

Task for lecture 11

Consider the following set of differential equations

$$\begin{cases} u'(x) = \cos[-1 + x + u(x) + 3v(x)], & u(0) = 1 \\ v'(x) = -u(x)^2 + 2 \sin[v(x)], & v(0) = 0 \end{cases} \quad (1)$$

Your tasks:

- a) Perform one step with the Midpoint method(2nd order Runge-Kutta) with step size h . Clearly illustrate each step

We denote the solution from a) with u_1, v_1 .

- b) How does the error $\|u(h) - u_1, v(h) - v_1\|$ depend on the step size h (use $O()$ notation).
- c) We now want to estimate $u(x)$ at $x = 1$. Implement the Midpoint method and Richardson extrapolation to achieve a proven accuracy on $u(1)$ of 10^{-6} . Present the output in a table form similar to the ones used in class.
- d) Determine analytically the Jacobian used for the Trapezoidal method for the above problem.