CPF-AS Exercises (Systems of Systems)

Mohamad Al Ahdab

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1 Exercise

Consider the state space model

$$\begin{aligned} x_1(t+1) &= 0.5x_1(t) + w(t) + 0.1x_2(t), \quad w(t) \sim \mathcal{N}(0, \sigma_w^2) \\ x_2(t+1) &\sim Ber(q), \\ y(t) &= x_1(t) + v(t), \quad v(t) \sim \mathcal{N}(0, \sigma_v^2) \end{aligned}$$

with initial conditions:

$$\begin{aligned} x_1(1) &\sim \mathcal{N}(0, \sigma_w^2), \\ x_2(1) &\sim Ber(q), \end{aligned}$$

and a beta prior for $q \sim Beta(1, 1)$.
Given $\sigma_w = 0.05$, $\sigma_v = 0.1$, and a sequence of measurements $y(1:T)$, it is desired to approximate $p(x(1:T), q|y(1:T))$.

- 1) Write the transition density $\mathrm{x}(t+1) \sim f(x(t+1)|x(t), q)$.
- 2) Write the measurement density $\mathrm{y}(t) \sim g(y(t)|x(t))$.
- 3) Download and extract the zip file CPFASEx.zip. Write the Gibbs sampling algorithm in the script GibbsCPFAS.m to obtain samples from $p(x(1:T), q|y(1:T))$. Use the function CPFAS.m to run a CPF-AS with the model.