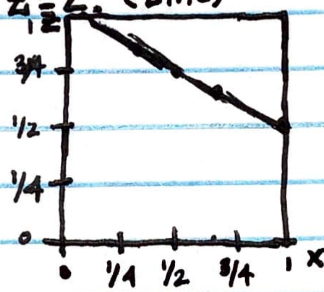
MATH 532 — Homework #2 — Mathew Houser

1. Image Plane: $y=0$
Object Plane: $z=0$
Viewing Point: $(0,1,1)$

(A) Line defined by $z=0$ and $x=2$.
Parametric: $\{(2,t,0) : t\in\mathbb{R}\}$
Equation: $a(0,1,1)+(1-a)(2,t,0) = (2-2a,\ t+a-ta,\ a)$ $(a\in\mathbb{R})$
Solve, $t+a-ta=0$ for $a \rightarrow$ $a=\frac{-t}{t-1}$
Intersection Point: $\left(\frac{2}{1-t}, 0, \frac{-t}{t-1}\right)$
Image: $y=0$ and $x-2z=2$. (Line)

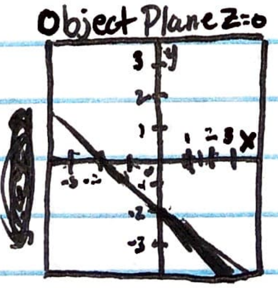
t : (x,	y,	z)
-1 : (1,	0,	1/2)
-2 : (2/3,	0,	2/3)
-3 : (1/2,	0,	3/4)
-4 : (2/5,	0,	4/5)



(B) Diagonal Line defined by $z=0$ and $x+y=-2$.
Parametric: $\{(t,-2-t,0) : t\in\mathbb{R}\}$
Equation: $a(0,1,1)+(1-a)(t,-2-t,0) = (t-ta,\ -2-t+3a+ta,\ a)$ $(a\in\mathbb{R})$
Solve, $-2-t+3a+ta=0$ for $a \rightarrow$ $a=\frac{t+2}{t+3}$
Intersection Point: $\left(\frac{t}{t+3}, 0, \frac{t+2}{t+3}\right)$
Image: Line given by $y=0$ and $3z-x=2$

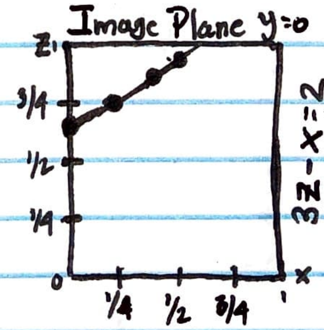
t : (x,	y,	z)
0 : (0,	0,	2/3)
1 : (1/4,	0,	3/4)
2 : (2/5,	0,	4/5)
3 : (1/2,	0,	5/6)

Object Plane $z=0$



$y=-x-2$

Image Plane $y=0$



(C) Circle defined by $z=0$ and $(x+2)^2+(y+1)^2=1$
Let $(x_0, y_0, 0)$ be a point on the circle. Then
Equation: $a(0,1,1)+(1-a)(x_0,y_0,0) = ((1-a)x_0,\ a+(1-a)y_0,\ a)$ $(a\in\mathbb{R})$
Solve, $a+(1-a)y_0=0$, for $a \rightarrow$ $a=\frac{-y_0}{1-y_0}$.
Intersection Point: $\left(\frac{x_0}{1-y_0}, 0, \frac{-y_0}{1-y_0}\right)$
The unique Point mapping to $(x_0', 0, z_0')$ is $\left(\frac{x_0'}{1-z_0'}, \frac{-z_0'}{1-z_0'}, 0\right)$

Image: $\left\{\left(\frac{x_0}{1-y_0}, 0, \frac{-y_0}{1-y_0}\right) : (x_0+2)^2+(y_0+1)^2=1\right\}$

Given $(x+2)^2+(y+1)^2=1$,

Plug in $(x_0', 0, z_0') \rightleftharpoons \left(\frac{x_0'}{1-z_0'}, \frac{-z_0'}{1-z_0'}, 0\right)$.

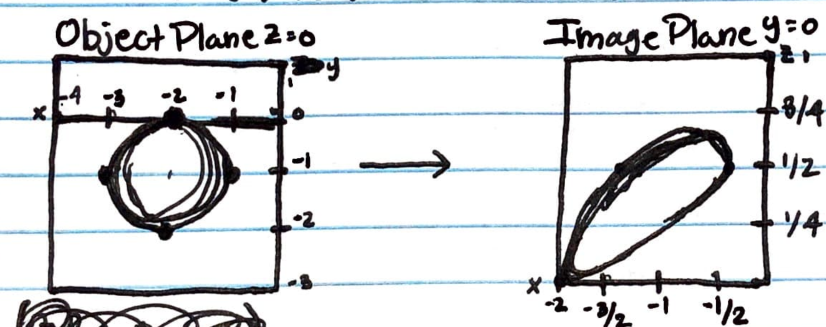
$$\left(\frac{x_0'}{1-z_0'}+2\right)^2+\left(\frac{-z_0'}{1-z_0'}+1\right)^2=1 \rightarrow \left(\frac{x_0'+2-2z_0'}{1-z_0'}\right)^2+\left(\frac{1-2z_0'}{1-z_0'}\right)^2=1$$

$$\rightarrow (x_0'+2-2z_0')^2+(1-2z_0')^2=(1-z_0')^2$$

$$\rightarrow (x_0'^2+4x_0'-4x_0'z_0'+4-8z_0'+4z_0'^2)+(1-4z_0'+4z_0'^2)=(1-2z_0'+z_0'^2)$$

$$\rightarrow x_0'^2-4x_0'z_0'+7z_0'^2+4x_0'-10z_0'+4=0$$

Since $(-4)^2-(4)(1)(7)=16-28=-12<0$, this is an ellipse.



$(-2,-2,0) \longrightarrow (-2/3, 0, 2/3)$

$(-1,-1,0) \longrightarrow (-1/2, 0, 1/2)$

$(-3,-1,0) \longrightarrow (-3/2, 0, 1/2)$

$(-2,0,0) \longrightarrow (-2, 0, 0)$

2. Image Plane: $z=0$

Object Plane: $y=0$

Viewing Point: $(0,1,1)$

(A) Parallel Lines defined by (1) $y=0$ and $x=1$, (2) $y=0$ and $x=-1$

Papametric: (1) $\{(1,0,t) : t\in\mathbb{R}\}$; (2) $\{(-1,0,r) : r\in\mathbb{R}\}$

Equation (1): $a(0,1,1)+(1-a)(1,0,t)=(1-a, a, a+t-ta)$ $(a\in\mathbb{R})$

Solve, $a+t-ta=0$, for $a \rightarrow a=\frac{-t}{1-t}$

Intersection Point (1): $\left(\frac{1}{1-t}, \frac{-t}{1-t}, 0\right)$

Image (1): $y=-x+1$

Equation (2): $b(0,1,1)+(1-b)(-1,0,r)=(b-1, b, b+r-rb)$ $(b\in\mathbb{R})$

Solve, $b+r-rb=0$, for $b \rightarrow b=\frac{-r}{1-r}$

Intersection Point (2): $\left(\frac{-1}{1-r}, \frac{-r}{1-r}, 0\right)$

Image (2): $y = x+1$

Object Plane $y=0$

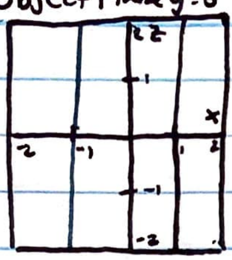
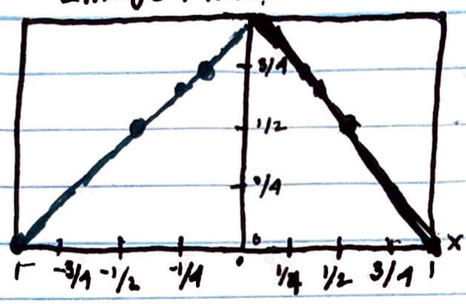


Image Plane $z=0$



$y=0,\ x=1$

$y=0,\ x=-1$

t : (x, y, z)	r : (x, y, z)
0 : (1, 0, 0)	0 : (-1, 0, 0)
-1 : (1/2, 1/2, 0)	-1 : (-1/2, 1/2, 0)
-2 : (1/3, 2/3, 0)	-2 : (-1/3, 2/3, 0)
-3 : (1/4, 3/4, 0)	-3 : (-1/4, 3/4, 0)

The Images of these Lines intersect at (0, 1, 0)

(B) Circle defined by $y=0$ and $x^2+(z-1)^2=1$

Let $(x_0, 0, z_0)$ be a point on the Circle.

Equation: $a(0,1,1)+(1-a)(x_0,0,z_0) = ((1-a)x_0,\ a,\ a+(1-a)z_0)$ $(a \in \mathbb{R})$

Solve, $a+(1-a)z_0 = 0$, for $a \rightarrow a = \frac{-z_0}{1-z_0}$

Intersection Point: $\left(\frac{x_0}{1-z_0}, \frac{-z_0}{1-z_0}, 0\right)$

Unique Point mapping to $(x_0', y_0', 0)$ is $\left(\frac{x_0'}{1-y_0'}, \frac{-y_0'}{1-y_0'}, 0\right)$

Image: $\left\{\left(\frac{x_0}{1-z_0}, \frac{-z_0}{1-z_0}, 0\right) : (x_0)^2 + (z_0-1)^2 = 1\right\}$

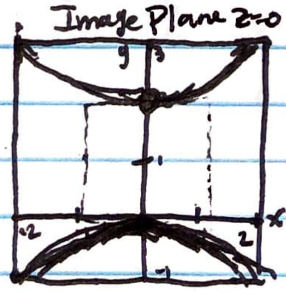
$$\left(\frac{x_0'}{1-y_0'}\right)^2 + \left(\frac{-y_0'}{1-y_0'} - \frac{1-y_0'}{1-y_0'}\right)^2 = \left(\frac{x_0'}{1-y_0'}\right)^2 + \left(\frac{-1}{1-y_0'}\right)^2 = 1$$

$\rightarrow x_0'^2 + 1 = (1-y_0')^2 \rightarrow (y_0'-1)^2 - (x_0')^2 = 1$

This is the Equation of a Hyperbola.

Object Plane $y=0$

Image Plane $z=0$



$(1,0,1) \rightarrow$ undefined

$(-1,0,1) \rightarrow$ undefined

$(0,0,2) \rightarrow (0,2,0) \rightarrow$ vertex

$(0,0,0) \rightarrow (0,0,0) \rightarrow$ vertex

3\. Image Plane: $y=0$

Object Plane: $z=0$

Viewing Point: $(2,1,1)$

(A) Parallel Lines defined by (1) $z=0$ and $x=1$; (2) $z=0$ and $x=-1$

Parametric: (1) $\{(1,t,0) : t\in\mathbb{R}\}$; (2) $\{(-1,r,0) : r\in\mathbb{R}\}$

Equation (1): $a(2,1,1)+(1-a)(1,t,0)=(1+a,\ a+t-ta,\ a)$ $(a\in\mathbb{R})$

Solve, $a+t-ta=0$, for $a \rightarrow$ $a=\frac{-t}{1-t}$

Intersection Point (1): $\left(\frac{2t-1}{t-1},\ 0,\ \frac{t}{t-1}\right)$

Image (1): Line defined by $y=0$ and $x-z=1$

Equation (2): $b(2,1,1)+(1-b)(-1,r,0)=(3b-1,\ b+r-rb,\ b)$ $(b\in\mathbb{R})$

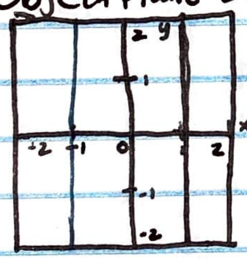
Solve, $b+r-rb=0$, for $b \rightarrow$ $b=\frac{-r}{1-r}$

Intersection Point (2): $\left(\frac{2r+1}{r-1},\ 0,\ \frac{r}{r-1}\right)$

Image (2): Line defined by $y=0$ and $3z-x=1$

These Images Intersect at the Point $(2,0,1)$.

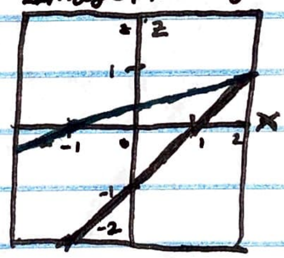
Object Plane $z=0$



(1) $x=1,\ z=0$

(2) $x=-1,\ z=0$

Image Plane $y=0$



(1) $y=0,\ x-z=1$

(2) $y=0,\ 3z-x=1$