

Supplemental Note on ard-NMF

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Derivation of joint half-normal exponential prior Given the formulation for the priors on w_{fk} and h_{kn} and λ_k from Tan et al. (2012) we derive \hat{b} for the mixed data model.

$$\begin{aligned}\mathbb{E}[w_{fk}, h_{kn}] &= \mathbb{E}[\mathbb{E}[w_{fk}, h_{kn} | \lambda_k]] \\ \mathbb{E}[\mathbb{E}[w_{fk}, h_{kn} | \lambda_k]] &= \mathbb{E}[\mathbb{E}[w_{fk} | \lambda_k] \mathbb{E}[h_{kn} | \lambda_k]] = \mathbb{E}\left[\lambda_k \sqrt{\frac{2\lambda_k}{\pi}}\right] \\ &= \sqrt{\frac{2}{\pi}} \int_0^\infty \lambda_k^{\frac{3}{2}} f_{\lambda_k} d\lambda_k = b^{\frac{3}{2}} \frac{\sqrt{2}\Gamma(a - \frac{3}{2})}{\sqrt{\pi}\Gamma(a)}\end{aligned}$$

Finally we solve for \hat{b} :

$$\hat{b} = \frac{\mu_V \sqrt{2}\Gamma(a - \frac{3}{2})}{K \sqrt{\pi}\Gamma(a)}$$