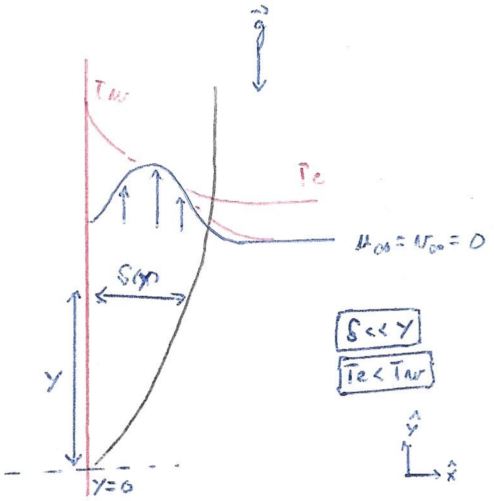
Blasius (2D)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$



simplifications:

$\boxed{\delta << y}$ ✓

$$\underbrace{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0$$

$$O\left(\frac{U}{\delta}\right) = O\left(\frac{V}{y}\right) \Rightarrow \boxed{U = V\frac{\delta}{y} << V}$$

négligeable

$$\rho\left[\underbrace{u\frac{\partial u}{\partial x}}_{\frac{U^2}{\delta},\ \frac{V^2}{y^2}\cdot\delta} + \underbrace{v\frac{\partial u}{\partial y}}_{\frac{VU}{y},\ \frac{V^2}{y^2}\cdot\delta}\right] = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \underbrace{\left(\frac{\mu}{\rho}\right)}_{O(\nu)}\left[\underbrace{\frac{\partial^2 u}{\partial x^2}}_{\frac{U}{\delta^2}} + \cancel{\frac{\partial^2 u}{\partial y^2}}\right]$$

$$\frac{U}{\delta^2} >> \frac{U}{y^2}$$

INERTIES — VISQUEUX

$\boxed{\frac{V\delta}{y}}$

$$\Rightarrow \cancel{\frac{V^2}{y^2}\cdot\delta = \nu\frac{U}{\delta^2} \Rightarrow \frac{U^2}{y^2}\cdot\delta = \nu\frac{V\delta}{y}\cdot\frac{1}{\delta^2} \Rightarrow \frac{\delta^2}{y^2} = \frac{\nu V}{U^2 y}}$$

couche laminaire : termes d'inerties $\approx$ termes visqueux

$$\boxed{\frac{\delta}{y} = \sqrt{\frac{1}{Re_y}}} \Rightarrow \boxed{Re_y \text{ grand}}$$

$$\rho\left[\underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}_{\frac{UV}{\delta},\ \frac{V^2}{y},\ \frac{V^2}{y}}\right] = -\frac{\partial P}{\partial y} + \mu\left[\underbrace{\frac{\partial^2 v}{\partial x^2}}_{\frac{V}{\delta^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right]$$

$$\frac{V}{\delta^2} >> \frac{V}{y^2}$$

Hypothèse de Prandtl : $\rho\frac{V^2}{y} = \mu\frac{V}{\delta^2} \Rightarrow \frac{\delta^2}{y^2} = \frac{\mu}{V\rho y} = \frac{1}{Re_y}$

(INERTIE $\approx$ VISQUEUX)

$$\Rightarrow \boxed{\frac{\delta}{y} = \sqrt{\frac{1}{Re_y}}}$$

$\boxed{\text{Pression dans la couche laminaire } x \sim O(\delta)}$ ✓

$$[P] = \left[\frac{kg}{m\cdot s^2}\right]$$

pas de $\frac{\partial P}{\partial x}$

$$P(x,y) - P_0 \approx \underbrace{P(\delta,y) - P_0}_{= y\frac{\partial P}{\partial y},\ O(y)\, O\left(\rho\frac{V^2}{y}\right) \;=\; O(\rho V^2)} + \underbrace{\cancel{\left.\frac{\partial P}{\partial x}\right|_{x=\delta}^{(x,y)}\underbrace{(x-\delta)}_{O(\delta)}}}_{O\left(\frac{\rho U V}{y}\right)} + \ldots$$

$$= O(\rho V^2) >> O\left(\rho V^2\frac{\delta^2}{y^2}\right)$$

$$\cancel{\frac{\partial P}{\partial x}}$$

$\Rightarrow$ P = cste sur une tranche horizontale ($P(y)$)

$$\Rightarrow \boxed{P(x,y) \approx O(\rho V^2) \approx \rho v^2}$$

→ on supprime l'équation contenant $\frac{\partial P}{\partial x}$ car celui-ci est négligeable.

on obtient les équations de Prandtl :

$$\boxed{\begin{gathered}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{dP}{dy} + \nu\frac{\partial^2 v}{\partial x^2}\end{gathered}}$$

P(y)

note : si l'écoulement est uniforme : $\frac{dP}{dy} = 0$

introduction des fonctions de courant :

$$u = \frac{\partial \psi}{\partial y} \; ; \; v = -\frac{\partial \psi}{\partial x}$$

on remplace dans les équations de Prandtl :

$$\cancel{\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0}$$

$$\boxed{-\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y \partial x} = -\frac{1}{\rho}\frac{dP}{dy} + \nu\frac{\partial^3 \psi}{\partial x^3}}$$

L'énoncé ~~nous~~ propose la variable de similitude suivante :

$$\boxed{\eta(x,y) = \frac{x}{\delta(y)}} = \frac{x}{y}\left(\frac{Gr(y)}{4}\right)^{1/4} \qquad \frac{1}{\delta(y)}$$

avec $Gr(y) = \frac{\beta \Delta T g y^3}{\nu^2}$

ansatz :

La fonction de courant peut s'écrire : ~~$\psi(x,y) = \ldots$~~

$$\psi(x,y) = -4\nu f(\eta)\left[\frac{1}{4}(Gr(y))^{\ldots}\right]^{1/4} \quad \text{(TP8)}$$

$$\frac{\delta'}{\delta} = \frac{1}{y} + \frac{3}{4y} = \frac{7}{4y}$$

$$\boxed{\frac{\partial \eta}{\partial x} = \frac{1}{y}\left(\frac{Gr(y)}{4}\right)^{1/4} = \frac{1}{\delta(y)}}$$

$$\frac{1}{\delta^2} = \frac{1}{2y^2}\sqrt{Gr}$$

$$\delta(y) = y\left(\frac{4}{Gr}\right)^{1/4}$$

$$\delta'(y) = \left(\frac{4}{Gr}\right)^{1/4} + y\frac{1}{4}\left(\frac{4}{Gr}\right)^{-3/4}\cdot\left(-\frac{4}{Gr^2}\right)Gr'$$

$$\boxed{\frac{\partial \eta}{\partial y} = -\frac{x}{y^2}\left(\frac{Gr(y)}{4}\right)^{1/4} + \frac{x}{y}\cdot\frac{1}{4}\left(\frac{Gr(y)}{4}\right)^{-3/4}\cdot\frac{1}{4}\frac{\beta\Delta T g 3y^2}{\nu^2}}$$

$$= -x\frac{\delta'(y)}{\delta(y)^2} = -\eta\frac{\delta'(y)}{\delta(y)}$$

on trouve donc (via l'ansatz) :

$$-\frac{\partial \psi}{\partial y} = \nu\delta'(y)f(\eta) + \nu\delta(y)f'(\eta)\frac{\partial \eta}{\partial y}(x,y)$$

$$= \nu\left(\delta'(y)f + \delta(y)f'\left[\frac{x}{4y}\left(\frac{Gr(y)}{4}\right)^{-3/4}\cdot\frac{3}{4}\frac{\beta\Delta T y^2}{\nu^2} - \frac{x}{y^2}\left(\frac{Gr(y)}{4}\right)^{1/4}\right]\right) = \nu\left(\delta'(y)f - f'\cdot\eta\,\delta'(y)\right)$$

$$= \nu\delta'(y)\left(f(\eta) - f'(\eta)\cdot\eta\right)$$

$$\frac{\partial \psi}{\partial x} = -\nu\delta(y)f'(\eta)\frac{\partial \eta}{\partial x}$$

$$= -\nu\delta(y)f'\cdot\frac{1}{\delta(y)} = -\nu f'(\eta)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\nu f''(\eta)\frac{\partial \eta}{\partial x}$$

$$= -\frac{\nu f''(\eta)}{\delta(y)}$$

$$\frac{\partial^3 \psi}{\partial x^3} = -\frac{\nu f'''}{\delta^2(y)}$$

$$\frac{\partial^2 \psi}{\partial x \partial y} = \frac{\partial}{\partial x}\left(-\nu\left[\delta'(y)f + \cancel{\delta(y)}\left[-\eta\frac{\delta'(y)}{\cancel{\delta(y)}}\right]\right]\right) = -\nu\left(\delta'(y)f'(\eta)\cdot\frac{1}{\delta(y)} - \delta'(y)\cdot\frac{1}{\delta(y)}\right)$$

$$= +\nu\frac{\delta'(y)}{\delta(y)}\left(1 - f'(\eta)\right)$$

Blasius

Si on est dans le cas de la convection naturelle, on ajoute le terme d'Archimède : (et la force à distance) $\leftarrow \rho g$

$\rho = \rho_0(1 - \underbrace{\beta(T-T_0)}_{<<1})$

$P(y) = \rho_0 . g y \Rightarrow \frac{dP}{dy} = \rho_0 . g$

On trouve les équations de Prandtl pour la convection naturelle :

$$\boxed{\theta(\eta) = \frac{T - T_e}{T_w - T_e}} \qquad T_w - T_e = \Delta T$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho_0(1 - \cancel{\beta(T-T_0)})\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = \cancel{-\rho_0 g} + \mu\frac{\partial^2 v}{\partial x^2} + \rho_0(\cancel{1} - \beta(T-T_0))g$$

$$\boxed{\begin{aligned}&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\\ &u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \beta(T-T_0)g + \nu\frac{\partial^2 v}{\partial x^2}\end{aligned}} \Longrightarrow \boxed{\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} = \beta(T-T_0)g - \nu\frac{\partial^3\psi}{\partial x^3}}$$

On trouve les termes de calcul avec variable de similitude :

$$\Rightarrow \cancel{-Vf'(\eta).V\frac{\delta'(y)}{\delta(y)}(1-f'(\eta)) * V\delta'(y)(f(\eta)-f'(\eta)\eta).\frac{Vf''(\eta)}{\delta(y)}} = \cancel{\beta(T-T_0)g + \nu\frac{Vf'''}{\delta^2(y)}}$$

on dérive :

$$\frac{\partial\psi}{\partial x} = -4\nu\left[\frac{Gr(y)}{4}\right]^{1/4}.f'(\eta).\frac{1}{\delta(y)} \; ; \; \frac{\partial^2\psi}{\partial x^2} = -4\nu\left[\frac{Gr(y)}{4}\right]^{1/4}.f''(\eta).\frac{1}{\delta^2(y)}$$

$$\frac{\partial^2\psi}{\partial y\partial x} = -4\nu.\frac{1}{4}\left(\frac{Gr(y)}{4}\right)^{-3/4}.\frac{1}{4}Gr'(y).\frac{f'(\eta)}{\delta(y)} + 4\nu\left[\frac{Gr(y)}{4}\right]^{1/4}.f''(\eta).\eta\frac{\delta'(y)}{\delta(y)}.\frac{1}{\delta(y)} + 4\nu\left[\frac{Gr(y)}{4}\right]^{1/4}f'(\eta).\frac{1}{\delta^2(y)}.\delta'(y)$$

$$\frac{\partial\psi}{\partial y} = +4\nu f'(\eta)\eta\frac{\delta'(y)}{\delta(y)}\overset{\left[\frac{Gr}{4}\right]^{1/4}}{\vee} - 4\nu f(\eta)\frac{1}{4}\left[\frac{Gr(y)}{4}\right]^{-3/4}.\frac{1}{4}Gr'(y)$$

$$\boxed{\begin{aligned}Gr(y) &= \frac{\beta\Delta T g y^3}{\nu^2}\\ Gr'(y) &= \frac{3\beta\Delta T g y^2}{\nu^2}\end{aligned}}$$

$$\hookrightarrow = \frac{3}{y}Gr(y)$$

$$\frac{\partial^3\psi}{\partial x^3} = -4\nu\left[\frac{Gr(y)}{4}\right]^{1/4}.f'''(\eta).\frac{1}{\delta^3(y)}$$

on trouve :

$$16\nu^2.\frac{2}{4}\left[\frac{Gr(y)}{4}\right]^{-2/4}.Gr'(y)\left[\frac{f'(\eta)}{\delta(y)}\right]^2 \cancel{- 16\nu^2\left[\frac{Gr(y)}{4}\right]^{2/4}\frac{f'(\eta)}{\delta(y)}\frac{f''(\eta)}{\delta(y)^2}\eta\delta'(y)} - 16\nu^2\left[\frac{Gr(y)}{4}\right]^{2/4}\left[\frac{f'(\eta)}{\delta(y)}\right]^2.\frac{\delta'(y)}{\delta(y)}$$

$$\cancel{+16\nu^2\left[\frac{Gr(y)}{4}\right]^{2/4}\left[\frac{f'(\eta)}{\delta(y)}\right].\eta f''(\eta)\frac{\delta'(y)}{\delta^2(y)}} - 16\nu^2.\frac{2}{4}\left[\frac{Gr(y)}{4}\right]^{-2/4}.Gr'(y)f(\eta)f''(\eta).\frac{1}{\delta^2(y)}$$

$$-\underbrace{\beta(T-T_e)g}_{=\beta\theta(\eta)\Delta T g = \theta(\eta)Gr(y)\frac{\nu^2}{y^3}} - 4\nu^2\left[\frac{Gr(y)}{4}\right]^{1/4}.\frac{f'''(\eta)}{\delta^3(y)} = 0$$

problème thermique :

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

condition d'interface

$$\boxed{Nu=\frac{q_w}{k\Delta T/2R}}$$

$Pr=\frac{\nu}{\alpha}$

Flux de chaleur dans l'écoulement

$$\boxed{Pe_u=\frac{UL}{\alpha}}$$

simplification :

toujours : $\boxed{\delta \ll Y}$

toujours : $U=\frac{V\delta}{Y} \ll V$

$$\underbrace{\boxed{\overset{\text{transport}}{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}}}}_{\mathcal{O}\left(\frac{V\Delta T}{Y}\right)}=\boxed{\overset{\text{diffusion}}{\alpha\left(\frac{\partial^2 T}{\partial x^2}+\cancel{\frac{\partial^2 T}{\partial y^2}}\right)}}$$

$u\frac{\partial T}{\partial x}\sim U.\frac{\Delta T}{\delta}\sim\frac{V\Delta T}{Y}$, $\quad v\frac{\partial T}{\partial y}\sim V.\frac{\Delta T}{Y}$

$\frac{\Delta T}{\delta^2} \gg \frac{\Delta T}{Y^2}$

$$1=\frac{\cdot}{\cdot}=\frac{V\Delta T}{Y}\cdot\frac{\delta^2}{\alpha\Delta T}\Rightarrow\frac{VY}{\alpha}\cdot\frac{\delta^2}{Y^2}$$

$$\Rightarrow\frac{\delta^2}{Y^2}=\frac{\alpha}{VY}$$

$$\Rightarrow\frac{\delta}{Y}=\sqrt{\frac{\alpha}{VY}}=\sqrt{\frac{1}{Pe_Y}}$$

transfert de chaleur d'un écoulement

On a donc :

$$\boxed{u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}}$$

$$\theta(\eta)=\frac{T-Te}{\Delta T}\Rightarrow\boxed{T=\Delta T.\theta(\eta)+Te}$$

→ calculer les dérivée... pour retrouver l'équation de similitude.

Flux de chaleur : (à la paroi)

$\alpha=\frac{k}{\rho c}\Rightarrow\rho c\alpha=k$

$$q_w=-k\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

$$=-k\left.\frac{\partial}{\partial x}(\Delta T.\theta(\eta)+Te)\right|_{\eta=0}$$

$$=-k\Delta T\,\theta'(\eta)\Big|_{\eta=0}\cdot\frac{\partial\eta}{\partial x}$$

$$=-k\Delta T\,\theta'(\eta)\Big|_{\eta=0}\cdot\boxed{\frac{1}{\delta(y)}}\sim\boxed{\frac{1}{y}\left(\frac{Gr}{4}\right)^{1/4}}$$

$$=-k\Delta T\,\theta'\eta\Big|_{\eta=0}\cdot\frac{1}{y}\left(\frac{Gr}{4}\right)^{1/4}$$

cours : (pour $Pr>0{,}01$) ok

$$\theta'(\eta)\Big|_{\eta=0}=0{,}332\sqrt{2}(Pr)^{1/3}$$

Blasius

$$\boxed{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \beta(T-T_e)g + \nu\frac{\partial^2 v}{\partial x^2}}$$

où $\boxed{\theta(\eta) = \frac{T-T_e}{T_w-T_e} = \frac{T-T_e}{\Delta T}}$

$$\boxed{\beta(T-T_e)g = \theta\, Gr\,\frac{\nu^2}{y^3}}$$

Fonction de courant :

$$\boxed{u = \frac{\partial \psi}{\partial y} \quad ; \quad v = -\frac{\partial \psi}{\partial x}}$$

avec $\boxed{\psi(x,y) = -4\nu f(\eta)\left[\frac{Gr(y)}{4}\right]^{1/4}}$

Variable de similitude :

$$\boxed{\eta(x,y) = \frac{x}{\delta(y)} = \frac{x}{y}\left[\frac{Gr(y)}{4}\right]^{1/4}}$$

avec $\begin{cases} Gr(y) = \dfrac{\beta\Delta T g y^3}{\nu^2} \\ Gr'(y) = \dfrac{3\beta\Delta T y^2}{\nu^2} = \dfrac{3}{y}Gr(y) \end{cases}$

Quelques calculs préliminaires :

$\rightarrow \delta(y) = y\left[\frac{4}{Gr(y)}\right]^{1/4}$

val. au bord visqueux → ϕ_0 ϕ_1

vrai val. au bord →

$$\rightarrow \delta'(y) = \left[\frac{4}{Gr(y)}\right]^{1/4} - \frac{y\,4}{4}\left[\frac{4}{Gr(y)}\right]^{-3/4}\cdot\frac{1}{Gr^2(y)}\cdot\overbrace{Gr'(y)}^{\frac{3}{y}Gr(y)}$$

$$= \left[\frac{4}{Gr(y)}\right]^{1/4} - \frac{3}{Gr(y)}\left[\frac{4}{Gr(y)}\right]^{-3/4}$$

$$\rightarrow \frac{1}{\delta^2(y)} = \frac{1}{2y^2}\sqrt{Gr(y)}$$

$$\rightarrow \frac{\delta'(y)}{\delta(y)} = \frac{1}{y} - \frac{\frac{3}{Gr(y)}\overbrace{\left[\frac{4}{Gr(y)}\right]^{-3/4}\left[\frac{4}{Gr(y)}\right]^{-1/4}}^{\left[\frac{Gr(y)}{4}\right]^{1}}}{y} = \frac{1}{y} - \frac{3}{4y} = \frac{4-3}{4y} = \boxed{\frac{1}{4y}}$$

$$\rightarrow \frac{\partial \eta}{\partial x} = \frac{1}{\delta(y)} = \frac{1}{y}\left[\frac{Gr(y)}{4}\right]^{1/4}$$

$$\rightarrow \frac{\partial \eta}{\partial y} = -\eta\frac{\delta'(y)}{\delta(y)} = -\frac{\eta}{4y} = -\frac{x}{4y^2}\left[\frac{Gr(y)}{4}\right]^{1/4}$$

On calcule ensuite que : (brouillon A)

$$\rightarrow u = \frac{\partial \psi}{\partial y} = \cancel{4\nu f'\eta\frac{\delta'(y)}{\delta(y)}\left[\frac{Gr}{y}\right]^{1/4} - 4\nu f\frac{1}{16}\left[\frac{Gr}{4}\right]^{-3/4}\cdot Gr'} = -4\nu f'\frac{\partial \eta}{\partial y}\left[\frac{Gr}{4}\right]^{1/4} - \frac{\nu}{4}f\left[\frac{Gr}{4}\right]^{-3/4}Gr'$$

$$= -4\nu f'\frac{(-x)}{4y^2}\left[\frac{Gr(y)}{4}\right]^{1/4}\left[\frac{Gr}{4}\right]^{1/4} - \frac{3\nu}{4y}f\left[\frac{Gr}{4}\right]^{-3/4}\cdot Gr$$

$$= \frac{\nu x f'}{2y^2}\sqrt{Gr} - \frac{3\nu}{4y}f\left[\frac{Gr}{4}\right]^{-3/4}\cdot Gr$$

$$\rightarrow \frac{\partial u}{\partial x}\quad v = -\frac{\partial \psi}{\partial x} = 4\nu f' \frac{\partial \eta}{\partial x}\left[\frac{Gr}{4}\right]^{1/4}$$

$$= 4\nu f' \cdot \frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4} \cdot \left[\frac{Gr}{4}\right]^{1/4}$$

$$= \frac{2\nu f' \sqrt{Gr}}{y}$$

$$\left[\frac{Gr}{4}\right]^{1/4} \cdot \left[\frac{Gr}{4}\right]^{1/4}$$

$$= \frac{V}{2}$$

$$\rightarrow \frac{\partial v}{\partial x} = \left(-\frac{\partial^2 \psi}{\partial x^2}\right) = \frac{2\nu}{y} f'' \frac{\partial \eta}{\partial x}\sqrt{Gr}$$

$$= \frac{2\nu}{y} f'' \cdot \frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}$$

$$= \frac{2\nu}{y^2}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f''$$

$$\rightarrow \frac{\partial v}{\partial y} = \frac{\left(2\nu f'' \frac{\partial \eta}{\partial y}\sqrt{Gr} + 2\nu f' \cdot \frac{1}{2}(Gr)^{-1/2} Gr'(y)\right) y - \left(2\nu f'\sqrt{Gr} \cdot 1\right)}{y^2}$$

$$= \frac{\left(-2\nu f'' \frac{x}{4y^2}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr} + \nu f'(Gr)^{-1/2} \cdot \frac{3}{y} Gr\right) y - 2\nu f'\sqrt{Gr}}{y^2}$$

$$= -\frac{1}{2}\frac{\nu x}{y^3}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr} \cdot f'' + \frac{\nu 3}{y^2} f' \cdot \sqrt{Gr} - \frac{2\nu}{y^2} f'\sqrt{Gr}$$

$$= -\frac{1}{2}\nu \cdot \frac{x}{y^3}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f'' + \frac{\nu}{y^2} f'\sqrt{Gr}$$

$$\rightarrow \frac{\partial^2 v}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right) = \frac{2\nu}{y^2}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f''' \frac{\partial \eta}{\partial x}$$

$$= \frac{2\nu}{y^2}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f''' \cdot \frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4}$$

$$= \frac{\nu}{y^3} Gr\, f'''$$

There we go, on remplace dans l'équation :

$$\left(\frac{\nu x f'}{2y^2}\sqrt{Gr} - \frac{3\nu}{4y} f\left[\frac{Gr}{4}\right]^{-3/4} \cdot Gr\right) \cdot \frac{2\nu}{y^2}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f'' + \frac{2\nu f'\sqrt{Gr}}{y}\left(-\frac{\nu}{2}\frac{x}{y^3}\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}\, f'' + \frac{\nu}{y^2} f'\sqrt{Gr}\right)$$

$$= \Theta\, Gr \frac{\nu^2}{y^3} + \nu \cdot \frac{\nu}{y^3} Gr\, f'''$$

Blasius :

Goal : $f''' + 3ff'' - 2(f')^2 + \theta = 0$

continue :

✓ $3\nu^2/y^3$ Manque un Gr ok

Gr

$\sqrt{Gr}$

$$\frac{2\nu^2}{2y^4}\cancel{\times}\sqrt{Gr}\sqrt{Gr}\left[\frac{Gr}{4}\right]^{1/4} f f'' - \frac{6\nu^2}{4y^3}\cdot\frac{2}{\sqrt{Gr}}\cancel{\sqrt{Gr}}\, f f'' - \frac{2\nu^2}{2y^4}\cdot x\cdot\sqrt{Gr}\left[\frac{Gr}{4}\right]^{+1/4} f' f''$$

✓ ✓ ✓

$$+ \frac{2\nu^2}{y^3}\sqrt{Gr}\sqrt{Gr}\,(f')^2 = \theta\, Gr\,\frac{\nu^2}{y^3} + \frac{\nu^2}{y^3}\,Gr\, f'''$$

Manque $\frac{1}{y}$ ok

On trouve finalement :

$$\boxed{f''' + 3ff'' - 2(f')^2 + \theta = 0}$$

Température

$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}} \quad \text{où} \quad \boxed{T = \Delta T\,.\,\theta(\eta) + T_e}$$

On calcule que :

$$\longrightarrow \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\Delta T\,\theta(\eta) + T_e\right)$$

$$= \Delta T\,\theta'(\eta)\frac{\partial \eta}{\partial x}$$

$$= \Delta T\,\theta'(\eta)\,\frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4}$$

$$\longrightarrow \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\Delta T\,.\,\theta(\eta) + T_e\right)$$

$$= \Delta T\,\theta'(\eta)\frac{\partial \eta}{\partial y}$$

$$= -\Delta T\,\theta'(\eta)\,\frac{x}{4y^2}\left[\frac{Gr}{4}\right]^{1/4}$$

$$\longrightarrow \frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\Delta T\,\theta'(\eta)\,\frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4}\right)$$

$$= \Delta T\,\theta''(\eta)\frac{\partial \eta}{\partial x}\cdot\frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4}$$

$$= \Delta T\,\theta''(\eta)\,\frac{1}{y^2}\left[\frac{Gr}{4}\right]^{1/2} = \Delta T\,\theta''(\eta)\cdot\frac{1}{2y^2}\sqrt{Gr}$$

On remplace dans l'équation :

$$\left(\frac{\nu x f'}{2y^2}\sqrt{Gr} - \frac{3\nu}{4y} f\left[\frac{Gr}{4}\right]^{-3/4}\cdot Gr\right)\cdot\Delta T\,\theta'(\eta)\cdot\frac{1}{y}\left[\frac{Gr}{4}\right]^{1/4} + \frac{2\nu f'\sqrt{Gr}}{y}\cdot\left(-\Delta T\,\theta'(\eta)\,\frac{x}{4y^2}\left[\frac{Gr}{4}\right]^{1/4}\right)$$

$$= \alpha\,\Delta T\,\theta''(\eta)\cdot\frac{1}{2y^2}\sqrt{Gr}$$

$$\Rightarrow \cancel{\frac{\nu x \overbrace{f'}^{\Delta T\theta'(\eta)}}{2y^3}\sqrt{Gr}\left[\frac{Gr}{4}\right]^{1/4}} - \frac{3\nu \overbrace{f}^{\Delta T\theta'(\eta)}}{4y^2}\overbrace{\underbrace{\left[\frac{Gr}{4}\right]^{-3/4}\left[\frac{Gr}{4}\right]^{1/4}}_{2/\sqrt{Gr}}\cdot Gr}^{2\sqrt{Gr}} - \cancel{\frac{8\nu x}{24 y^3} f'\Delta T\theta'(\eta)\left[\frac{Gr}{4}\right]^{1/4}\sqrt{Gr}}$$

$$= \alpha\,\Delta T\,\theta''(\eta)\cdot\frac{1}{2y^2}\sqrt{Gr}$$

$$\boxed{Pr = \frac{\nu}{\alpha}}$$

$$\boxed{\Rightarrow \alpha = \frac{\nu}{Pr}}$$

$$\Rightarrow -\frac{\nu\Delta T}{2y^2}\sqrt{Gr}\cdot 3f\theta'(\eta) = \frac{1}{Pr}\cdot\frac{\nu\Delta T}{2y^2}\sqrt{Gr}\,\theta''(\eta)$$

On trouve finalement :

$$\boxed{3Pr\,f\theta' + \theta'' = 0}$$