The Equal-Effect and Random-Effects Model
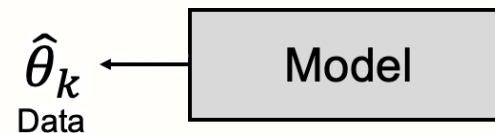
What Are Statistical Models?

- Statistics is full of "models": linear models, generalized linear models, mixture models, gaussian additive models, structural equation models, …

- There is a model behind t-tests, ANOVAs, and regression, …

- Every hypothesis test has its corresponding statistical model.

The Equal-Effect and Random-Effects Model

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What Are Statistical Models?

- Defining a statistical model, we start with the information that is already <u>given</u> to us: our **data** (*lat*: "datum": a thing that is given)

- In meta-analysis, these are the **observed effect sizes** in each study

- In our model, we try to describe the **"data-generating process"** through which these observed effect sizes were generated

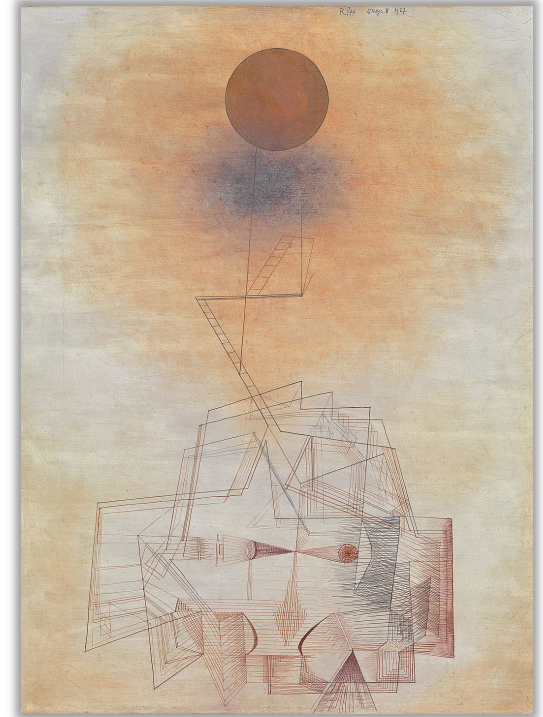- The data are seen as the product of a black box, and we want to illuminate what is going on inside it

$$\hat{\theta}_k \longleftarrow \boxed{\text{Model}}$$

Data

The Equal-Effect and Random-Effects Model

What Are Statistical Models?

"*Statistics […] is not really about facts; it is about how we know, or suspect, or believe, that something is a fact.*"

"*This science has much more in common with philosophy (in particular epistemology) than it does with accounting. Statisticians are applied philosophers.*"

– Senn, 2022, chap. 1.

Paul Klee (1927): *The Bounds of Reason*

The Equal-Effect and Random-Effects Model

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What Are Statistical Models?

- A statistical model is like a special type of theory, an **"imitation of life"** that tries to describe processes in the world around us in an idealized way

- This explanatory character of models is deeply ingrained in modern statistics (see Breiman, 2001 for a criticism of this "culture")

$\hat{\theta}_k \longleftarrow$ Model

Data

Statistical Modeling: The Two Cultures

Leo Breiman

Abstract. There are two cultures in the use of statistical modeling to reach conclusions from data. One assumes that the data are generated by a given stochastic data model. The other uses algorithmic models and treats the data mechanism as unknown. The statistical community has been committed to the almost exclusive use of data models. This commitment has led to irrelevant theory, questionable conclusions, and has kept statisticians from working on a large range of interesting current problems. Algorithmic modeling, both in theory and practice, has developed rapidly in fields outside statistics. It can be used both on large complex data sets and as a more accurate and informative alternative to data modeling on smaller data sets. If our goal as a field is to use data to solve problems, then we need to move away from exclusive dependence on data models and adopt a more diverse set of tools.

The Equal-Effect and Random-Effects Model

What Are Statistical Models?

- In meta-analysis, we need a model that **explains how we can find the true effect size** underlying our studies, based on their **observed results.**

- Our goal is to find one numerical value that characterizes our studies *as a whole*.

- Therefore, our model should explain why **the observed effect sizes differ** from study to study, even though there is **only one overall, true effect**.

- There are **two competing models** that try to answer this question: the equal-effect and random-effects model.

$\hat{\theta}_k \longleftarrow$ Model

Data