Referee report for the manuscript **SEIO-D-25-00070** entitled
"A simple and useful regression model for bimodal extreme data"

1 Summary

This manuscript proposes a new regression model for extreme values, based on a reparametrisation of the Bimodal Generalised Extreme Value (BGEV) distribution. The model introduces a median-based formulation, with parameter estimation carried out via maximum likelihood. Simulation studies are used to explore the performance of the estimators, and a real-data application to meteorological variables demonstrates the model's empirical behaviour.

The main strengths of the paper are:

- it addresses an underexplored area, that of regression modelling for extreme data with potential bimodality;
- the use of a median-based reparametrisation enhances interpretability; and
- the paper presents a complete pipeline, from model formulation to implementation using maximum likelihood.

2 Main comments

1. **Model structure limitation**: Only the median parameter is modelled as a function of covariates. While the focus on interpretability is appreciated, it would be useful to comment on the potential benefits or drawbacks of extending the regression structure to the other natural parameter for regression: σ. For this, another reparametrisation might be needed, probably in terms of a *spread* that aids in the interpretation of it, since, as clarified by the authors, σ is not a scale parameter. One way to do this is to follow ideas in Castro-Camilo *et al.* (2022) and define a *spread* parameter as the difference between quantiles (such as the IQR).
2. **Connections to other papers**: The idea of reparametrising in terms of quantiles has been around in extreme value theory since the work of Coles and Tawn in 1996. Another example is the paper of *Castro-Camilo et al.* already cited above.
3. **Uncertainty quantification (UQ):** While the manuscript reports asymptotic standard errors and z-statistics for parameter estimates, the treatment of uncertainty is otherwise limited. In particular, there is no discussion of how parameter uncertainty propagates to model predictions or forecasts. The UQ presented is restricted to inference on point estimates and does not extend to the predictive or

applied context; this is a substantial shortcoming for a model intended for real-world extreme value analysis. To enhance the practical relevance of the work, the authors should consider incorporating additional UQ tools, such as:

- Predictive intervals for the response variable;
- Interval estimates for model-based forecasts;
- Assessment of coverage probabilities through simulation;
- Bootstrap-based methods for quantifying uncertainty;
- Sensitivity analysis to evaluate the impact of parameter uncertainty on fitted values and predictions.

4. **Identifiability**: The authors do not explicitly discuss the identifiability of the parameters or regression coefficients in the manuscript. Although the simulation results include estimates of bias and RMSE (which can be indirectly related to identifiability), the authors do not link these to identifiability or estimation stability. Given that the model is nonlinear and involves a reparametrisation, some discussion would be expected, but it is absent. Specifically, it would strengthen the paper if the authors add a formal assessment of identifiability, such as:

   - A discussion of potential confounding between parameters.
   - Evaluation of the uniqueness of maximum likelihood estimates.
   - Numerical issues or convergence problems during estimation.

5. **Model under misspecification**: In the simulation studies, there is no examination of estimator bias under model misspecification, no sensitivity analysis to weak bimodality, and no coverage evaluation for confidence intervals.

6. **Covariate treatment**: All covariates are assumed to be fixed and measured without error. Given the environmental context and potential collinearity (e.g., between humidity and pressure), some discussion on the conclusion section about robustness or sensitivity to covariate specification would be helpful.

7. **Code and reproducibility**: There is no mention of publicly available code or data, which limits reproducibility. Sharing code for estimation and plotting would greatly benefit readers.

8. **Discussion of model diagnostics**: While quantile residuals are discussed and plotted, further diagnostic tools (e.g., PIT histograms, leverage plots) could enhance the assessment of model adequacy.

3 Minor comments

1. **Page 1, line 36**: "We employ the maximum likelihood method to estimate the model parameters..."

   Suggestion: "We use maximum likelihood estimation to fit the model parameters..."

2. **Page 12, line 1**: Clarify that “DTP” refers to minimum dew point temperature. The acronym is not defined at its first use.
3. **Page 12, line 40**: “the variable DTP **as** not influenced by HUM, P, WS, S.”

   Suggestion: “the variable DTP **is** not influenced...”
4. **Page 15, Table 3**: The formatting of the table could be improved for readability; the comparison of AIC values would benefit from highlighting the best model(s) under each ξ setting.
5. **Page 15, line 27**: “’the model with $\xi = 0$ **is adequate**”

   Suggestion: “**appears** adequate” (more cautious scientific tone)
6. **Page 15, line 38**: “(**maintained** at their respective sample means)”

   Suggestion: do you mean **fixed**?
7. **References**: Ensure consistent formatting (e.g., some article entries include URLs/DOIs, others do not; unify these).

References

Castro-Camilo, D., Huser, R. and Rue, H. (2022) Practical strategies for generalized extreme value-based regression models for extremes. *Environmetrics* **33**(6), e2742.

Coles, S. G. and Tawn, J. A. (1996) A bayesian analysis of extreme rainfall data. *Journal of the Royal Statistical Society Series C: Applied Statistics* **45**(4), 463–478.