Mandatory 5

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May 30, 2025

Exercise 4

Code used for the exercises can be found in "mandatory_5/", "utils/", and "Numerical-Recipes/".

The equations is set up as follows:

```
// V'' = F
double F(double v_prime, double v, double u) {
  return 48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
         (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) /
         (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};

// d/dV F = F_v
double F_v(double v_prime, double v, double u) {
  return (48 *
              (3 * pow(v, 2) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
               (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
                   (-2 * v * pow(v_prime, 2))) *
              (1 + 64 * pow(u, 6) + 16 * pow(v, 6)) -
          48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
              (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) *
              (96 * pow(v, 5))) /
         pow(1 + 64 * pow(u, 6) + 16 * pow(v, 6), 2);
};

// d/d(V') F = F_v_prime
double F_v_prime(double v_prime, double v, double u) {
  return 48 *
         (2 * pow(u, 3) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
          (pow(v, 3) + 2 * pow(u, 3) * v_prime) * (-2 * pow(v, 2) * v_prime)) /
         (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};
```

4.1

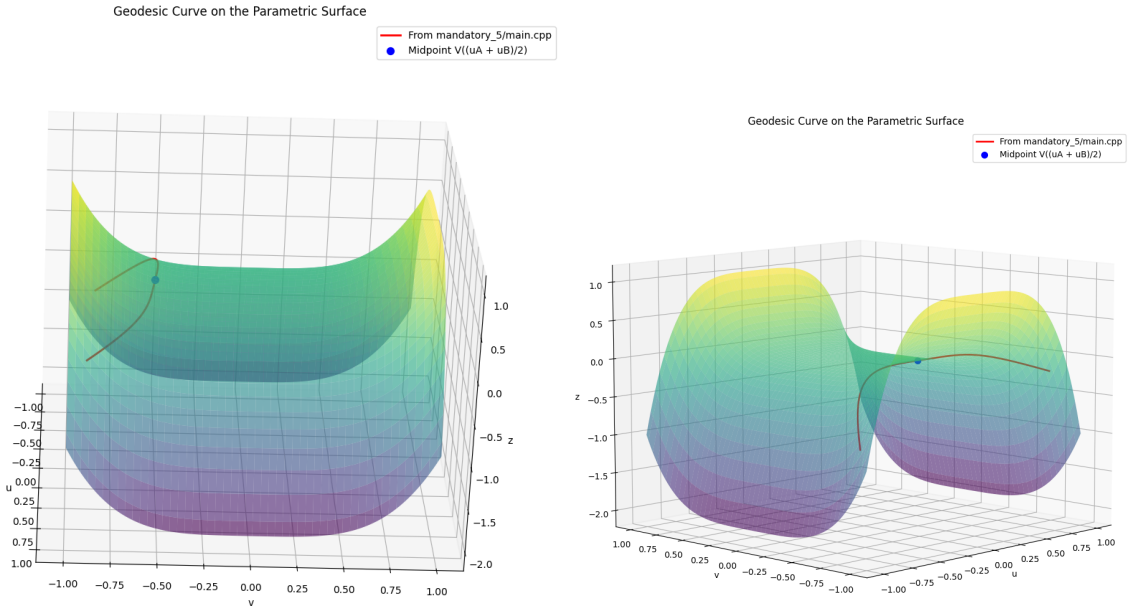
N	A(N)	A(N/2)-A(N)	alpha^k	Rich error
2	-0.86872569807			
4	-0.21646267914	-0.652263018932		0.217421006311
8	-0.10093715052	-0.115525528625	5.6460509	0.0385085095417
16	-0.52998401181	0.429046861297	-0.2692608	-0.143015620432
32	-0.50272420575	-0.02725980606	-15.739175	0.00908660202
64	-0.52565138564	0.0229271798843	-1.1889733	-0.00764239329475
128	-0.52923104286	0.00357965722644	6.4048534	-0.00119321907548
256	-0.52916814457	-6.28982936317e-05	-56.911833	2.09660978772e-05
512	-0.52916397817	-4.1663974627e-06	15.096565	1.38879915423e-06
1024	-0.52916368265	-2.95528367e-07	14.098130	9.85094556668e-08

The error is calculated using Richardson extrapolation. For N=1024 the error is $1 * 10^{-6}$ or better.

The estimate for $V\left(\frac{u_A+u_B}{2}\right)$ is -0.52916368265

4.2

The plot is made using Matplotlib in python: "mandatory_5/plot.py":



From the plot of the surface with the calculated values overlayed, it seems that they match the curve of the surface. Also the points seems to correctly show the shortest path of the geodesic curve.