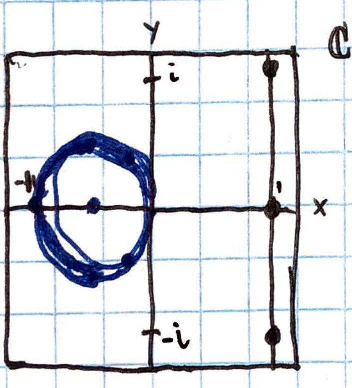
$T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

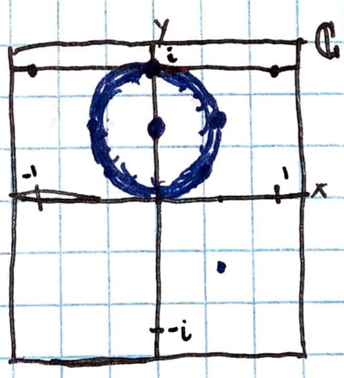
• = center of circle

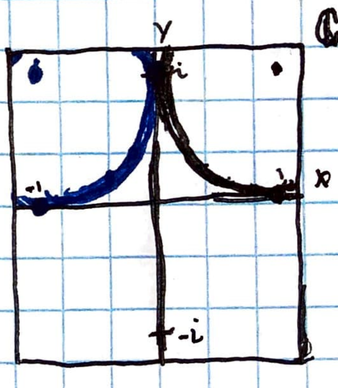
Line $x=1$	
$Z = 1 + yi$	$\lvert z + 1/2 + 1/2 i \rvert^2 = \frac{1}{4}$
$1 - 2i$	$-1/5 - 2/5 i$
$1 - i$	$-1/2 - 1/2 i$
1	-1
$1 + i$	$-1/2 + 1/2 i$
$1 + 2i$	$-1/5 + 2/5 i$

Line $y=1$	
$Z = X + i$	$\lvert z - 1/2 i \rvert^2 = 1/4$
$-2 + i$	$2/5 + 1/5 i$
$-1 + i$	$1/2 + 1/2 i$
i	i
$1 + i$	$-1/2 + 1/2 i$
$2 + i$	$-2/5 + 1/5 i$

Circle: $(x-1)^2 + (y-1)^2 = 1$	
$\lvert Z - 1 - i \rvert^2 = 1$	$\lvert z + 1 - i \rvert^2 = 1$
$2 + i$	$-2/5 + 1/5 i$
$1 + 2i$	$-1/5 + 2/5 i$
1	-1
i	i
$1 + i$	$-1/2 + 1/2 i$







$Z = 1 + yi \mapsto$
$\lvert Z + 1/2 + 1/2 i \rvert^2 = 1/4$
which is the circle
$(x + 1/2)^2 + (y + 1/2)^2 = 1/4$

$Z = X + i \mapsto$
$\lvert Z - 1/2 i \rvert^2 = 1/4$
which is the circle
$x^2 + (y - 1/2)^2 = 1/4$

$\lvert Z - 1 - i \rvert^2 = 1$
which is the circle
$(x-1)^2 + (y-1)^2 = 1$
$\mapsto \lvert Z + 1 - i \rvert^2 = 1$
which is the circle
$(x+1)^2 + (y-1)^2 = 1$

2. Let "T" be the Möbius Transformation

$z \mapsto \frac{(3z+2)}{(4z+3)}$. Then we see:

$T = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$	$T^2 = \begin{bmatrix} 17 & 12 \\ 24 & 17 \end{bmatrix}$	$T^3 = \begin{bmatrix} 99 & 70 \\ 140 & 99 \end{bmatrix}$	$T^{-1} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$
$T(\infty) = 3/4$	$T^2(\infty) = 17/24$	$T^3(\infty) = 99/140$	$T^{-1}(\infty) = -3/4$
$T(-3/4) = \infty$	$T^2(-17/24) = \infty$	$T^3(-99/140) = \infty$	$T^{-1}(-3/4) = \infty$.

3. (a) Given the points $1, i, -1, (3/5)+(4/5)i$;

the cross ratio is given by:

$$(1, i; -1, 3/5+4/5i) = \left(\frac{(1-(-1))}{((1)-(3/5+4/5i))}\right) \Big/ \left(\frac{((i)-(-1))}{((i)-(3/5+4/5i))}\right)$$

$$= \left(\frac{2}{2/5-4/5i}\right) \cdot \left(\frac{-3/5+1/5i}{1+i}\right)$$

$$= \frac{-6/5+2/5i}{6/5-2/5i} = -1$$

All possible values of this cross ratio:

$\{-1, -1, 2, 1/2, -1/2, -2\}$.

(b). Given the points $1, i, -1, (3/5)+(6/5)i$;

the cross ratio is given by:

$$(1, i; -1, 3/5+6/5i) = \frac{(1-(-1))}{(1-(3/5+6/5i))} \Big/ \frac{(i-(-1))}{(i-(3/5+6/5i))}$$

$$= \left(\frac{2}{2/5-6/5i}\right) \cdot \left(\frac{-3/5-1/5i}{i+1}\right)$$

$$= \frac{-6/5-2/5i}{8/5-4/5i} = \frac{3+i}{-4+2i} = \frac{-1-i}{2}$$

All possible values of this Cross Ratio:

$\{\frac{-1-i}{2}, 1-i, \frac{1-i}{2}, 1+i, -i, i\}$.

(Hanh) 4. By computation it is clear that:

$$\lambda = (z_0, z_1; z_2, z_3) = ((z_0 - z_2)(z_1 - z_3))/((z_0 - z_3)(z_1 - z_2))$$
$$= (z_1, z_0; z_3, z_2) = (z_2, z_3; z_0, z_1) = (z_3, z_2; z_1, z_0);$$
$$1/\lambda = (z_0, z_1; z_3, z_2) = ((z_0 - z_3)(z_1 - z_2))/((z_0 - z_2)(z_1 - z_3))$$
$$= (z_1, z_0; z_2, z_3) = (z_2, z_3; z_1, z_0) = (z_3, z_2; z_0, z_1);$$

Similarly, and WLOG, (Mutatis Mutandis)

$$1 - \lambda = (z_0, z_2; z_1, z_3) = (z_1, z_3; z_0, z_2) = (z_2, z_0; z_3, z_1) = (z_3, z_1; z_2, z_0);$$
$$\frac{1}{1-\lambda} = (z_0, z_2; z_3, z_1) = (z_1, z_3; z_2, z_0) = (z_2, z_0; z_1, z_3) = (z_3, z_1; z_0, z_2);$$
$$\lambda/1-\lambda = (z_0, z_3; z_1, z_2) = (z_1, z_2; z_0, z_3) = (z_2, z_1; z_3, z_0) = (z_3, z_0; z_2, z_1);$$
$$\frac{1-\lambda}{\lambda} = (z_0, z_3; z_2, z_1) = (z_1, z_2; z_3, z_0) = (z_2, z_1; z_0, z_3) = (z_3, z_0; z_1, z_2). \blacksquare$$

(work shown on attatched Page).

8(a). Suppose the Möbius Transformation "T" has fixed points $0, 1, \infty$.

Fixed points must satisfy the equation

$$cz^2 + (d-a)z - b = 0.$$

Suppose $c \neq 0$, then $T(\infty) = a/c$, which is a contradiction.

Thus $c = 0$, and we can simplify our equation to

$dz - az + b = 0 \Rightarrow z = \frac{b}{a-d}$. It is now clear that

since $T(0) = 0$, it must be that $b = 0$. Moreover,

since $T(1) = 1$, $a - d = b$, and since we know that

$b = 0$, it follows that $a = d$. This yields the

Matrix $T = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, and $\forall a \in \mathbb{C}$, $\mathrm{rref}(T) = I$. $\blacksquare$

10. Translation: $w = z+b$, for some constant $b \in \mathbb{C}$. Then,

$$(w_1, w_2; w_3, w_4) = \frac{(w_1 - w_3)(w_2 - w_4)}{(w_1 - w_4)(w_2 - w_1)}$$

$$= \frac{((z_1+b)-(z_3+b))((z_2+b)-(z_4+b))}{((z_1+b)-(z_4+b))((z_2+b)-(z_3+b))}$$ (the 'b's cancel)

$$= \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}.$$ ■

Dialation: $w = aZ$, for some constant $a \in \mathbb{C}$, $a \neq 0$. Then

$$(w_1, w_2; w_3, w_4) = \frac{(w_1 - w_3)(w_2 - w_4)}{(w_1 - w_4)(w_2 - w_3)}$$

$$= \frac{(az_1 - az_3)(az_2 - az_4)}{(az_1 - az_4)(az_2 - az_3)}$$ (factor out 'a')

$$= \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}.$$ ■

Reciprication: $w = 1/z$

$$(w_1, w_2; w_3, w_4) = \frac{(w_1 - w_3)(w_2 - w_4)}{(w_1 - w_4)(w_2 - w_3)}$$

$$= \frac{(1/z_1 - 1/z_3)(1/z_2 - 1/z_4)}{(1/z_1 - 1/z_4)(1/z_2 - 1/z_3)}$$ (Distribute)

$$= \frac{\frac{1}{z_1 z_2} - \frac{1}{z_1 z_4} - \frac{1}{z_2 z_3} + \frac{1}{z_3 z_4}}{\frac{1}{z_1 z_2} - \frac{1}{z_1 z_3} - \frac{1}{z_2 z_4} + \frac{1}{z_3 z_4}}$$ (multiply by $\frac{z_1 z_2 z_3 z_4}{z_1 z_2 z_3 z_4}$)

$$= \frac{z_3 z_4 - z_2 z_3 - z_1 z_4 + z_1 z_2}{z_3 z_4 - z_2 z_4 - z_1 z_3 + z_1 z_2}$$ (factor)

$$= \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}.$$ ■