

Mandatory 4

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Exercise 3

Code for the exercises can be found in "mandatory_4/main.cpp" and "utils/".

The equation is set up as follows:

```
VecDoub derivs(const Doub t, VecDoub_I &v) {  
    VecDoub_0 dvdt(3);  
    dvdt[0] = exp(-t) * cos(v[1]) + pow(v[2], 2) - v[0];  
    dvdt[1] = cos(pow(v[2], 2)) - v[1];  
    dvdt[2] = cos(t) * exp(-pow(v[0], 2)) - v[2];  
    return dvdt;  
}
```

3.1

$$\begin{aligned}v_1'(0) &= 7.583853163452858 \\v_2'(0) &= -2.911130261884677 \\v_3'(0) &= -2.6321205588285577\end{aligned}$$

3.2

```
v1'(5), v2'(5), v3'(5) with N=50, h=0.1 , vec_size 3  
0.22036334741857197      0.974731402942015      -0.2058170170760096  
  
v1'(5), v2'(5), v3'(5) with N=100, h=0.05 , vec_size 3  
0.22930763450443528      0.973473649310275      -0.23381689689962976  
  
v1'(5), v2'(5), v3'(5) with N=200, h=0.025 , vec_size 3  
0.2378207610883643      0.9722033785617273      -0.260787727843097  
  
v1'(5), v2'(5), v3'(5) with N=400, h=0.0125 , vec_size 3  
0.23784281379424982      0.9722015341574787      -0.26079977212077576  
  
v1'(5), v2'(5), v3'(5) with N=800, h=0.00625 , vec_size 3  
0.23680841941101274      0.9723602284478439      -0.25748963470881164
```

3.3

For v(1)

N	A(N)	A(N/2)-A(N)	Rich error
50	0.220363347419		
100	0.229307634504	-0.00894428708586	0.00298142902862
200	0.237820761088	-0.00851312658393	0.00283770886131
400	0.237842813794	-2.20527058855e-05	7.35090196184e-06
800	0.236808419411	0.00103439438324	-0.000344798127746

For v(2)

N	A(N)	A(N/2)-A(N)	Rich error
50	0.974731402942		
100	0.97347364931	0.00125775363174	-0.00041925121058
200	0.972203378562	0.00127027074855	-0.000423423582849
400	0.972201534157	1.84440424866e-06	-6.14801416221e-07
800	0.972360228448	-0.000158694290365	5.28980967884e-05

For v(3)

N	A(N)	A(N/2)-A(N)	Rich error
50	-0.205817017076		
100	-0.2338168969	0.0279998798236	-0.00933329327454
200	-0.260787727843	0.0269708309435	-0.00899027698116
400	-0.260799772121	1.20442776788e-05	-4.01475922625e-06
800	-0.257489634709	-0.00331013741196	0.00110337913732

The error is calculated using Richardson extrapolation. For N=800 the error for each unknown is calculated as follows:

$$\text{Rich error} = \frac{A(800) - A(400)}{2^2 - 1}$$

2^2 is used since the expected order is 2 and N is doubled for each iteration.

The accuracy for the unknown variables at N=800 is:

$$\text{error}(v_1(5)) = -0.000344798127746$$

$$\text{error}(v_2(5)) = 5.28980967884e - 05$$

$$\text{error}(v_3(5)) = 0.00110337913732$$