Shape-from-Template in Flatland

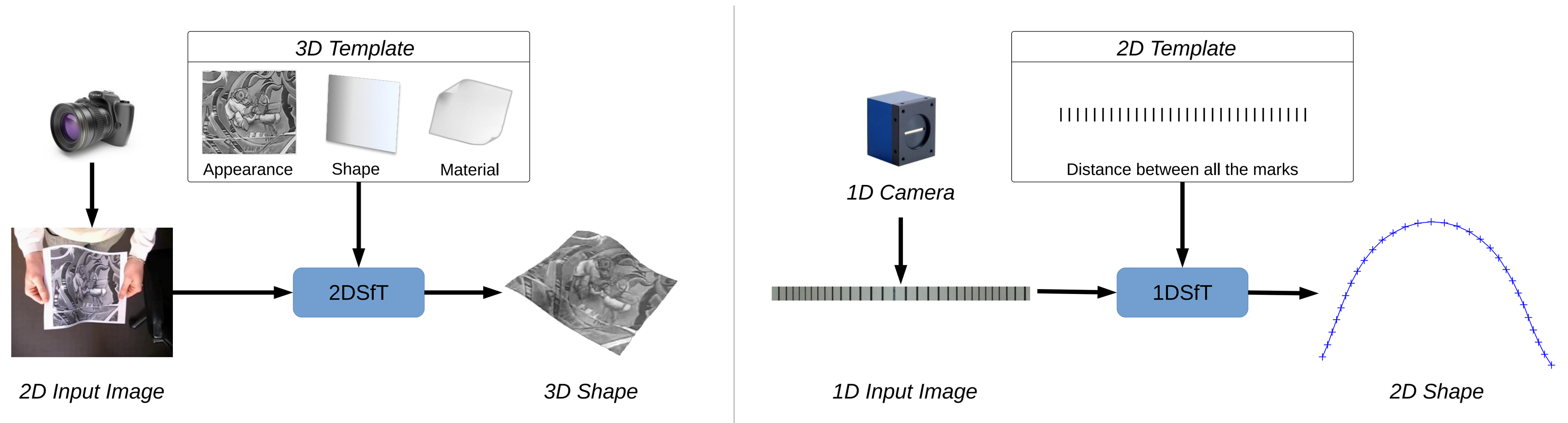
Mathias Gallardo, Daniel Pizarro, Adrien Bartoli and Toby Collins,
ALCoV-ISIT, UMR6284 CNRS / Université d'Auvergne, Clermont-Ferrand, France

IEEE 2015 Conference on Computer Vision and Pattern Recognition
CVPR 2015 BOSTON

Introduction

Context: 2D Shape-from-Template (SfT):
recovers the 3D shape of a surface from a 3D template and a 2D image

Challenge: 1DSfT, a novel instance of SfT:
recovers the 2D shape of a curve from a 2D template and a 1D image



Existing 2DSfT | *Proposed 1DSfT*

Goal: To solve 1DSfT for **isometric** deformations and **calibrated perspective** camera

Contributions:

- Theoretical study of 1DSfT through a differential formulation
- Established two main properties: *(i)* no local exact solutions and *(ii)* no unique solution, but the number of possible solutions is bounded
- Introduced the notion of critical points
- Proposed an angle-base parameterization to enforce isometry
- Two convex initializations and one non-convex refinement

Differential Formulation

Perspective reprojection constraint: $\varphi_y \eta = \varphi_x$

$$\xrightarrow[\text{Algebraic Manipulations}]{} \quad (\varphi_y' \eta)^2 + 2\varphi_y \varphi_y' \eta\eta' + (\varphi_y \eta')^2 + (\varphi_y')^2 = 1. \qquad (2)$$

1st order non-linear ODE

$\longrightarrow$ Under-constrained at each order: no local exact solutions

Change of Variable

We define $\varepsilon = \sqrt{1+\eta^2}$, $\xi = \frac{\eta'^2}{\varepsilon^4}$ and $\theta = \varphi_y \varepsilon$

$$(2) \longrightarrow \theta'^2 + \xi\theta^2 = 1 \quad \text{or} \quad \theta' = \pm\sqrt{1-\xi\theta^2} \qquad (3)$$

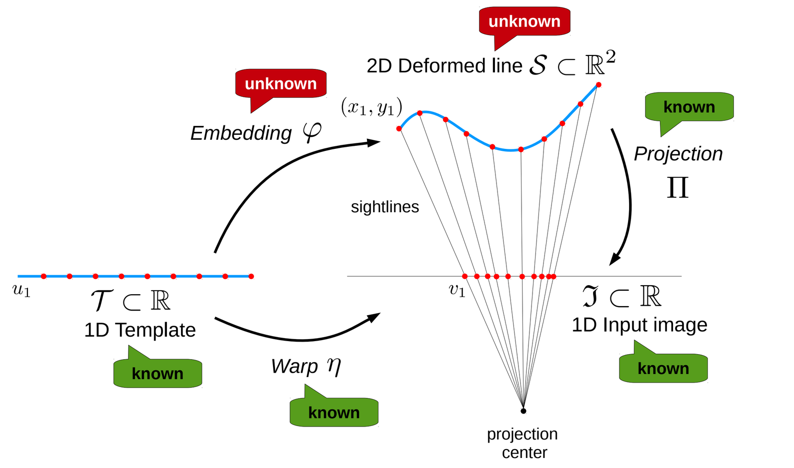
No closed-form solution, but introduces special points at $\theta'(u_c) = 0$, the so-called critical points

Modelling

Problem formulation:

$$\text{Find} \quad \varphi \in C^\infty(\mathcal{T}, \mathbb{R}^2)$$

$$s.t. \quad \begin{cases} \eta = \Pi \circ \varphi & \text{(reprojection)} \\ \|\varphi'\|_2^2 = 1 & \text{(isometry)} \end{cases} \qquad (1)$$



Approach

1. From equation (3), between two consecutive **critical points**, the sign of θ' is constant:

$$\theta' = \{\theta'_- \leq 0, \theta'_+ \geq 0\}$$

2. At each critical point, $\theta(u_c) = \frac{1}{\sqrt{\xi(u_c)}}$

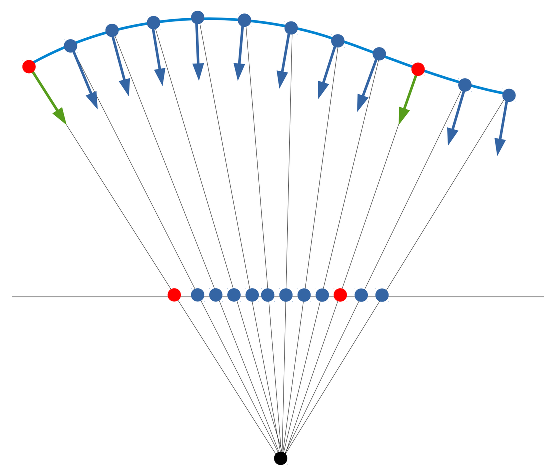
3. Considering θ'_- or θ'_+, we can apply **Picard-Lindelöf theorem** between two consecutive critical points

4. Considering N critical points and the two solutions θ'_- and θ'_+, the maximum number of solutions is $2^N - 1$

Critical Points

Provided by the warp η, these points are another type of priors.

- $u_c \in \mathcal{T}$ is a critical point if $\theta'(u_c) = 0$
- All solutions sharing a critical point are constrained to intersect in 2D
- All critical points are not necessarily shared by all solutions
- We can find all critical points from the warp η and some differential manipulations of equation (3)
- At critical point, θ is recoverable, so the depth φ_y is



Geometric Interpretation of critical points :
at these points, normal and optical ray are colinear

Picard-Lindelöf Theorem

Consider the following initial value problem in a general ODE with θ as dependent variable and u as independent variable,

$$\begin{cases} \theta' = \psi(\theta, u) \\ \theta(u_0) = \theta_0. \end{cases}$$

If ψ is a Lipschitz continuous function in θ and continuous in u for all $u \in [u_0 - \varepsilon; u_0 + \varepsilon]$ with $\varepsilon > 0$ then there exists a unique solution to equation (3) in the interval $[u_0 - \varepsilon; u_0 + \varepsilon]$.

$\longrightarrow$ Existence and Uniqueness of a solution for each interval bounded by two consecutive critical points

Computational Solutions

Pipeline Algorithm:

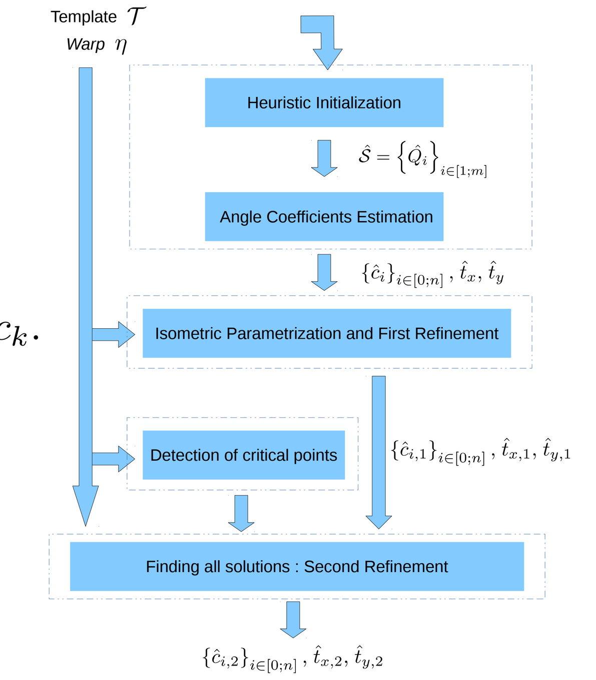
Initialization: two strategies, one based on the assumption of an infinitesimal planar solution and a second one based on the Maximum Depth Heuristic [1] which consists on a convex SOCP.

Refinement: we minimize the reprojection error for a solution defined by this angle-based parameterization:

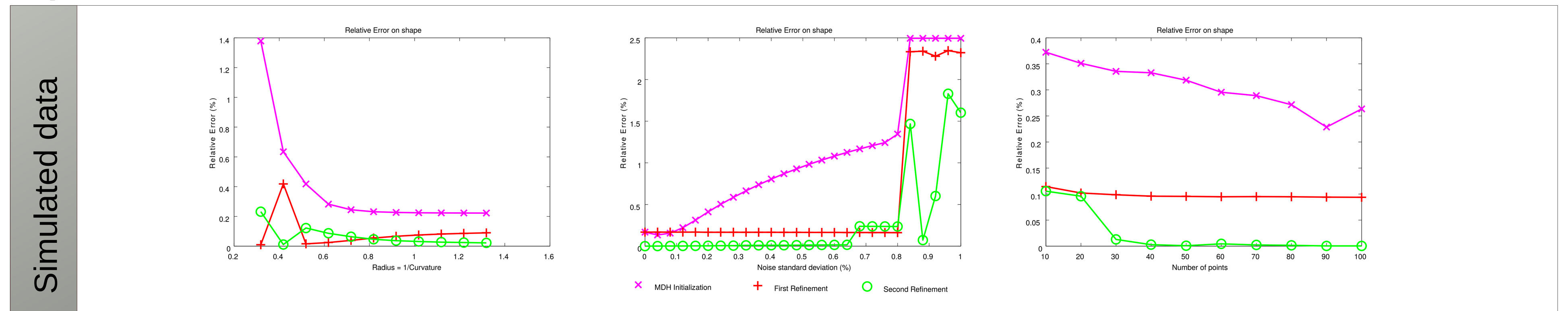
$$\forall u_i \in \mathcal{T}, \quad \varphi(u_i) = \sum_{j=1}^{i} (u_j - u_{j-1}) \begin{pmatrix} \cos\alpha(u_j) \\ \sin\alpha(u_j) \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}, \text{ with } \alpha(u_j) = \sum_{k=0}^{n} u_j^k c_k.$$

Critical points: we find the critical points in the refined solution and divide the template in interval bounded by two consecutive critical points.

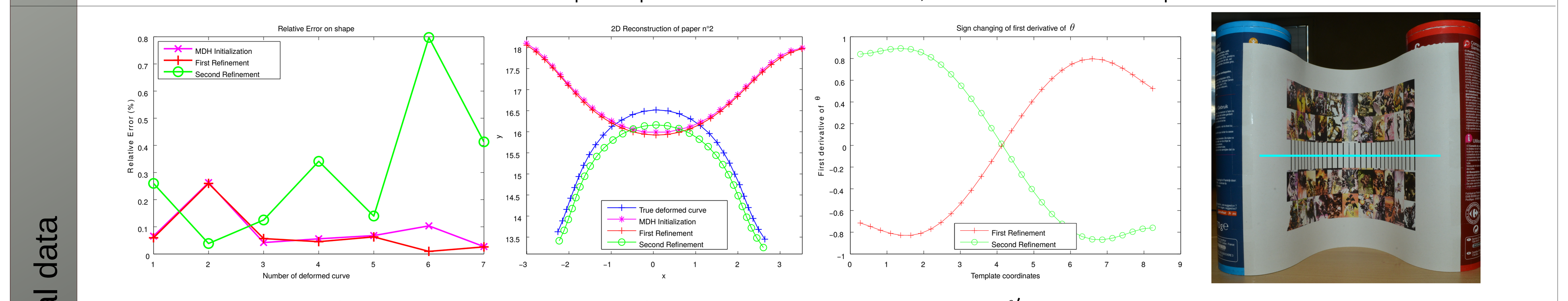
Finding all solutions: we minimize the reprojection error and enforce the change of θ' sign for each interval.



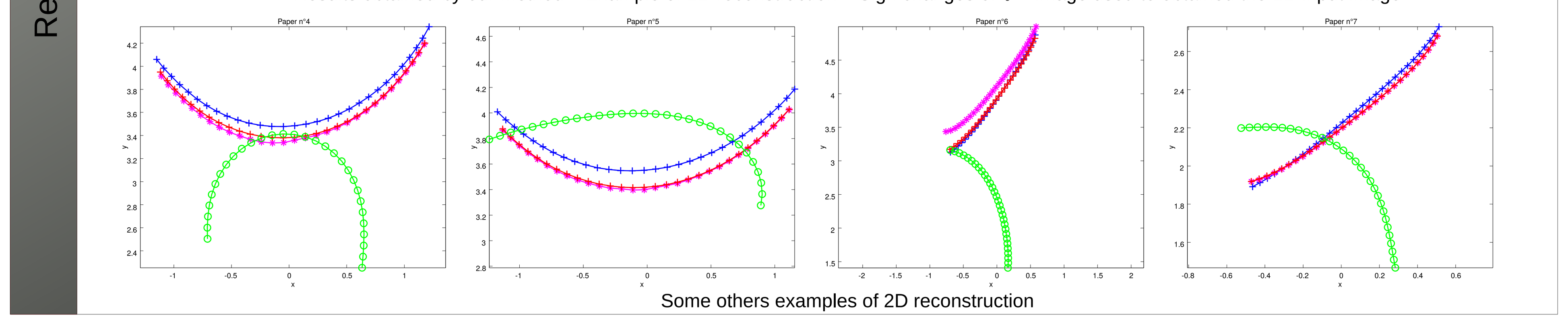
Experiments:



Influence of three important parameters for 1DSfT : curvature, noise and number of correspondences



Results obtained by our method – Example of 2D reconstruction – Sign changes of θ' - Image used to obtained the 1D input image



Some others examples of 2D reconstruction

Conclusion

- An ODE analysis reveals the complexity of this simple case of 1DSfT
- A theoretical study introduce the notion of critical points which are necessary to compute 1DSfT solutions
- A method to detect critical points from the knowledge of the warp
- Encouraging results with simulated and real data

[1] M. Perriollat, R. Hartley, and A. Bartoli. *Monocular Template-Based Reconstruction of Inextensible Surfaces*. In *IJCV*, 95(2):124-137, November 2011