Math Basics

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1 SinCosTan

Relationer for SinCosTan

$$\begin{aligned}
\textbf{SOH} \quad \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\textbf{CAH} \quad \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\textbf{TOA} \quad \tan\theta &= \frac{\text{opposite}}{\text{adjacent}}
\end{aligned}$$

2 Brøkregler

$$a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

$$\frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b \cdot c}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

3 Kvadratsætninger

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$(a+b)(a-b) = a^2 - b^2$$

4 Potensregneregler

$$a^r \cdot a^s = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$(a^r)^s = a^{r \cdot s}$$

$$(a \cdot b)^r = a^r \cdot b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$a^0 = 1$$

$$a^{-r} = \frac{1}{a^r}$$

$$a^{-1} = \frac{1}{a}$$

$$\sqrt[r]{a} = a^{\frac{1}{r}}$$

$$\sqrt[s]{a^r} = a^{\frac{r}{s}}$$

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

5 Logaritmeregneregler

Husk formler fra matundervisning med log

$$\log_n(a^x) = x \cdot \log_n(a)$$

$$\log_n(a \cdot b) = \log_n(a) + \log_n(b)$$

$$\log_n\left(\frac{a}{b}\right) = \log_n(a) - \log_n(b)$$

$$\log_n(a) = b \quad \Leftrightarrow \quad n^b = a$$

$$\log_n(n) = 1$$

$$n^{\log_n(x)} = x$$

$$n^{\frac{a}{b} \cdot \log_n(x)} = \sqrt[b]{x^a} = x^{\frac{a}{b}}$$

$$n^{\log_n(x) \pm c} = x \cdot n^{\pm c}$$

Naturlig logaritme

$$\ln = \log_e$$

$$\ln(e) = 1$$

$$e^{\ln(x)} = x$$

$$e^{\frac{a}{b} \cdot \ln(x)} = \sqrt[b]{x^a} = x^{\frac{a}{b}}$$

$$e^{\ln(x) \pm c} = x \cdot e^{\pm c}$$

6 Vektorer i planen

Koordinatsættet for $\vec{a}$

$$\vec{a} = a_1 \cdot \vec{i} + a_2 \cdot \vec{j} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Enhedsvektor

$$\vec{e} = \begin{pmatrix} \cos(v) \\ \sin(v) \end{pmatrix}$$

Længden af $\vec{a}$

$$|\vec{a}| = \left| \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \right| = \sqrt{a_1^2 + a_2^2}$$

Multiplikation af $\vec{a}$ med tallet k

$$k \cdot \vec{a} = k \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} k \cdot a_1 \\ k \cdot a_2 \end{pmatrix}$$

Summen af to vektorere

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

Differensen af to vektorere

$$\vec{a} - \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \end{pmatrix}$$

Koordinatsættet for $\overrightarrow{AB}$

$$\overrightarrow{AB} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

Skalarproduktet (prikproduktet) af $\vec{a}$ og $\vec{b}$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(v)$$

$$\cos(v) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

Ortogonale vektorer

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}$$

Kvadratet på en vektor

$$\vec{a} \cdot \vec{a} = \vec{a}^2 = |\vec{a}|^2$$

Projektion af $\vec{b}$ på $\vec{a}$

$$\vec{b}_a = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\cdot\vec{a}$$

Længden af projektionen

$$\vec{b}_a = \frac{|\vec{a}\cdot\vec{b}|}{|\vec{a}|}$$

Tværvektoren til $\vec{a}$

$$\hat{\vec{a}} = \widehat{\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}} = \begin{pmatrix} -a_2 \\ a_1 \end{pmatrix}$$

Determinanten for vektorparret $(\vec{a},\vec{b})$

$$\det(\vec{a},\vec{b}) = \hat{\vec{a}}\cdot\vec{b} = a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

$$\det(\vec{a},\vec{b}) = |\vec{a}|\cdot|\vec{b}|\cdot\sin(v)$$

Parallelle vektorer

$$\det(\vec{a},\vec{b}) = 0 \quad \Leftrightarrow \quad \vec{a}\|\vec{b}$$

Arealet af parallelogrammet udspændt af $\vec{a}$ og $\vec{b}$

$$A = |\det(\vec{a},\vec{b})|$$

https://www.webmatematik.dk/lektioner/matematik-a/vektorfunktioner

7 Vektorer i rummet

Vektorprodukt (Krydsprodukt):

$$\vec{a}\times\vec{b}=\begin{pmatrix}\begin{vmatrix}a_2 & b_2\\ a_3 & b_3\end{vmatrix}\\ \\ \begin{vmatrix}a_3 & b_3\\ a_1 & b_1\end{vmatrix}\\ \\ \begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{pmatrix}$$

Længden af vektorproduktet:

$$\left|\vec{a}\times\vec{b}\right|=|\vec{a}|\left|\vec{b}\right|\sin v$$

Længden af vektorproduktet er også lig arealet af det parallellogram der udspændes af $\vec{a}$ og $\vec{b}$

$$A=\left|\vec{a}\times\vec{b}\right|$$

Parameterfremstilling for linjen l gennem punktet:$P_0(x_0,y_0,z_0)$ med retningsvektor $\vec{r}=\begin{pmatrix}r_1\\ r_2\\ r_3\end{pmatrix}$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix}+t\begin{pmatrix}r_1\\ r_2\\ r_3\end{pmatrix}$$

Afstand fra punktet P til linjen l der går gennem punktet P_0 med retningsvektor $\vec{r}$

$$dist(P,l)=\frac{\left|\vec{r}\times\overrightarrow{P_0P}\right|}{|\vec{r}|}$$

Ligningen for planen α gennem punktet $P_0(x_0,y_0,z_0)$ med normalvektor $\vec{n}=\begin{pmatrix}a\\ b\\ c\end{pmatrix}$

$$\vec{n}\cdot\begin{pmatrix}x-x_0\\ y-y_0\\ z-z_0\end{pmatrix}=0$$

$$a\cdot(x-x_0)+b\cdot(y-y_0)+c\cdot(z-z_0)=0$$

Afstand fra punktet $P(x_1,y_1,z_1)$ til planen α gennem punktet $P_0(x_0,y_0,z_0)$ med normalvektor

$$\vec{n}=\begin{pmatrix}a\\ b\\ c\end{pmatrix}$$

$$dist(P,\alpha)=\frac{\left|\vec{n}\cdot\begin{pmatrix}x_1-x_0\\ y_1-y_0\\ z_1-z_0\end{pmatrix}\right|}{|\vec{n}|}$$

Afstand fra punktet $P(x_1,y_1,z_1)$ til planen α med ligningen: $ax+by+cz+d=0$

$$dist(P,\alpha)=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Ligningen for en kugle med centrum $C(x_0, y_0, z_0)$ og radius r:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

https://www.webmatematik.dk/lektioner/matematik-a/vektorer-i-3d

8 Rummet

Cartesian koordinater

$(x,\ y,\ z)$

Cylindrisk koordinater

$[r, \theta, z]$

$$x = r \cdot \cos\theta$$

$$y = r \cdot \sin\theta$$

$$z = z$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

Sfæriske koordinater

$[R,\ \phi,\ \theta]$

$$x = R \cdot \sin\phi\cos\theta$$

$$y = R \cdot \sin\phi\sin\theta$$

$$z = R \cdot \phi$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\tan\phi = \frac{\sqrt{x^2 + y^2}}{z}$$

$$\tan\theta = \frac{y}{x}$$

9 Complex Numbers

Cartesian coordinate system

$$z = x + yi$$

$$w = a + bi$$

$a, b, x, y \in \mathbb{R}$ og $z, w \in \mathbb{C}$

$$i^2 = -1$$

$$|i| = 1$$

$$\frac{1}{i} = -j$$

$$\arg(i) = \frac{\pi}{2}$$

$$Re(z) = x$$

$$Im(z) = y$$

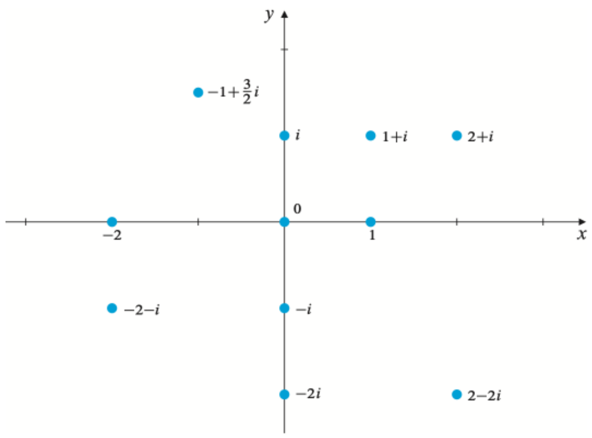
$$w + z = (a + x) + (b + y)i$$

$$w - z = (a - x) + (b - y)i$$

$$wz = (a + bi)(x + yi) = (ax - by) + (ay + bx)i$$

$$\frac{w}{z} = \frac{a + ib}{x + iy} = \frac{ax + by + i(bx - ay)}{x^2 + y^2}$$

Argand diagram:



Modulus

$$|w| = M = r = |a+bi| = \sqrt{a^2+b^2}$$

$$|wz| = |w|\,|z|$$

$$\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$$

Argument

$$\arg(w) = \theta$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \pm p \cdot \pi$$

$$p \in \{-1, 0, 1\}$$

$$\arg(wz) = \arg(w) + \arg(z)$$

$$\arg\frac{z}{w} = \arg(z) - \arg(w)$$

Polar representation

$$z = r \cdot \cos\theta + i \cdot r \cdot \sin\theta$$

$$z = r \cdot (\cos\theta + i \cdot \sin\theta)$$

$$z = r \cdot e^{i\theta}$$

Konjugeret

$$\overline{w} = a - bi$$

$$\overline{w + z} = \overline{w} + \overline{z}$$

$$\overline{wz} = \overline{w} \cdot \overline{z}$$

De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

$$e^z = e^{x+iy} = e^x e^{iy} = e^x(\cos y + i\sin y)$$

$$r^n = (r \cdot e^{i\theta})^n = r^n \cdot e^{i\theta n} = r^n \cdot (\cos(\theta n) + i \cdot \sin(\theta n))$$

$$r^{\frac{p}{q}} = (r \cdot \cos\theta + i \cdot \sin\theta)^{\frac{p}{q}} = r^{\frac{p}{q}} \cdot (\cos(\frac{p}{q} \cdot \theta) + i \cdot \sin(\frac{p}{q} \cdot \theta))$$

10 Differentialligninger

11 Differential- og integralregning

Definition af differentialkvotient

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Ligning for tangenten i punktet $P(x_0, f(x_0))$

$$y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

Konstantreglen

$$(k \cdot f(x))' = k \cdot f'(x)$$

Sumreglen

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

Produktreglen

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Kvotientreglen

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Kædereglen

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Regneregler for integration

$$\int f(x)dx = F(x) + c$$

$$\int k \cdot f(x)dx = k \cdot \int f(x)dx$$

$$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$$

$$\int f(g(x)) \cdot g'(x)dx = \int f(t)dt, \quad t = g(x)$$

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_a^b k \cdot f(x)dx = k \cdot \int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(b)} f(t)dt = [F(t)]_{g(a)}^{g(b)} = F(g(b) - F(g(a))$$

12 Funktioner

13 Procent- og rentesregning

14 SinCosTan relationer

$$\cos^2 \cdot \sin^2 = 1^2 \quad \tan x = \frac{\sin x}{\cos x}$$

$$\cos(x + 2\pi) = \cos(x) \qquad \sin(x + 2\pi) = \sin(x)$$

$$\cos(-x) = \cos(x) \qquad \sin(-x) = -\sin(x)$$

$$\cos(\pi - x) = -\cos(x) \qquad \sin(\pi - x) = \sin(x)$$

$$\cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$\sin(x + y) = \cos x \cdot \sin y - \sin x \cdot \cos y$$

$$\sin(x - y) = \cos x \cdot \sin y + \sin x \cdot \cos y$$

$$\cos(x)^2 - \sin(x)^2 = \cos(2x)$$

15 SI prefixes

Yotta (**Y**): 10^{24}
Zetta (**Z**): 10^{21}
Exa (**E**): 10^{18}
Peta (**P**): 10^{15}
Tera (**T**): 10^{12}
Giga (**G**): 10^{9}
Mega (**M**): 10^{6}
Kilo (**k**): 10^{3}
Hecto (**h**): 10^{2}
Deka (**da**): 10^{1}
Deci (**d**): 10^{-1}
Centi (**c**): 10^{-2}
Milli (**m**): 10^{-3}
Micro (**µ**): 10^{-6}
Nano (**n**): 10^{-9}
Pico (**p**): 10^{-12}
Femto (**f**): 10^{-15}
Atto (**a**): 10^{-18}
Zepto (**z**): 10^{-21}
Yocto (**y**): 10^{-24}