Simulating Digital Circuits Using Wang Cubes [under construction]

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Abstract

This report describes a method by which Wang cubes tiling 3D space can be used to simulate digital circuits, and presents a web-app to run such simulations. It also describes an efficient algorithm used to compute adjacent planes along the z-axis, along with the software architecture and technology used to create the web-app. Common elements of digital circuits (such as clocks, wires, logic gates, *et cetera*), along with the challenges of instantiating them using Wang cubes, are discussed here.

Introduction

The primary goal of this project is to develop software to design and tile R^3 with Wang cubes, which are the 3D analogue for Wang tiles. Since 2D Wang tiles are Turing-complete, 3D Wang tiles possesses the expressive power required to simulate a Von Neumann machine built out of simple circuit elements like clocks, wires, logic-gates and flip-flops.

Movement along the z-axis is treated as equivalent to stepping forward and backward in time, which is treated as a distinct quantity. At each stage, the current plane contains a finite number of tiles, which are used to compute the preceding and succeeding plane in real time. The algorithm to do so is described in this report. A user manual [1], the web-app [2], its source [3] and files describing common circuit elements [4] are available online.

Architecture

A "tile type" refers to an entity that describes the adjacency rules which determine the legality of a tiling. A tile is a combination of a tile type with a position vector (a vector being a 2-tuple of real numbers). All tiles have positions which are aligned with the grid - each number in the 2-tuple is an integer.

A tiling is a collection of tiles in the x-y plane. The function mapping the tiles in a plane tiling to the set of tile types is surjective, but not necessarily injective. The function mapping the tiles in a tiling to the set of all possible positions is surjective, and not injective (since there are an infinite possible number of positions, and the number of tiles is finite).

Instead of using colors, the adjacency rules for tiles are enforced using strings. A significant advantage of using unicode strings instead of RGB 3-tuples is that a string is self-documenting. Another advantage is that strings are easy to tell apart from each other, unlike close shades of colors. Furthermore, the system is designed such that one can use it even if completely color-blind, albeit not as efficiently as if one can perceive colors perfectly.

A Wang file consists of a global set of tile types, a main tiling (on which most of the work is done), a cached tiling (used to implement resets while editing), a collection of sub tilings (which can be copied from and pasted into the main plane tiling) and a global color map that maps strings to colors.

The entire state of the data is stored in the Wang file, and the Wang file can be exported and imported via JSON [15] serialization. This process presents the following challenges:

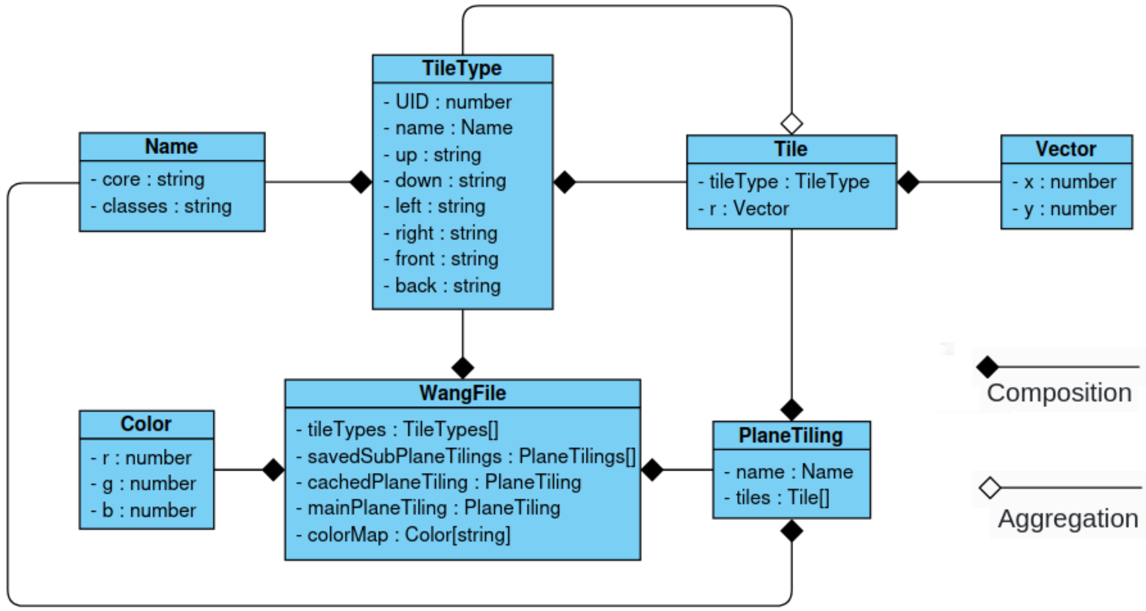

Figure 1: *A UML class diagram describing the ontology underlying the system*

- JSON objects do not preserve references, so if multiple tiles have references to the same tile type object, deserialization creates two distinct but identical tile type objects. This is resolved using a readonly static UID associated with the tile type class, which is used to collapse references to identical tile types into a single object.
- JSON objects carry only data and do not record non-static functions. This is resolved by replacing all non-static functions with static functions bound to the class, that takes an additional argument in place of ”this” that the non-static function operates on.

Technology

The software was designed to be a web-app, since this would allow it to be used by anyone with a browser. This also saves a lot of developer effort that would otherwise be spent in packaging the software for various operating systems and platforms.

The following technology was used in the web-app:

- HTML5 [8], which is the industry standard for web development, used for the high-level structure of the web-app
- NPM [13] (Node package manager), used to install and maintain dependencies
- SASS [9] (Syntactically Awesome StyleSheets), which is a superset of CSS with additional syntactic sugar, used for styling the elements of the web-app
- TypeScript [10], which is a superset of JavaScript with a typing system, used for the describing the algorithms, data structures and UI interactions of the web-app
- three.js [17], a package used to render objects in 3D using a WebGL context, without having to write any WebGL code directly

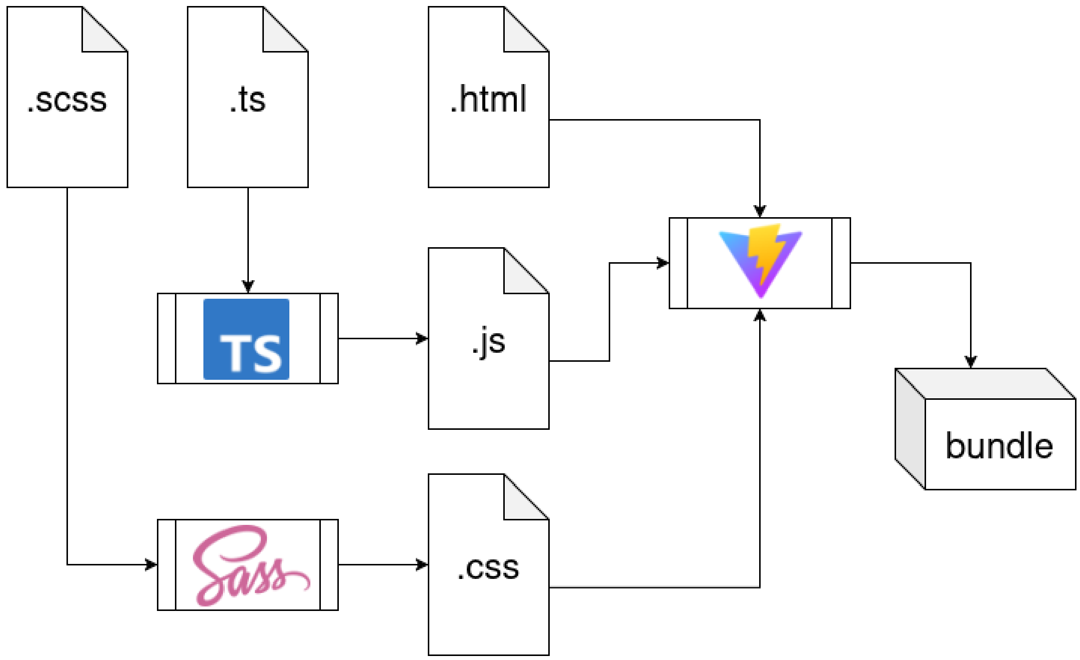

Figure 2: *The build pipeline, from the source files to the bundle*

- Vite [11], used to manage the compilation to CSS/JavaScript, minification [7], tree-shaking [6] and bundling the web-app into a small, portable form
- Vitepress [16], a static site generator [12] used to generate the user manual from markdown [14] source files
- GitHub actions, used as a CI/CD solution to build and publish the web-app incrementally
- GitHub pages, used to host the web-app online

In addition, the icons for the web-app were sourced from Google Material [18] icons, downloaded as SVG images.

LLM disclosure

No LLMs were used in the generation of this report or the user manual for the web-app. However, when writing the code for the web-app, the involvement of LLMs is difficult to quantify and disentangle from the code written by a human, for the following reasons:

- Development often involves copying and pasting code from online forums such as StackOverflow. The source of the content on such forums cannot be guaranteed to be written by a human, unless accompanied with a timestamp preceding the release of popular LLMs like ChatGPT.
- Since search engines integrate AI assistance with their service, searching for a topic results in the user being exposed to an AI-generated answer that appears before every search result. This could subtly influence the thought-process of the user, guiding them toward certain implementations and solutions.

- Popular code-editors like Copilot now integrate agentic coding assistants powered by LLMs, which autocomplete code snippets and generate functions from comments. Though no such editor has been used in the development of this web-app, it is often the case that the coder does not have direct control over what editor they are allowed to use.

Algorithm

Results

Future work

The following circuits can be added to the project:

- D-flip-flops, constructed either as a standalone tile or from two D-latches. This requires a consistent translation of the notion of rising and falling edges to a system with discrete time, which requires research into variants of flip-flops
- shift-registers, built from T-flip-flops. This allows one to build clocks of arbitrary frequencies without creating a large number of clock tiles to represent each state
- Single-tile wires with more than two states, which can be used to transmit signals effectively, when coupled with multiplexers and demultiplexers
- A 7-segment display, using a $3x5$ grid of screen tiles, with each tile having 10 states, though making 150 manually using the current UI is tedious

The following UI improvements can be made to the project:

- An implementation of the pause/play functionality, which is currently left unimplemented due to difficulties involved with managing the animation framerate with the system clock (for security reasons, most browsers do not allow scripts to access the exact time on the system clock)
- A custom console, to display errors and warnings at different levels of verbosity (at present, all such messages show up in the console provided with the developer tools of the browser)
- An input console to allow the programmatic manipulation of the Wang file, to automate tedious tasks
- Collapsible classes in the search results, making a tree-like structure that's easier to navigate than scrolling over all the results
- A graphical representation of the illegality of the tiling (for instance, using red circles to point out areas where the adjacency rules are violated)
- Integrating the main editor with three.js, allowing the user to manipulate the main tiling using orbit controls (currently unimplemented due to problems with mixing dynamic resizing with the WebGL API of three.js)

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Regular papers should be either 8 or 6 pages, including references. Every paper should nicely fill an even number of pages without a lot of wasted white space, so that we can make optimal use of the Proceedings pages and start every paper on a right-hand page.

Short papers, which have a later submission deadline, should be 4 or 2 pages long, including references. Here it is particularly important to focus on just one or two novel ideas and results. Short papers are not a good medium to give tutorial introductions or cursory reviews over a domain that could be the subject of one or more books. Also, this is not the place to give preliminary ideas on new teaching experiments, or to present intuitive hunches how some classical artwork might be analyzed in a novel way. A Bridges paper should only be submitted after the experiments or novel analysis have been performed and when concrete results are available.

The program committee has found that certain types of papers almost always need to be rejected: papers on numerology or work that extracts numbers or ratios from artwork or architecture. A somewhat "fuzzy" search for the Golden Ratio (or any other ratio) will almost always produce some hits (a simple matter of statistics). However, such coincidences do not tell us anything about the method or intent of the creator of those artifacts, unless there is some other compelling evidence, such as auxiliary lines or notes that explicitly state what the artist or architect was doing.

Please write your paper in such a way that attendees with a general education can follow your discourse without the need to look up several references to find out what the main gist is of your paper. Skip lengthy sections on *background* and *previous work* and instead give clear and detailed explanations on your novel contributions, ideally well-supported with diagrams and/or images.

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Figure 3: *An individual figure [**?**]. Make it large enough so that necessary details can be seen. Fine-tune its size, so that you obtain convenient page breaks.*

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The font is *Times New Roman*. The text of the main body, as well as the figure captions use size 11 pt. Section headings, authors, and affiliations are size 12 pt. Paper title is 16 pt. Abstract is 9 pt. Use this template to see what items are printed in **bold face** or in *italics*. Use italics for emphasis, rather than blood face (which reads like yelling.) Mathematical expressions are typeset in the same font as the body text, as in $\sqrt{b^2 - 4ac}$.

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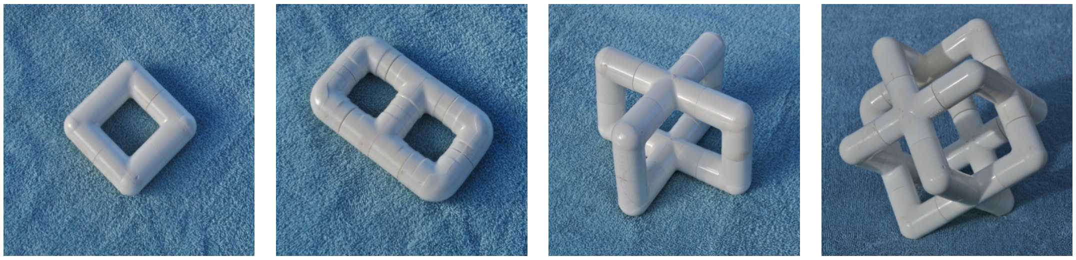
Figure 4 shows how to format and caption a figure with multiple panels. Here we use the *subcaption* package, which enables the `\subcaption` command. Leave the subfigure captions blank to automatically label the panels (a), (b), etc. We ensure that the full figure does not exceed the width of a full line of text by setting the width of each individual panel to be an appropriate proportion of the `\textwidth`.



(a) **(b)** **(c)** **(d)**

Figure 4: *Orientable handle-bodies made from PVC pipe components: (a) simple torus of genus 1, (b) 2-hole torus of genus 2, (c) handle-body of genus 3, (d) handle-body of genus 7.*

Tables are mostly treated just like figures, except that the caption is is placed *above* the table body. This is accomplished by placing the `\caption` before the table data. In this example we reduce the font size in the *tabular* environment so the table fits within the margins. All figures and tables must be referenced and explained in the main text, and they must not bleed into the margins.

Table 1: *Number of Topologically Different Linkings of Two Cube-Frames.*

	2 edge-loops	3 edge-loops	4 edge-loops	5 edge-loops	6 edge-loops	total
2 edge-loops	2sm, 1dm					3
3 edge-loops	0	1sm, 1sp, 2sc, 2dm				6
4 edge-loops	0	2dc	1sm, 10 sc, 2dm, 10dc			25
5 edge-loops	0	0	1dm, 20dc	16sc, 3dm, 1dp, 24dc		65
6 edge-loops	0	0	3dm, 2dc	26dc	14sc	45
					total:	144
	s: swap symmetry		m: mirror symmetry			
	d: different roles for		c: chiral configuration		date: 1/20/2016	
	the two frames		p: pseudo chirality			

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Important words should be capitalized in your paper's title and section headings, while conjunctions (e.g., "and" / "or") and prepositions (e.g., "of" / "with") begin with a lower case letter. Avoid most punctuation (one colon or comma is acceptable). Do not end a lone title or section heading with a period. For section headings use `\section*`. Do not use numbered sections unless you refer to a section by number rather than title in the text. If you must have your sections numbered use the `\section` command. For any Bridges paper, it would probably be excessive to use more than two levels of hierarchy in your document. In all papers the **Abstract**, **Acknowledgments**, and **References** headings are unnumbered sections.

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There is no need to use only the passive voice, often encountered in some formal academic journals. It is perfectly acceptable to use the first person "I" when talking about your own artistic endeavors, and the inclusive, academic "we" works well for more formal statements.

To submit your paper via EasyChair [**?**], you will have to convert it into a PDF (Portable Document Format) file and that file must not exceed 10 MB. Papers will appear in the online Bridges Archives and in print in full color.

Summary and Conclusions

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Acknowledgements

Suggestions on how the clarity and usefulness of this guide could be enhanced would be gladly accepted.

References

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[17] three.js https://threejs.org/

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