Math 527 Homework 3

Mathew Houser - Written Responses and Conclusions are included in the written portion of this assignment.

CE 4.1.2

Find the polynomial of degree 10 that interpolates the function arctan(x) at 11 equally spaced points in the interval [1,6].

$$p_{10}(x) =$$

```
x=linspace(1,6,11);
y=atan(x);
mx=length(x);
my=length(y);
      if mx ~= my, error('Input vectors must be the same length');
end
T=zeros(mx,mx);
T(:,1)=y';
for j = 2:mx
      for i = 1:(mx-j+1)
            T(i,j)=(T(i+1,j-1)-T(i,j-1))/(x(i+j-1)-x(i));
      end
end
a=T(1,:);
fprintf('Coefficient of the Newton Interpolating Polynomial\n\n')
for i = 1:length(a)
      fprintf('%5.0f\t%15.10f\n', i-1,a(i));
end
```

Print the coefficients in the Newton form of the polynomial.

```
Coefficient of the Newton Interpolating Polynomial
       n    Coefficient
       0     0.7853981634
       1     0.3947911197
       2    -0.1460811306
       3     0.0424357369
       4    -0.0099989661
       5     0.0019334873
       6    -0.0003029475
       7     0.0000359312
       8    -0.0000022713
       9    -0.0000002850
      10     0.0000001379
```

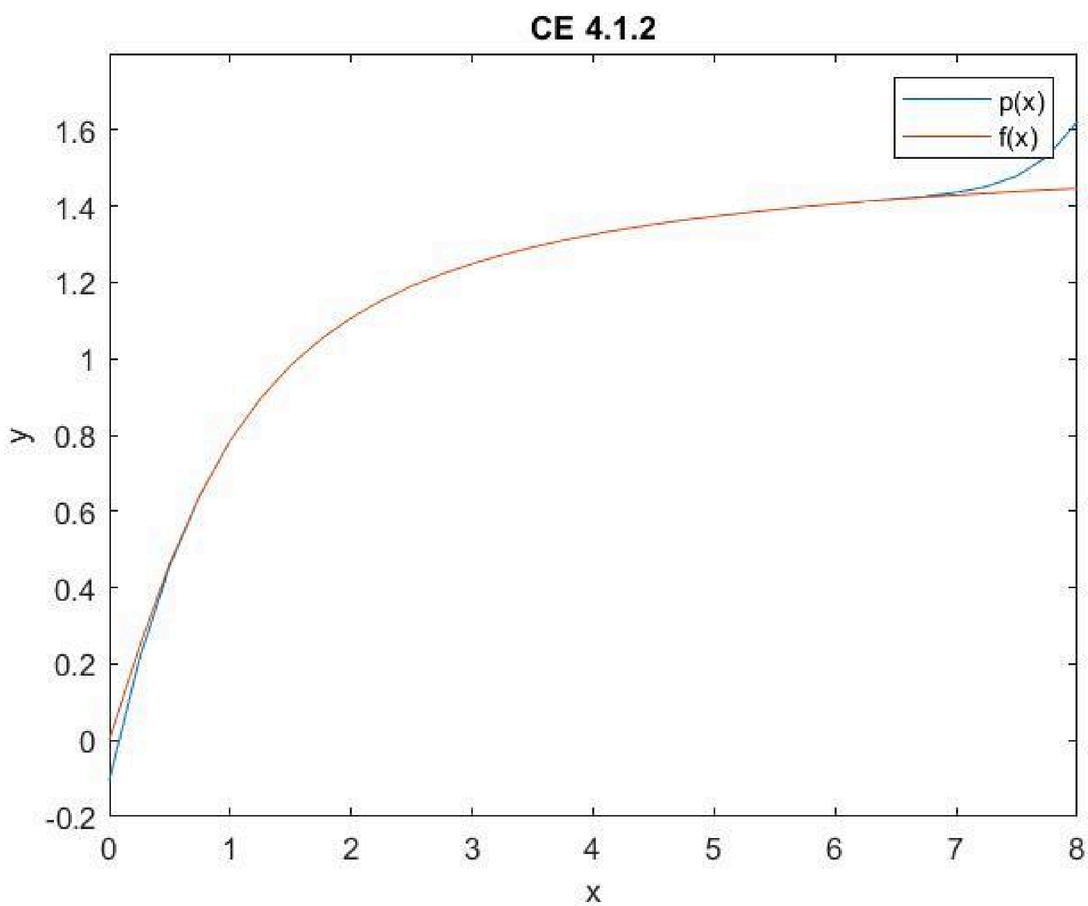
Compute and print the difference between the polynomial and the function at 33 equally spaced points in the interval [0,8].

```
xp=linspace(0,8,33);
fprintf('Approximate Value of y(x)\n')
fprintf('n x p(x)');
fprintf('atan(x) Difference\n');
x2=zeros(1,length(xp));
p2= zeros(1,length(xp));
px= zeros(1,length(xp));
for i = 1:length(xp)
      p=xp(i);
      funval=atan(p);
      m = length(a);
      sum = 0;
      for k = 1:m
            prodx = 1;
            for j = 1 : k-1
                  prodx=prodx*(p-x(j));
            end
            sum=sum+a(k)*prodx;
      end
      pn = sum;
      px(i)=pn;
      p2(i)=funval;
      x2(i)=p;
      Diff=abs(funval-pn);
      fprintf('%5.0f\t%5.10f\t%15.10f\t%15.10f\t%15.10f\n' , i, p, pn,
      funval, Diff);
end
```

n	x	p(x)	atan(x)	Difference
1	0.0000000000	-0.1053163147	0.0000000000	0.1053163147
2	0.2500000000	0.2130806365	0.2449786631	0.0318980266
3	0.5000000000	0.4563962815	0.4636476090	0.0072513275
4	0.7500000000	0.6424669733	0.6435011088	0.0010341355
5	1.0000000000	0.7853981634	0.7853981634	0.0000000000
6	1.2500000000	0.8960899620	0.8960553846	0.0000345774
7	1.5000000000	0.9827937232	0.9827937232	0.0000000000
8	1.7500000000	1.0516462667	1.0516502125	0.0000039459
9	2.0000000000	1.1071487178	1.1071487178	0.0000000000
10	2.2500000000	1.1525728741	1.1525719972	0.0000008769
11	2.5000000000	1.1902899497	1.1902899497	0.0000000000
12	2.7500000000	1.2220250013	1.2220253232	0.0000003219
13	3.0000000000	1.2490457724	1.2490457724	0.0000000000
14	3.2500000000	1.2722975763	1.2722973952	0.0000001811
15	3.5000000000	1.2924966678	1.2924966678	0.0000000000
16	3.7500000000	1.3101937839	1.3101939350	0.0000001511
17	4.0000000000	1.3258176637	1.3258176637	0.0000000000
18	4.2500000000	1.3397058456	1.3397056596	0.0000001860
19	4.5000000000	1.3521273809	1.3521273809	0.0000000000
20	4.7500000000	1.3632997547	1.3633001004	0.0000003457

```
21 5.0000000000    1.3734007669       1.3734007669       0.0000000000
22 5.2500000000    1.3825758554       1.3825748215       0.0000010339
23 5.5000000000    1.3909428270       1.3909428270       0.0000000000
24 5.7500000000    1.3985996839       1.3986055123       0.0000058284
25 6.0000000000    1.4056476494       1.4056476494       0.0000000000
26 6.2500000000    1.4122511123       1.4121410646       0.0001100477
27 6.5000000000    1.4187694760       1.4181469984       0.0006224776
28 6.7500000000    1.4260133110       1.4237179714       0.0022953396
29 7.0000000000    1.4356992391       1.4288992722       0.0067999669
30 7.2500000000    1.4512050950       1.4337301525       0.0174749425
31 7.5000000000    1.4787596104       1.4382447945       0.0405148159
32 7.7500000000    1.5292396036       1.4424730991       0.0867665045
33 8.0000000000    1.6207929289       1.4464413322       0.1743515967
```

```
plot(x2,px);
hold on;
plot(x2,p2);
plot(x,y);
hold off
```



What conclusion can be drawn?

CE 4.2.1

Using 21 equally spaced nodes on the interval [-5,5], find the interpolating polynomial p of degree 20 for the function $f(x) = (x^2 + 1)^{-1}$.

$p_{20}(x) =$

```
f = @(x) (x.^2+1).^(-1);
x = linspace(-5,5,21);
y = f(x);
mx=length(x);
my=length(y);
if mx ~= my, error('Input vectors must be the same length');
end
T=zeros(mx,mx);
T(:,1)=y';
for j = 2:mx
      for i = 1:(mx-j+1)
            T(i,j)=(T(i+1,j-1)-T(i,j-1))/(x(i+j-1)-x(i));
      end
end
a=T(1,:)
```

Print the values of $f(x)$ and $p(x)$ at 41 equally spaces points, including the nodes.

```
xp = linspace(-5,5,41);
fprintf(' n xi f(xi) ' );
fprintf(' p(xi) |f(xi) - p(xi)|\n\n' );
x2=zeros(1,length(xp));
y2=zeros(1,length(xp));
p2=zeros(1,length(xp));
for i = 1: 1: length(xp)
      xpi = xp(i);
      x2(i)=xpi;
      ype = f(xpi);
      y2(i)=ype;
      m = length(a);
      sum = 0;
      for k = 1:m
            prodx = 1;
            for j = 1 : k-1
                  prodx=prodx.*(xpi-x(j));
            end
            sum=sum+a(k).*prodx;
      end
```

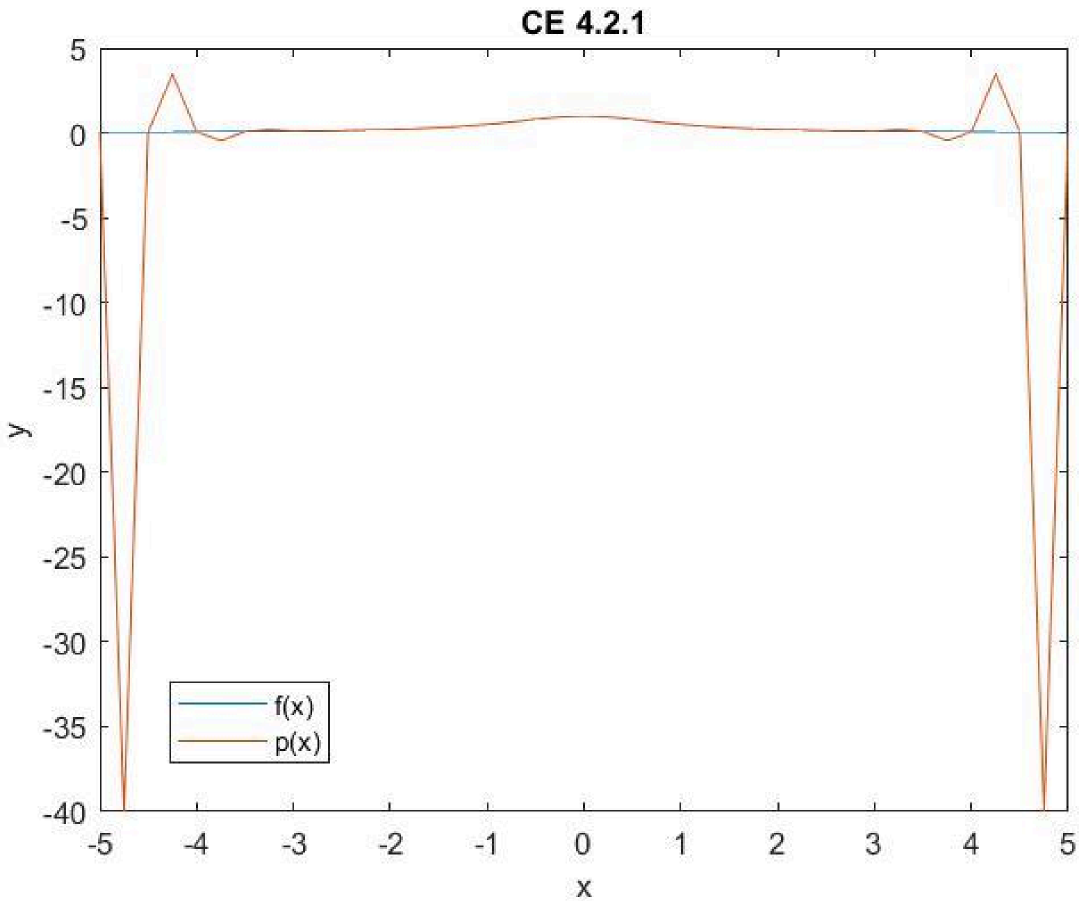
```
        pn = sum;
        ypi = pn;
        p2(i)=ypi;
        fprintf('%5.0f %10.5f %15.10f %15.10f %15.10f\n' , i,xpi,ype,ypi,
        abs(ype-ypi));
end
```

```
  n      x         f(xi)           p(xi)     |f(xi) - p(xi)|
  1 -5.00000 0.0384615385   0.0384615385  0.0000000000
  2 -4.75000 0.0424403183 -39.9524490330 39.9948893513
  3 -4.50000 0.0470588235   0.0470588235  0.0000000000
  4 -4.25000 0.0524590164   3.4549577999  3.4024987835
  5 -4.00000 0.0588235294   0.0588235294  0.0000000000
  6 -3.75000 0.0663900415  -0.4470519607  0.5134420022
  7 -3.50000 0.0754716981   0.0754716981  0.0000000000
  8 -3.25000 0.0864864865   0.2024226157  0.1159361292
  9 -3.00000 0.1000000000   0.1000000000  0.0000000000
 10 -2.75000 0.1167883212   0.0806599934  0.0361283277
 11 -2.50000 0.1379310345   0.1379310345  0.0000000000
 12 -2.25000 0.1649484536   0.1797626299  0.0148141763
 13 -2.00000 0.2000000000   0.2000000000  0.0000000000
 14 -1.75000 0.2461538462   0.2384459337  0.0077079124
 15 -1.50000 0.3076923077   0.3076923077  0.0000000000
 16 -1.25000 0.3902439024   0.3950930537  0.0048491512
 17 -1.00000 0.5000000000   0.5000000000  0.0000000000
 18 -0.75000 0.6400000000   0.6367553359  0.0032446641
 19 -0.50000 0.8000000000   0.8000000000  0.0000000000
 20 -0.25000 0.9411764706   0.9424903797  0.0013139092
 21  0.00000 1.0000000000   1.0000000000  0.0000000000
 22  0.25000 0.9411764706   0.9424903797  0.0013139092
 23  0.50000 0.8000000000   0.8000000000  0.0000000000
 24  0.75000 0.6400000000   0.6367553359  0.0032446641
 25  1.00000 0.5000000000   0.5000000000  0.0000000000
 26  1.25000 0.3902439024   0.3950930537  0.0048491512
 27  1.50000 0.3076923077   0.3076923077  0.0000000000
 28  1.75000 0.2461538462   0.2384459337  0.0077079124
 29  2.00000 0.2000000000   0.2000000000  0.0000000000
 30  2.25000 0.1649484536   0.1797626299  0.0148141763
 31  2.50000 0.1379310345   0.1379310345  0.0000000000
 32  2.75000 0.1167883212   0.0806599934  0.0361283277
 33  3.00000 0.1000000000   0.1000000000  0.0000000000
 34  3.25000 0.0864864865   0.2024226157  0.1159361292
 35  3.50000 0.0754716981   0.0754716981  0.0000000000
 36  3.75000 0.0663900415  -0.4470519607  0.5134420022
 37  4.00000 0.0588235294   0.0588235294  0.0000000000
 38  4.25000 0.0524590164   3.4549577999  3.4024987835
 39  4.50000 0.0470588235   0.0470588235  0.0000000000
 40  4.75000 0.0424403183 -39.9524490330 39.9948893513
 41  5.00000 0.0384615385   0.0384615385  0.0000000000
```

Observe the large discrepancy between $f(x)$ and $p(x)$.

```
plot(x2,y2);
```

```
hold on;
plot(x2,p2);
hold off
```

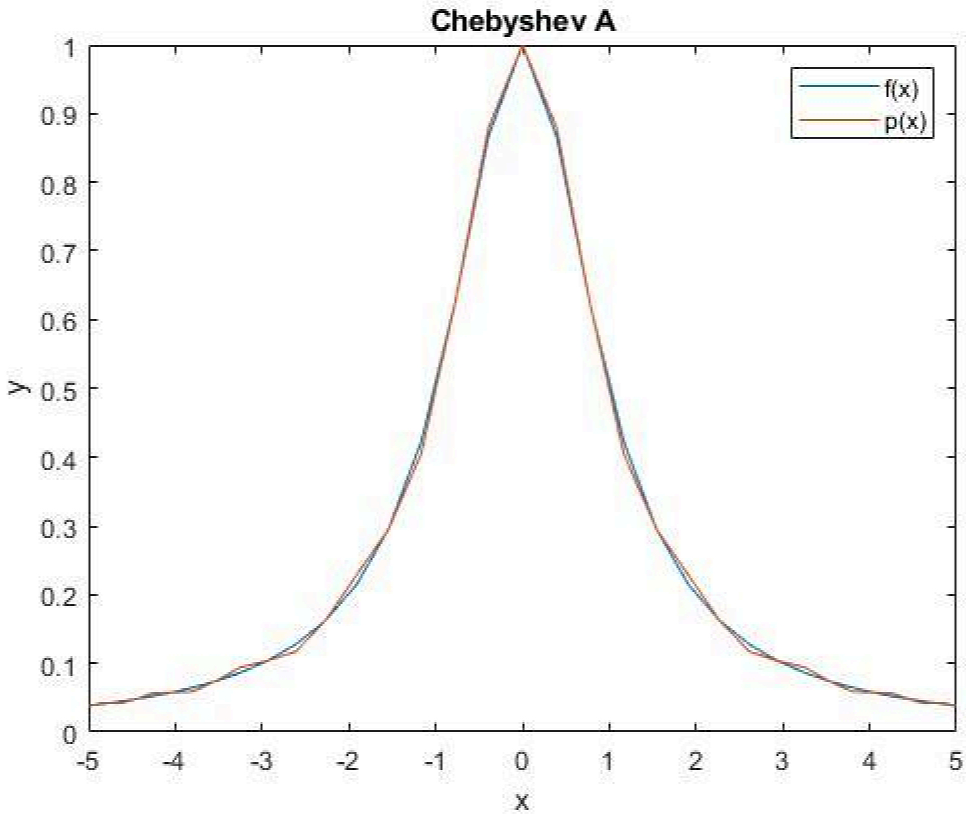


CE 4.2.2

(Continuation) Perform the experiment in the preceding computer problem, using Chebyshev nodes $x_i = 5cos(\frac{i\pi}{20})$, where $0 \leq i \leq 20$.

```
fprintf('x = 5*cos(i*pi/20)\n');
f = @(x) (x.^2+1).^(-1);
i = 0:1:20;
x = 5*cos(i*pi/20);
y = f(x);
mx=length(x);
my=length(y);
if mx ~= my, error('Input vectors must be the same length');
end
T=zeros(mx,mx);
T(:,1)=y';
for j = 2:mx
```

```
        for i = 1:(mx-j+1)
            T(i,j)=(T(i+1,j-1)-T(i,j-1))/(x(i+j-1)-x(i));
        end
    end
    a=T(1,:);
    i = 0: 0.5: 20
    xp = 5*cos(i*pi/20);
    fprintf(' n xi f(xi) ');
    fprintf(' p(xi) |f(xi) - p(xi)|\n\n' );
    x2=zeros(1,length(xp));
    p2=zeros(1,length(xp));
    y2=(1,length(xp));
    for i = 1: 1: length(xp)
        xpi = xp(i);
        x2(i)=xpi;
        ype = f(xpi);
        y2(i)=ype;
        m = length(a);
        sum = 0;
        for k = 1:m
            prodx = 1;
            for j = 1 : k-1
                prodx=prodx*(xpi-x(j));
            end
            sum=sum+a(k)*prodx;
        End
        pn = sum;
        ypi = pn;
        p2(i)=ypi;
        fprintf('%5.0f %10.5f %15.10f %15.10f %15.10f\n', i,xpi,ype,ypi,
        abs(ype-ypi));
    end
    plot(x2,y2);
    hold on;
    plot(x2,p2);
    hold off;
```



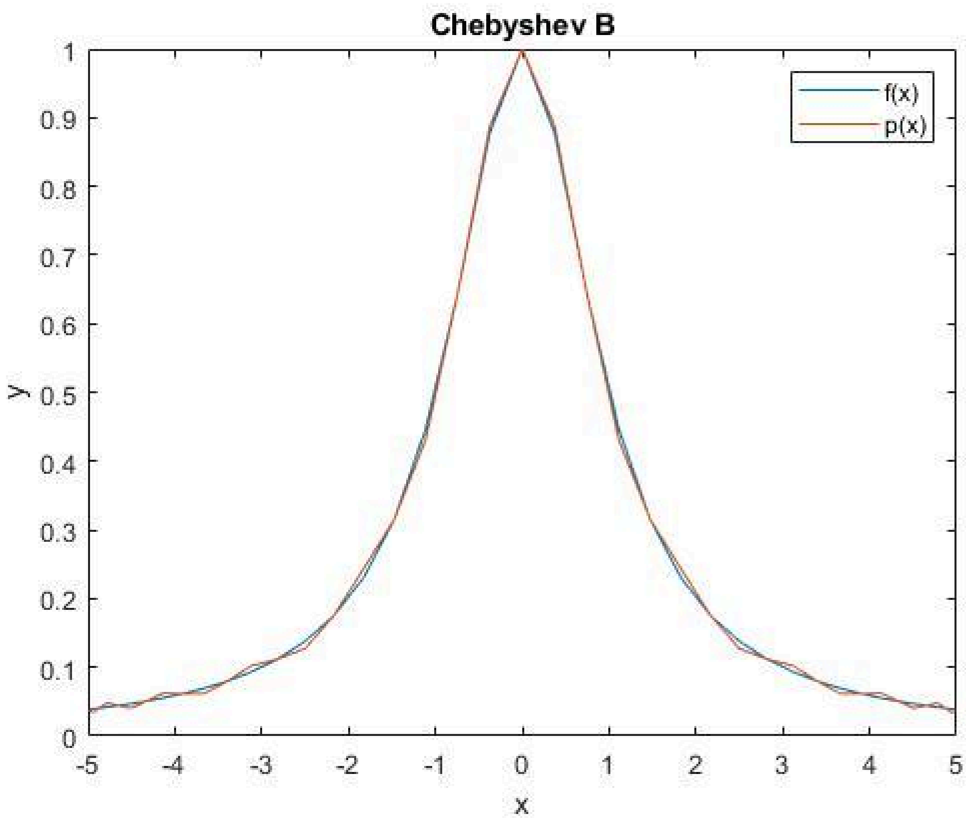
n	xi	f(xi)	p(xi)	|f(xi) - p(xi)|
1	5.00000	0.0384615385	0.0384615385	0.0000000000
2	4.98459	0.0386905504	0.0381324095	0.0005581409
3	4.93844	0.0393883673	0.0393883673	0.0000000000
4	4.86185	0.0405883994	0.0422876413	0.0016992419
5	4.75528	0.0423500690	0.0423500690	0.0000000000
6	4.61940	0.0447650923	0.0418461131	0.0029189792
7	4.45503	0.0479678063	0.0479678063	0.0000000000
8	4.26320	0.0521515616	0.0564366000	0.0042850384
9	4.04508	0.0575946877	0.0575946877	0.0000000000
10	3.80203	0.0647021757	0.0588090551	0.0058931206
11	3.53553	0.0740740741	0.0740740741	0.0000000000
12	3.24724	0.0866208157	0.0945103018	0.0078894861
13	2.93893	0.1037636361	0.1037636361	0.0000000000
14	2.61249	0.1277935877	0.1172934145	0.0105001732
15	2.26995	0.1625306848	0.1625306848	0.0000000000
16	1.91342	0.2145386292	0.2285279626	0.0139893334
17	1.54508	0.2952214653	0.2952214653	0.0000000000
18	1.16723	0.4232950352	0.4055736993	0.0177213359
19	0.78217	0.6204268545	0.6204268545	0.0000000000
20	0.39230	0.8666294216	0.8791312171	0.0125017954
21	0.00000	1.0000000000	1.0000000000	0.0000000000
22	-0.39230	0.8666294216	0.8791312171	0.0125017954
23	-0.78217	0.6204268545	0.6204268545	0.0000000000
24	-1.16723	0.4232950352	0.4055736993	0.0177213359
25	-1.54508	0.2952214653	0.2952214653	0.0000000000

26	-1.91342	0.2145386292	0.2285279626	0.0139893334
27	-2.26995	0.1625306848	0.1625306848	0.0000000000
28	-2.61249	0.1277935877	0.1172934145	0.0105001732
29	-2.93893	0.1037636361	0.1037636361	0.0000000000
30	-3.24724	0.0866208157	0.0945103018	0.0078894861
31	-3.53553	0.0740740741	0.0740740741	0.0000000000
32	-3.80203	0.0647021757	0.0588090551	0.0058931206
33	-4.04508	0.0575946877	0.0575946877	0.0000000000
34	-4.26320	0.0521515616	0.0564366000	0.0042850384
35	-4.45503	0.0479678063	0.0479678063	0.0000000000
36	-4.61940	0.0447650923	0.0418461131	0.0029189792
37	-4.75528	0.0423500690	0.0423500690	0.0000000000
38	-4.86185	0.0405883994	0.0422876413	0.0016992419
39	-4.93844	0.0393883673	0.0393883673	0.0000000000
40	-4.98459	0.0386905504	0.0381324095	0.0005581409
41	-5.00000	0.0384615385	0.0384615385	0.0000000000

(Continuation) Perform the experiment in the preceding computer problem, using Chebyshev nodes $x_i = 5cos(\frac{(2i+1)\pi}{42})$, where $0 \le i \le 20$.

```
fprintf('x = 5*cos((2.*i+1)*pi/42)\n');
f = @(x) (x.^2+1).^(-1);
i = 0:1:20;
x = 5*cos((2.*i+1)*pi/42);
y = f(x);
mx=length(x);
my=length(y);
if mx ~= my, error('Input vectors must be the same length');
end
T=zeros(mx,mx);
T(:,1)=y';
for j = 2:mx
    for i = 1:(mx-j+1)
        T(i,j)=(T(i+1,j-1)-T(i,j-1))/(x(i+j-1)-x(i));
    end
end
a=T(1,:);
i = 0: 1: 40;
xp = 5*cos((i+1)*pi/42);
fprintf('     n          xi           f(xi)             ');
fprintf('  p(xi)      |f(xi) - p(xi)|\n\n' );
x3=zeros(1,length(xp));
p3=zeros(1,length(xp));
y3=zeros(1,length(xp));
for i = 1: 1: length(xp)
      xpi = xp(i);
      x3(i)=xpi;
      ype = f(xpi);
      y3(i)=ype;
      m = length(a);
```

```
        sum = 0;
        for k = 1:m
              prodx = 1;
              for j = 1 : k-1
                    prodx=prodx*(xpi-x(j));
              end
              sum=sum+a(k)*prodx;
        End
        pn = sum;
        ypi = pn;
        p3(i)=ypi;
        fprintf('%5.0f   %10.5f  %15.10f  %15.10f  %15.10f\n',
        i,xpi,ype,ypi, abs(ype-ypi));
  end
  plot(x3,y3);
  hold on
  plot(x3,p3);
  hold off
```



n	xi	f(xi)	p(xi)	|f(xi) - p(xi)|
1	4.98602	0.0386691841	0.0386691841	0.0000000000
2	4.94415	0.0393009770	0.0333094413	0.0059915357
3	4.87464	0.0403842793	0.0403842793	0.0000000000
4	4.77786	0.0419674603	0.0481503183	0.0061828580
5	4.65437	0.0441244960	0.0441244960	0.0000000000
6	4.50484	0.0469624179	0.0404390325	0.0065233854

7	4.33013	0.0506329114	0.0506329114	0.0000000000
8	4.13119	0.0553502590	0.0624010531	0.0070507941
9	3.90916	0.0614193583	0.0614193583	0.0000000000
10	3.66526	0.0692802590	0.0614503446	0.0078299144
11	3.40086	0.0795806176	0.0795806176	0.0000000000
12	3.11745	0.0932967383	0.1022650071	0.0089682689
13	2.81660	0.1119415032	0.1119415032	0.0000000000
14	2.50000	0.1379310345	0.1272983041	0.0106327304
15	2.16942	0.1752425254	0.1752425254	0.0000000000
16	1.82671	0.2305820193	0.2435698520	0.0129878327
17	1.47378	0.3152569893	0.3152569893	0.0000000000
18	1.11260	0.4468496572	0.4315195529	0.0153301042
19	0.74521	0.6429462667	0.6429462667	0.0000000000
20	0.37365	0.8774895799	0.8875995600	0.0101099801
21	0.00000	1.0000000000	1.0000000000	0.0000000000
22	-0.37365	0.8774895799	0.8875995600	0.0101099801
23	-0.74521	0.6429462667	0.6429462667	0.0000000000
24	-1.11260	0.4468496572	0.4315195529	0.0153301042
25	-1.47378	0.3152569893	0.3152569893	0.0000000000
26	-1.82671	0.2305820193	0.2435698520	0.0129878327
27	-2.16942	0.1752425254	0.1752425254	0.0000000000
28	-2.50000	0.1379310345	0.1272983041	0.0106327304
29	-2.81660	0.1119415032	0.1119415032	0.0000000000
30	-3.11745	0.0932967383	0.1022650071	0.0089682689
31	-3.40086	0.0795806176	0.0795806176	0.0000000000
32	-3.66526	0.0692802590	0.0614503446	0.0078299144
33	-3.90916	0.0614193583	0.0614193584	0.0000000000
34	-4.13119	0.0553502590	0.0624010531	0.0070507941
35	-4.33013	0.0506329114	0.0506329114	0.0000000000
36	-4.50484	0.0469624179	0.0404390325	0.0065233854
37	-4.65437	0.0441244960	0.0441244960	0.0000000000
38	-4.77786	0.0419674603	0.0481503183	0.0061828580
39	-4.87464	0.0403842793	0.0403842793	0.0000000000
40	-4.94415	0.0393009770	0.0333094413	0.0059915357
41	-4.98602	0.0386691841	0.0386691841	0.0000000000