Mandatory 2

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Exercise 5

NR3's implementation of `DErule` from "derule.h" is used as a base for solving the integral in question 5.3

The equations for Extended Midpoint and DErule are set up as follows:

```
double eqn(double x) { return (cos(pow(x, 3)) * exp(-x)) / sqrt(x); }

double eqn_derule(double x, double delta) {
  // If x is small use delta instead to avoid division by zero
  if (abs(x) < 1e-6) {
    return (cos(pow(x, 3)) * exp(-x)) / sqrt(delta);
  } else {
    return (cos(pow(x, 3)) * exp(-x)) / sqrt(x);
  }
}
```

5.1

The analytical solution using Extended Midpoint is:

$$\int_a^b \frac{\cos(x^3)\exp(-x)}{\sqrt{x}}dx$$

$$\approx \frac{(b-a)}{N-1}\cdot\sum_{i=1}^{N-1}\left(\frac{\cos\left((a+\Delta x\cdot i)^3\right)\exp\left(a+\Delta x\cdot i\right)}{\sqrt{a+\Delta x\cdot i}}\right)+O\left(\frac{1}{N^2}\right) \quad \text{where } \Delta x = \frac{(b-a)}{N}$$

$$= (b-a)\cdot\left(\frac{\cos\left(\left(a+\frac{b-a}{2}\right)^3\right)\exp\left(a+\frac{b-a}{2}\right)}{\sqrt{a+\frac{b-a}{2}}}\right)+O\left(\frac{1}{4}\right) \quad \text{for } N = 2$$

5.2

5.3