Mixed-Effects Models, Spring 2022

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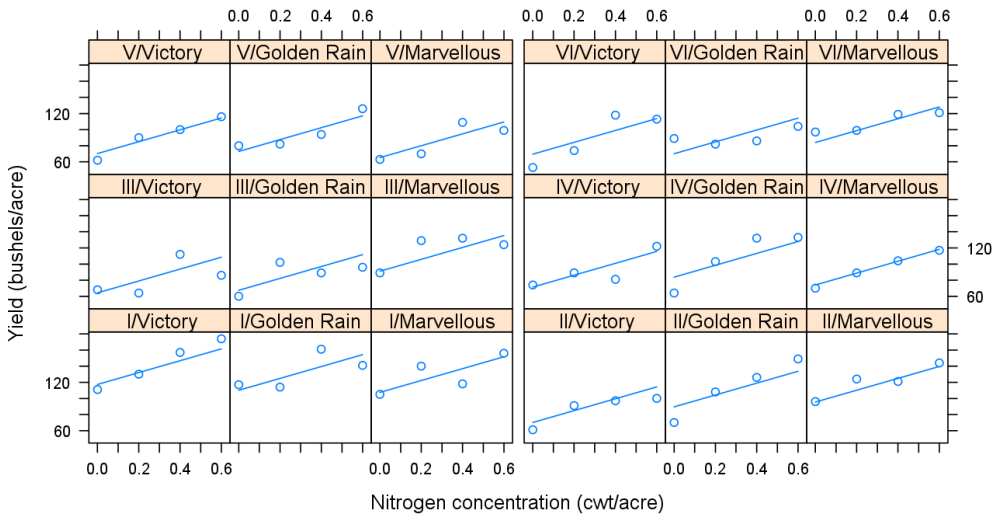
Exercise 1

(a)

Steps reproduced until the plot



(b)

```
lme(yield~nitro, data=Oats, random=~1|Block/Variety)
```

$$\mathbf{y}_{ij} = \mathbf{X}_{ij}\boldsymbol{\beta} + \mathbf{Z}_{i,j}\mathbf{b}_i + \mathbf{Z}_{ij}\mathbf{b}_{ij} + \boldsymbol{\epsilon}_{ij}$$

Fixed effect:

$\boldsymbol{\beta}$	:	Nitrogen level	$p = 2$ (intercept + slope)

Random effects:

$\mathbf{b}_i$	:	Block	$q_1 = 1$ (intercept)
$\mathbf{b}_{ij}$	:	Variety within block	$q_2 = 1$ (intercept)

Matrices:

$\mathbf{X}_{ij}$	$\mathbf{Z}_{i,j}$	$\mathbf{Z}_{ij}$	$\boldsymbol{\Psi}_1$	$\boldsymbol{\Psi}_2$
$\begin{bmatrix} 1 & 0.0 \\ 1 & 0.2 \\ 1 & 0.4 \\ 1 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} \sigma_1^2 \end{bmatrix}$	$\begin{bmatrix} \sigma_2^2 \end{bmatrix}$

(c)

Equivalent model specification:

```
library(nlme)
lme(yield~nitro, data=Oats, random=list(Block=~1, Variety=~1))
```

Or

```
library(lme4)
lmer(yield~nitro + (1|Block) + (1|Variety:Block), data=Oats)
```

Exercise 2

$Var(\mathbf{y}_i) = \mathbf{Z}_i \mathbf{\Psi} \mathbf{Z}_i^T + \sigma^2 \mathbf{I}_{n_i} =$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} & C_{18} & C_{19} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & C_{27} & C_{28} & C_{29} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} & C_{37} & C_{38} & C_{39} \\ & & & & & & & & \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} & C_{47} & C_{48} & C_{49} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} & C_{57} & C_{58} & C_{59} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} & C_{67} & C_{68} & C_{69} \\ & & & & & & & & \\ C_{71} & C_{72} & C_{73} & C_{74} & C_{75} & C_{76} & C_{77} & C_{78} & C_{79} \\ C_{81} & C_{82} & C_{83} & C_{84} & C_{85} & C_{86} & C_{87} & C_{88} & C_{89} \\ C_{91} & C_{92} & C_{93} & C_{94} & C_{95} & C_{96} & C_{97} & C_{98} & C_{99} \end{bmatrix}$$

(a)

$$\begin{bmatrix} C_1 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 \\ C_2 & C_1 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 \\ C_2 & C_2 & C_1 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 \\ & & & & & & & & \\ C_2 & C_2 & C_2 & C_1 & C_2 & C_2 & C_2 & C_2 & C_2 \\ C_2 & C_2 & C_2 & C_2 & C_1 & C_2 & C_2 & C_2 & C_2 \\ C_2 & C_2 & C_2 & C_2 & C_2 & C_1 & C_2 & C_2 & C_2 \\ & & & & & & & & \\ C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_1 & C_2 & C_2 \\ C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_1 & C_2 \\ C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_2 & C_1 \end{bmatrix}$$

(b)

$$\left[\begin{array}{ccc|ccc|ccc} C_1 & C_2 & C_2 & C_3 & C_3 & C_3 & C_3 & C_3 & C_3 \\ C_2 & C_1 & C_2 & C_3 & C_3 & C_3 & C_3 & C_3 & C_3 \\ C_2 & C_2 & C_1 & C_3 & C_3 & C_3 & C_3 & C_3 & C_3 \\ \hline C_3 & C_3 & C_3 & C_1 & C_2 & C_2 & C_3 & C_3 & C_3 \\ C_3 & C_3 & C_3 & C_2 & C_1 & C_2 & C_3 & C_3 & C_3 \\ C_3 & C_3 & C_3 & C_2 & C_2 & C_1 & C_3 & C_3 & C_3 \\ \hline C_3 & C_3 & C_3 & C_3 & C_3 & C_3 & C_1 & C_2 & C_2 \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_3 & C_2 & C_1 & C_2 \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_3 & C_2 & C_2 & C_1 \end{array}\right]$$

(c)

$$\left[\begin{array}{ccc|ccc|ccc} C_1 & C_4 & C_4 & C_7 & C_7 & C_7 & C_8 & C_8 & C_8 \\ C_4 & C_1 & C_4 & C_7 & C_7 & C_7 & C_8 & C_8 & C_8 \\ C_4 & C_4 & C_1 & C_7 & C_7 & C_7 & C_8 & C_8 & C_8 \\ \hline C_7 & C_7 & C_7 & C_2 & C_5 & C_5 & C_9 & C_9 & C_9 \\ C_7 & C_7 & C_7 & C_5 & C_2 & C_5 & C_9 & C_9 & C_9 \\ C_7 & C_7 & C_7 & C_5 & C_5 & C_2 & C_9 & C_9 & C_9 \\ \hline C_8 & C_8 & C_8 & C_9 & C_9 & C_9 & C_3 & C_6 & C_6 \\ C_8 & C_8 & C_8 & C_9 & C_9 & C_9 & C_6 & C_3 & C_6 \\ C_8 & C_8 & C_8 & C_9 & C_9 & C_9 & C_6 & C_6 & C_3 \end{array}\right]$$