Supplemental Note on ard-NMF

Derivation of joint half-normal exponential prior Given the formulation for the priors on w_{fk} and h_{kn} and λ_k from Tan et al. (2012) we derive $\hat{b}$ for the mixed data model.

$$\mathbb{E}\left[w_{fk}, h_{kn}\right] = \mathbb{E}\left[\mathbb{E}\left[w_{fk}, h_{kn}|\lambda_k\right]\right]$$

$$\mathbb{E}\left[\mathbb{E}\left[w_{fk}, h_{kn}|\lambda_k\right]\right] = \mathbb{E}\left[\mathbb{E}\left[w_{fk}|\lambda_k\right]\mathbb{E}\left[h_{kn}|\lambda_k\right]\right] = \mathbb{E}\left[\lambda_k\sqrt{\frac{2\lambda_k}{\pi}}\right]$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \lambda_k^{\frac{3}{2}} f_{\lambda_k} d\lambda_k = b^{\frac{3}{2}}\frac{\sqrt{2}\Gamma(a-\frac{3}{2})}{\sqrt{\pi}\Gamma(}$$

Finally we solve for $\hat{b}$:

$$\hat{b} = \frac{\mu_V\sqrt{2}\Gamma(a-\frac{3}{2})}{K\sqrt{\pi}\Gamma(a)}$$