Nerve and Graph Induced Complex

Mathieu Carrière

Inria Saclay, 24/10/2017

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Mathieu Carrière + Marc Glisse

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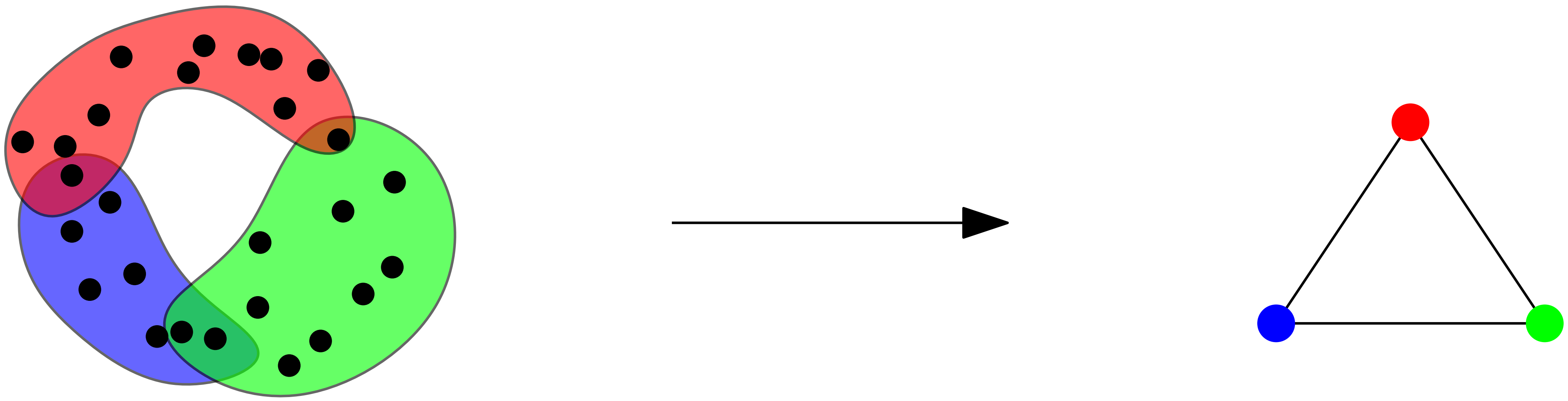
What is a Nerve?



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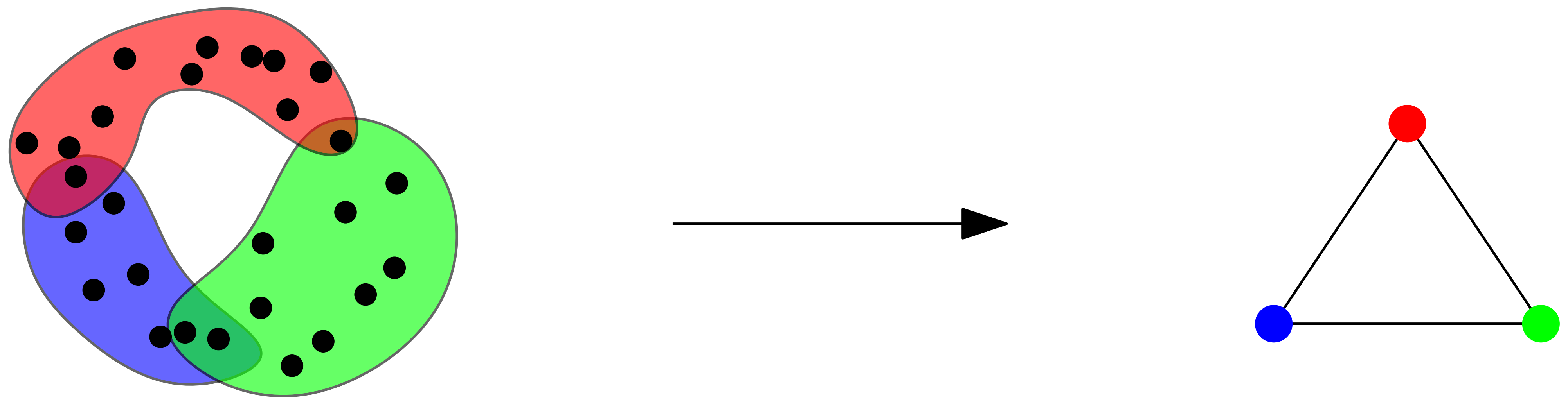


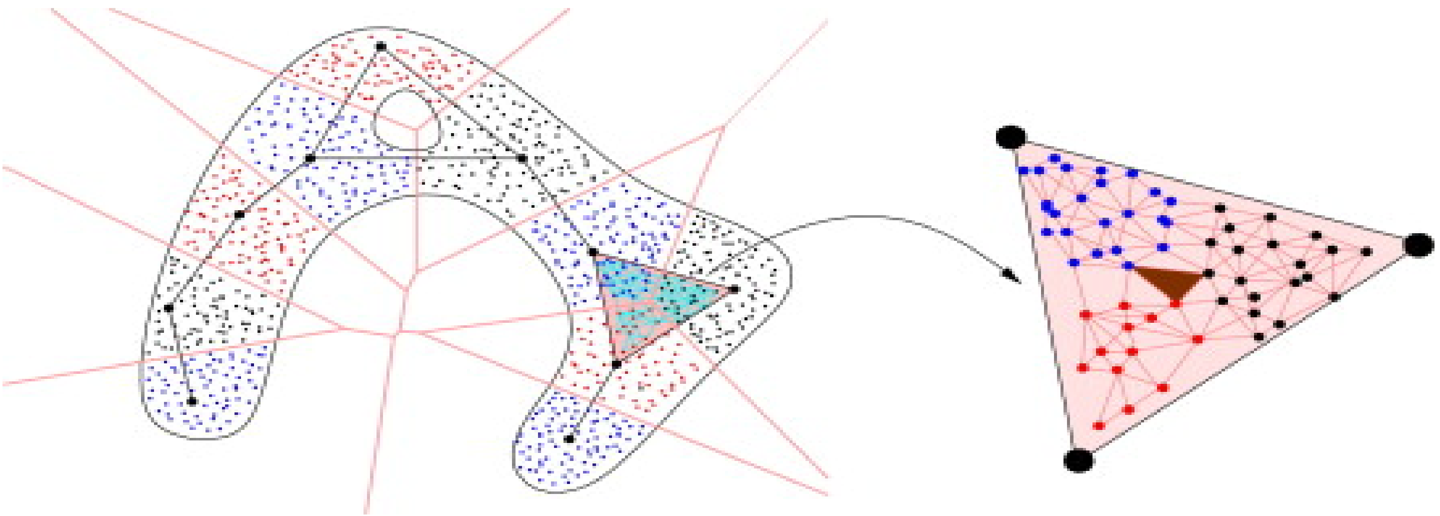
Def: Let: $P =$ point cloud
$C = \{C_n\}_{1\leq i\leq n}$ cover of P, i.e. $P = \bigcup_{i=1}^{n} C_i$

Then, the *Nerve* S is a simplicial complex s.t.

$$\text{Vertices} = \{C_1, ..., C_n\}$$

$$\{C_{i_1}, ..., C_{i_k}\} \in S \iff \bigcap_{j=1}^{k} C_{i_j} \neq \emptyset$$

What is a Graph Induced Complex?



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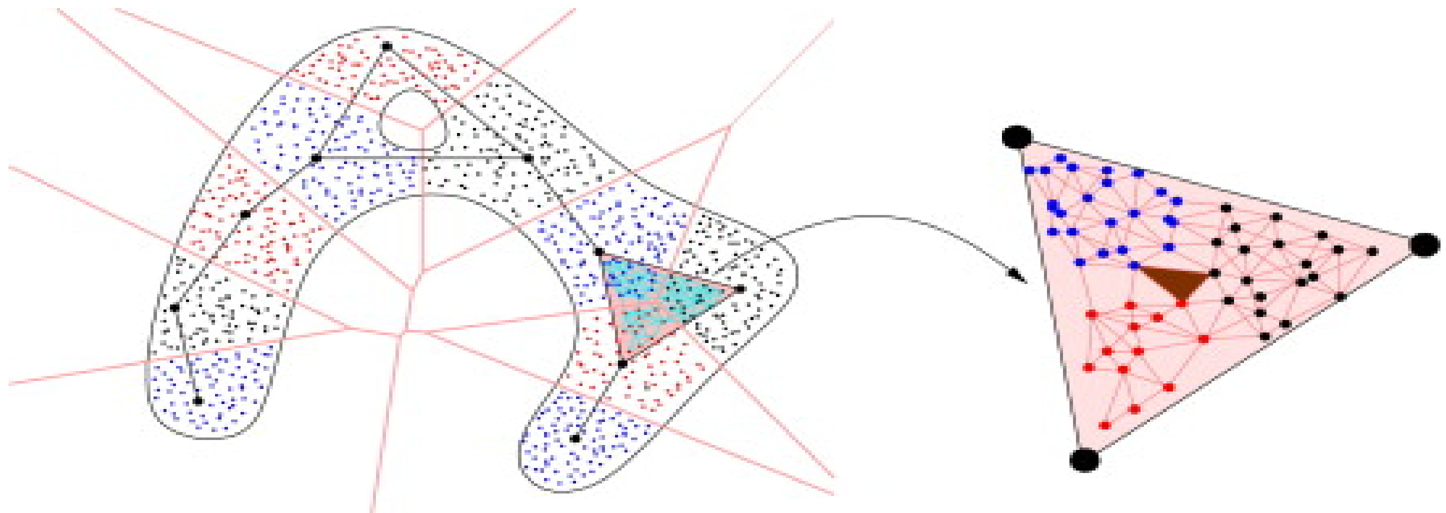

Def: Let: $P =$ point cloud

$C = \{C_n\}_{1 \leq i \leq n}$ cover of P, i.e. $P = \bigcup_{i=1}^{n} C_i$

$G =$ graph with vertex set P

Then, the *Graph Induced Complex* S is a simplicial complex s.t.

$$\text{Vertices} = \{C_1, ..., C_n\}$$

$$\{C_{i_1}, ..., C_{i_k}\} \in S \iff \exists (p_{i_1}, ..., p_{i_k}) \in C_{i_1} \times ... \times C_{i_k} \text{ s.t.}$$

$$\{p_{i_1}, ..., p_{i_k}\} \text{ is a } \textit{clique} \text{ in } G$$

Common case: Voronoi cover

Subsample P with m points: $\{q_1, ..., q_m\} \subset P$

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Not well defined...

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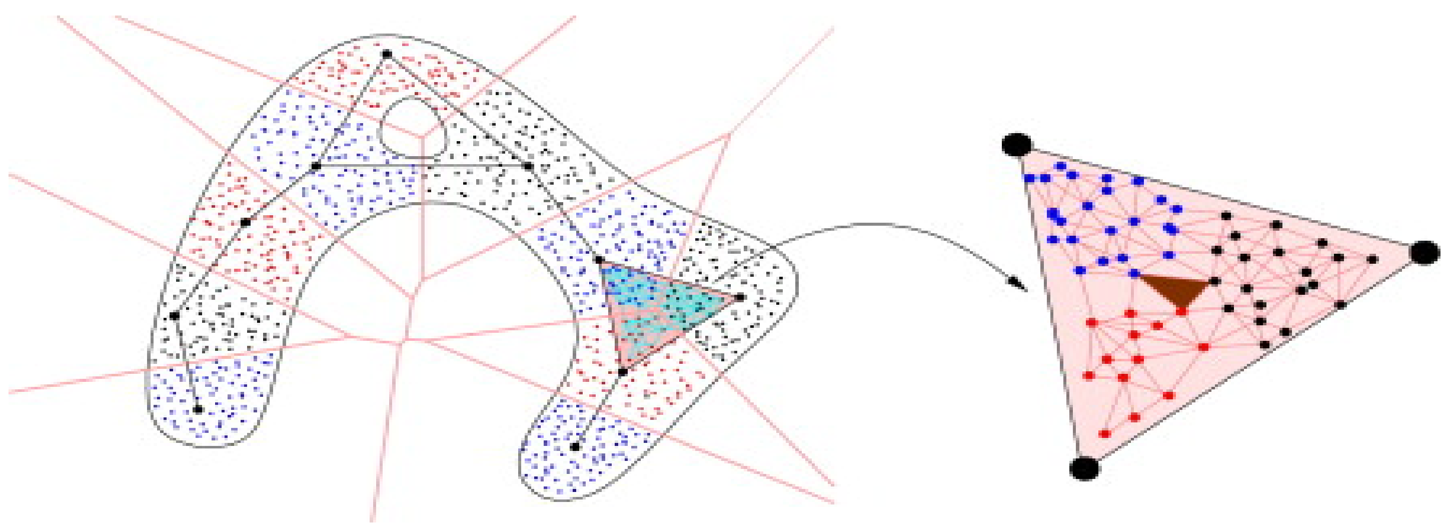
Computed with shortest path distance on neighborhood graph



Link with Mapper?

The Mapper is a particular nerve

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Let $f : P \to \mathbb{R}^d$ and $\tilde{C}$ a cover of $\mathrm{im}(f)$ with open sets

$C = f^{-1}(\tilde{C})$ is a cover of P

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Let $f : P \to \mathbb{R}^d$ and $\tilde{C}$ a cover of $\mathrm{im}(f)$ with open sets

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C can be refined into its different connected components

The Mapper is the nerve of the refined C

Note: $d = 1$ and $\tilde{C}$ minimal $\Longrightarrow$ the Mapper is a graph

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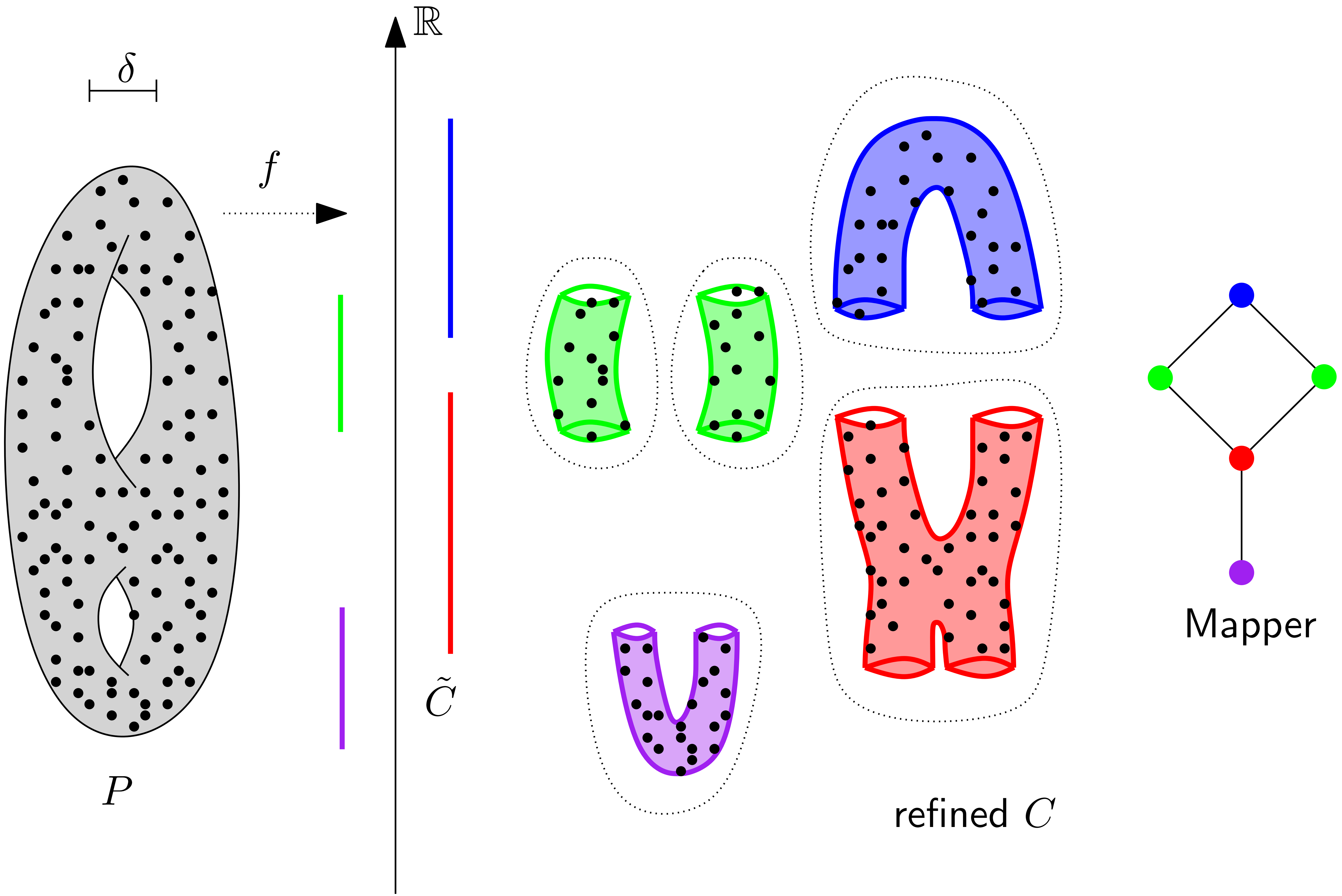
Computed with connected components of δ-neighborhood graph

Link with Mapper?



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$\tilde{C}$ is usually parametrized by:

resolution $r > 0$ (interval length)

gain $0 < g < 1$ (overlap percentage): length$(I \cap J) = gr$

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Statistical Analysis and Parameter Selection for Mapper,
C., Michel, Oudot, 2017

Heuristic for a choice of δ and r that ensures

fast convergence to Reeb graph

no discretization artifacts

What is implemented in the package?

1. Graph
   a. From file
   b. From 1-skeleton of Rips complex

2. Cover
   a. From file
   b. From preimages of a function
   c. From Voronoi

3. Nerve

4. GIC

5. Visualization
   a. txt file (kepler mapper)
   b. dot file (neato)
   c. off file (geomview)

What is implemented in the package?

1. Graph
   a. From file
   b. From 1-skeleton of Rips complex

```
set_graph_from_file (string filename)
set_graph_from_OFF (string filename)
set_graph_from_rips (double delta, distance d)
set_graph_from_automatic_rips (distance d)
```

What is implemented in the package?

2. Cover
   a. From file
   b. From preimages of a function
   c. From Voronoi

```
set_cover_from_file (string filename)

set_cover_from_function ()
   set_function_from_file (string filename)
   set_function_from_coordinate (int k)
   set_gain (double gain)
   set_resolution (double reso)
   set_automatic_resolution ()

set_cover_from_Voronoi (int m)
```

What is implemented in the package?

3. Nerve

4. GIC

`set_type (string type)` "Nerve" or "GIC"

`find_simplices ()`

What is implemented in the package?

5. Visualization
   a. txt file (kepler mapper)
   b. dot file (neato)
   c. off file (geomview)

```
set_color_from_file (string filename)
set_color_from_coordinate (int k)
```

```
plot_DOT ()
plot_OFF ()
write_info ()
```

Examples