Tasks for lecture 10

Consider the following system of differential equations

$$u'(x) = u(x)v(x), \quad u(0) = 1 \tag{1}$$
$$v'(x) = -u^2(x), \quad v(0) = 1 \tag{2}$$

Approximate the solution using (in order recommended, helps understands 4th order Runge-Kutta better):

- Euler method
- Midpoint method
- Trapezoidal method
- 4th order Runge-Kutta method
- Leap-Frog method

for

$$0 \leq x \leq 10 \quad , \quad n = 5, 10, 20, 40 \ldots \tag{3}$$

then evaluate the error and order.

Disclaimer: $u^2(x) + v^2(x)$ shall remain constant for the exact solution.