

# Mandatory 5

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## Exercise 4

Code used for the exercises can be found in "mandatory\_5/main.cpp", "utils/", and "Numerical-Recipes/".

The equations is set up as follows:

```
// V'' = F
double F(double v_prime, double v, double u) {
    return 48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
        (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) /
        (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};

// d/dV F = F_v
double F_v(double v_prime, double v, double u) {
    return (48 *
        (3 * pow(v, 2) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
        (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
        (-2 * v * pow(v_prime, 2))) *
        (1 + 64 * pow(u, 6) + 16 * pow(v, 6)) -
        48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
        (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) *
        (96 * pow(v, 5))) /
        pow(1 + 64 * pow(u, 6) + 16 * pow(v, 6), 2);
};

// d/d(V') F = F_v_prime
double F_v_prime(double v_prime, double v, double u) {
    return 48 *
        (2 * pow(u, 3) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
        (pow(v, 3) + 2 * pow(u, 3) * v_prime) * (-2 * pow(v, 2) * v_prime)) /
        (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};
```

### 4.1

N	A(N)	A(N/2)-A(N)	alpha^k	Rich error
2	-0.86872569807			
4	-0.21646267914	-0.652263018932		0.217421006311
8	-0.10093715052	-0.115525528625	5.6460509	0.0385085095417
16	-0.52998401181	0.429046861297	-0.2692608	-0.143015620432

	32		-0.50272420575		-0.02725980606		-15.739175		0.00908660202	
	64		-0.52565138564		0.0229271798843		-1.1889733		-0.00764239329475	
	128		-0.52923104286		0.00357965722644		6.4048534		-0.00119321907548	
	256		-0.52916814457		-6.28982936317e-05		-56.911833		2.09660978772e-05	
	512		-0.52916397817		-4.1663974627e-06		15.096565		1.38879915423e-06	
	1024		-0.52916368265		-2.95528367e-07		14.098130		9.85094556668e-08	

The error is calculated using Richardson extrapolation. For  $N=1024$  the error is  $1 * 10^{-6}$  or better.

The estimate for  $V\left(\frac{u_A+u_B}{2}\right)$  is  $-0.52916368265$

## 4.2

