Task for lecture 12

Consider the following differential equation

$$\left.\begin{array}{l} y''(x) = 2x + \sin[y'(x)] - \cos[y(x)] \quad \text{for} \quad 0 < x < 2 \\ y(0) = 0 \quad , \quad y(2) = 1 \end{array}\right\} \tag{1}$$

Solve the equation, numerically, following the procedure outlined by HGP today. That is

- Set Initial Guess, $y_i^{(0)}$ for $i = 0, ..., N$
- Define $\boldsymbol{F}$, $\boldsymbol{F_y}$, $\boldsymbol{F_{y'}}$
- Define $\boldsymbol{J}_{i,j}(y_0, ..., y_N)$ HINT:(Consider tridag function!)
- Define $\boldsymbol{\phi}_i(y_0, ..., y_N)$
- Solve the system of equations $\boldsymbol{J}(\boldsymbol{y^k})\Delta\boldsymbol{y} = -\boldsymbol{\phi}(\boldsymbol{y^{(k)}})$
- Update your $\boldsymbol{y}$, using $\boldsymbol{y^{(k+1)}} = \boldsymbol{y^{(k)}} + \boldsymbol{\Delta y}$
- Run iterations until a "satisfying" result appears at $y(1)$

Finally

- Use the above method to estimate y(1) with a proven accuracy better than 10^{-4}.
- Perform Richardson to verify error estimate and order. Start with $h = 1$, then $h = 0.5$ ...