MATH 554 Homework #9-Mathew Houser Chapter 4 # 3,6,11,14,15,16

3. Use the definition to find the derivative of $f(x) = \sqrt{x}$, for x > 0. Is f differentiable at

Zero? Explain. Let $f'(0,\infty) \to \mathbb{R}$ be defined by $f(x) = \sqrt{x}$.

Let $x_0 \in (0, \infty)$ be an accumulation point of $(0, \infty)$, then for each $x \in (0, \infty)$, with $x \neq x_0$ we have

 $T(x) = \frac{f(x) - f(x_0)}{x - x_0} = \frac{\sqrt{x} - \sqrt{x_0}}{x - x_0} \cdot \frac{(\sqrt{x} + \sqrt{x_0})}{(\sqrt{x} + \sqrt{x_0})}$

 $\frac{-(x-x_0)}{(x-x_0)(\sqrt{x}+\sqrt{x_0})} = \frac{1}{\sqrt{x}+\sqrt{x_0}} \quad \text{and} \quad$

 $\lim_{x\to\infty} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} \quad \text{if } x=0 \text{ then the limit is undefined}$ $\lim_{x\to\infty} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} \quad \text{hence } f \text{ is not differentiable at zero.}$

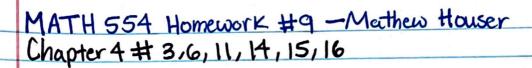
6. Suppose f: (a,b)→R is differentiable at x ∈ (a,b).

Prove that lim f(x+h)-f(x-h) exists and equals f'(x). h→0 Zh Give an example of a function where this limit exists, but the function is not differentiable.

 $\lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h} = \lim_{h\to 0} \frac{f(x+h)-f(x)+f(x)-f(x+h)}{2(x+h-x)}$

= $\lim_{h\to 0} \frac{f(x+h)-f(x)}{2(x+h-x)} + \lim_{h\to 0} \frac{f(x)-f(x-h)}{2(x-(x-h))}$

 $=\frac{1}{2}f'(x)+\frac{1}{2}f'(x)=f'(x).$



- 11. Prove $f'(0,1) \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{2x^2 3x + 6}$ is differentiable on (0,1) and compute the derivative. Let $f(x) = \sqrt{2x^2 - 3x + 6}$. Then (fower Rule Schuin Rule) $f'(x) = \frac{1}{2}(2x^2 - 3x + 6)^{-\frac{1}{2}} \cdot (4x - 3)$ which is defined $\forall x \in (0,1)$ $f'(x) = \frac{4x - 3}{2\sqrt{2x^2 - 3x + 6}}$. Thus f is differentiable on (0,1).
- 14. Suppose $f: R \to R$ is differentiable and define $g(x) = x^2 f(x^3)$. Show that g is differentiable and compute g'. (product Rule)

 9 is composed of differentiable functions, therefore $g'(x) = x^2(3x^2) + (x^3)(2x) = 3x^4 + 2x^4 = 5x^4$.
- 15. Define $f(x) = \sqrt{x+\sqrt{x}+\sqrt{x}}$ for $x \ge 0$. Determine where f is differentiable and compute the derivative. $f(x) = \sqrt{x+\sqrt{x}+\sqrt{x}} = (x+(x+(x)^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}. \text{ Then (Power Rule A Chain Fulle)}$ $f'(x) = \frac{1}{2}(x+(x+(x)^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}} \cdot (1+\frac{1}{2}(x)^{\frac{1}{2}}) \cdot (1)$ $f'(x) = \frac{1}{2\sqrt{x+\sqrt{x}+\sqrt{x}}} \left(1+\frac{1}{2\sqrt{x}+\sqrt{x}}\right) \left(1+\frac{1}{2\sqrt{x}}\right)$

which is defined $\forall x>0$. Hence f is differentiable on $(0,\infty)$



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16. Define f: [0,2] → R by f(x) = √2x-x²,

Show that f satisfies the conditions of

Rolle's Theorem and find c such that f'(c) = 0.

Let g(x) = 2x-x², g(x) is a poly numial so it is

continuous ∀x ∈ R. Further more g(x) ≥ 0 ∀x∈[0,2].

As we showed in chapter 2.3, All [1] [1]

I'm √9(x) = √10, g(x) = 10, f(x). Thus fis continuous

on [0,2] and differentiable. f(o) = 0 = f(2). Also,

f'(x) = 2√2x-x² = 0 => 2 = 2x => x = 1. Then

f'(1) = 2√2x-x² = 0.

