Inequality and Environmental Taxation

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Abstract

This paper examines optimal environmental taxation under an inequality-averse government. Empirical studies show that the necessity of pollution intensive goods often leads to environmental taxation being regressive in the absence of compensation schemes. Building on a framework proposed by Klenert et al. (2018b), which models the pollution intensive good as a necessity, we show that adequate redistribution mechanisms can ensure the progressiveness of the environmental tax-and-transfer system at low taxation levels. This can prompt an inequality-averse government - without the ability to modify income taxation - to use environmental taxes to achieve increases in equality. Under a rising preference for a clean environment, however, further revenues achieved as a consequence of increased taxation will not be able to make up for poor households' welfare loss, constraining the government's ability to compensate the poor. This fact is likely to make inequality-averse governments reluctant to further increase environmental taxation whenever the ability to counteract regressiveness by adjusting income taxation is absent. We extend the framework with a clean technology input and find that an unequal distribution of rents from such technology limits the government's ability to ensure progressiveness of the environmental tax-and-transfer system.

Keywords: Environment, inequality, optimal taxation.
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Contents

1 Introduction

Environmental change and rising income inequality are some of the most discussed topics in the world today. In 2023, the poorest half of individuals earned just 8.4% of the world's total income (World Inequality Lab, 2023). In that same year, worldwide carbon dioxide emissions rose to the highest levels in human existence (Friedlingstein et al., 2023).

In this paper, we examine the interaction between environmental taxation - aimed at reducing environmental damage from emission of pollutants - and income taxation, which remains most governments' primary tool for addressing inequality. We find that at low levels of environmental taxation, such policies can reduce inequality, provided that revenues are redistributed through uniform lump-sum transfers. However, as society's preferences for environmental quality increase - leading to higher environmental taxes and less pollution - this "equality dividend" fades as additional revenues from such taxation diminish. We show that this dynamic can lead inequality-averse policymakers to opt for lower environmental taxes when adjustments to income taxation are not available to counteract regressiveness. Finally, we extend our analysis with a clean technology input and show that the ability to achieve a redistribution dividend at low levels of environmental taxation fades when the input is cheap and its ownership is unequal.

Our analysis is motivated by recent data showing an increase in consumer preference for a clean environment (McKinsey & Company, 2025; PWC, 2024) as well as 196 countries' commitment to the Paris Agreement in 2015, aimed at major pollution reductions in order to contain global temperature increases (Council on Foreign Relations, 2023). We interpret both as indicating that governments increasingly prioritize reductions in emissions of pollutants, reflecting an increase in society's environmental preference.

Policy discussions around mitigation of environmental change and rising income inequality are not new; most developed countries have been experiencing a rising Gini coefficient since the 1990s, and the negative effects of increasing pollution emission levels have long been known. In accordance with The Tinbergen Rule (Tinbergen, 1952), stating that a government should only use one policy instrument for each policy objective, policymakers have largely treated these policy discussions separately. However, recent studies have found that environmental taxes disproportionately affect low-income households because they devote a larger share of their income to consuming pollution-intensive goods. Hence, some argue that environmental taxes are regressive (Bento et al., 2009; Wier et al., 2005). This creates an interaction between environmental and inequality concerns, where an inequality-averse government risks compromising equity in pursuit of its environmental goals. Consequently, this trade-off has resulted in significant political barriers for environmental tax policies and has led to increased demand for more sophisticated policies that incorporate these additional concerns into their design (Klenert et al., 2018a).

The question of how these policies should be designed is a relatively recent focus in academic literature. Historically, the focus has primarily been on the interaction between environmental taxes and the efficiency of the overall tax system, commonly known as the "double dividend" of environmental taxation (first conceptualized by Tullock, 1967). This theory suggests that taxes can be used not only as a tool to improve the environment, but also to improve the efficiency of the tax system by using the revenue to reduce distortionary taxes. However, the extent of this double dividend is still contentious (Bovenberg and de Mooij, 1994; Goulder, 1995), and while it appears in our analysis, it is not the main focus of this paper.

Instead, we focus on a more recently discovered third dividend of environmental taxation and how this can affect the optimal design of tax policies. Studies such as Fried et al. (2024) and Chiroleu-Assouline and Fodha (2014) have identified this additional "equality dividend" by designing tax revenue recycling schemes that in some cases reduce overall inequality, counteracting the regressive effect from the environmental taxes.[1]

These studies are based on general equilibrium models that account for income heterogeneity and use Stone-Geary preferences, which ensure that low-income households spend a larger share of their budget on pollution-intensive goods and are thus more exposed to price increases of such goods.

Our contribution to this branch of literature is to provide further insights into the theoretical mechanisms that connect inequality and environmental as well as income taxation, when redistribution of environmental tax revenue through lump-sum transfers is possible. Furthermore, we contribute by analyzing this interaction at high levels of environmental taxation, while prior work only focuses on such at lower levels.

Building on the framework proposed by Klenert et al. (2018b), we employ a general equilibrium model consisting of three modules: households, divided into income quintiles by differentiated productivities; firms, representing two sectors producing goods of distinct pollution intensities; and finally an inequality-averse government, which selects optimal policy instruments to maximize social welfare.

We solve the model numerically in Python, using a two-layer code structure. The inner layer determines the general equilibrium allocation for a given tax policy, while the outer layer calculates the optimal tax policy resulting in the allocation that maximizes the government's social welfare function.

[1]This third dividend of environmental taxation is also widely referred to as the "double dividend of redistribution".

2 The model

Our model includes three modules: firms, households, and the government. In this section, we describe the modules, highlighting features that are important in the later analysis.

2.1 Firms

The model contains two representative firms in two different sectors, each producing a different consumption good: one firm produces a clean good, which we call good c; the other firm produces a dirty good, which we call good d.

The two firms have their own unique (but similar) production technologies, $f_j(t_j, z_j)$, $j \in \{c, d\}$, that take two inputs: a finite resource t_j and a pollution amount z_j. We refer to t_j as effective labor, but it can generally be interpreted as any resource that can be purchased from households, including land or capital.

Production technology

Firms produce using a constant elasticity of substitution (CES) technology with constant returns to scale:

$$f_j(t_j, z_j) = \left(\epsilon_j t_j^{\frac{\sigma-1}{\sigma}} + (1 - \epsilon_j) z_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.1.1}$$

where σ is the elasticity of substitution between the inputs, ϵ_j denotes the factor share of effective labor, and $1 - \epsilon_j$ denotes the factor share of pollution.

Note that we slightly simplify the production technology in comparison to Klenert et al. (2018b), who include an upper bound on z_j, such that $z_j \leq t_j$ at any choice of inputs. The original paper includes this constraint to ensure that an equilibrium with close to zero environmental taxation is feasible. We do not consider any equilibrium with close to zero environmental taxation in our analysis and hence choose to suppress this constraint (as it is always respected).

Firm optimality

The firms pay a wage w for effective labor, a tax τ_z for pollution, and sell their respective good at price p_j. We assume that firms operate in perfectly competitive output and input markets. The former assumption is a simplification, since each firm is the sole supplier of its respective good and should, in theory, have price-setting power. To avoid this inconsistency, we may think of the firms as representative of sectors in a two-sector economy.

Under the perfect competition assumption, each firm maximizes profits and solves:

$$\max_{t_j,z_j} \pi_j(t_j,z_j) = p_j f_j(t_j,z_j) - wt_j - \tau_z z_j.$$

Profit maximization implies that marginal factor costs equal the value of each factor's marginal product:

$$w = \epsilon_j t_j^{-\frac{1}{\sigma}} f_j(t_j,z_j)^{\frac{1}{\sigma}} p_j, \tag{2.1.2}$$

$$\tau_z = (1-\epsilon_j) z_j^{-\frac{1}{\sigma}} f_j(t_j,z_j)^{\frac{1}{\sigma}} p_j. \tag{2.1.3}$$

In Appendix A, we show that the optimal ratio of effective labor to pollution is increasing in both the ratio between τ_z and w as well as in ϵ_j whenever $\sigma > 0$. Hence, we model a sector that produces a clean good and a sector that produces a dirty good by calibrating the model such that $\epsilon_c > \epsilon_d$ (and $\sigma > 0$). Appendix A also includes the full derivation of the household FOCs.

Finally, Euler's theorem on homogeneous functions ensures zero profits, so there is no need to specify firm ownership rights in the model.

2.2 Households

The model accommodates N heterogeneous households that differ in their labor productivity. We denote productivity class by $i \in \{1, 2, \ldots, N\}$, where productivity increases with i. Households derive utility from consuming two goods - the dirty and the clean good - as well as from leisure, optimally choosing labor supply and allocating their income between the two goods.

The labor market is assumed to be perfectly flexible, allowing households to freely move between the two sectors. We discuss the implications of this assumption in Section 6.

Budget

Households' income is derived from labor compensation such that the pre-tax income of household i is

$$m_i = \phi_i w(t - \ell_i),$$

where ϕ_i (ranked such that $\phi_i > \phi_j \Leftrightarrow i > j$) denotes the productivity of the ith household and ℓ_i denotes leisure choice, such that $t - \ell_i$ characterizes total labor units supplied. We note that the model is constructed such that one labor unit for household i translates into ϕ_i effective labor units at production level. This feature ensures heterogeneity in labor compensation.

Households are subject to a proportional income tax, $\tau_{w,i}$, which differs across productivity class. Thus, we can write the disposable labor income of household i as

$$m_i^d = (1 - \tau_{w,i})\phi_i w(t - \ell_i).$$

As opposed to most modern tax systems, the assumed system implies the absence of tax brackets, such that the average income tax rate for household i always equals its marginal income tax rate. We allow for a simpler tax system to ease the solution of the model. Another, perhaps more realistic way to model the system, would be to assume a linear or exponential marginal tax schedule (to which all households would be subject). This, however, would constrain the government's ability to redistribute through the income tax system *and* complicate interpretation of our results. Hence, we assume that the government can target different productivity classes *perfectly* through the individual income tax rates.

Households allocate disposable income across the clean good, c_i, priced at p_c and the dirty good, d_i, priced at p_d, facing the following budget constraint:

$$c_i p_c + d_i p_d = m_i^d + l,$$

where l denotes a uniform lump-sum transfer from the government.

Utility

Households derive utility from consumption of the dirty good, the clean good, and leisure. Following Klenert et al. (2018b), we adopt Stone-Geary preferences and let the utility of household i be given by

$$u_i(c_i, d_i, \ell_i) = c_i^\alpha (d_i - d_0)^\beta \ell_i^\gamma + (e_0 - \xi(z_c + z_d)^\theta),$$

where total pollution is given by $z \equiv z_c + z_d$ and per-household utility from a clean environment is given by $e(z) \equiv e_0 - \xi(z_c + z_d)^\theta$.

ξ is a latent variable representing the rise in the preference for a clean environment. Is serves as a proxy for both increased willingness to pay for less pollution intensive good from consumers and countries' increased commitment to environmental goals.

d_0 denotes the "subsistence" level of dirty good consumption, i.e. the level of dirty good consumption at which marginal utility from obtaining more of the good is infinite. Due to this feature any household will always consume at least d_0 units of the dirty good.

We define $\tilde{u}_i \equiv c_i^{\alpha}(d_i - d_0)^{\beta}\ell_i^{\gamma}$ as total utility from consumption of goods and leisure and refer to it as "blue" utility. Furthermore, we refer to utility from a clean environment as "green" utility. The assumed functional form implies that pollution affects all households' utility equally. We discuss the implications of this simplifying assumption in Section 6.

Household optimality

Household i's utility maximization problem is

$$\max_{c_i, d_i, \ell_i} u_i(c_i, d_i, \ell_i) \text{ s.t. } c_i p_c + d_i p_d = m_i^d + l.$$

In Appendix B we show that the optimal resource allocation for household i implies:

$$\ell_i = \frac{\gamma}{\alpha+\beta+\gamma}\left(\frac{h_i}{(1-\tau_{w,i})\phi_i w}\right), \tag{2.2.1}$$

$$c_i = \frac{\alpha}{\alpha+\beta+\gamma}\left(\frac{h_i}{p_c}\right), \tag{2.2.2}$$

$$d_i = \frac{\beta}{\alpha+\beta+\gamma}\left(\frac{h_i}{p_d}\right) + d_0, \tag{2.2.3}$$

where $h_i \equiv (1-\tau_{w,i})\phi_i w t + l - p_d d_0$.

Necessity of the dirty good

We note that the elasticity of demand for the dirty good with regard to potential disposable income (defined as $(1-\tau_{w,i})\phi_i w t + l = h_i + p_d d_0$) is:

$$\varepsilon^{d_i}_{h_i + p_d d_0} = \frac{1}{1 + \frac{(\alpha+\gamma) d_0 p_d}{\beta(h_i + p_d d_0)}},$$

which is lower than unity whenever subsistence income d_0 is strictly positive such that the pollution good is a necessity for the households. See Appendix B for a derivation of the elasticity.

The necessity assumption implies that poorer households devote a larger share of their income to purchase of the dirty good. As increased environmental taxation makes the dirty good more expensive by driving up production costs, the necessity feature is crucial in modeling regressivity of a raise in environmental taxes (see Andersson and Atkinson (2020) for a proof of the former and a discussion of the latter).

2.3 Government

The government acts as a Stackelberg leader, and is thus the first one to "move" in the model by choosing its policy instruments. The households and the firms act as followers who observe the government policies and then maximize their utility or profit. Importantly, we assume that the government has perfect information on how the households and firms will react to a given choice of policy.

Social welfare

We consider alternative policy scenarios that vary in terms of instruments (taxes) available to the government. In any scenario the government chooses the available policy instruments to maximize the following social welfare function (SWF):

$$\mathcal{W} = \sum_{i=1}^{N} \left[\log \tilde{u}_i + e(z) \right].$$

We employ a different SWF than the original framework, which utilizes a utilitarian SWF. The main reason for our departure from the original framework is the wish to clarify and increase the government's inequality-aversiveness in order to achieve a clearer interpretation of results.

Looking at the original study's results (mainly at optimal income taxation), it becomes clear that the strictly utilitarian SWF assumed by the authors implies (at least some degree) of an equality preference. This is evident in the government's optimal choice of income taxes, which are heavily skewed in a progressive manner in the original paper. Due to the utilitarian SWF (and income taxes being inherently distortionary) this choice of income taxation can only be a consequence of a decreasing marginal utility of income for households, such that the utilitarian SWF implies no inequality-aversiveness in utility, but inequality-aversiveness in income.[2] In Appendix B we show the necessary condition for income inequality aversion with a utilitarian SWF. This condition is not automatically fulfilled with our calibration, and we strengthen the assumption of inequality-aversiveness by utilizing a SWF, which explicitly prioritizes equality in blue utilities by rescaling them logarithmically.

We choose to exclude the green part of utility from the logarithmic transformation as its inclusion would imply a diminishing marginal disutility of pollution

[2]See p. 827 in *Microeconomic Theory* by Mas-Colell, Whinston and Green for details.

per household.[3] This implication would move us further from the original paper and change the interpretation of our results, which is why we choose to avoid it and let green welfare enter additively.

Budget

We assume that the government runs a balanced budget, financing its constant spending requirement $\mathcal{G}$ with revenues from income- and environmental taxation, and distributing remains through uniform lump-sum transfers to households.

Hence, the government's budget constraint is:

$$\mathcal{G} + Nl = \sum_{i=1}^{N} \tau_{w,i}\phi_i w(t - \ell_i) + \tau_z(z_c + z_d). \tag{2.3.1}$$

We note that the government's spending requirement has to be financed at all times. Besides lump-sum transfers and income taxes, environmental taxation is one (potential) instrument to finance the spending. Consequently, the spending requirement might have an effect on the optimal environmental tax level.

The vector of policy instruments that the government chooses is

$$\mathbf{p} = (\tau_z, \tau_{w,1}, \ldots, \tau_{w,N}),$$

where the size of the lump-sum transfer resulting for a given $\mathbf{p}$ can be backed out of (2.3.1).

Whenever a given income tax package is imposed, τ_z becomes the only choice variable in $\mathbf{p}$. Note that we sometimes denote a given income tax system as a vector, i.e. $\tau_w = (\tau_{w,1}, \tau_{w,2}, \ldots, \tau_{w,N})$, such that $\mathbf{p} = (\tau_z, \tau_w)$.

Incentive compatibility constraints

We assume that the government does not have information about the individual productivity ϕ_i of each household unless the household itself chooses to provide it. Following Mirrlees (1971), we implement incentive compatibility constraints

$$u_i \geq u_i^j,$$

which ensure that household i always prefers its consumption bundle over all other households' consumption bundles and thus has the incentive to provide the government correct information regarding its productivity.

[3] $\frac{\partial \log(\tilde{u}_i + e(z))}{\partial z} = \frac{\partial e(z)}{\partial z} u_i^{-1}$.

u_i^j denotes the utility that household i would receive if it pretended to be household j, and is therefore given by

$$u_i^j = u_i\left(c_j, d_j, t - \frac{m_j^d}{(1-\tau_{w,j})\phi_i w}\right).$$

Household i can pretend to be household j by copying household j's consumption bundle and by supplying the required amount of labor to earn household j's optimal income. If i is more productive than j, and the incentive compatibility constraints are not respected, i might have an incentive to pretend to be j (i.e. inform the government that its productivity equals ϕ_j).

The incentive compatibility constraints place an upper bound on how progressive (meaning skewed to the poor households' benefit) an income tax system imposed by the government can be. A very skewed tax system, with large labor subsidies to the poor and high income taxes for the rich, can create an incentive for rich households to substitute good consumption for leisure by misrepresenting their productivity to achieve a lower income tax rate.

We assume that the government has perfect information on the households and hence ensures that the incentive compatibility constraints are respected for any optimal (or given) income tax package.

2.4 General equilibrium

We define the general equilibrium as an allocation of resources for which labor as well as the goods markets are cleared, while both firms and all households maximize profit and utility, respectively, and the government either chooses or obeys a vector of policy instruments $\mathbf{p}$.

We assume that government consumption is split equally (in nominal terms) between the clean and the dirty good. Hence market clearing for the clean good implies

$$p_c \sum_{i=1}^{N} c_i + \frac{1}{2}\mathcal{G} = f_c(t_c, z_c) p_c,$$

while market clearing for the dirty good implies:

$$p_d \sum_{i=1}^{N} d_i + \frac{1}{2}\mathcal{G} = f_d(t_d, z_d) p_d. \tag{2.4.1}$$

The labor market clears whenever

$$t_c + t_d = \sum_{i=1}^{n} \phi_i (t - l_i) \tag{2.4.2}$$

holds.

Note that labor supplied by households is not measured by the same metric as effective labor, which is used in production. E.g., one unit of labor supplied by household 1 translates to ϕ_1 units of effective labor. This assumption is necessary for existence of a general equilibrium allocation whenever pre-tax wage per labor unit supplied by household i is equal to $\phi_i w_i$ as each household has to be paid its marginal product (under the perfect competition and constant return to scale assumptions). Hence, each labor unit supplied by household i should be equal to ϕ_i effective labor units.

3 Numerical implementation

We solve the model numerically in Python. Our solution computes the optimal taxation in two layers. The inner layer finds the general equilibrium allocation for a given policy vector $\mathbf{p}$, while the outer layer solves for the optimal policy vector, i.e., the $\mathbf{p}$ that results in the general equilibrium allocation preferred by the government.

Our code is accessible in a repository on GitHub.[4]

3.1 Inner layer

As described in Section 2, general equilibrium implies an allocation where firms' FOCs, the government's budget constraint, market clearing conditions, and the optimal allocation equations for all households are fulfilled simultaneously.

We insert households' optimal allocation equations into market clearing conditions (insert (2.2.1)-(2.2.2) for each of the households into (2.4.1)-(2.4.2)) and rewrite the equilibrium conditions, firms' FOCs and the government's budget constraint such that they have zero on one side of the equality. The equilibrium condition for the clean good market is excluded, as it holds due to Walras's law. Furthermore, we set the clean good price as the numeraire (i.e., $p_c = 1$) and transform the equations to be written in terms of $\tilde{z} \equiv \log z$ rather than z to avoid potential solutions with $z \leq 0$.

The steps described above yield a system of seven equations with seven unknowns. This system comprises two market equilibrium conditions, four firm

[4] github.com/mathiasdmikkelsen/bachelors-thesis.

FOCs, and the government's budget constraint, and we write it as:

$$f(\mathbf{x}, \mathbf{g}) = \mathbf{0},$$

where $\mathbf{x} \equiv (t_c, t_d, \tilde{z}_c, \tilde{z}_d, w, p_d, l)$ is a vector of unknowns and $\mathbf{g}$ is a vector of exogenous variables/parameters.

The Levenberg-Marquardt solver

We solve the inner layer system of equations using the Levenberg-Marquardt (LM) solver, contained in the SciPy library. The LM solver is an iterative algorithm for solving non-linear least squares problems.[5]

The iterative process starts with an initial solution guess, and then improves the guess until it is acceptably close to the actual solution, i.e., when the objective function is minimized. The solver calculates each next guess by combining the Gauss-Newton algorithm and the method of gradient descent.

Below, we explain the solver's iterative adjustments towards the solution.

We let

- n denote the number of equations and unknowns,
- $\mathbf{x} = (x_1, \ldots, x_n)^T$ be the current solution guess for the vector of unknown variables,
- $\mathbf{x}_{\text{candidate}} = (x_{1,\text{candidate}}, \ldots, x_{n,\text{candidate}})^T$ be the candidate replacement guess for the vector of unknown variables,
- $\mathbf{g}$ be the vector of exogenous variables/parameters given in our model,
- $\mathbf{y} = (y_1, \ldots, y_n)^T$ be the target output values (in our model, $\mathbf{y} = \mathbf{0}$),
- $f(\mathbf{x}, \mathbf{g})$ be the non-linear model function that predicts the output values given $\mathbf{x}$ and $\mathbf{g}$.

Then, $\mathbf{r}(\mathbf{x}) = f(\mathbf{x}, \mathbf{g})$ is the residual vector (defined this way because $\mathbf{y} = \mathbf{0}$), and $S(\mathbf{x}) = \mathbf{r}^T\mathbf{r}$ is thus the sum of squared residuals that we want to minimize (also called the objective function).

The purpose of each iteration is to find the candidate solution guess $\mathbf{x}_{\text{candidate}}$ by adding a step $\mathbf{d}$ to the current solution guess $\mathbf{x}$ (such that $\mathbf{x}_{\text{candidate}} = \mathbf{x} + \mathbf{d}$). This step is found as the solution to

[5] In our implementation, we use it as a root-finding algorithm.

$$(\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I})\mathbf{d} = -\mathbf{J}^T\mathbf{r},$$

where $\mathbf{J}$ is the Jacobian matrix of the residuals, $\mathbf{J}^T\mathbf{J}$ is an approximation of the Hessian matrix of the objective function, $\mathbf{I}$ is the identity matrix, and λ is the *damping parameter* that controls which method the LM algorithm behaves more like; the Gauss-Newton method or the gradient descent method.

When λ is very large, the term $\lambda\mathbf{I}$ dominates the matrix on the left-hand side of the equation, resulting in $\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I} \approx \lambda\mathbf{I}$. The system of linear equations is then reduced to $\lambda\mathbf{I}\mathbf{d} \approx -\mathbf{J}^T\mathbf{r} \Leftrightarrow \mathbf{d} \approx -\lambda^{-1}\mathbf{J}^T\mathbf{r}$. In this case, the step is proportional to the negative gradient of the objective function, i.e. the direction of the steepest descent of this function. Hence, when λ is large, the LM algorithm behaves like the gradient descent method.

If λ is very small, the term $\lambda\mathbf{I}$ becomes insignificant, resulting in $\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I} \approx \mathbf{J}^T\mathbf{J}$. The system of linear equations then simplifies to $\mathbf{J}^T\mathbf{J}\mathbf{d} \approx -\mathbf{J}^T\mathbf{r} \Leftrightarrow \mathbf{d} = -(\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{r}$. In this case, the step is not only proportional to the negative gradient of the objective function, it is also scaled based on the local curvature of the squared residual function (as $\mathbf{J}^T\mathbf{J}$ is a local quadratic approximation of the squared residual function). When the local residual "landscape" is relatively curved, $(\mathbf{J}^T\mathbf{J})^{-1}$ will be small, and thus the step size will be small, as the solver expects to be close to the minimum. When the local residual landscape is relatively flat, $(\mathbf{J}^T\mathbf{J})^{-1}$ will be larger, and thus the step size will be larger, as the solver expects to be farther away from the minimum. Hence, when λ is small, the LM algorithm behaves like the Gauss-Newton method.

Each method has its own strengths. The gradient descent method is guaranteed to decrease the objective function, but converges slowly. The Gauss-Newton method converges faster when close to the minimum and the local linear approximation is accurate, but can be unstable and lead to increases in the objective function instead. As such, the LM algorithm increases λ when the step taken fails to decrease the Euclidean norm[6] of the residual vector and decreases λ when the step taken decreases this norm.

The iteration process stops when convergence is reached, which is when the length of the iteration steps has become extremely small, or once the maximum number of iterations has been reached.

Detailed pseudo-code for the LM-solver is provided below.

[6] $||\mathbf{r}|| = \sqrt{r_1^2 + \cdots + r_n^2}$

Algorithm 1 LM solver

Require: Function $f(\mathbf{x}, \mathbf{g})$ that returns the residual vector, initial guess $\mathbf{x}_0$, maximum iterations *max_iter*, tolerance *tol*, initial damping parameter λ_{init}, arguments $\mathbf{g}$

$\mathbf{x} \leftarrow \mathbf{x}_0$ ▷ Initialize $\mathbf{x}$ as the starting guess $\mathbf{x}_0$
$\lambda \leftarrow \lambda_{init}$ ▷ Initialize λ as the starting guess λ_{init}
for $i \leftarrow 1$ **to** *max_iter* **do** ▷ Begin main loop
    $r \leftarrow f(\mathbf{x}, \mathbf{g})$ ▷ Compute residual vector
    $J \leftarrow$ `compute_jacobian`$(f, \mathbf{x}, \mathbf{g})$ ▷ Compute Jacobian matrix
    Solve $(\mathbf{J}^T\mathbf{J} + \lambda\mathbf{I})\mathbf{d} = -\mathbf{J}^T\mathbf{r}$ for $\mathbf{d}$ ▷ Solve linear system of equations for step $\mathbf{d}$
    $\mathbf{x}_{\text{candidate}} \leftarrow \mathbf{x} + \mathbf{d}$ ▷ Calculate candidate solution vector
    $r_{\text{candidate}} \leftarrow f(\mathbf{x}_{\text{candidate}}, \mathbf{g})$ ▷ Calculate candidate residual vector
    if $\|r_{\text{candidate}}\| < \|r\|$ **then** ▷ If Euclidean norm of $r_{\text{candidate}}$ is lower, then;
        $\mathbf{x} \leftarrow \mathbf{x}_{\text{candidate}}$ ▷ Update $\mathbf{x}$ as $\mathbf{x}_{\text{candidate}}$
        $\lambda \leftarrow \lambda/10$ ▷ Decrease the dampening parameter
    else ▷ If Euclidean norm of $r_{\text{candidate}}$ isn't lower, then;
        $\lambda \leftarrow \lambda \times 10$ ▷ Increase the dampening parameter
    end if
    if $\|\mathbf{d}\| < tol$ **then** ▷ Check if convergence is achieved
        break ▷ Break loop if converged
    end if
end for
return x ▷ Return solution when convergence or maximum iterations is reached

3.2 Outer layer

The policy vector preferred by the government is the one that maximizes the government's social welfare function.

The Sequential Least Squares Programming solver

We solve for the optimal policy vector using SciPy's Sequential Least Squares Programming (SLSQP) solver together with the `NonlinearConstraint` class.

The SLSQP solver is an iterative algorithm used for constrained non-linear optimization problems.

The iterative process starts with an initial guess for the optimal policy vector, and then improves the guess such that it moves toward the minimum of the objective function (in our case, the negative of social welfare), respecting the problem's constraints (in our case, the IC-constraints). Unlike the LM algorithm, the SLSQP algorithm combined with the `NonlinearConstraint` class can handle both equality and inequality constraints, making it suitable for our outer layer optimization problem.

Below, we explain the solver's iterative adjustments toward the solution.

We let

- $\mathbf{p} = (\tau_z, \tau_{w,1}, \ldots, \tau_{w,N})^T$ be the current solution guess for the vector of policy instruments,

- $\mathbf{p}_{\text{candidate}} = (\tau_{z,\text{candidate}}, \tau_{w,1,\text{candidate}}, \ldots, \tau_{w,N,\text{candidate}})^T$ be the candidate replacement guess for the vector of policy instruments,

- $f(\mathbf{p})$ be the negative of the government's social welfare function,

- $\mathbf{c}(\mathbf{p})$ be the vector of constraint functions (rewritten to have zero on one side of the equality), i.e., the incentive compatibility constraints that must be respected.

At each iteration the current guess $\mathbf{p}$ is updated with a step $\mathbf{d}$ such that the candidate replacement guess is $\mathbf{p}_{\text{candidate}} = \mathbf{p} + \mathbf{d}$. The step is found as the solution to the following quadratic programming (QP) subproblem

$$
\begin{aligned}
\min_{\mathbf{d}} \quad & (\nabla f)^T \mathbf{d} + \frac{1}{2} \mathbf{d}^T \mathbf{B} \mathbf{d} \\
\text{s.t.} \quad & \mathbf{A}\mathbf{d} + \mathbf{c}(\mathbf{p}) \geq 0 \\
& a \leq \mathbf{p} + \mathbf{d} \leq b,
\end{aligned}
$$

where ∇f is the gradient of the objective function, $\mathbf{B}$ is an approximation of the Hessian matrix of the Lagrangian function, $\mathbf{A}$ is the Jacobian of the inequality constraint, and a and b denote the lower and upper bounds on the policy instruments.

The step is taken in the direction that you have to follow to reach the lowest point in the approximated local shape of the objective function, while (approximately) satisfying the constraints.

To realize this, notice that $\mathbf{d}$ is chosen to minimize the approximate change in objective function caused by that step. The change is approximated by a second order Taylor series such that $f(\mathbf{p} + \mathbf{d}) - f(\mathbf{p}) \approx (\nabla f)^T \mathbf{d} + \frac{1}{2} \mathbf{d}^T \mathbf{B} \mathbf{d}$. However, this explanation is slightly simplifying as $\mathbf{B}$ also includes the constraint function curvature.[7]

Similarly, $\mathbf{A}\mathbf{d}$ is a first order Taylor approximation of the change in constraint functions caused by the step. As seen above, $\mathbf{d}$ is chosen such that the constraints still hold after the step is taken (note that $\mathbf{A}\mathbf{d} + \mathbf{c}(\mathbf{p}) \approx \mathbf{c}(\mathbf{p} + \mathbf{d})$).

After solving the QP problem and finding the candidate solution $\mathbf{p}_{\text{candidate}}$, the algorithm evaluates whether the candidate should replace the current guess. To do so, it calculates the merit function:

[7] As it is the Hessian of $\mathcal{L}(\mathbf{p}, \lambda) = f(\mathbf{p}) + \sum_j \lambda c_j(\mathbf{p})$ rather than the Hessian of $f(\mathbf{p})$.

$$\Phi(\mathbf{p}) = f(\mathbf{p}) + \rho \sum_j \phi(c_j(\mathbf{p})),$$

where $\phi(c_j(\mathbf{p}))$ is a penalty term that penalizes constraint residuals, and ρ is a penalty parameter that is adjusted throughout the iteration process to change the importance of satisfying the constraints; if constraint violations are very large, then ρ is increased, and vice versa.

If the merit function decreases sufficiently with the step (i.e., if $\Phi(\mathbf{p}_{\text{candidate}}) - \Phi(\mathbf{p})$ is sufficiently small; follows the Armijo condition), the candidate solution is accepted as the next guess. Otherwise, the algorithm performs a backtracking line search on $\mathbf{d}$, shrinking its length by a factor $\eta \in (0, 1)$ until the Armijo condition holds. If the line search fails to identify an acceptable step size, the algorithm will discard the step and reset the Hessian approximation to find a new search direction.

After each (successful) step, the Hessian approximation, $\mathbf{B}$, containing local curvature information, is updated using a quasi-Newton BFGS formula, which improves the speed of the optimizer as it avoids recalculating the Hessian in each iteration.

Detailed pseudo-code for the SLSQP solver is provided below.

Algorithm 2 SLSQP solver

Require: Objective function $f(\mathbf{p})$, constraint function $\mathbf{c}(\mathbf{p})$, bounds (a, b), initial guess $\mathbf{p}_0$, maximum iterations max_iter, tolerance tol, initial Hessian approximation $\mathbf{B}_0 = \mathbf{I}$

$\mathbf{p} \leftarrow \mathbf{p}_0$ ▷ Initialize $\mathbf{p}$ as starting guess
$\mathbf{B} \leftarrow \mathbf{B}_0$ ▷ Initialize Hessian approximation
$\rho \leftarrow \rho_0$ ▷ Initialize penalty parameter to some $\rho_0 > 0$
for $i = 1$ to *max_iter* **do** ▷ Begin main loop
  Evaluate $f(\mathbf{p})$, $\nabla f(\mathbf{p})$ ▷ Evaluate objective and gradient
  Evaluate constraints $\mathbf{c}(\mathbf{p})$ ▷ Evaluate constraint vector
  Compute constraint Jacobian $\mathbf{A}$ ▷ Linearize constraints
  Solve QP subproblem: ▷ Solve problem to find step $\mathbf{d}$
  minimize $(\nabla f)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{B}\mathbf{d}$
  subject to $\mathbf{A}\mathbf{d} + \mathbf{c}(\mathbf{p}) \geq 0$
  $a \leq \mathbf{p} + \mathbf{d} \leq b$
  $\mathbf{p}_{\text{candidate}} \leftarrow \mathbf{p} + \mathbf{d}$ ▷ Calculate candidate solution (initially with step length $\eta = 1$)
  Evaluate merit function $\Phi(\mathbf{p}_{\text{candidate}})$ ▷ Check merit function improvement
  if $\Phi(\mathbf{p}_{\text{candidate}})$ sufficiently decreases **then** ▷ If line search with $\eta = 1$ succeeded;
    $\mathbf{p} \leftarrow \mathbf{p}_{\text{candidate}}$ ▷ Accept candidate
    Update $\mathbf{B}$ via BFGS ▷ Update Hessian approximation
  else ▷ If step $\eta = 1$ failed; begin backtracking line search
    ▷ If backtracking line search also fails to find suitable $\eta \in (0, 1)$;
    $\mathbf{B} \leftarrow \mathbf{I}$ ▷ Reset Hessian approximation to the identity matrix $\mathbf{I}$
    ▷ Keep current $\mathbf{p}$ unchanged
  end if
  if $\|\mathbf{d}\| <$ tol **then** ▷ Check if convergence is achieved
    break ▷ Break loop if converged
  end if
end for
return p ▷ Return solution when convergence or maximum iterations is reached

4 Results

This section considers an analysis of the model's dynamics and optimal taxation. First, we present the calibration. Then, to illustrate the model's dynamics, we describe changes in key variables and distributional effects that follow variations in environmental taxation. Finally, we discuss optimal taxation in our model, detailing the trade-off between efficiency and inequality faced by the inequality-averse government.

4.1 Calibration

We calibrate our model in line with Klenert et al. (2018b), whose calibration matches parameters to data for the United States from 2012 and 2013.

Pre-existing tax rates

A substantial part of our analysis explores optimal environmental taxation under a fixed, pre-existing income tax system. The pre-existing income tax rates are calibrated to match a composite tax burden measure for the United States in 2013, calculated using individual income taxes as well as corporate income taxes, social insurance taxes, and excise taxes (CBO, 2013). The tax rates are seen in Table 1.

Table 1: Pre-existing income tax rates by quintile

	Quintile (i)				
	1	2	3	4	5
Tax rate ($\tau^0_{w,i}$)	0.015	0.072	0.115	0.156	0.240

Household productivities

Household productivities are calibrated to match data from the U.S. Census Bureau on the income shares of different quintiles (DeNavas-Walt et al., 2012).

Table 2: Productivity by quintile

	Quintile (i)				
	1	2	3	4	5
Productivity (ϕ_i)	0.03	0.0825	0.141	0.229	0.511

Additional parameters

Calibration of additional parameters is stated in Table 3. For a detailed treatment of the applied calibration, we refer to the original study.

4.2 Key variables

In this section, we consider the effects of environmental taxation on key economic variables, which are illustrated in Figure 1. In the figure, solid lines illustrate the effects under a calibration with fixed pre-existing tax rates, while dashed lines illustrate the effects under a fixed income tax system, which is optimal at the baseline environmental preference (a tax system that maximizes social welfare when $\xi = 0.1$).[8] First, we consider the effects under the pre-existing income tax system and then compare them with the effects under the baseline optimal income tax system.

[8] $\tau_w^{opt} = (-1.130, -0.066, 0.204, 0.383, 0.632)$.

Table 3: Additional parameters

Firms		
ϵ_c	Labor intensity clean production	0.995
ϵ_d	Labor intensity poll. production	0.92
σ	Elasticity of substitution between labor and pollution	0.5
Households		
α	Clean consumption share in utility	0.7
β	Polluting consumption share in utility	0.2
γ	Leisure share in utility	0.2
d_0	Subsistence level pollution consumption	0.5
e_0	Green utility under zero pollution	0.0
θ	Damage exponent	1.0
ξ	Preference for environmental quality (baseline)	0.1
t	Time endowment per household	24*
Government		
$\mathcal{G}$	Government spending	5

* retrieved in private correspondence with David Klenert.

Production

We note that higher environmental taxation depresses pollution in both sectors. Due to positive cross-products between the inputs, this decline reduces labor productivity and lowers the wage per effective labor unit. Due to the necessity of the dirty good, this aggregate income decrease prompts an all-else-equal shift in demand from the clean to the polluting sector, which, together with an increase in the marginal cost of the dirty good, leads to a higher relative price between the dirty and the clean good. The relative price change shifts labor demand from the clean to the dirty sector, so that absolute production falls less in the latter, eventually (with further increases in τ_z) plateauing at the total subsistence demand of $d_0 \cdot N = 0.5 \cdot 5 = 2.5$, while absolute production in the clean sector steadily declines.

Households' labor choice

Total leisure is concave in the environmental tax, increasing at low levels of taxation and decreasing at high levels. This is due to households' labor choice being subject to multiple confounding effects of increased environmental taxation.

On the one hand, increases in the tax rate decrease wages and increase the lump-sum transfer, leading to an ambiguous income change. Whenever total income decreases, households become poorer, which prompts them to "consume" less leisure and hence to work more (i.e., an income effect). On the other hand,

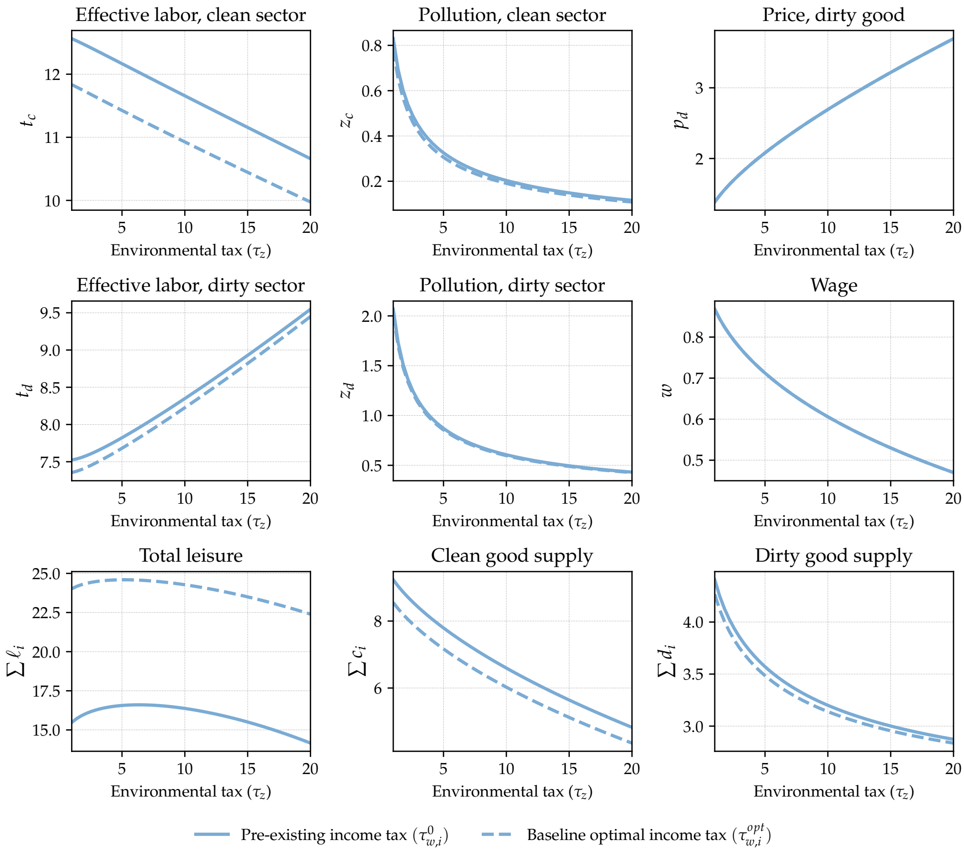

Figure 1: Key variables at alternative levels of environmental taxation

Notes: The figure shows the levels of key model variables at alternative levels of the environmental tax rate (τ_z). The left column displays effective labor in the clean and polluting sectors as well as total household leisure. The middle column displays pollution in the clean and dirty sectors as well as the supply of the clean good. The right column displays the dirty good price, the wage, and the supply of the dirty good. Solid lines show the variables under a calibration with a pre-existing income tax system, while dashed lines show the variables under a calibration with a baseline optimal income tax system.

lower wages decrease the additional income received from supplying an extra labor unit; this, in turn, encourages households to work less (i.e., a substitution effect). Lastly, increases in the dirty good price prompt households to work more due to an increased cost of subsistence dirty good consumption. At low levels of the tax, the effect from increases in the lump-sum transfer and decreases in the wage received for an extra labor unit dominates, leading to an upward-sloping schedule. At higher levels, however, the other effects triumph, resulting in a negative relationship. This feature of the model is introduced through Stone-Geary preferences and the presence of a lump-sum transfer. In the absence of both (i.e., $l = 0$ and $d_0 = 0$), labor supply would be inelastic.

Effect of an alternative income tax system

Figure 1 also shows the difference in schedules for the key variables under two different tax systems: fixed pre-existing income tax rates and fixed income tax rates, which are optimal at the baseline. Generally, the schedules show similar behavior, although at different levels for some variables.

Effective labor in both sectors is lower under the fixed optimal income tax system, which is due to this tax system being more progressive, reducing the incentive to work for the richer households and increasing it for the poorer households. Because richer households are more productive than poorer households, this leads to a negative effect on the total effective labor supply at any given environmental tax rate.

Lower effective labor in both sectors decreases the marginal product of pollution. Hence, firms choose to employ less pollution at any level of the environmental tax.

A lower level of both inputs decreases the supply of the clean good as well as the dirty good.

Furthermore, the input changes result in two opposing effects on the wage: the decrease in effective labor creates upward pressure on the wage, while the decrease in pollution presses the wage down. As is shown in Figure 1, this results in the wage being identical between the two income tax systems.

Lastly, we note that the dirty good price remains unchanged. This result is due to identical marginal productivity of inputs as well as identical factor prices between the tax systems.

Relevance to the government

The changes in key variables highlight the "obvious" downside of environmental taxation, as it depresses overall consumption as well as leisure for households, leading to (at least some) households experiencing a utility loss, even when revenue from such taxation is redistributed back to households.[9] Any government interested in the overall well-being of households will take this effect into account when picking an optimal taxation level.

4.3 Distributional effects

Under our specification, the government also cares about inequality (in blue utility). In this section, we consider distributional effects of environmental taxation in the applied framework. We decompose the total effect of environmental taxation

[9] This is further evident from Figure 2 in Section 4.3.

on inequality into two distinct channels, one working through the dirty good price and one through households' income, and discuss them. Finally, we evaluate the total inequality effect by looking at changes in households' utilities, which enter directly into the government's objective function and thus form the basis for tax system choice.

We evaluate the effects of environmental taxation by increasing it from a baseline level of one (i.e., $\tau_z = 1.0$). This choice is convenient, as the optimal environmental tax rate always lies above this level for our calibration.

Price channel

A consequence of increases in environmental taxation is growth of the dirty good price. When households' income is held constant, this results in environmental taxation being regressive.

The left panel of Figure 2 shows the relative CV, short for compensating variation, as a function of environmental tax for each of the five households. Solid lines account for a calibration with fixed pre-existing income tax system, while dashed lines account for a calibration with fixed baseline optimal income tax system.

CV measures the income increase a household needs in order to achieve the same level of blue utility as it achieved at baseline when the dirty good price is increased, but all else is held equal.[10]

As seen in the figure, relative CV increases faster and maintains a consistently higher level for poorer households than for richer households, indicating that poorer households indirectly "pay" a larger share of their income as a consequence of a tax rate increase. This heterogeneity is introduced through Stone-Geary preferences, which ensure that low-income households devote a higher share of their income to the purchase of the dirty good at baseline.

We call this channel, connecting environmental taxation with inequality, the *price channel*. This channel is of interest to an inequality-averse government and might result in an all-else-equal reluctance to increase environmental taxation much even when such an increase is due (e.g. under positive shocks to the environmental preference).

Income channel

Besides the *price channel*, inequality is affected through the *income channel*. The right panel in Figure 2 shows the absolute change in disposable income (relative to baseline) as a function of the environmental tax for each of the five households. The total change in income for each household is due to two confounding effects:

[10] A formula for CV for applied preferences is derived in Appendix B.

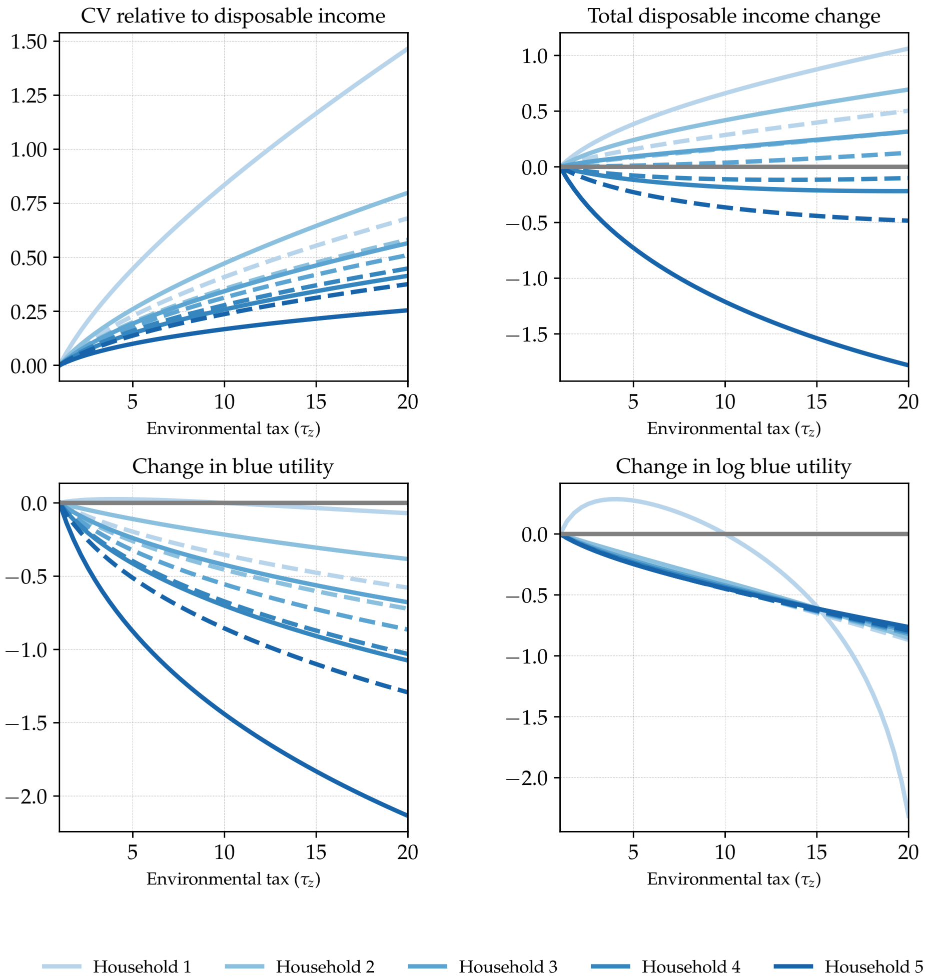

Figure 2: Relative CV, income, blue utility and log blue utility change across households

Notes: Upper left panel of the figure shows the compensating variation (CV_i) relative to baseline disposable income ($(1 - \tau_{w,i})m_i^d + l$) when $\tau_z = 1$ for each of the five households as a function of the environmental tax rate. Upper right panel shows the total disposable income change for each of the households relative to baseline disposable income. Lower left panel shows the change in blue utility ($\tilde{u}_i$) for each of the five households as a function of the environmental tax rate relative to baseline, while lower right panel shows that same change in log blue utility ($\log \tilde{u}_i$). All panels are color-coded such that more productive households are illustrated by a darker shade of blue and vice versa.

changes in total labor income and changes in the lump-sum transfer. The lump-sum transfer is an increasing function of environmental tax and homogeneous across households and hence does not explain heterogeneity of income changes. Thus, the differences are explained by changes in labor income. Labor income falls more for richer households as they receive a higher wage per labor unit supplied. At low levels of taxation, this effect is counteracted by a smaller decrease in labor supply for rich households. However, the latter effect is too small to dominate the former. At higher levels of taxation, rich households decrease their labor supply more, which further magnifies the income change differences.

Effects' dependence on initial income tax system

Comparing results between the alternative tax systems, we note that the overall heterogeneity of environmental taxation's effect is smaller for the more progressive baseline optimal tax system. This system implies a relatively smaller disposable income inequality at baseline, which dampens the difference between households' relative CV. The logic behind this is straightforward: when less productive (poorer) households receive larger labor subsidies, they become richer, spending a lower proportion of their income on the dirty good at baseline and hence being hit less by increases in the dirty good price. The converse is true for more productive (richer) households, who become poorer as a consequence of higher labor taxes. The difference between disposable income changes is dampened too as the post-tax wage differences at baseline become smaller.

Inequality effects

To conclude on the overall effect of environmental taxation on inequality, we have to combine the two channels. Due to lack of an objective inequality metric, there are multiple ways to do so.

One approach, taken by Klenert et al. (2018b), relies on the Gini coefficient in blue utility. Even though this metric is arguably more useful than the "classic" income Gini, it will not necessarily be in line with the way that the government *views* inequality (and might not be in line with how the reader views it either). In order to evaluate the government's policy choices, we have to look closer at the social welfare function, which shows that all government decisions are taken against the backdrop of households' utilities. Being aware of this fact, we choose to illustrate changes in blue utility for each of the five households. These are seen in the lower left panel of Figure 2.

We focus on changes under the pre-existing income taxation and try to, initially, stay away from *cardinal* interpretations of utility. We note that environmental taxation initially (while being at low levels) increases blue utility for the poorest household, but decreases it for the rest. Hence, if inequality were judged by comparing only the poorest and the richest, we could conclude that increases in environmental taxation, whenever such is at low levels, are inequality-reducing.

The logic behind this result is as follows. At low levels of the tax rate, the tax base is large (i.e. pollution is widely used in production), such that increases in the tax rate bring in relatively large revenues. When redistributed through lump-sum transfers, these revenues more than outbalance the income reduction suffered due to lower wages for the poorest three households, such that these receive an income bonus. For the poorest household, this bonus more than compensates for the increased dirty good price, making them better off as a result of increases in environmental tax.

To view the inequality effects from the government's perspective, we have to employ an *cardinal* utility interpretation; after all, the SWF directly compares the magnitude of (blue) utility changes between households. As discussed, the "blue welfare" part of the government's social welfare function is a sum of logarithmically transformed household utilities. The logarithmic transformation introduces inequality-aversion, rescaling changes in blue utility such that increases in it for poor households are magnified, while decreases in it for rich households are dampened upon entering social welfare.

Logarithmic rescaling of households' blue utilities and an inspection of the change in these rescaled utilities, shown in the lower right panel of Figure 2, makes it evident that an inequality-averse government will receive a social welfare bonus from increasing environmental taxation when such is at a low level, even if it fails to reduce pollution (as the increase for the poorest household more than compensates for the other households' losses). This somewhat surprising result depends highly on the lump-sum transfer size and its increase as consequence of a larger environmental tax as well as the initial distribution of disposable income.

Looking at the lower right panel of Figure 2 we also notice that log blue utility for the poorest household only increases as consequence of increased environmental taxation when such is at a low level (in the pre-existing income tax case). Namely, the increase depends on the relationship between the compensation that poor households need for decreased wages and the increased dirty good price and the compensation itself, which works through the lump-sum transfer. As seen in Figure 3, the latter increases less and less in the environmental tax because the tax base is narrowed as firms substitute away from pollution. This limits the government's ability to compensate the poorest and might have an effect on how the government chooses its environmental policy.

In the economy with an initially more equal distribution of disposable income

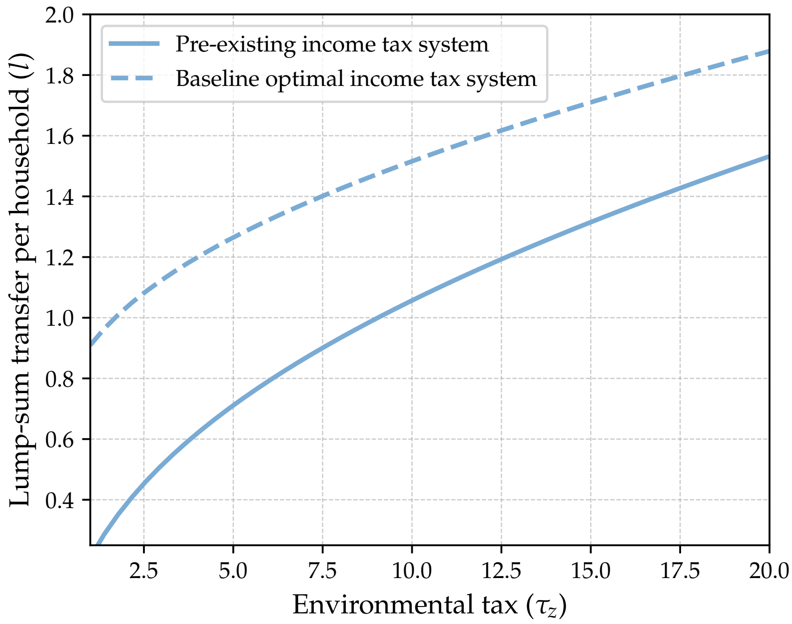

Figure 3: Lump-sum transfer at alternative environmental tax

Notes: The figure shows the per-household lump-sum transfer as a function of the environmental tax. The dashed line indicates the transfer under a fixed baseline optimal income tax system, while the solid line indicates the transfer under a fixed pre-existing income tax system.

(i.e. the economy with the more progressive baseline optimal income tax rates), the inequality-reducing character of environmental taxation (when such is at low levels) disappears. This essentially happens as the households are "equalized": Although poor households still achieve the highest income bonus, it is not nearly enough to compensate them for increases in the price of the dirty good, as these households become richer at baseline and thus require higher compensation.

4.4 Optimal taxation

Having established the baseline dynamics of our model, we proceed with the analysis of optimal government policy.

Our analysis from Section 4.3 shows that the government faces a series of trade-offs when choosing the environmental tax rate. On the one hand, when there is a strictly positive environmental preference, pollution negatively affects all households' utilities. This motivates environmental taxation, as it can reduce the use of the polluting input and thereby improve households' green utility. On the other hand, higher environmental taxation decreases the total production of goods and compels households to work more (at higher tax rates), reducing blue utility for at least some households. Lastly, the government's aversion to inequality also plays a role. As discussed, it can receive a so-called "equality dividend" from increased environmental taxation when baseline tax rates are low. Such distributional considerations also influence optimal policy choices.

In this section, we explain how the government navigates these trade-offs and chooses optimal tax packages under varying circumstances and policy instrument availability.

We examine optimal taxation choices under an increasing environmental preference. Our analysis considers three scenarios that vary in the availability of policy instruments:

1. In the **first scenario**, we consider optimal environmental taxation under the assumption of a fixed pre-existing income tax system. In this scenario, the government only has one instrument to influence the economy, namely the environmental tax. This analysis is motivated by income tax reforms being politically demanding, and as such, they might not be included in the design of environmental tax policy packages.

2. The **second scenario** considers a slightly different environment. Here, we assume that the government can adjust the income tax system once, when environmental preference is at baseline (i.e., $\xi = 0.1$), but cannot adjust it further thereafter, leaving the environmental tax as the only remaining instrument. The baseline optimal income tax system turns out to be more progressive than the pre-existing one, effectively making the comparison between the first and second scenarios a comparison between optimal environmental taxation under less and more progressive income tax systems. Such a comparison yields insights into how the disposable income distribution affects the way governments tax pollution.

3. In the **third scenario**, the government is allowed to adjust both the income tax system *and* the environmental tax *continuously* as environmental preference increases. Such policy flexibility highlights the optimal interplay between the two forms of taxation and emphasizes how a government's environmental policy reform might differ when simultaneous reshaping of the income tax system is possible.

We divide our analysis into three parts. First, we focus on the revenues collected and lump-sum transfers issued under optimal policy. Second, we use the revenue analysis to explain optimal income taxation when the income tax system is variable (i.e., in the third scenario). Third, we compare optimal environmental taxation across the three scenarios.

Tax revenue and lump-sum transfers

Figure 4 illustrates how total and individual tax revenues respond to a rising environmental preference. The revenues are illustrated for the second and third policy scenarios. The illustration of revenues for the first scenario has been moved to the Appendix C, as the takeaways are qualitatively identical to ones from the second scenario.

We note that revenues from environmental tax increase in both scenarios as the tax rate rises. This rise in environmental tax revenue is caused by a change in environmental tax rates, which are optimally increased as environmental preference rises (see Figure 6, to be discussed in detail later). This revenue increase occurs even though the environmental tax reduces total pollution (i.e., the tax base), indicating that the effect of the higher tax rate on revenue outweighs the impact of the shrinking tax base.

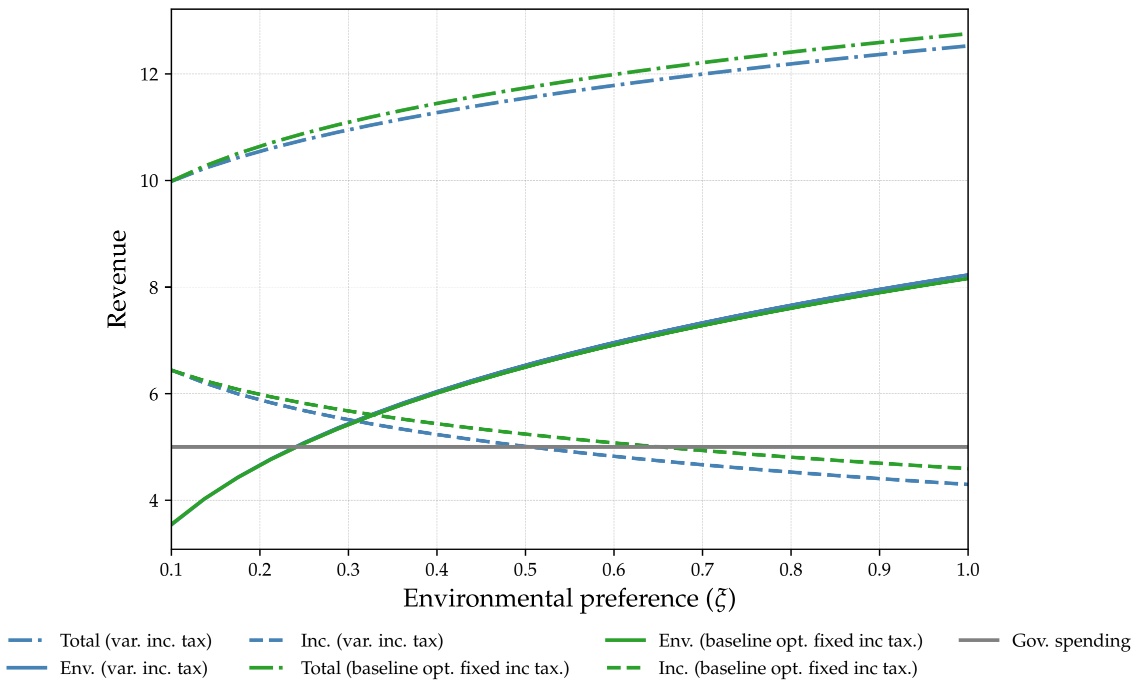

Figure 4: Tax revenue under optimal taxation

Notes: The figure shows revenues from environmental taxation, income taxation, and total revenues, collected as a consequence of optimal taxation at each level of environmental preference (ξ). Income tax revenue are indicated by dotted lines, environmental tax revenue by solid lines, and total revenues by dash-dotted lines. Green lines represent revenues under baseline optimal but fixed income taxation, while blue lines represent revenues under optimal variable income taxation. Government spending requirement ($\mathcal{G}$) is illustrated by the grey horizontal line.

Even though the optimal environmental tax is almost identical between the two scenarios (as seen in Figure 6), the rise in environmental tax revenue happens slightly faster under variable income taxation. This outcome reflects the government's optimal choice to lower income taxes across all households (as seen in Figure 5) when given the opportunity. Lower income taxes stimulate effective labor supply and increase the productivity of pollution inputs, prompting firms to pollute more, which widens the environmental tax base and increases revenues.

The income tax revenue falls in both scenarios because of tax base reductions. These reductions arise from decreases in the wage, which follow increased environmental taxation and are not offset by increased working hours. In the case of variable income taxation, income tax revenue falls even more, as the government continuously lowers income taxes for all households.

We note that the increase in environmental tax revenue more than compensates for the decrease in income tax revenue, such that total revenues in both scenarios increase as environmental preference rises. In both scenarios, this primarily happens because the increasing environmental tax does not distort the income tax base enough to completely crowd out the increase in the environmental tax revenue.

Furthermore, we note that the total revenue schedule is concave, such that the total revenues increase by less and less as the environmental preference rises. Importantly, this concavity is not solely a consequence of the relationship between the environmental preference and optimal environmental taxation and would also be the case for the direct relationship between environmental taxation and revenues (see Figure 3, noting that multiplying the lump-sum by five and adding the spending requirement yields total revenue). The concavity is explained by diminishing marginal revenues from environmental taxation, caused by a narrowing tax base, and decreasing revenues from income taxation.

At a close look, Figure 4 also illustrates the lump-sum transfer under optimal taxation. As the government runs a balanced budget with a constant spending requirement, the transfer size can be inferred as the difference between total revenues and the spending requirement (divided by the number of households). We also note that increases in the lump-sum transfer are diminishing (in environmental preference as well as environmental tax, the latter relationship being illustrated in Figure 3). This concavity illustrates an important point already mentioned in Section 4.3: the government's ability to compensate households for income loss and price increases becomes increasingly limited as environmental preference and environmental tax are increased.

Finally, we note that the total tax revenue under optimal policy lies well above the level required to finance the government spending (in all scenarios), suggesting that the spending requirement itself is not the primary driver of the optimal tax path. Later in our analysis, we show that the government sets the optimal environmental tax according to the marginal trade-off between blue and green welfare, further showing that the government's budget constraint is not binding at this level of spending.

Optimal income taxation

As mentioned, the optimal income tax reform at the baseline level of environmental preference implies an increase in the progressivity of the income tax system. As the environmental preference rises from the baseline, a government with full policy flexibility optimally shifts the tax burden by lowering income tax rates for all households. This section explains the mechanics behind that adjustment.

Figure 5 shows the impact of an increasing environmental preference on optimal income taxation.

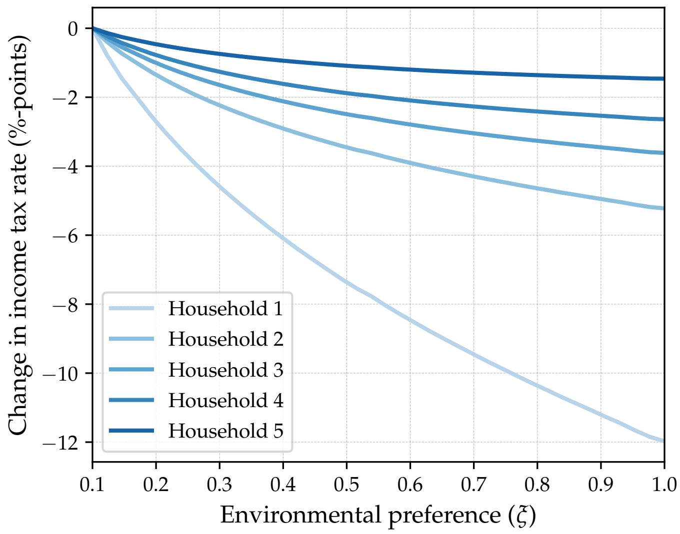

Figure 5: Change in optimal income taxation

Notes: The figure shows changes in optimal income tax rates ($\tau_{w,i}$) relative to optimal income tax at baseline, where $\xi = 0.1$. Changes are in %-points. The figure is color-coded such that more productive households are indicated by a darker shade of blue.

We note that the government chooses to decrease all income tax rates, including those for rich households, as the environmental preference rises. This outcome reflects the increasing role of environmental tax revenue in financing government spending and lump-sum transfers, which reduces the need for distortionary income taxation.[11] Essentially, this is an appearance of the efficiency double dividend, achieved by shifting public financing from distortionary income taxes to the non-distortionary environmental tax to increase welfare.

However, distortionary income subsidies for low-productivity (and hence low-income) households are increased. Here the government's inequality concern tri-

[11]One might think that this decrease can be explained by incentive compatibility constraints, i.e., that taxes for the rich must be lowered to increase subsidies to the poor. However, the income tax rates for the rich households are lowered regardless of their presence.

umphs over its desire for the efficiency dividend. As environmental preference - and thus environmental taxation - increases, the government continuously redistributes some of the additional revenues through income subsidies instead of lump-sum transfers. Income subsidies can be seen as a more direct (but also more distortionary) redistribution tool than lump-sum transfers, as the latter go to all households, while income subsidies specifically target the poor. As environmental preference and the environmental tax rise, poor households require greater compensation for increases in the price of the dirty good. The government chooses to provide with labor subsidies as compensation through transfers is limited, as these transfers increase at a diminishing rate when environmental taxes rise.

Consequently, while environmental taxation alone tends to be regressive and the ability to compensate poor households through transfers diminishes at higher environmental tax rates, a fully flexible government has the means to counteract this regressiveness through labor subsidies, thereby decreasing the undesirable inequality effect of higher environmental taxation.

Optimal environmental taxation

Figure 6 illustrates the optimal environmental tax for alternative levels of the environmental preference. The optimal tax rate is shown for all three income taxation scenarios.

We note that optimal environmental taxation is strictly rising in the environmental preference in all scenarios. Furthermore, the figure shows that continuous optimization of income tax rates has a minimal impact on the optimal environmental tax when the income tax rates are optimized at baseline (i.e., there is a minimal difference between the blue and the green lines). However, we note that optimal taxation is highly dependent on the baseline income tax system.

For small values of the environmental preference, the optimal environmental tax is higher under the pre-existing income tax system than under the other two scenarios. However, for higher values of the environmental preference (around $\xi = 0.60$ and above), this picture reverses: Optimal environmental taxation becomes higher in the second and third scenarios, i.e., when the initial income tax system is more progressive (and, in the third scenario, it also increases in progressivity as the environmental preference rises).

To understand this disparity, we turn to Figure 7. This figure shows two key measures:

1. The *marginal blue welfare loss*, which arises when the environmental tax is increased from a given (on the x-axis) level. This loss represents the total fall in the sum of households' log blue utilities, when the environmental

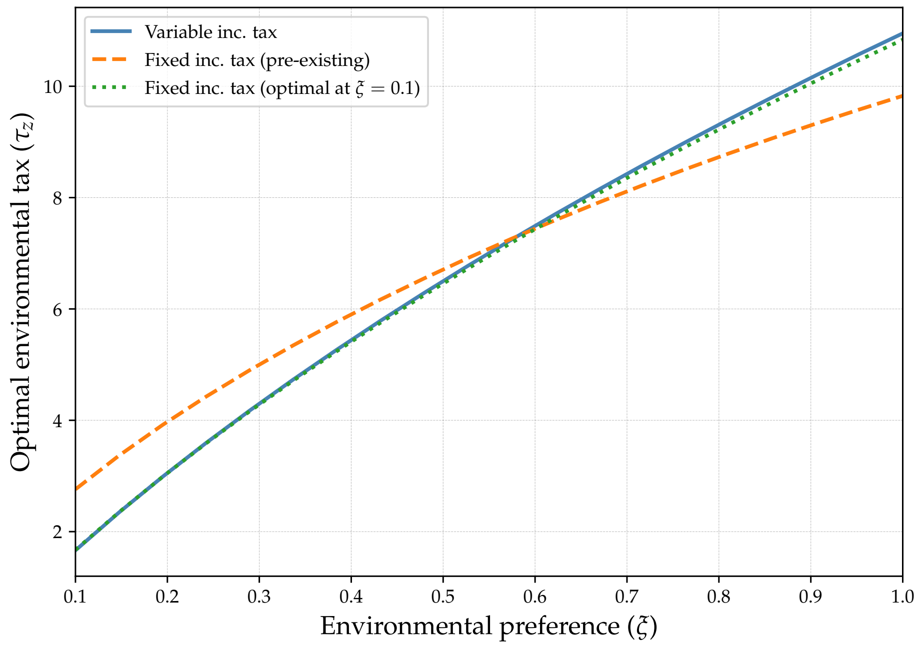

Figure 6: Optimal environmental tax

Notes: The figure shows the optimal environmental tax rate as a function of the environmental preference. The optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

tax is increased marginally. This measure varies between the scenarios, and we illustrate it for the first and second scenarios, omitting it for the third scenario.[12]

2. The *marginal green welfare benefit*, which arises when the environmental tax is increased from a given level. This benefit represents the total increase in the green welfare when the environmental tax is increased marginally. This measure depends on the given scenario *as well as* the level of the environmental preference. We illustrate it for the first and second scenarios at $\xi \in \{0.1, 0.55, 1.0\}$.

As such, Figure 7 illustrates the trade-off faced by the government. For a given environmental preference, the government will set the environmental tax rate at the level where the marginal blue welfare loss *equals* the marginal green welfare benefit. If the tax rate is set at any other level, the government can (and thus will) change it to increase social welfare, provided its revenue requirement is met.

Regardless of the tax system, the marginal benefit is convex and decreasing

[12]The third scenario is omitted as the schedule is largely identical to the one for the second scenario.

with the environmental tax. A higher environmental preference shifts the curve upward, indicating an increased demand for measures that can restrict pollution. We note that disparities in the marginal benefit are minimal across the scenarios.[13]

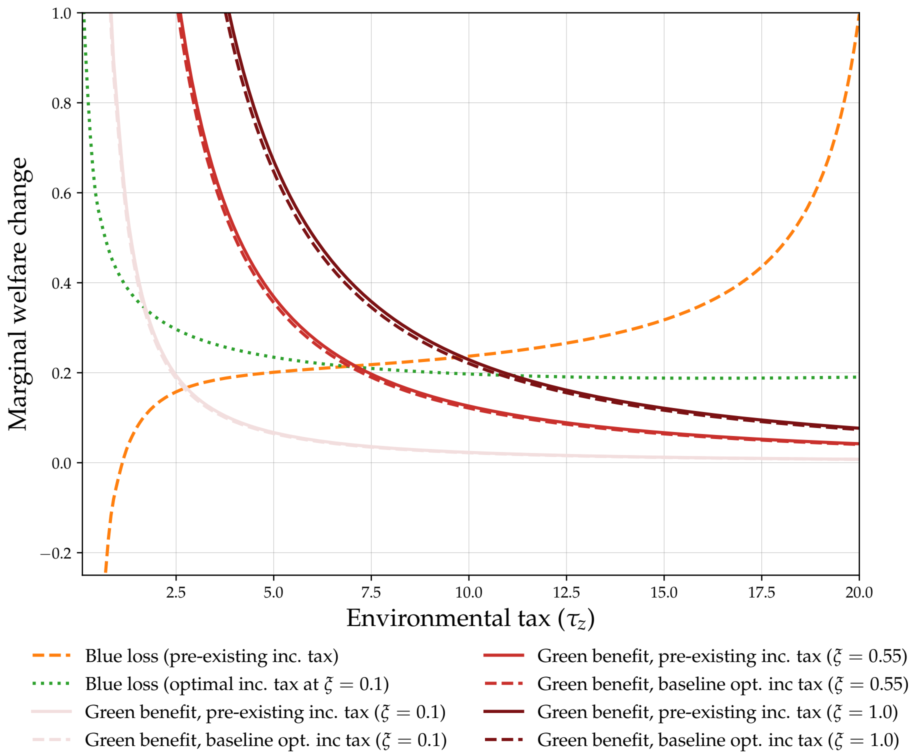

Figure 7: Welfare trade-off, fixed income tax scenarios

Notes: The figure shows the marginal blue welfare loss and the marginal green welfare benefit from increasing the environmental tax rate when it is at a given level. The dotted green line indicates the marginal blue welfare loss in case of fixed optimal income taxes, while dashed maroon lines show the marginal green welfare benefit in this scenario. The dashed orange line indicates the marginal blue welfare loss in case of fixed pre-existing income taxes, while solid maroon lines show the marginal green welfare benefit in this scenario. Maroon lines are color-coded such that a darker shade of maroon indicates the welfare benefit for a higher rate of environmental preference (ξ).

This, however, is not true for the marginal loss. Under the pre-existing income tax system, the marginal loss is negative for low environmental tax rates. In this case, the environmental tax increases social welfare even when the government has zero environmental preference. As discussed in Section 4.3, this occurs because the poorest household gains more in log blue utility than the other households collectively lose in log blue utility. Even when the marginal loss becomes positive at higher tax rates, it (initially) remains relatively low. This too can be attributed to the aforementioned dynamic: Lump-sum transfers restrict the fall in the poorest household's utility, dampening increases in the marginal loss.

In contrast, the marginal loss under the baseline optimal income tax system is

[13] Although this has a minor effect on optimal taxation, we choose to focus on the larger effect introduced through disparities in the marginal loss.

positive for all values of the environmental tax and especially large at low taxation levels.

As a result, the marginal benefit equals the marginal loss at a *higher* environmental tax rate under the pre-existing income tax system when the environmental preference is low. The intuition behind this result is as follows. At low environmental preferences, the environmental tax base is large, and substantial revenue gains from environmental tax increases turn into large increases in lump-sum transfers. These transfers compensate the poorest household. Since this household is worse off under the pre-existing income tax system, a lump-sum transfer has a greater impact on its log utility, making environmental taxation substantially *cheaper* (in terms of blue welfare). Under the more progressive optimal tax system, the poorest require a higher compensation *and* become "less important" for the government (due to higher utility, increases in their log utility become smaller). As a result, environmental taxation proves more *costly* at low environmental preferences if income taxes are optimized.

Having explained the differences in optimal taxation at low levels of environmental preference, we turn to the disparities observed at higher levels. Here, the optimal environmental tax is higher when income taxation is optimal (baseline optimal as well as continuously optimal) compared to when income taxation is set to the pre-existing level. The main reason why this happens can again be seen in Figure 7, which shows that as the environmental tax increases, the marginal loss under the pre-existing income tax system continues to increase, while the marginal blue welfare loss under the baseline optimal income tax system continues to decrease. Thus, at high levels of the environmental preference, marginal benefit equals the marginal loss at a *lower* environmental tax rate under the fixed pre-existing income tax system. The intuition behind this result is as follows. As the government's environmental preference rises, its ability to compensate the poorest household through lump-sum transfers becomes limited due to diminishing increases in environmental tax revenue and falling income tax revenue. Under pre-existing income tax rates, the poorest household is (substantially) poorer than under the more progressive optimal income tax rates. Hence, decreases in this household's utility translate into large decreases in social welfare (due to the logarithmic transformation). As such, the inability to adequately compensate this household at higher levels of the environmental preference renders environmental taxation more *costly* under the pre-existing income tax system, creating the pattern observed in Figure 6.

To conclude, at low levels of environmental preference and under high disposable income inequality (i.e., in the case of the pre-existing income tax system), increasing environmental taxes generate additional revenues that can be redistributed through lump-sum transfers, providing more than sufficient compensation to poorer households for the higher price of the dirty good and lower labor income. This redistribution creates an equality dividend, incentivizing the inequality-averse government unable to change the income tax system to increase environmental taxation to make the poorest better off. However, as the environmental preference and hence the optimal environmental taxation rises further, the dividend diminishes due to declining increases in revenue and increased compensation requirements for poorer households. Here, the inherent regressiveness of environmental taxation places a constraint on its optimal level in scenarios with an initially unequal disposable income distribution (such as the pre-existing income tax scenario). Consequently, at higher environmental preferences, a government unable to adjust income taxes ultimately sets a *lower* environmental tax rate than a government capable of adjusting income taxes - either continuously or initially - to counteract the regressiveness of increased environmental taxation.

Notably, some of our results depend on the applied calibration. However, the underlying mechanisms described in this section remain consistent upon changes in parameter values. In Appendix E, we check the robustness of our results to changes in the government spending requirement, elasticity of substitution and the subsistence dirty good consumption level.

5 Extension with clean technology

Our analysis thus far shows that the level of the optimal environmental tax depends on the government's ability to adequately compensate poorer households through revenue recycling. In the baseline formulation, where the substitutability between labor and pollution is low, firms have limited ability to substitute away from pollution as its price increases, resulting in substantial additional revenues from environmental taxation. In reality, however, clean technologies that offer abatement options may be available. To test the robustness of our baseline results, we extend the model to include a clean technology input.

5.1 Changes

We introduce the clean technology (abatement) input factor with minimal departure from the baseline model. This choice is motivated by our desire to clearly isolate the effect of introducing such technology on optimal taxation, holding other factors constant.

Production technology

We extend the production technology by assuming that the "dirty" portion of production is provided jointly by pollution (z_j) *and* clean technology (abatement, a_j), contrary to the baseline scenario where it was solely dependent on pollution. This extension is captured by employing a nested constant elasticity of substitution (nested CES) production function:

$$f_j(t_j, z_j, a_j) = \left[\epsilon_j t_j^{\frac{\sigma-1}{\sigma}} + (1-\epsilon_j) \left\{ \left[\varepsilon_j z_j^{\frac{\varsigma-1}{\varsigma}} + (1-\varepsilon_j) a_j^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma-1}} \right\}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (5.1.1)$$

where ε_j represents the share of pollution within the dirty component of production, and ς is the elasticity of substitution between pollution and clean technology. Under the assumption that $\varsigma > 0$, firms will substitute clean technology for pollution when environmental taxes increase, provided the price of abatement remains constant or increases relatively less than the pollution tax rate τ_z.

Market for clean technology

We assume that the supply of clean technology is perfectly elastic at a fixed exogenous price p_a. While recognizing that assuming a constant p_a is a simplification - and that endogenous determination of p_a would be preferable in a more detailed analysis - our main objective is to examine the qualitative implications of introducing an abatement input that firms can substitute towards in response to higher environmental taxation. Hence, any p_a that does not drastically alter optimal taxation or resource allocation at the baseline level ($\xi = 0.1$) is considered acceptable.

Ownership of clean technology and budget constraint

Given the constant p_a, total revenue from the sale of clean technology is:

$$\mathcal{A} = p_a(a_c + a_d).$$

We further assume private, heterogeneous ownership of the rents from clean technology, such that total rents received by household i are:

$$\varphi_i \mathcal{A},$$

where φ_i denotes the ownership share of household i, and feasibility in general equilibrium requires that $\sum_{i=1}^{N} \varphi_i = 1$.

Thus, the after-tax income for household i is given by:

$$m_i^d = (1 - \tau_{w,i}) \left[\phi_i w(t - \ell_i) + \varphi_i \mathcal{A} \right].$$

Note that income derived from ownership of clean technology is assumed to be taxed at the same rate as labor income, which is reasonable, as pre-existing income tax rates are calibrated to match taxation of *total* income, including potential technology ownership rents.

Solving the model

The extended model is solved numerically, using the same methods as applied to the baseline model. The inner equation system is extended with two FOCs (for clean technology) and two unknowns (optimal levels of clean technology). The new FOCs are derived in Appendix D. The outer layer remains unchanged.

5.2 Further calibration

Clean technology ownership

We calibrate the ownership share of clean production technology across productivity to match the wealth distribution between income quintiles in the US in 2019 (CBO, 2022). The calibration is stated in Table 4.

Table 4: Clean technology ownership by quintile

	Income quintile (i)				
	1	2	3	4	5
Clean tech. ownership share (φ_i)	0.03	0.04	0.08	0.15	0.70

Additional new parameters

Elasticity of substitution between clean technology and pollution (ς) is calibrated to match empirical estimates from Jo (2020). Price of the clean technology is set to 5.0, while ε_j is calibrated such that the share of expenditure on pollution in total expenditure on the "dirty" side of production (i.e., the nest, which includes pollution and clean technology) matches the share of energy extracted from fossil fuels out of total energy extracted in the US in 2024 (Center for Sustainable Systems, University of Michigan, 2024) when τ_z is set to 5.0.[14] The choice of 5.0 as a benchmark is somewhat arbitrary, and we recognize that a more elaborate treatment of

[14] See Appendix D for a derivation of this share.

the chosen parameters is warranted. Varying p_a changes some results, which are discussed in Section 5.3.

Table 5: Extension parameters

p_a	Price of clean technology	5.0
ε_j	Pollution intensity	0.82
ς	Elasticity of substitution between pollution and clean tech	2.0

5.3 Extension results

Figure 8 illustrates optimal environmental taxation across various levels of environmental preference. We note that the changes introduced to the production technology affect the overall production capacity of the economy, making direct quantitative comparisons with baseline results challenging. Consequently, our analysis will primarily focus on qualitative insights.

The similarities

From Figure 8, it is evident that the general dynamics remain consistent with the baseline model upon introduction of the new input. Specifically, a government unable to modify the existing income tax system will, for lower values of ξ, choose a higher level of environmental taxation compared to a government capable of optimally adjusting the income tax system (either initially or continuously). This picture, again, reverses at higher levels of the preference. The underlying mechanism remains the same.

Importantly, consistency with the baseline depends heavily on our calibration of the clean technology price, p_a. If this price is set at a lower level, the optimal environmental tax decreases under the pre-existing income tax system, and if the downward adjustment in the price is large enough, the tax is set lower in case of the pre-existing income tax system for all levels of environmental preference considered. The logic behind this result is as follows. When clean technology is cheap, firms demand more of it when the environmental tax rises. This means that the shift in demand from pollution to clean technology following such increases is larger. This contributes to a larger reduction in environmental tax revenues, which depresses lump-sum transfers more, reducing the government's ability to compensate the poor and making it more reluctant to increase the tax. Sensitivity of optimal environmental taxation to the clean technology price is illustrated in Figure 9.

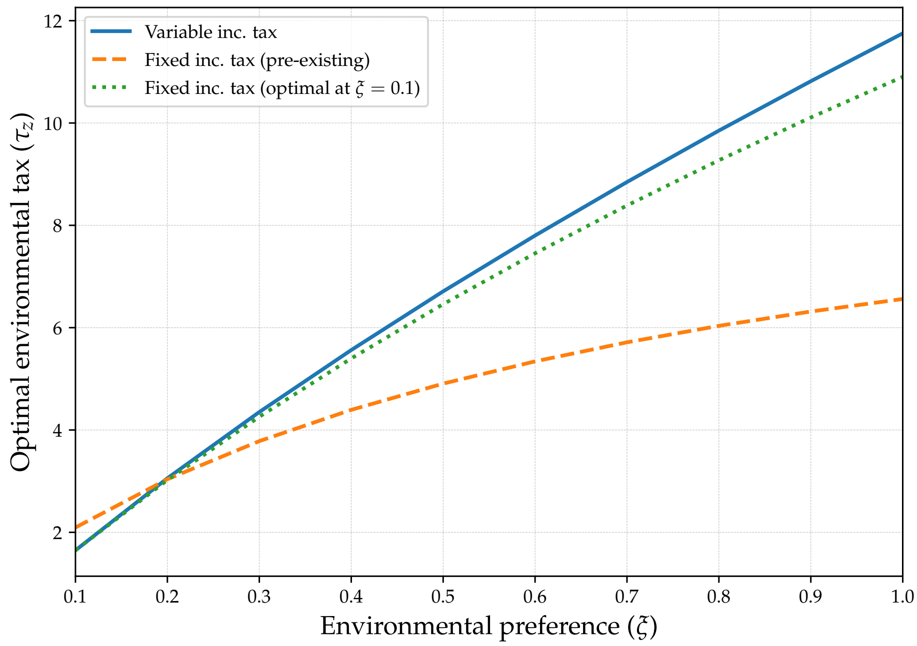

Figure 8: Optimal environmental taxation, clean tech. extension

Notes: The figure shows the optimal environmental tax rate as a function of the environmental preference for clean tech. extension. The optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

Notably, our results also depend on the calibration of the elasticity of substitution between the clean technology and the polluting good. In Appendix E, we provide a robustness check where we vary ς and briefly discuss the implications.

The differences

Contrary to the baseline model, we note an increasing divergence (with respect to ξ) between optimal environmental tax rates set by governments that can adjust income taxes initially versus continuously. As shown in Figure 8, optimal environmental taxation is increasingly higher when the government can continuously adjust income taxation. The intuition is as follows. When the environmental tax is increased, firms substitute away from pollution toward clean technology. This reduces the environmental tax base, diminishing tax revenues and limiting lump-sum transfers relative to the scenario without clean technology. The income derived from ownership of clean technology can thus be viewed as an indirect revenue stream from environmental taxation. However, instead of being evenly distributed among households (as implicitly assumed in the baseline model), this indirect revenue predominantly benefits richer households. Consequently, this dy-

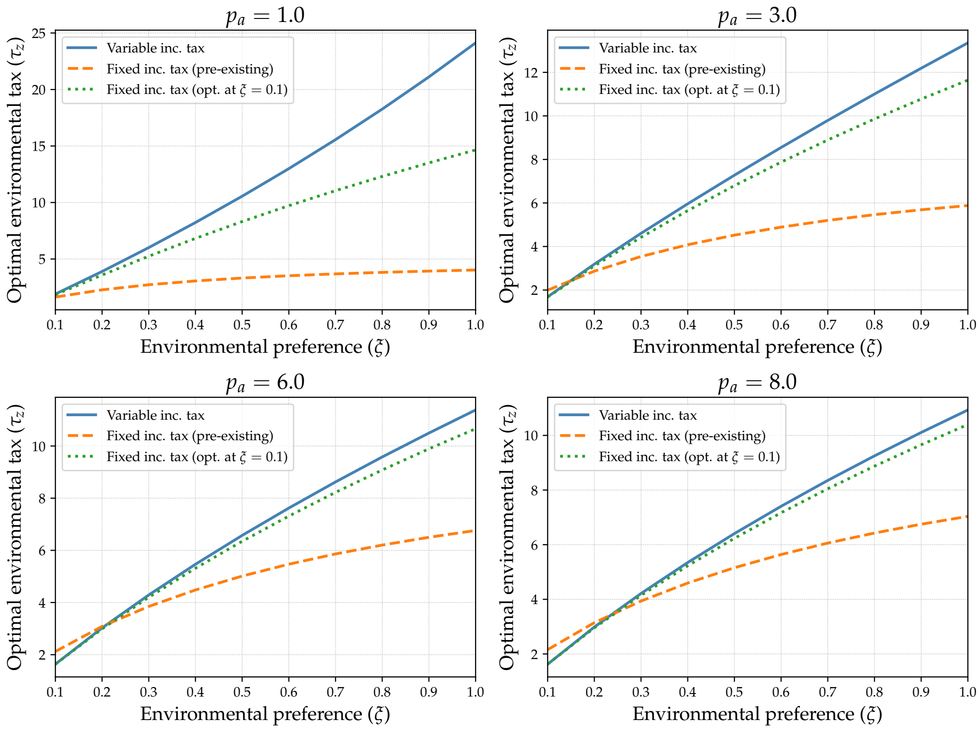

Figure 9: Optimal environmental taxation, extension, alternative clean tech. prices

Notes: The figures show the optimal environmental tax rates as a function of the environmental preference for different levels of clean tech. price (p_a) in the clean tech. extension. In each panel, the optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

namic exacerbates inequality effects of increasing environmental taxation, restraining its optimal level in economies managed by policymakers who value equality but lack the ability to implement progressive adjustments in income taxation to counteract this effect.

6 Discussion

In this section, we discuss our results. We critique the qualitative manner of our findings as well as the assumptions made in the applied framework. We propose extensions and changes to the model that can mitigate these issues. Additionally, we compare our results with relevant empirical findings and outline our contribution to the literature on the matter.

Limits of our calibration

A natural question to pose regarding our analysis is: Are our results quantitatively tractable, i.e., are the illustrated optimal environmental and income tax rates predictive of actual environmental and income tax rates in the US in the 2010s? The short answer is *no*, which is also a central limit of our study.

The main reason for this outcome is the highly stylized nature of the applied framework. Our social welfare function is designed to exhibit certain realistic traits such as inequality-aversion, but almost certainly does not represent how the US government makes decisions. This is clearly shown by the optimal income taxation in our model being significantly different from the pre-existing US income tax rates in 2013. A way to improve on this is to calibrate the environmental preference and inequality aversion parameter in a constant relative risk aversion (CRRA) social welfare function and introduce more constraints on the choice of income taxation such that the implied optimal taxation matches data on environmental tax revenues as a share of GDP and pre-existing income tax rates:

However, instead of providing quantitative predictive insights, the aim of this study is to bring qualitative insights into the dynamics of the tax system under different circumstances, making the simpler logarithmic social welfare function sufficient for our purposes.

Model assumptions

The assumption of a perfectly flexible labor market leaves out many potential effects from our model. As shown by Yang and Tang (2023), environmental taxes might have heterogeneous effects on the wage in different labor markets. This introduces a possible third channel that might affect inequality, namely, a wage channel. This channel is currently not present in our model.

Another assumption in our model is that pollution has a homogeneous effect on households' utility. This assumption can be criticized, as some studies argue for heterogeneous effects of pollution, as it is found to disproportionately affect households residing in (poorer) industrial districts (see Hajat et al., 2015). This means that while the *costs* of environmental taxation are regressively distributed, the *benefits* of environmental taxation might be progressively distributed. As such, this could alter how the government views the environmental tax and thus how it designs its policy packages.

We also assume that the government only has access to two specific revenue recycling instruments; uniform lump-sum transfers and labor income subsidies. Allowing the government to have access to more revenue recycling instruments, as is done by Klenert et al. (2018b), might change how much the government can raise the environmental tax, as the diminishing ability to compensate poorer households for the cost of the environmental tax is the key reason why governments with fixed income tax systems are reluctant to raise the environmental tax further.

Empirical relevance

Some of our findings are supported by empirical studies of interactions between environmental taxes, revenue redistribution mechanisms, and the progressiveness of the income tax system. However, we note that empirical literature on this topic is limited, a fact that in itself motivates further applied research.

Andersson and Atkinson (2020) study the incidence of energy and carbon taxes across the income distribution in OECD countries. Regressiveness is defined as the implicit share of income paid due to environmental taxation, and income inequality is measured by the Gini coefficient. They find that environmental taxes are less regressive in countries with lower income inequality and more regressive where inequality is higher. As shown in Section 4.3, our model reproduces this pattern. The authors further argue that greater inequality increases the political resistance to expanding environmental taxation. As anecdotal evidence, they mention the lack of political opposition in relatively equal Nordic countries, where carbon taxation was introduced in the 1990s. In our framework, this mechanism manifests through the government's reluctance to raise environmental taxes unless redistribution through the income tax system has first been achieved.

Oueslati et al. (2016) perform a correlation study between environmental tax revenues as a share of GDP and income inequality measured by the Gini coefficient for 34 OECD countries (1995-2011). The study finds no significant overall relationship between the variables. However, when the sample is reduced to countries with revenue recycling (e.g., through labor subsidies), a negative relationship appears. Additionally, Metcalf (1999) documents the possibility of achieving increased equality through adequate redistribution of revenues that materialize as a consequence of increased environmental taxation in a hypothetical green reform in the American context of the 1990s. These studies serve as an empirical backdrop to the theoretical literature on the double dividend of redistribution (see Klenert et al., 2018b; Chiroleu-Assouline and Fodha, 2014), which appears in our findings when environmental preference is low.

Our contribution

Our analysis provides insight into the mechanisms that connect inequality and (optimal) environmental taxation as well as (optimal) income taxation, when redistribution of environmental tax revenue through lump-sum transfers is possible. Furthermore, we contribute an analysis of the equality double dividend at high levels of environmental preference, in contrast to Klenert et al. (2018b), who only look at such dividends at low levels of the latter.

7 Conclusion

We study optimal environmental taxation for an inequality-averse government, as well as the interaction between environmental and income taxation.

We apply a theoretical framework in which the "dirty good" (i.e., the more pollution-intensive good) is modeled as a necessity, consistent with empirical findings. Our main results are as follows.

Under an initially unequal distribution of disposable income (i.e., under a relatively less progressive pre-existing income tax system), an inequality-averse government has the means to achieve a more egalitarian distribution of welfare by increasing environmental taxation, thereby obtaining an equality dividend. This result critically depends on several factors: the government's ability to redistribute revenues through lump-sum transfers, an initially unequal income (and welfare) distribution, and substantial revenues from the environmental tax (i.e., a broad tax base resulting from extensive pollution). When the latter condition fails (i.e., at high levels of taxation), the ability to achieve an equality dividend vanishes, and the inequality-averse government optimally selects lower levels of environmental taxation when unable to adjust the progressivity of the income tax.

When the government can make an initial adjustment to the income tax system to achieve a more egalitarian income distribution, the regressiveness of the environmental tax-and-transfer system is mitigated. Consequently, at high levels of environmental preference, optimal environmental taxation becomes higher than it would be under an initially less egalitarian income distribution, as poorer households are less impacted by the tax.

When the income tax system can be adjusted continuously, the government further mitigates regressiveness by increasing labor subsidies for poorer households, thereby rendering even higher levels of environmental taxation optimal.

Our findings are generally robust to an extension of the framework that includes unequally owned clean technology. However, the possibility of an equality dividend under pre-existing income tax rates is highly sensitive to the calibration of the clean technology price; if clean technology is relatively inexpensive, this dividend becomes questionable.

Although our findings are supported by some empirical evidence, the literature remains limited, highlighting the need for more empirical analysis to inform models such as ours.

Finally, our findings should be interpreted in light of the framework's limitations. Specifically, the model's predictions are not quantitatively tractable, necessitating more elaborate calibration to data, such as further calibration of the social welfare function, to yield optimal decisions that are empirically comparable.

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8 Appendix

A Firms

Firms' first order conditions

The firms' profit maximization problem is given by (2.1.1). We let $\psi = \frac{\sigma-1}{\sigma}$. Then, the production function can be written as

$$f_j(t_j, z_j) = \left(\epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi}\right)^{\frac{1}{\psi}}$$

Firstly, we find the FOC with respect to t_j:

$$\begin{aligned} \frac{\partial \pi_j}{\partial t_j} = p_j \frac{\partial f_j}{\partial t_j} - w = 0 \Leftrightarrow \\ w = p_j \frac{\partial f_j}{\partial t_j} \end{aligned} \tag{A.1}$$

Focusing on $\frac{\partial f_j}{\partial t_j}$, we get:

$$\begin{aligned} \frac{\partial f_j}{\partial t_j} &= \frac{1}{\psi}\left(\epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi}\right)^{\frac{1}{\psi}-1} \cdot \epsilon_j \psi t_j^{\psi-1} \\ &= \left(\epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi}\right)^{\frac{1-\psi}{\psi}} \cdot \epsilon_j t_j^{\psi-1} \end{aligned}$$

We know that $f_j(t_j, z_j) = \left(\epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi}\right)^{\frac{1}{\psi}} \Leftrightarrow \epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi} = f_j^{\psi}$. Inserting this, we get:

$$\begin{aligned} \frac{\partial f_j}{\partial t_j} &= \left(f_j^{\psi}\right)^{\frac{1-\psi}{\psi}} \cdot \epsilon_j t_j^{\psi-1} \\ &= f_j^{1-\psi} \cdot \epsilon_j t_j^{\psi-1} \end{aligned}$$

We also know that $1-\psi = 1 - \frac{\sigma-1}{\sigma} = \frac{\sigma-(\sigma-1)}{\sigma} = \frac{1}{\sigma}$ and $\psi - 1 = \frac{\sigma-1}{\sigma} - 1 = \frac{\sigma-1-\sigma}{\sigma} = -\frac{1}{\sigma}$. Inserting this, we get

$$\frac{\partial f_j}{\partial t_j} = f_j^{\frac{1}{\sigma}} \cdot \epsilon_j t_j^{-\frac{1}{\sigma}}$$

We can substitute this back into (A.1), yielding

$$w = p_j f_j^{\frac{1}{\sigma}} \epsilon_j t_j^{-\frac{1}{\sigma}}$$

Rearranging this, we finally obtain equation (2.1.2):

$$w = \epsilon_j t_j^{-\frac{1}{\sigma}} f_j(t_j, z_j)^{\frac{1}{\sigma}} p_j$$

Secondly, we find the FOC with respect to z_j:

$$\begin{aligned} \frac{\partial \pi_j}{\partial z_j} = p_j \frac{\partial f_j}{\partial z_j} - \tau_z = 0 \Leftrightarrow \\ \tau_z = p_j \frac{\partial f_j}{\partial z_j} \end{aligned} \tag{A.2}$$

Focusing on $\frac{\partial f_j}{\partial z_j}$, we get:

$$\begin{aligned} \frac{\partial f_j}{\partial z_j} &= \left(\epsilon_j t_j^{\psi} + (1-\epsilon_j) z_j^{\psi}\right)^{\frac{1-\psi}{\psi}} \cdot (1-\epsilon_j) z_j^{\psi-1} \\ &= \left(f_j^{\psi}\right)^{\frac{1-\psi}{\psi}} \cdot (1-\epsilon_j) z_j^{\psi-1} \\ &= f_j^{1-\psi} \cdot (1-\epsilon_j) z_j^{\psi-1} \\ &= f_j^{\frac{1}{\sigma}} (1-\epsilon_j) z_j^{-\frac{1}{\sigma}} \end{aligned}$$

We can substitute this back into (A.2), yielding

$$\tau_z = p_j f_j^{\frac{1}{\sigma}} (1-\epsilon_j) z_j^{-\frac{1}{\sigma}}$$

Rearranging this, we finally obtain equation (2.1.3):

$$\tau_z = (1-\epsilon_j) z_j^{-\frac{1}{\sigma}} f_j(t_j, z_j)^{\frac{1}{\sigma}} p_j$$

Input ratio

Dividing (2.1.2) with (2.1.3) yields

$$\begin{aligned} \frac{w}{\tau_z} = \frac{\epsilon_j}{1-\epsilon_j} \left(\frac{t_j}{z_j}\right)^{-\frac{1}{\sigma}} \Leftrightarrow \\ \frac{t_j}{z_j} = \left(\frac{\tau_z}{w} \frac{\epsilon_j}{1-\epsilon_j}\right)^{\sigma}. \end{aligned}$$

Hence, when $\sigma > 0$, $\frac{t_j}{z_j}$ is an increasing function of $\frac{\tau_z}{w}$ and ϵ_j (as $\frac{\partial}{\partial \epsilon_j} \frac{\epsilon_j}{1-\epsilon_j} > 0$).

B Households

Optimal allocation

The Lagrangian for the household problem is

$$\mathcal{L} = u_i(c_i, d_i, \ell_i) + \lambda(m_i^d + l - p_c c_i - p_d d_i)$$

FOCs are

$$\frac{\partial \mathcal{L}}{\partial c_i} : \alpha \frac{u_i}{c_i} = \lambda p_c \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial d_i} : \beta \frac{u_i}{d_i - d_0} = \lambda p_d \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_i} : \gamma \frac{u_i}{\ell_i} = \lambda (1 - \tau_{w,i}) \phi_i w \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : m_i^d + l = p_c c_i + p_d d_i \tag{B.4}$$

Combining (B.1) and (B.2) yields

$$\begin{aligned} \frac{\alpha}{\beta} \frac{d_i - d_0}{c_i} &= \frac{p_c}{p_d} \Leftrightarrow \\ d_i - d_0 &= \frac{p_c \beta}{p_d \alpha} c_i \end{aligned} \tag{B.5}$$

Combining (B.1) and (B.3) yields

$$\begin{aligned} \frac{\alpha}{\gamma} \frac{\ell_i}{c_i} &= \frac{p_c}{(1 - \tau_{w,i}) \phi_i w} \Leftrightarrow \\ \ell_i &= \frac{p_c \gamma}{\alpha (1 - \tau_{w,i}) \phi_i w} c_i \end{aligned} \tag{B.6}$$

Inserting (B.5) and (B.6) into (B.4) yields

$$\begin{aligned} (1 - \tau_{w,i}) \phi_i w \left(t - \frac{p_c \gamma}{\alpha (1 - \tau_{w,i}) \phi_i w} c_i \right) + l &= p_c c_i + p_d \left(\frac{p_c \beta}{p_d \alpha} c_i + d_0 \right) \Leftrightarrow \\ (1 - \tau_{w,i}) \phi_i w t - \frac{p_c \gamma}{\alpha} c_i + l &= p_c c_i + \frac{p_c \beta}{\alpha} c_i + p_d d_0 \Leftrightarrow \\ p_c c_i \left(\frac{\gamma}{\alpha} + 1 + \frac{\beta}{\alpha} \right) &= (1 - \tau_{w,i}) \phi_i w t + l - p_d d_0 \Leftrightarrow \\ c_i &= \frac{\alpha}{\alpha + \beta + \gamma} \frac{(1 - \tau_{w,i}) \phi_i w t + l - p_d d_0}{p_c} \end{aligned} \tag{B.7}$$

Inserting (B.7) into (B.5) yields

$$d_i = \frac{\beta}{\alpha+\beta+\gamma} \frac{(1-\tau_{w,i})\phi_i wt + l - p_d d_0}{p_d} + d_0$$

Inserting (B.7) into (B.6) yields

$$\ell_i = \frac{\gamma}{\alpha+\beta+\gamma} \frac{(1-\tau_{w,i})\phi_i wt + l - p_d d_0}{(1-\tau_{w,i})\phi_i w}$$

Necessity of the dirty good

The elasticity of the optimal consumption level of the dirty good d_i with regards to potential disposable income (i.e. $h_i + p_d d_0$) is

$$\begin{aligned}
\epsilon^{d_i}_{h_i+p_d d_0} &= \frac{\partial d_i}{\partial\{h_i + p_d d_0\}} \frac{h_i + p_d d_0}{d_i} \\
&= \frac{\beta}{(\alpha+\beta+\gamma)\, p_d} \frac{h_i + p_d d_0}{\frac{\beta}{(\alpha+\beta+\gamma)\, p_d}(h_i + p_d d_0) + \frac{\alpha+\gamma}{\alpha+\beta+\gamma} d_0} \\
&= \frac{\beta\,(h_i + p_d d_0)}{\beta\,(h_i + p_d d_0) + (\alpha+\gamma)\, d_0\, p_d} \\
&= \frac{1}{1 \; + \; \frac{(\alpha+\gamma)\, d_0\, p_d}{\beta\,(h_i + p_d d_0)}}.
\end{aligned}$$

which is lower than one whenever $d_0 > 0$.

Marginal utility of income

The indirect utility function v_i is obtained by substitution of (2.2.1)–(2.2.3) into u_i:

$$v_i = \left(\frac{\alpha}{\alpha+\beta+\gamma} \frac{h_i}{p_c}\right)^{\alpha} \left(\frac{\beta}{\alpha+\beta+\gamma} \frac{h_i + d_0}{p_d}\right)^{\beta} \left(\frac{\gamma}{\alpha+\beta+\gamma} \frac{h_i}{(1-\tau_{i,w})\phi_i w}\right)^{\gamma} + \left(e_0 - \xi(z_c + z_d)^{\theta}\right).$$

Double differentiating with regard to potential income $(h_i + p_d d_0)$ yields

$$\begin{aligned}
\frac{\partial^2 v_i}{\partial\{h_i + p_d d_0\}^2} &= \frac{\partial}{\partial\{h_i + p_d d_0\}} \Big[- v_i \Big(\frac{\alpha+\gamma}{h_i} + \frac{\beta}{h_i + d_0} \Big) \Big] \\
&= -\Big[v_i' \Big(\frac{\alpha+\gamma}{h_i} + \frac{\beta}{h_i + d_0} \Big) + v_i \left(-\frac{\alpha+\gamma}{h_i^2} - \frac{\beta}{(h_i + d_0)^2} \right) \Big] \\
&= -\Big[- v_i \Big(\frac{\alpha+\gamma}{h_i} + \frac{\beta}{h_i + d_0} \Big)^2 - v_i \Big(\frac{\alpha+\gamma}{h_i^2} + \frac{\beta}{(h_i + d_0)^2} \Big) \Big] \\
&= v_i \Big[\Big(\frac{\alpha+\gamma}{h_i} + \frac{\beta}{h_i + d_0} \Big)^2 - \Big(\frac{\alpha+\gamma}{h_i^2} + \frac{\beta}{(h_i + d_0)^2} \Big) \Big].
\end{aligned}$$

Such that marginal utility of income is diminishing (i.e. $\frac{\partial^2 v_i}{\partial \{h_i + p_d d_0\}^2} < 0$), whenever

$$\Big(\frac{\alpha+\gamma}{h_i^2} + \frac{\beta}{(h_i+d_0)^2}\Big) > \Big(\frac{\alpha+\gamma}{h_i} + \frac{\beta}{h_i+d_0}\Big)^2.$$

This condition can be evaluated for a given parameter vector $(h_i, \alpha, \gamma, \beta, d_0)$. As $\alpha + \gamma + \beta > 1$ in the applied calibration, we can't be sure that this condition holds.

Compensating variation

We define $\tilde{u}_i \equiv u_i - e(z)$. Lagrangian for the expenditure minimization problem is

$$\mathcal{L} = p_c c_i + p_d d_i + \ell_i(1-\tau_{w,i})\phi_i w + \lambda(\bar{u} - \tilde{u}_i(c_i, d_i, \ell_i))$$

FOCs are

$$\frac{\partial \mathcal{L}}{\partial c_i} : p_c = \lambda\alpha\frac{\tilde{u}_i}{c_i} \tag{B.8}$$
$$\frac{\partial \mathcal{L}}{\partial d_i} : p_d = \lambda\beta\frac{\tilde{u}_i}{d_i - d_0} \tag{B.9}$$
$$\frac{\partial \mathcal{L}}{\partial \ell_i} : (1-\tau_{w,i})\phi_i w = \lambda\gamma\frac{\tilde{u}_i}{\ell_i} \tag{B.10}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} : \tilde{u}_i = \bar{u} \tag{B.11}$$

Combining (B.8) and (B.9) yields

$$d_i - d_0 = \frac{p_c\beta}{p_d\alpha}c_i. \tag{B.12}$$

Combining (B.8) and (B.10) yields

$$\ell_i = \frac{p_c\gamma}{\alpha(1-\tau_{w,i})\phi_i w}c_i. \tag{B.13}$$

Inserting (B.12) and (B.13) into (B.11) yields

$$c_i^\alpha\left(\frac{p_c\beta}{p_d\alpha}c_i\right)^\beta\left(\frac{p_c\gamma}{\alpha(1-\tau_{w,i})\phi_i w}c_i\right)^\gamma = \bar{u} \Leftrightarrow$$
$$c_i = \left(\bar{u}\left(\frac{p_d\alpha}{p_c\beta}\right)^\beta\left(\frac{\alpha(1-\tau_{w,i})\phi_i w}{p_c\gamma}\right)^\gamma\right)^{\frac{1}{\alpha+\beta+\gamma}}$$

Inserting into (B.12) yields

$$d_i = \left(\bar{u} \left(\frac{p_d \alpha}{p_c \beta} \right)^{-\alpha-\gamma} \left(\frac{\alpha(1-\tau_{w,i})\phi_i w}{p_c \gamma} \right)^{\gamma} \right)^{\frac{1}{\alpha+\beta+\gamma}} + d_0.$$

Inserting into (B.13) yields

$$\ell_i = \left(\bar{u} \left(\frac{p_d \alpha}{p_c \beta} \right)^{\beta} \left(\frac{\alpha(1-\tau_{w,i})\phi_i w}{p_c \gamma} \right)^{-\alpha-\beta} \right)^{\frac{1}{\alpha+\beta+\gamma}}.$$

Hence the expenditure function is

$$E(\bar{u}, p_c, p_d, w) = p_d\, d_0 + \frac{\alpha+\beta+\gamma}{\alpha}\, p_c \left[\bar{u} \left(\frac{p_d \alpha}{p_c \beta} \right)^{\beta} \left(\frac{\alpha(1-\tau_{w,i})\phi_i w}{p_c \gamma} \right)^{\gamma} \right]^{\frac{1}{\alpha+\beta+\gamma}}.$$

The compensating variation for a price change $p_d \to p_d'$ (where optimal allocation given p_d' yields blue utility $\bar{u}$) is calculated as

$$CV = E(\bar{u}, p_c, p_d', w) - E(\bar{u}, p_c, p_d, w).$$

C Results

Tax revenues in case of a fixed pre-existing income tax system

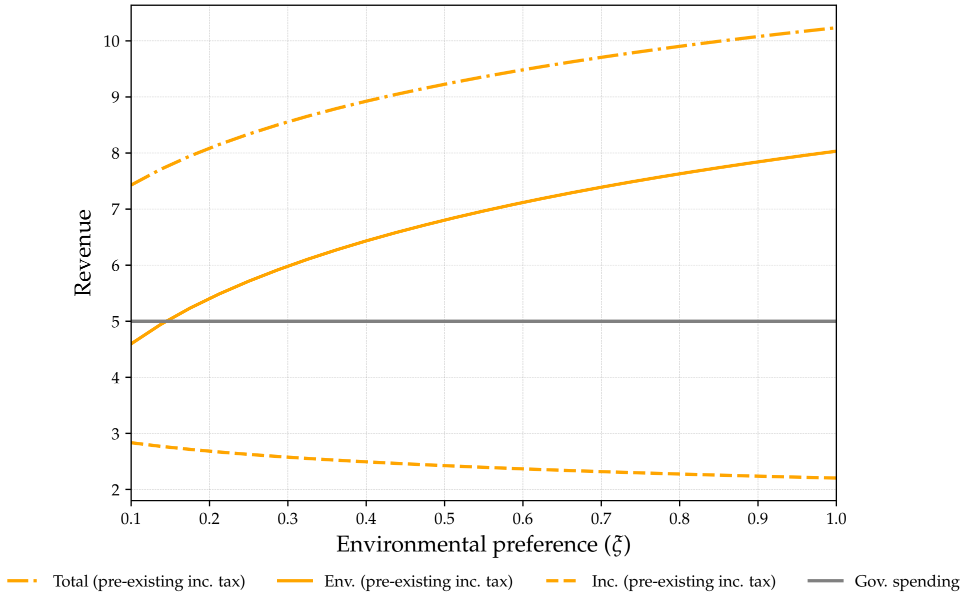

Figure 10: Tax revenue under optimal taxation, pre-existing income tax system

Notes: The figure shows revenues from environmental taxation, income taxation, and total revenues, collected as a consequence of optimal taxation at each level of environmental preference (ξ) in case of a fixed pre-existing income tax system. Income tax revenues are indicated by dotted lines, environmental tax revenues by solid lines, and total revenues by dash-dotted lines. Government spending requirement ($\mathcal{G}$) is illustrated by the gray horizontal line.

D Extension

Firms' first order conditions

The firms' profit maximization problem is:

$$\max_{t_j, z_j, a_j} \pi_j = p_j f_j(t_j, z_j, a_j) - w t_j - \tau_z z_j - p_a a_j$$

The nested CES production function is given by (5.1.1). We define $\psi_\sigma = \frac{\sigma-1}{\sigma}$ and $\psi_\varsigma = \frac{\varsigma-1}{\varsigma}$. We also let B_j represent the composite input from pollution and clean technology: $B_j \equiv \left[\varepsilon_j z_j^{\psi_\varsigma} + (1-\varepsilon_j) a_j^{\psi_\varsigma}\right]^{\frac{1}{\psi_\varsigma}}$.

Thus, the production function can be written as

$$f_j(t_j, z_j, a_j) = \left[\epsilon_j t_j^{\psi_\sigma} + (1-\epsilon_j) B_j^{\psi_\sigma}\right]^{\frac{1}{\psi_\sigma}}$$

To derive the three FOCs, we use the derivative rule for a standard CES function $Y = \left(\sum \kappa_i x_i^\nu\right)^{\frac{1}{\nu}}$, which is $\frac{\partial Y}{\partial x_k} = Y^{1-\nu} \kappa_k x_k^{\nu-1}$.

Firstly, we find the FOC with respect to t_j:

$$\frac{\partial \pi_j}{\partial t_j} = p_j \frac{\partial f_j}{\partial t_j} - w = 0 \Leftrightarrow$$

$$w = p_j \frac{\partial f_j}{\partial t_j} \tag{D.1}$$

Focusing on $\frac{\partial f_j}{\partial t_j}$, we apply the CES derivative rule and get

$$\frac{\partial f_j}{\partial t_j} = (f_j)^{1-\psi_\sigma} \cdot \epsilon_j t_j^{\psi_\sigma - 1}$$

We know that $1 - \psi_\sigma = 1 - \frac{\sigma-1}{\sigma} = \frac{\sigma-(\sigma-1)}{\sigma} = \frac{1}{\sigma}$ and $\psi_\sigma - 1 = \frac{\sigma-1}{\sigma} - 1 = \frac{\sigma-1-\sigma}{\sigma} = -\frac{1}{\sigma}$. Inserting this, we get

$$\frac{\partial f_j}{\partial t_j} = (f_j)^{\frac{1}{\sigma}} \cdot \epsilon_j t_j^{-\frac{1}{\sigma}}$$

We can substitute this back into (D.1), yielding

$$\begin{aligned} w &= p_j (f_j)^{\frac{1}{\sigma}} \cdot \epsilon_j t_j^{-\frac{1}{\sigma}} \\ &= \epsilon_j f_j(t_j, z_j, a_j)^{\frac{1}{\sigma}} t_j^{-\frac{1}{\sigma}} p_j, \end{aligned} \tag{D.2}$$

Which is the FOC with respect to t_j.

Secondly, we find the FOC with respect to z_j:

$$\frac{\partial \pi_j}{\partial z_j} = p_j \frac{\partial f_j}{\partial z_j} - \tau_z = 0 \Leftrightarrow$$
$$\tau_z = p_j \frac{\partial f_j}{\partial z_j} \tag{D.3}$$

Using the chain rule, we know that $\frac{\partial f_j}{\partial z_j} = \frac{\partial f_j}{\partial B_j}\frac{\partial B_j}{\partial z_j}$. Focusing on $\frac{\partial f_j}{\partial B_j}$ and $\frac{\partial B_j}{\partial z_j}$ separately, we apply the CES derivative rule and get:

$$\frac{\partial f_j}{\partial B_j} = \left(f_j\right)^{1-\psi_\sigma} \cdot (1-\epsilon_j) B_j^{\psi_\sigma - 1}$$
$$= (1-\epsilon_j) f_j^{\frac{1}{\sigma}} B_j^{-\frac{1}{\sigma}} \tag{D.4}$$

and

$$\frac{\partial B_j}{\partial z_j} = \left(B_j\right)^{1-\psi_\varsigma} \cdot \varepsilon_j z_j^{\psi_\varsigma - 1}$$
$$= \varepsilon_j B_j^{\frac{1}{\varsigma}} z_j^{-\frac{1}{\varsigma}} \tag{D.5}$$

Combining (D.4) and (D.5), we get:

$$\frac{\partial f_j}{\partial z_j} = (1-\epsilon_j) f_j^{\frac{1}{\sigma}} B_j^{-\frac{1}{\sigma}} \cdot \varepsilon_j B_j^{\frac{1}{\varsigma}} z_j^{-\frac{1}{\varsigma}}$$
$$= (1-\epsilon_j)\varepsilon_j f_j^{\frac{1}{\sigma}} B_j^{-\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}} z_j^{-\frac{1}{\varsigma}}$$
$$= (1-\epsilon_j)\varepsilon_j f_j^{\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}} z_j^{-\frac{1}{\varsigma}}$$

We can substitute this back into (D.3), yielding

$$\tau_z = p_j(1-\epsilon_j)\varepsilon_j f_j^{\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}} z_j^{-\frac{1}{\varsigma}}$$
$$= \ (1-\epsilon_j)\, \varepsilon_j\, f_j(t_j, z_j, a_j)^{\frac{1}{\sigma}}\, B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}}\, z_j^{-\frac{1}{\varsigma}} p_j \tag{D.6}$$

Which is the FOC with respect to z_j.

Thirdly, we find the FOC with respect to a_j:

$$\frac{\partial \pi_j}{\partial a_j} = p_j \frac{\partial f_j}{\partial a_j} - p_a = 0 \Leftrightarrow$$
$$p_a = p_j \frac{\partial f_j}{\partial a_j} \tag{D.7}$$

Using the chain rule, we know that $\frac{\partial f_j}{\partial a_j} = \frac{\partial f_j}{\partial B_j}\frac{\partial B_j}{\partial a_j}$. We have already derived $\frac{\partial f_j}{\partial B_j}$ in (D.4). Focusing on $\frac{\partial B_j}{\partial a_j}$, we apply the CES derivative rule and get:

$$
\begin{aligned}
\frac{\partial B_j}{\partial a_j} &= \left(B_j\right)^{1-\psi_\varsigma} \cdot (1-\varepsilon_j) a_j^{\psi_\varsigma - 1} \\
&= (1-\varepsilon_j) B_j^{\frac{1}{\varsigma}} a_j^{-\frac{1}{\varsigma}}
\end{aligned} \tag{D.8}
$$

Combining (D.4) and (D.8), we get:

$$
\begin{aligned}
\frac{\partial f_j}{\partial a_j} &= (1-\epsilon_j) f_j^{\frac{1}{\sigma}} B_j^{-\frac{1}{\sigma}} \cdot (1-\varepsilon_j) B_j^{\frac{1}{\varsigma}} a_j^{-\frac{1}{\varsigma}} \\
&= (1-\epsilon_j)(1-\varepsilon_j) f_j^{\frac{1}{\sigma}} B_j^{-\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}} a_j^{-\frac{1}{\varsigma}} \\
&= (1-\epsilon_j)(1-\varepsilon_j) f_j^{\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}} a_j^{-\frac{1}{\varsigma}}
\end{aligned}
$$

We can substitute this back into (D.7), yielding

$$
\begin{aligned}
p_a &= p_j (1-\epsilon_j)(1-\varepsilon_j) f_j^{\frac{1}{\sigma}} B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}} a_j^{-\frac{1}{\varsigma}} \\
&= \; (1-\epsilon_j)\,(1-\varepsilon_j)\, f_j(t_j, z_j, a_j)^{\frac{1}{\sigma}}\, B_j^{\frac{1}{\varsigma}-\frac{1}{\sigma}}\, a_j^{-\frac{1}{\varsigma}}\, p_j
\end{aligned}
$$

Which is the FOC with respect to a_j.

Share of spending on clean technology

Dividing (D.6) with (D.2) yields

$$
\frac{z_j}{a_j} = \left[\frac{1-\varepsilon_j}{\varepsilon_j}\frac{p_z}{p_a}\right]^{\frac{\varsigma}{\varsigma-1}}. \tag{D.9}
$$

Share of spending on clean tech. out of total spending on clean tech. and pollution is

$$
s_a = \frac{p_a\, a_j}{\tau_z\, z_j + p_a\, a_j}.
$$

Plugging in (D.9) yields

$$
\begin{aligned}
s_a &= \frac{(1-\varepsilon_j)\,(p_a)^{1-\varsigma}}{\varepsilon_j\,(\tau_z)^{1-\varsigma} \;+\; (1-\varepsilon_j)\,(p_a)^{1-\varsigma}} \\
&= \frac{(1-\varepsilon_j)\,(p_a)^{1-\varsigma}}{\varepsilon_j\,(\tau_z)^{1-\varsigma} + (1-\varepsilon_j)\,(p_a)^{1-\varsigma}}.
\end{aligned}
$$

For $\varsigma = 2.0$ and $p_a = \tau_z$, $s_a = 0.18 \Leftrightarrow \varepsilon_j = 0.82$.

E Robustness checks

Varying $\mathcal{G}$

This robustness check examines whether the level of government spending influences our core results on the environmental tax schedules.

Figure 11 shows that across finance requirements both higher and lower than the level of $\mathcal{G} = 5$ used in the analysis, the qualitative properties of of the optimal environmental tax schedules under the different income tax scenarios remain consistent. More specifically, the relative sizes of the three tax schedules as well as the crossing point between the schedules as the environmental preference increases are aligned with our findings in the main text.

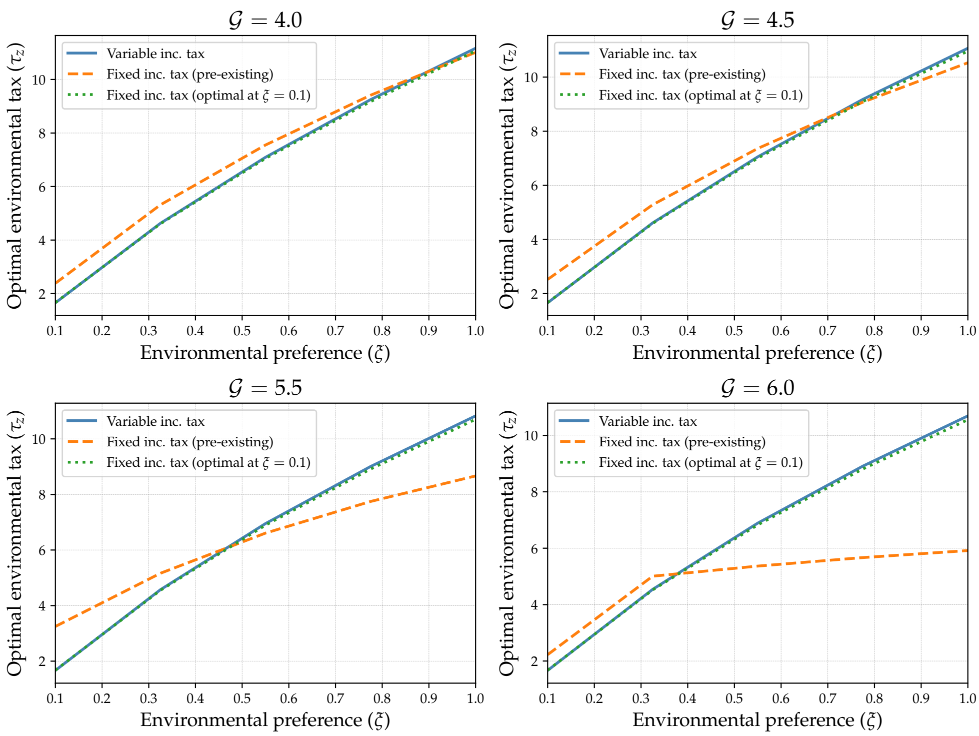

Figure 11: Optimal environmental taxation, alternative government spending

Notes: The figures show the optimal environmental tax rates as a function of the environmental preference for different levels of government spending ($\mathcal{G}$). In each panel, the optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

Varying d_0

This robustness check examines whether the subsistence level of the dirty good influences our core results on the environmental tax schedules.

Figure 12 shows that as the the effects of both increases and decreases in the subsistence level aligns with the mechanisms described in the main text. As the subsistence level increases, the environmental tax becomes increasingly more regressive, and thus compensation of the poorer households is more expensive for the government. As the governments tax revenue does not increase adequately to afford this increased demand for redistribution, the government is increasingly more reluctant to raise the environmental tax under the pre-existing income taxation scheme. Conversely, when the subsistence level decreases, the environmental tax is less regressive, thus making the government more inclined to increase the environmental tax, even under the less progressive pre-existing income taxation scheme.

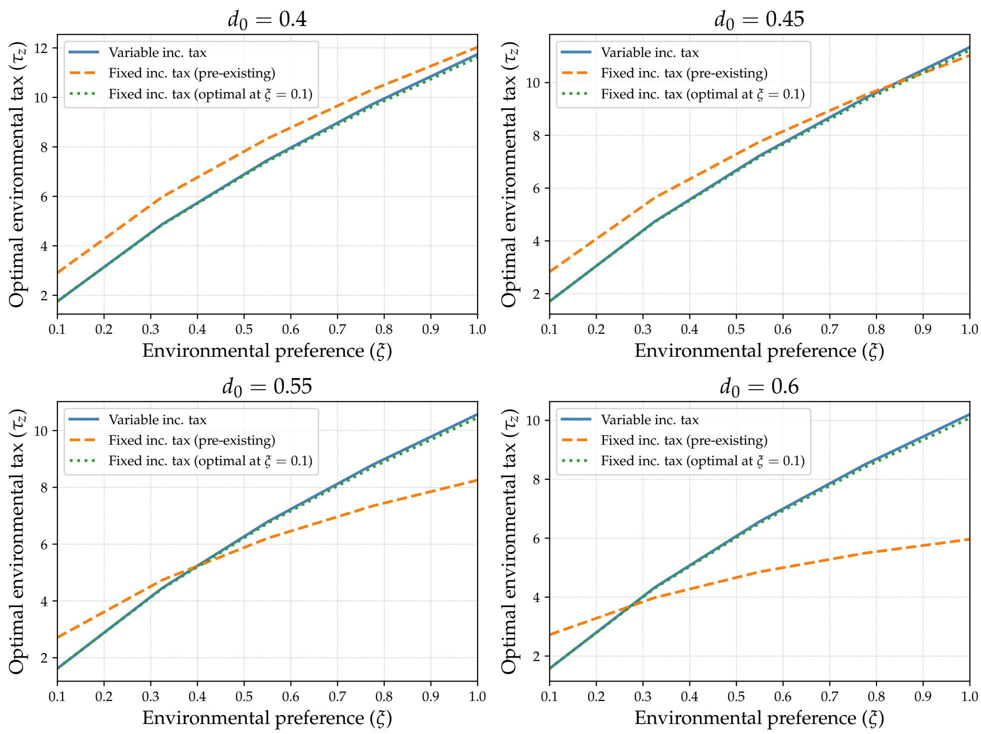

Figure 12: Optimal environmental taxation, alternative subsistence consumption level

Notes: The figures show the optimal environmental tax rates as a function of the environmental preference for different levels of subsistence level dirty good consumption (d_0). In each panel, the optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

Varying σ

This robustness check examines whether the elasticity of substitution between labor input and pollution input influences our core results on the environmental tax schedules.

The figures show that as the elasticity of substitution increases, meaning it is becoming easier for the firms to substitute between labor input and pollution input, the "equality dividend" at low environmental tax preferences under the fixed pre-existing income tax system vanishes. Additionally, the optimal environmental tax schedule is now consistently lower than under the two other scenarios. This suggests that when firms can easily substitute away from the pollution input, the tax base is reduced in such a degree that the potential for revenue recycling to offset the regressivity of the tax diminishes, making the inequality-averse government reluctant to raise the environmental tax. When the elasticity of substitution decreases, the results align with our findings presented in the main text, although the crossing point is at increasingly higher environmental tax rates, as the tax base is reduced at a slower rate.

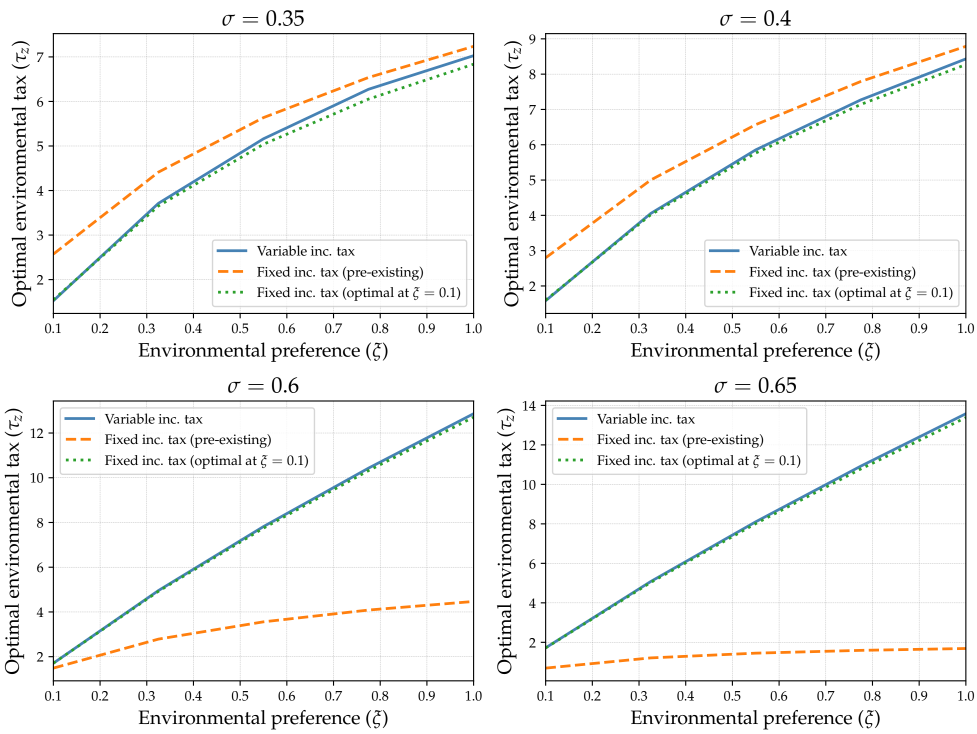

Figure 13: Optimal environmental taxation, alternative elasticity of substitution

Notes: The figures show the optimal environmental tax rates as a function of the environmental preference for different levels of the elasticity of substitution between labor and pollution (σ). In each panel, the optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.

Varying ς

This robustness check examines whether the elasticity of substitution between pollution input and clean technology input influences our core results.

Many of the effects shown in Figure 14 mirror those seen in Figure 13. As the elasticity of substitution increases, the potential "equality dividend" at low environmental preferences under the fixed pre-existing income tax system vanishes, and the optimal environmental tax schedule becomes very low and flat. This aligns with the mechanisms discussed in Section 5.3: easier substitution reduces revenue recycling potential, while the unequal distribution of clean technology rents (Section 5.2) exacerbates regressivity, making the government even more reluctant to raise the environmental tax than in the baseline model. Conversely, a decreasing elasticity of substitution aligns more with our findings in the baseline model, showing that as the elasticity of substitution between the two new input factors decreases, the extension model should behave more and more like the baseline model.

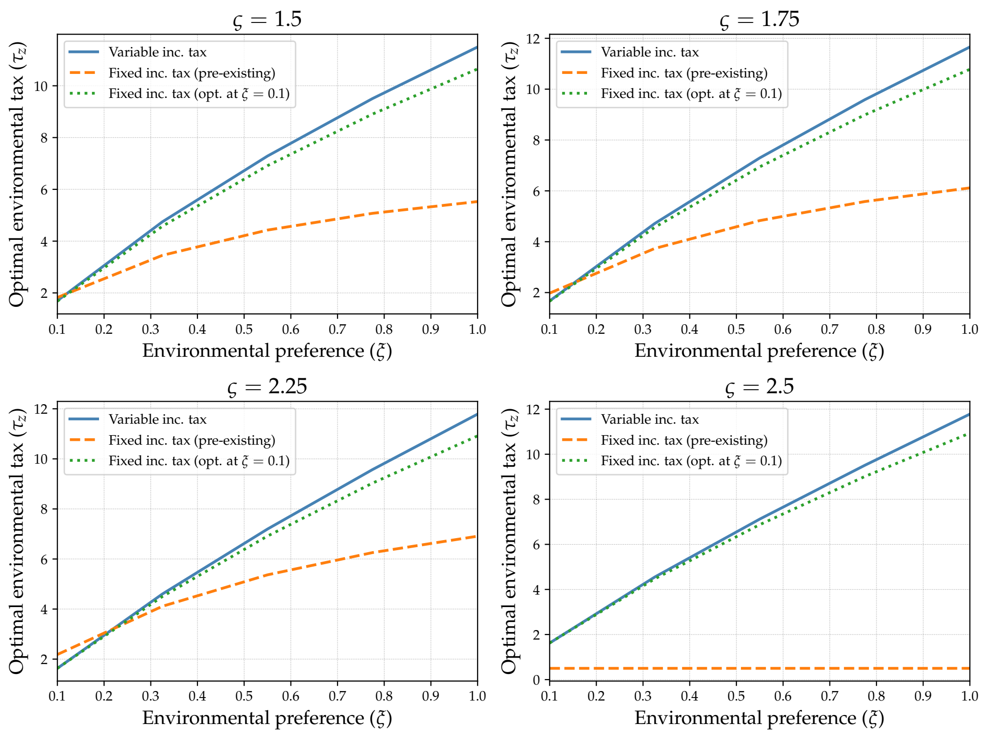

Figure 14: Optimal environmental taxation, extension, alternative elasticity of substitution

Notes: The figures show the optimal environmental tax rates as a function of the environmental preference for different levels of the elasticity of substitution between clean tech. and pollution (ς) in clean tech. extension. In each panel, the optimal environmental tax in the scenario with a fixed pre-existing income tax system is indicated by the dashed orange line, while the optimal environmental tax in the scenario with a fixed baseline optimal income tax system is indicated by the dotted green line. Finally, the optimal environmental tax in the scenario with variable income taxation is indicated by the solid blue line.