Mandatory 5

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Exercise 4

Code used for the exercises can be found in "mandatory_5/main.cpp", "utils/", and "Numerical-Recipes/".

The equations is set up as follows:

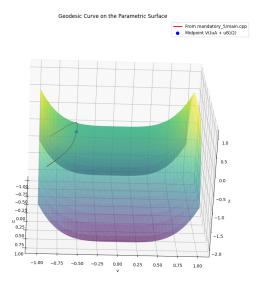
```
//V'' = F
double F(double v_prime, double v, double u) {
 return 48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
         (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) /
         (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};
// d/dV F = F_v
double F_v(double v_prime, double v, double u) {
 return (48 *
              (3 * pow(v, 2) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
               (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
                   (-2 * v * pow(v_prime, 2))) *
              (1 + 64 * pow(u, 6) + 16 * pow(v, 6)) -
          48 * (pow(v, 3) + 2 * pow(u, 3) * v_prime) *
              (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) *
              (96 * pow(v, 5))) /
         pow(1 + 64 * pow(u, 6) + 16 * pow(v, 6), 2);
};
// d/d(V') F = F_v_prime
double F_v_prime(double v_prime, double v, double u) {
 return 48 *
         (2 * pow(u, 3) * (2 * pow(u, 2) - pow(v, 2) * pow(v_prime, 2)) +
          (pow(v, 3) + 2 * pow(u, 3) * v_prime) * (-2 * pow(v, 2) * v_prime)) /
         (1 + 64 * pow(u, 6) + 16 * pow(v, 6));
};
```

4.1

-	N	A(N)	A(N/2)-A(N)		alpha^k	1	Rich error	
-		-				-		
	2	-0.86872569807		1		1		
-	4	-0.21646267914	-0.652263018932			1	0.217421006311	
-	8	-0.10093715052	-0.115525528625	5	5.6460509	1	0.0385085095417	
	16	-0.52998401181	0.429046861297	-(.2692608	1	-0.143015620432	-

The error is calculated using Richardson extrapolation. For N=1024 the error is $1*10^{-6}$ or better. The estimate for $V\left(\frac{u_A+u_B}{2}\right)$ is -0.52916368265

4.2



From mandatory_5/main.cpp
 Midpoint V((uA + uB)/2)

