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Rapport N°1:

1. Implementation of the triad wave-wave interaction approximation of Eldeberky into WW3.

When the waves approaching shallow water the nonlinearities enhances and the waves do not obey anymore the linear characteristics. Weak-resonant interactions in shallow water, the so called “Triad wave-wave interactions” become an important mechanism. The nonlinear regime changes from deep the shallow water from quartet interactions to triad interactions as the waves loose their dispersivity (see e.g. Cavaleri et al. (2007)). In shallow water the strong nonlinear coupling between three wave components in weak resonance leads to the growth of higher harmonics above the peak frequency of the spectrum (e.g. at 2*fp, 3*fp…). Deterministic wave models such as. Boussinesq models (e.g. Madsen & Sorensen (1993)) describe this kind of interactions inherently in the framework of there theory. In phase-averaged models, this process can only be parameterized since for the estimation of the nonlinear fluxes knowledge about the wave phases is necessary. In the WWM the LTA (Lumped Triad Approximation) of Elderberky (1996) was implemented for the approximation of the Triad wave-wave interactions. The LTA is applied to every spectral direction separately and consists of a positive and a negative part (Booij et al., 1999).

$$S_{nl3}(\sigma,\theta) = S^{-}_{nl3}(\sigma,\theta) + S^{+}_{nl3}(\sigma,\theta) \tag{1.1}$$

with

$$S^{+}_{nl3}(\sigma,\theta) = \max\left\{0,\ \alpha_{EB} 2\pi\, c_p\, c_g J^2 \left|\sin\left(\beta_{ph}\right)\right| E^2\left(\frac{\sigma}{2},\theta\right) - 2\, E\left(\frac{\sigma}{2},\theta\right) E\left(\sigma,\theta\right)\right\} \tag{1.2}$$

The negative component equals twice the positive component at the location of the higher harmonic in the wave spectrum.

$$S^{-}_{nl3}(\sigma,\theta) = -2 S^{+}_{nl3}(2\sigma,\theta) \tag{1.3}$$

The bi-phase β is defined according to Eldeberky (1996)

$$\beta_{ph} = -\frac{\pi}{2} + \frac{\pi}{2} \tanh\left(\frac{0.2}{U_{rsell}}\right) \quad 1.4$$

as function of the Ursell[1] number

$$U_{rsell} = \frac{g}{8\sqrt{2}\pi^2}\frac{H_s\overline{T}^2}{d^2} \quad 1.5$$

J is the interaction coefficient defined according to Madsen and Soerensen (1993):

$$J = \frac{k_{\sigma/2}^2\left(gd + 2c_{\sigma/2}^2\right)}{k_\sigma d\left(gd + \frac{2}{15}gd^3k_\sigma^2 - \frac{2}{5}\sigma^2 d^2\right)} \quad 1.6$$

The LTA is approximates the nonlinear transfer only for the case of horizontal bottom and unidirectional sea state (see Dingemans, 1998). According to Groeneweg et al. (2002) the "Lumped Triad Approximations" is one main reason for the overestimation of the higher harmonics in shallow waters. Becq-Girard et al. (1999) compared different nonlinear wave evolution models and showed also that the LTA overestimates the higher harmonics in there "wave breaking" case. Moreover, there is no approximation of the Triad wave-wave interactions that can reproduce the harmonic release; e.g. after the waves have broken over a bar. Here the LTA do not reproduces the backward energy transfer to the first harmonic (see e.g. Becq-Girad et al., 1999).

In the WWM-II the LTA is implemented using the SWAN source code and the suggested parameterization as defined in the SWAN 40.51AB source code. The main problem of this approximation is that the parametrization is uncertain (see e.g. Dingemans, 1998).

In this part of the project the weak resonant interaction approximation called "Lumped Triad Approximation" (hereinafter) LTA, was implemented into the ww3. The source code of the subroutine is attached. During the implementation procedure it was found that the source code in the SWAN model (Booij et al. 1999) is truncated version of the original work of Elderberky (1996a, 1996b). In SWAN the wave spectra is treated as one-dimensional and only the transfer to the higher harmonics is taken into account for this no justification is given and it has to be further investigated. The approximation of Elderberky is for a flat bottom (actually bragg-0 resonance). The biggest problem is that it is not conservative, which is the most stringent limitation factor of this approximation beside the fact that the proportionality coefficient is not a constant.

Moreover, it is questionable if it was taken into account the in spectral wave models the freq. bandwidths are exponentially distributed in freq. space, which leads to the problem that it is possible that some Jacobean transformation is missing the derivation of the discrete form, I am now looking into this and I hope that I can give some closure on this in the final report.

2. Validation of the triad wave-wave interaction approximation.

The validation of the implementation is done based on the experiments of Boers, which are experimentally investigating the wave transformation at a bared beach profile. The basic variable to look at is Tm02, which should clearly reduce as the waves are approaching the bar-through profile and finally break at the bar. If the nonlinear transfer to the higher harmonics is not considered and only depth induced wave breaking is taken into account the Tm02 is slowly increasing towards the beach, which is not physical, and poses then a problem for all

[1]The deep-water waves have a small Ursell number; as they propagate toward shallow water and become more nonlinear, the Ursell number increases. The Ursell number is in this way a measure of the nonlinearity of the waves.

quantities, related to the higher order spectral moments. 1st I recall the results presented in Roland et al. 2012, where a fully coupled 3d wave-current model was applied to this case.

In Fig. 1 the setup of the lab experiment is sketched. The vertical lines at the top of the picture show the hotwire stations where wave spectra have been computed. At this stage of the analysis we however just look at the downshift of the average period, which is quite indicative for the energy transfer to the higher harmonics.

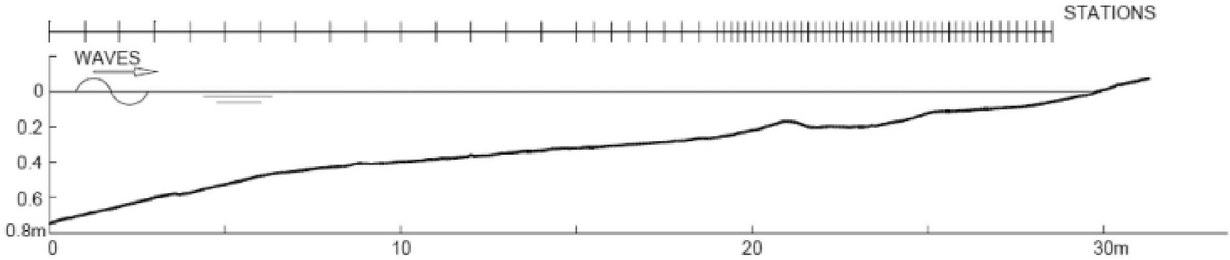

Figure 1: Boers (1996) Lab-setup

In Fig. 2 the results of the coupled model are run with two different parameterizations being one the default used in the SWAN code and the red is rather following the original literature, where these theories have been originally published or suggested in a review of the SWAN physics by Maarten Dingemans.

Similar runs will be done using the latest WW3 implementation of the triad source term listed in the appendix. The runs with the fully coupled model will serve as validation cases when also then next part of the project is finished and the wave setup can directly be estimated in WW3. The results should be comparable to the results given below, so we use the Boers test-case for both, validation of the near resonant wave-wave interactions and the computation of the wave setup.

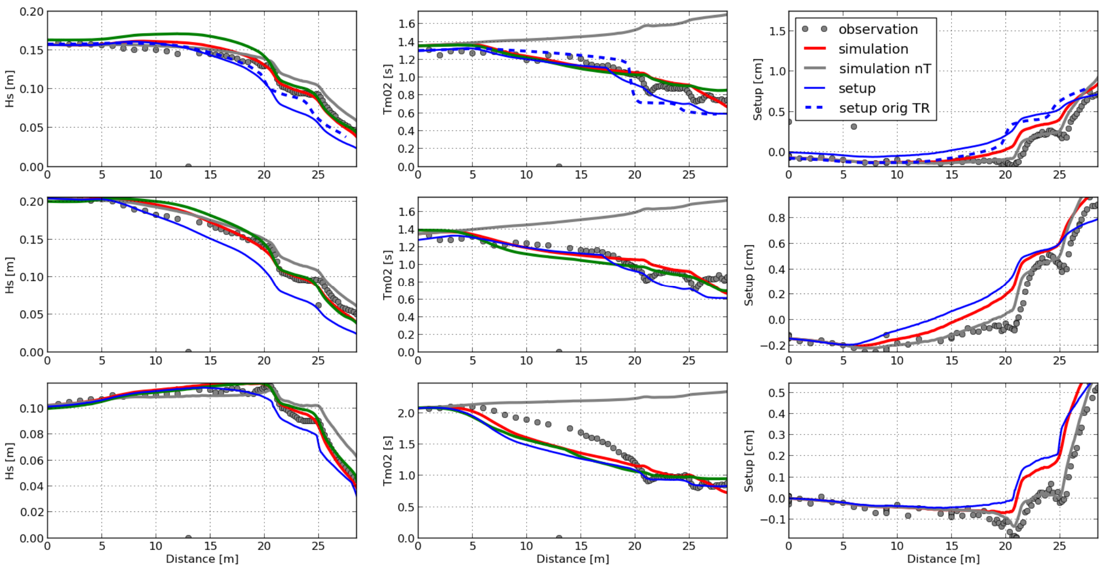

Figure 2: Comparison of wave heights, zero down crossing periods and wave induced setup for the three investigated cases between lab data (dots), model results using the default parameterization for triads and wave breaking (blue), and modified parameterization (red; Roland et al. 2012). 1st columns shows the wave heights, 2nd column zero down crossing periods, 3rd column the wave setup. The reference results are in red (wwm parameterization) and green (ww3 triads using wwm parameterization).

We do a similar setup using WW3 in order to reproduce the results obtained by Roland et al. (2012), however, we omit the computation of the wave setup, which will be done in the last stage of this project. As it can be seen the results compare quite well with the results of WWM. The differences can be explained by the fact that we have forced WW3 using a comparable JONSWAP spectra in contrast to the measured wave spectra that was prescribed in WWM in Roland et al. 2012

3. Literature:

Becq-Girard, F., Forget, P. and Benout, M., 1999. Non-linear propagation of unidirectional wave fields over varying topography. Coastal Engineering 38(2), pp. 91-113.

Booij, N., Ris, R.C. and Holthuijsen, L.H., 1999. A third-generation wave model for coastal regions. 1. Model description and validation. J. Geophys. Res., 104, 7649–7666.

Eldeberky, Y., 1996a. Nonlinear Transformation of Wave Spectra in the Nearshore Zone, Ph.D. Thesis, TU-Deft, The Netherlands

Eldeberky, Y. and J.A. Battjes, 1996b: Spectral modelling of wave breaking: Application to Boussinesq equations, J. Geophys. Res., 101, No. C1, 1253-1264

Roland, A., Zhang Y.J., Wang H.V., Meng Y., Teng Y-C, Maderich, V., Brovchenko, I., Dutour-Sikiric, M. and Zanke, U., 2012. A fully coupled 3D wave-current interaction model on unstructured grids, J. Geophys. Res. to appear.

4. Source code LTA

```
!/ ------------------------------------------------------------------- /
      MODULE W3STRXMD
!/
!/                  +-----------------------------------+
!/                  | WAVEWATCH III           NOAA/NCEP |
!/                  |           A. Roland (IT&E)        |
!/                  |                        FORTRAN 90 |
!/                  | Last update :         29-May-2012 |
!/                  +-----------------------------------+
!/
!/    15-Jul-2005 : Origination.                        ( version 3.07 )
!/    23-Jun-2006 : Formatted for submitting code for   ( version 3.09 )
!/                  inclusion in WAVEWATCH III.
!/    29-May-2009 : Preparing distribution version.     ( version 3.14 )
!/
!/    Copyright 2009 National Weather Service (NWS),
!/       National Oceanic and Atmospheric Administration.  All rights
!/       reserved.  WAVEWATCH III is a trademark of the NWS.
!/       No unauthorized use without permission.
!/
!  1. Purpose :
!
!     This peace of code computes the triad interaction term in the same way
!     as done in the SWAN model.
!
!  2. Variables and types :
!
!      Name      Type  Scope    Description
!     ----------------------------------------------------------------
!     ----------------------------------------------------------------
!
!  3. Subroutines and functions :
!
!      Name      Type  Scope    Description
```

```
!     ------------------------------------------------------------------
!      W3STRX    Subr. Public   User supplied triad interactions.
!      INSTRX    Subr. Public   Corresponding initialization routine.
!     ------------------------------------------------------------------
!
!  4. Subroutines and functions used :
!
!      Name      Type  Module   Description
!     ------------------------------------------------------------------
!      STRACE    Subr. W3SERVMD Subroutine tracing.
!     ------------------------------------------------------------------
!
!  5. Remarks : The approach is truncated version of the work of Elderberky.
!              In SWAN the wave spectra is treated as one-dimensional and
!              only the transfer to the higher harmoics is taken into account
!              for this no justification is given and it has to be further investigated.
!              The approximation of Elderberky is for a flat bottom (actually bragg-0 resonance)
!              The biggest problem is that it is not conservative, which is the biggest limitation factor.
!              Moreover it is questionable if it was taken into account the in spectral wave models the
!              freq. bandwidths are exponentially distributed in freq. space, which leads to the problem that
!              it is possible that some jacobian transformation is missing the derivation of hte discrete form,
!              I am now looking into this and I hope that I can give some closure soon.
!
!
!     See notes in the file below where to add these elements.
!
!  6. Switches :
!
!     !/S  Enable subroutine tracing.
!
!  7. Source code :
!/
!/ ------------------------------------------------------------------- /
!/
!     *****************************************
!     ***    Declare saved variables here   ***
!     ***  public or private as appropriate ***
!     *****************************************
!
      PUBLIC
!/
      CONTAINS
!/ ------------------------------------------------------------------- /
      SUBROUTINE W3STR2(A, CG, WN, DEPTH, S, D)
!/
!/                  +-----------------------------------+
!/                  | WAVEWATCH III           NOAA/NCEP |
!/                  |           H. L. Tolman            |
!/                  |                        FORTRAN 90 |
!/                  | Last update :         23-Jun-2006 |
!/                  +-----------------------------------+
!/
!/    15-Jul-2005 : Origination.                        ( version 3.07 )
!/    23-Jun-2006 : Formatted for submitting code for   ( version 3.09 )
!/                  inclusion in WAVEWATCH III.
!/
```

```
!  1. Purpose :
!
!     Slot for user-supplied triad interaction source term.
!
!  2. Method :
!
!  3. Parameters :
!
!     Parameter list
!     ----------------------------------------------------------------
!     ----------------------------------------------------------------
!
!  4. Subroutines used :
!
!      Name      Type  Module   Description
!     ----------------------------------------------------------------
!      STRACE    Subr. W3SERVMD Subroutine tracing.
!     ----------------------------------------------------------------
!
!  5. Called by :
!
!      Name      Type  Module   Description
!     ----------------------------------------------------------------
!      W3SRCE    Subr. W3SRCEMD Source term integration.
!      W3EXPO    Subr.   N/A    Point output post-processor.
!      GXEXPO    Subr.   N/A    GrADS point output post-processor.
!     ----------------------------------------------------------------
!
!  6. Error messages :
!
!       None.
!
!  7. Remarks :
!
!  8. Structure :
!
!     See source code.
!
!  9. Switches :
!
!     !/S  Enable subroutine tracing.
!
! 10. Source code :
!
!/ ------------------------------------------------------------------- /
      USE W3ODATMD, ONLY: NDSE
      USE W3SERVMD, ONLY: EXTCDE
!/S      USE W3SERVMD, ONLY: STRACE
      USE W3GDATMD, ONLY: NK, NTH, NSPEC, SIG, DTH, DDEN, RREF, &
                          REFPARS, ECOS, ESIN, EC2, MAPTH, MAPWN, &
        SIG2, DSII
!/S      USE W3SERVMD, ONLY: STRACE
!/
!
      IMPLICIT NONE
!/
```

```
!/ ------------------------------------------------------------------- /
!/ Parameter list
!/
      REAL, INTENT(IN)        :: CG(NK), WN(NK), DEPTH
      REAL, INTENT(IN)        :: A(NTH,NK)
      REAL, INTENT(OUT)       :: S(NSPEC), D(NSPEC)
!/
      IMPLICIT NONE
!/
!/ ------------------------------------------------------------------- /
!/ Parameter list
!/
!/
!/ ------------------------------------------------------------------- /
!/ Local parameters
!/
!/S      INTEGER, SAVE           :: IENT = 0
      LOGICAL, SAVE           :: FIRST = .TRUE.

      INTEGER                 :: I1, I2, IJ1, IJ2, IRES
      INTEGER                 :: ISP, ISP1, ISM, ISM1
      INTEGER                 :: ISMAX
      REAL                    :: FAC1, FACSCL, AUX1, AUX2, RINT, FT
      REAL                    :: DEP_2, DEP_3, XIS, XISLN
      REAL                    :: WISP, WISP1, WISM, WISM1
      REAL                    :: BIPH, SINBH
      REAL                    :: E0, W0, C0, WN0
      REAL                    :: EM, WM, CM, WNM

      REAL                    :: JAC(NK)
!/
!/ ------------------------------------------------------------------- /
!/
!/S      CALL STRACE (IENT, 'W3STRX')
!
! 0.  Initializations ------------------------------------------------ *
!
!     **********************************************************
!     ***    The initialization routine should include all    ***
!     *** initialization, including reading data from files. ***
!     **********************************************************
!
      IF ( FIRST ) THEN
         CALL INSTRX
         FIRST  = .FALSE.
        END IF

      IJ2    = INT (FLOAT(NK) / 2.)
      IJ1    = IJ2 - 1
      FAC1   = SIG(IJ2) / SPSIG(IJ1)
      IRES   = NINT ( LOG10( 2.) / LOG10( FAC1 ) )
      FACSCL = SIG(NK)/SIG(NK-IRES)

      IF (ABS(FACSCL-2.).GT.0.05) THEN
        FACRES = 10.**( LOG10(2.) / FLOAT(IRES) )
        SIGLOW   = SIG(NK) / ( FACRES**(FLOAT(NK-1) ) )
```

```
!        WRITE (*,*) 'CHECK RESOLUTION', IRES, FACSCL, FACRES, SIGLOW
       END IF

      DEP_2 = DEPTH**2
      DEP_3 = DEPTH**3
      I2     = INT (FLOAT(NK) / 2.)
      I1     = I2 - 1
      XIS    = SIG(I2) / SIG(I1)
      XISLN  = LOG( XIS )
      ISP    = INT( LOG(2.) / XISLN )
      ISP1   = ISP + 1
      WISP   = (2. - XIS**ISP) / (XIS**ISP1 - XIS**ISP)
      WISP1  = 1. - WISP
      ISM    = INT( LOG(0.5) / XISLN )
      ISM1   = ISM - 1
      WISM   = (XIS**ISM -0.5) / (XIS**ISM - XIS**ISM1)
      WISM1  = 1. - WISM
!
! 1.  Sets scattering term to zero
!
      D   = 0.
      S   = 0.
      JAC = 1. / (TPIINV / SIG * CG)
!
! 2.  Triad wave-wave interactions ---------------------------------- *
!
      ALLOCATE (E (1:NK))
      ALLOCATE (SA(1:NTH,1:NK+ISP1))
      E  = 0.
      SA = 0.
      ISMAX = 1
      DO IK = 1, NK
       IF ( SIG(IS) .LT. ( PTRIAD(2) * MEANT) ) THEN
          ISMAX = IS
        ENDIF
      ENDDO
      ISMAX = MAX ( ISMAX , ISP1 )

      IF ( URSELL .GT. PTRIAD(5) ) THEN
        BIPH   = (0.5*PI)*(TANH(PTRIAD(4)/URSELL)-1.)
        SINBPH = ABS( SIN(BIPH) )
        DO IT = 1, NTH
          DO IK = 1, NK
            E(K) = A(:,IK) * JAC(IK)
          END DO
          DO IK = 1, ISMAX
            E0  = E(IK)
            W0  = SIG(IK)
            WN0 = WN(IK)
            C0  = W0 / WN0
            IF ( IS.GT.-ISM1 ) THEN
              EM  = WISM * E(IK+ISM1)      + WISM1 * E(IK+ISM)
              WM  = WISM * SPSIG(IK+ISM1)  + WISM1 * SPSIG(IK+ISM)
              WNM = WISM * WK(IP,IK+ISM1)  + WISM1 * WK(IP,IK+ISM)
              CM  = WM / WNM
            ELSE
```

```
            EM  = 0.
            WM  = 0.
            WNM = 0.
            CM  = 0.
          END IF
          AUX1 = WNM**2 * ( G9 * DEPTH + 2.*CM**2 )
          AUX2 = WN0 * DEPTH * ( G9 * DEPTH + (2./15.) * GRAV *
     &          DEP_3 * WN0**2 -(2./ 5.) * W0**2 * DEP_2 )
          RINT = AUX1 / AUX2
          FT = PTRIAD(1) * C0 * CG(IK) * RINT**2 * SINBPH
          SA(IT,IK) = MAX(0., FT * ( EM * EM - 2. * EM * E0 ))
        END DO
      END DO
      DO IK = 1, NK
        SIGPI = SIG(IK) * ZPI
        DO IT = 1, NTH
          ISS    = ITH + (IK-1)*NTH
          STRI = SA(IT,IK) - 2.*(WISP * SA(IT,IK+ISP1) +
     &         WISP1 * SA(IT,IK+ISP ))
          D(ISS) =  D(ISP) + STRI /
     &                 MAX(1.E-18_rkind, A(IK,IT)*SIGPI)
          S(ISS) =  S(ISS) + STRI / SIGPI /
          END DO
        END DO
!
! 1.  .... ---------------------------------------------------------- *
!     .... conver action to energy
!
!T      CALL EXTCDE ( 99 )
!/
!/ End of W3STRX ---------------------------------------------------- /
!/
      END SUBROUTINE W3STR2
!/ ------------------------------------------------------------------- /
      SUBROUTINE INSTRX
!/
!/                  +-----------------------------------+
!/                  | WAVEWATCH III           NOAA/NCEP |
!/                  |           H. L. Tolman            |
!/                  |                        FORTRAN 90 |
!/                  | Last update :         23-Jun-2006 |
!/                  +-----------------------------------+
!/
!/    23-Jun-2006 : Origination.                        ( version 3.09 )
!/
!  1. Purpose :
!
!     Initialization for source term routine.
!
!  2. Method :
!
!  3. Parameters :
!
!     Parameter list
!     ----------------------------------------------------------------
!     ----------------------------------------------------------------
```

```
!
!  4. Subroutines used :
!
!      Name      Type  Module   Description
!     ----------------------------------------------------------------
!      STRACE    Subr. W3SERVMD Subroutine tracing.
!     ----------------------------------------------------------------
!
!  5. Called by :
!
!      Name      Type  Module   Description
!     ----------------------------------------------------------------
!      W3STRX    Subr. W3STRXMD Corresponding source term.
!     ----------------------------------------------------------------
!
!  6. Error messages :
!
!       None.
!
!  7. Remarks :
!
!  8. Structure :
!
!     See source code.
!
!  9. Switches :
!
!     !/S  Enable subroutine tracing.
!
! 10. Source code :
!
!/ ------------------------------------------------------------------- /
      USE W3ODATMD, ONLY: NDSE
      USE W3SERVMD, ONLY: EXTCDE
!/S      USE W3SERVMD, ONLY: STRACE
!/
      IMPLICIT NONE
!/
!/ ------------------------------------------------------------------- /
!/ Parameter list
!/
!/
!/ ------------------------------------------------------------------- /
!/ Local parameters
!/
!/S      INTEGER, SAVE           :: IENT = 0
!/
!/ ------------------------------------------------------------------- /
!/
!/S      CALL STRACE (IENT, 'INSTRX')
!
! 1.  .... ---------------------------------------------------------- *
!
!/
!/ End of INSTRX ----------------------------------------------------- /
!/
```

```
      END SUBROUTINE INSTRX
!/
!/ End of module INSTRXMD -------------------------------------------- /
!/
      END MODULE W3STRXMD
```

Dr.-Ing. A. Roland, 22.9.2013, Frankfurt, Germany