Homework 7.

3.105) A group of 8 candidates for 3 local teaching positions consisted of 5 who took paid internships and 3 who enrolled in student teaching programs. All 8 candidates appear to be equally qualified, so 3 are randomly selected to fill the spots. Let Y be the number of internship trained candidates who are hired

a. Does Y have a binomial or hypergeometric distribution? why?

answer -> Y has a hypergeometric distribution because the probability of a candidate who enrolled in an internship being selected for a teaching position is not constant for all 3 open spots. The probability changes based on which type of candidate was selected for the previous open slot.

b.) Find the probability that 2 or more internship trained candidates are hired.

$P(Y \geq 2)$

using R code: $P(Y \geq 2) = 0.714$

double checking

$$P(Y=1) = \frac{\binom{5}{1}\binom{3}{2}}{\binom{8}{3}} = 0.2678$$

$$P(Y=0) = \frac{\cancel{\binom{5}{0}}^{1}\cancel{\binom{3}{3}}^{1}}{\binom{8}{3}} = 0.0178$$

$P(Y \leq 1) = 0.2678 + 0.0178 = 0.2857$

$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.2857 = 0.7142$ ✓

c. Find mean and standard deviation of Y

mean for hyper-geometric distribution

$E(Y) = \frac{nr}{N}$ $n=3$, $r=5$, $N=8$

$= \left(\frac{3 \cdot 5}{8}\right) = 1.875$

standard deviation $= \sqrt{var(Y)}$

Var (Y) for hyper-geometric distribution

$var(Y) = n \cdot \frac{r}{N} \cdot \frac{(N-r)}{N} \cdot \frac{(N-n)}{(N-1)}$

$var(Y) = 3 \cdot \frac{5}{8} \cdot \frac{(8-5)}{8} \cdot \frac{(8-3)}{(8-1)} = 0.502$

Std dev $= \sqrt{var(Y)}$

$= \sqrt{0.502}$

$= 0.708$

Std $= 0.708$

3.122) Customers arrive at a checkout counter in a department store according to a Poison distribution at an average of 7/hr. During a given hour, what are the probabilities that: $\lambda = 7$

a.) no more than 3 customers arrive?
$P(Y \le 3)$
using R code $P(Y \le 3) = 0.0817$

b.) at least 2 customers arrive
$P(Y \ge 2) \to 1 - P(Y \le 1)$
using R code $P(Y \ge 2) = 0.9927$

c.) Exactly 5 five customers 5
$P(Y = 5)$

$$P(Y=5) \to \frac{\lambda^y}{y!} e^{-\lambda} = \frac{7^5}{5!} e^{-7} = 0.1277$$

3.134 Consider a binomial experiment for $n = 20$, $p = 0.05$. Calculate the binomial probabilities for $Y = 0, 1, 2, 3,$ and 4. Calculate the same probabilities by using the Poisson approximation with $\lambda = np$. Compare

$$P(Y) = \binom{n}{y} p^y q^{n-y}$$

$n = 20$
$p = 0.05, q = 0.95$

$$P(0) = \binom{20}{0} 0.05^0 \cdot 0.95^{20}$$

$$P(0) = 1 \cdot 1 \cdot = 0.3585$$

- $P(1) = \binom{20}{1} \cdot 0.05^1 \cdot 0.95^{19}$

$P(1) = 20 \cdot 0.05 \cdot 0.95^{19} = 0.377$

- $P(2) = \binom{20}{2} \cdot 0.05^2 \cdot 0.95^{18}$

$= 190 \cdot 0.0025 \cdot 0.95^{18} = 0.1887$

- $P(3) = \binom{20}{3} \cdot 0.05^3 \cdot 0.95^{17}$

$= 1140 \cdot 1.25E^{-4} \cdot 0.95^{17} = 0.06$

- $P(4) = \binom{20}{4} \cdot 0.05^4 \cdot 0.95^{16}$

$= 4845 \cdot 6.25E^{\cancel{-6}} \cdot 0.95^{16} = 0.0133$

- Poisson Approximation

$\lambda = n \cdot p = 0.05 \cdot 20 = 1$

$P(Y) = \frac{\lambda^y}{y!} e^{-\lambda}$

Binomial Values

$P(0) = \frac{1^0}{0!} e^{-1} = 0.368 \rightarrow 0.3585$

$P(1) = \frac{1^1}{1!} e^{-1} = 0.368 \rightarrow 0.377$

$P(2) = \frac{1^2}{2!} e^{-1} = 0.184 \rightarrow 0.1887$

Binom values

$P(3) = \frac{1^3}{3!} \cdot e^{-1} = 0.0613 \to 0.06$

$P(4) = \frac{1^4}{4!} \cdot e^{-1} = 0.0153 \to 0.0133$

4.11) Suppose Y possesses the density function

$$f(Y) = \begin{cases} cy, & 0 \le Y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

$$\int_0^2 cy\,dy = c\int_0^2 y\,dy = c\left(\frac{y^2}{2}\Big|_0^2\right) =$$

$$= c\left(\frac{(2)^2}{2} - 0\right) = 2c, \text{ set equal to } 1$$

$$2c = 1, \; c = 1/2$$

b. Find F(Y)

$$F(Y) = \int_0^y \frac{1}{2} t\,dt = \frac{1}{2}\int_0^y t\,dt$$

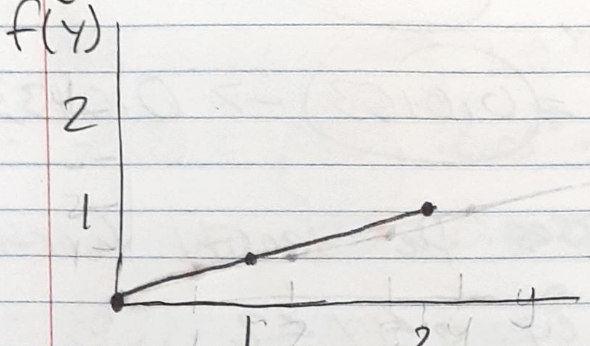
$$= \frac{1}{2}\left(\frac{t^2}{2}\Big|_0^y\right) = \frac{1}{2}\left(\frac{y^2}{2} - 0\right) = \frac{y^2}{4}$$

$$F(Y) = y^2/4$$

c. graph f(Y) and F(Y)

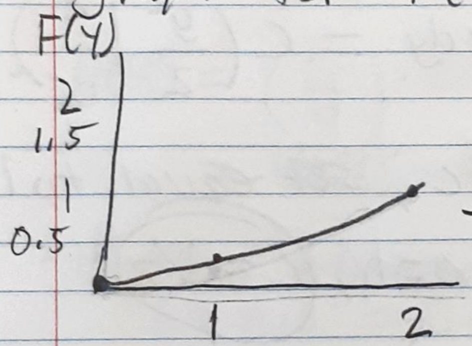
$f(Y) = \frac{1}{2}y$, $F(Y) = y^2/4$

graph for $f(Y) = \frac{1}{2}y$ (PDF) Probability density function



graph for $F(Y) = y^2/4$

$F(1) = 1/4$
$F(2) = 4/4 = 1$



→ curved because function is quadratic.

(CDF) Cumulative distribution function

d.) Use $F(Y)$ to find $P(1 \le Y \le 2)$

$P(a \le Y \le b) = F(b) - F(a)$

$P(1 \le Y \le 2) = F(2) - F(1)$

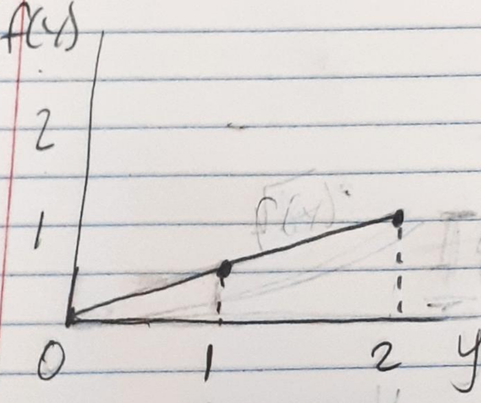
$= \frac{2^2}{4} - \frac{1^2}{4} = 1 - \frac{1}{4} = 0.75$

f.) Use $f(Y)$ and geometry to find $P(1 \leq Y \leq 2)$

$f(y) = \frac{1}{2}y$

$f(1) = 1/2$ $\quad$ $F(1) = 1/4$

$f(2) = 1$ $\quad$ $F(2) = 1$



Area of triangle $= 1/2\, bh$

area for whole triangle $\quad b=2, h=1$

$\text{area}_1 = \cancel{1/2} \cdot \cancel{2} \cdot 1 = 1$ ✓

area for triangle for $Y=0$ to $Y=1$, $b=1, h=0.5$

$\text{area}_2 = \frac{1}{2} \cdot 0.5 \cdot 1 = 1/4$

$\text{area}_T = a_1 - a_2 = 1 - 1/4 = 0.75$

Matt Williams

3/30/2022

PRV

Assignment 7 R code

```
#Author: Matt Williams
#Version: 3/28/2022

# Q 3.105 part b
# p(Y>=2) = 1 - p(y<=1)
#Y: number of people to choose from r
#N: Total number of people
#n: total number of people for the sample
#r: total number of people with a specific attribute

N = 8
n = 3
y = 1
r = 5

1 - phyper(y, r, N-r, n)
```

```
> #Author: Matt Williams
> #Version: 3/28/2022
>
> # Q 3.105 part b
> # p(Y>=2) = 1 - p(y<=1)
> #Y: number of people to choose from r
> #N: Total number of people
> #n: total number of people for the sample
> #r: total number of people with a specific attribute
>
> N = 8
> n = 3
> y = 1
> r = 5
>
> 1 - phyper(y, r, N-r, n)
[1] 0.7142857
>
```

```
# Q 3.122 part a
#P(Y<=3)

lambda = 7
y = 3
ppois(y, lambda)
```

```
> # Q 3.122 part a
> #P(Y<=3)
>
> lambda = 7
> y = 3
> ppois(y, lambda)
[1] 0.08176542
```

```
# Q 3.122 part b
#P(Y>=2) -> 1 - P(Y<=1)

lambda = 7
y = 1
1 - ppois(y, lambda)
```

```
> # Q 3.122 part b
> #P(Y>=2) -> 1 - P(Y<=1)
>
> lambda = 7
> y = 1
> 1 - ppois(y, lambda)
[1] 0.9927049
>
```