The Equal-Effect and Random-Effects Model

The Equal-Effect Model

- Also known as the **fixed-effect** or **common-effect model**
- Assumes that all effect sizes stem from a **single homogeneous population…**
- …and thus **share the same, true effect size** θ
- The observed effect size $\hat{\theta}_k$ of study k deviates from θ because of its sampling error ε_k

→ **The content of the "back box" is simple: all studies estimate the same true effect**

- But since every study can only draw bigger or smaller samples of the population, results are burdened by sampling error
- This sampling error causes the observed effect to differ from the true effect

$$\hat{\theta}_k = \theta + \varepsilon_k$$

Compare to the model for a single study:

$$\hat{\theta}_k = \theta_k + \varepsilon_k$$

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- Based on this model, we know that studies with a low standard error (high precision, high sample size) are better estimators of the true effect size
- Therefore, we should give them a **higher weight**
- These weights are determined as the inverse of the variance V_k of each study's effect size, where $V_k = \mathrm{SE}_k^2$:

$$w_k = \frac{1}{\mathrm{SE}_k^2} = \frac{1}{V_k}$$

- We then create a weighted average of the observed effect sizes $\hat{\theta}_k$
- This results in the EE-model estimate of the **true effect size:**

$$\hat{\theta} = \frac{\sum_{k=1}^{K} \hat{\theta}_k w_k}{\sum_{k=1}^{K} w_k}$$

→ Known as inverse-variance weighting or generic inverse variance meta-analysis

→ Can also be used for log-transformed odds ratios and relative risks, but other methods (Mantel-Haenszel) are preferable

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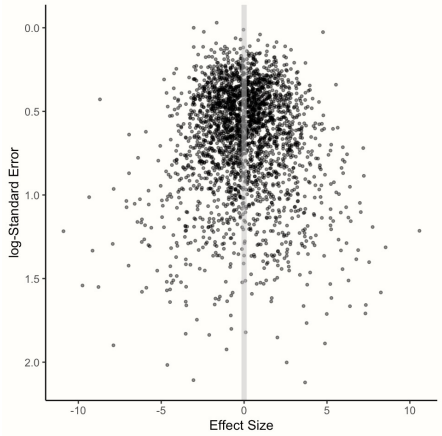

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→ Known as inverse-variance weighting or generic inverse variance meta-analysis

→ Can also be used for **log-transformed odds and risk ratios**, but other methods (e.g., the Mantel-Haenszel method) are preferable

$$\hat{\theta}_k = \theta + \varepsilon_k$$