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Question: Linear Regression (Gradient Descent)

Linear Equation $y = mx + b$

Initial $m = -1$ Initial $b = 1$

Learning Rate $\alpha = 0.1$ Given points $(1, 3)$ & $(3, 6)$

Cost Function: Mean Squared Error (MSE)

$$J(m, b) = \frac{1}{n} \sum (y_i - \hat{y}_i)^2$$

Gradients $\frac{dJ}{dm} = -\frac{2}{n} \sum (y_i - \hat{y}_i) x_i$

$$\frac{dJ}{db} = -\frac{2}{n} \sum (y_i - \hat{y}_i)$$

where $\hat{y} = mx + b$

$$(x_1, y_1) = (1, 3) \quad (x_2, y_2) = (3, 6)$$

Predictions (\hat{y}_i):

$$\hat{y}_1 = (-1)(0) + 1$$

$$= 0$$

$$\hat{y}_2 = (-1)(3) + (1)$$

$$= -2$$

2) Compute Gradients: (1,3) (3,6)

$$\frac{dJ}{dm} = -\frac{2}{2} [(3-0)(1) + (6-(-2))(3)]$$

$$= -1 [3 + 24]$$

$$= -27$$

$$\frac{dJ}{db} = -\frac{2}{2} [(3-0) + (6-(-2))]$$

$$= -1 (3 + 8)$$

$$= -11$$

$$m_{\text{new}} = m_{\text{old}} - a \frac{dJ}{dm}$$

~~$$m_{\text{old}} = b$$~~

$$b_{\text{new}} = b_{\text{old}} - a \frac{dJ}{db}$$

$$m_{\text{new}} = -1 - 0,1 (-27)$$

$$= \underline{\underline{1,7}}$$

$$b_{\text{new}} = 1 - 0,1 (-11)$$

$$= \underline{\underline{2,1}}$$

$$m_{\text{new}} = 1,7 \quad b_{\text{new}} = 2,1$$