Polynomial Regression
Solution

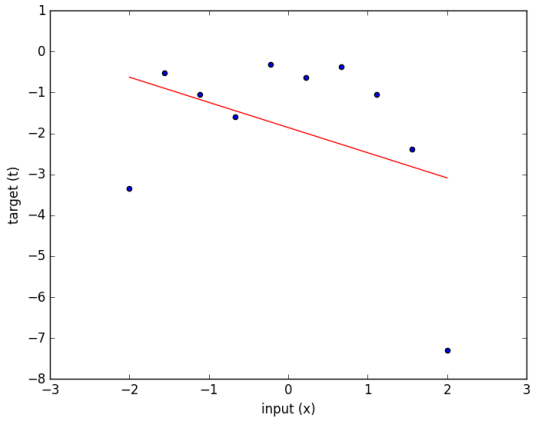
January 22, 2018

1 Polynomial Regression

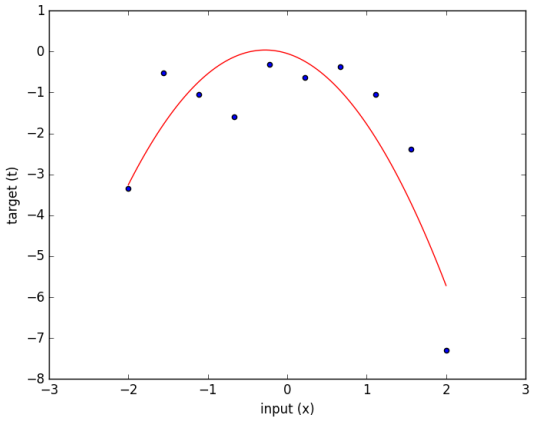
1. The optimum weights are provided in Table 1.
2. Figure 1 shows the prediction plots for the polynomial function with different order.
3. The models that best fit the data and capture the structure of the data are the models of order 4, 5, 6, and 7 however, higher orders start to overfit the data which is obvious for the model of order 9 as it passes with all the given data points, i.e., it models the data and the noise.
4. **Regularization:**
   Figure 1 shows the prediction plots for the polynomial function with order 9 and with a regularization parameter $\lambda = 0.1$. The optimum weights in this case are [-0.58 0.63 -0.59 -0.084 0.05 -0.45 0.12 0.15 -0.043 -0.013].

Table 1: Optimum weights for polynomial function

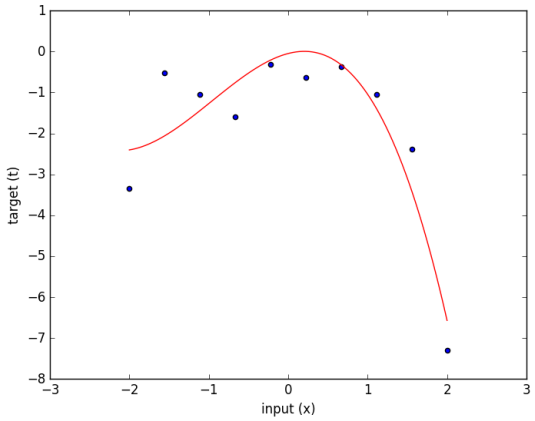
w	1	2	3	4	5	6	7	8	9
w_0	-1.86	-0.047	-0.047	-0.86	-0.86	-0.45	-0.45	-0.37	-0.37
w_1	-0.62	-0.62	0.5	0.5	1	1	0.84	0.84	-1.09
w_2	-	-1.11	-1.11	0.63	0.63	-1.35	-1.35	-2.24	-2.24
w_3	-	-	-0.39	-0.39	-0.91	-0.91	-0.54	-0.54	8.4
w_4	-	-	-	-0.43	-0.43	0.95	0.95	2.26	2.26
w_5	-	-	-	-	0.1	0.1	-0.095	-0.095	-10.48
w_6	-	-	-	-	-	-0.23	-0.23	-0.82	-0.82
w_7	-	-	-	-	-	-	0.029	0.029	4.21
w_8	-	-	-	-	-	-	-	0.079	0.079
w_9	-	-	-	-	-	-	-	-	-0.53



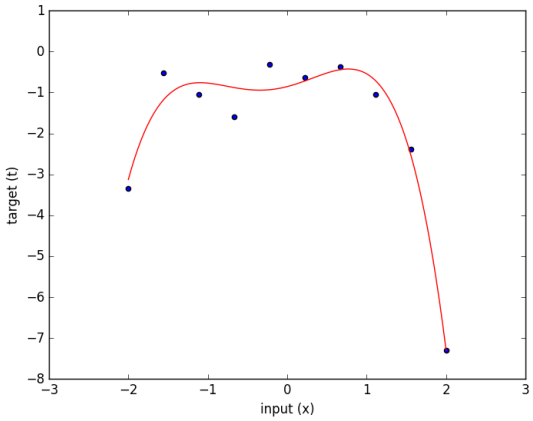
(a) Order = 1



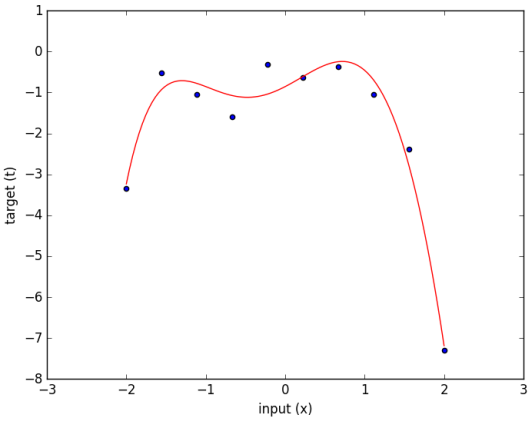
(b) Order = 2



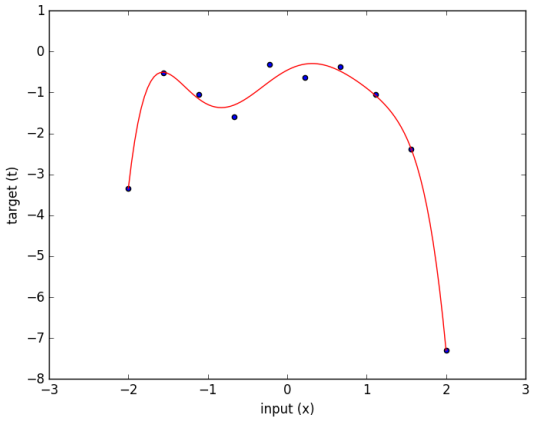
(c) Order = 3



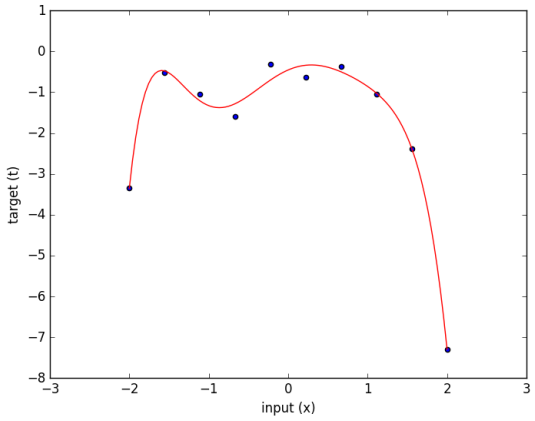
(d) Order = 4



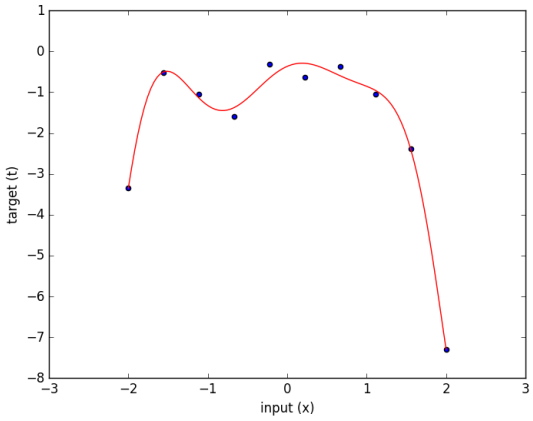
(e) Order = 5



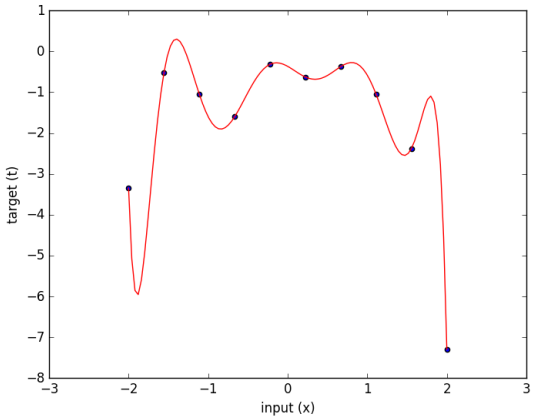
(f) Order = 6



(a) Order = 7



(b) Order = 8



(c) Order = 9

Figure 1: The prediction plot for the polynomial order with different order

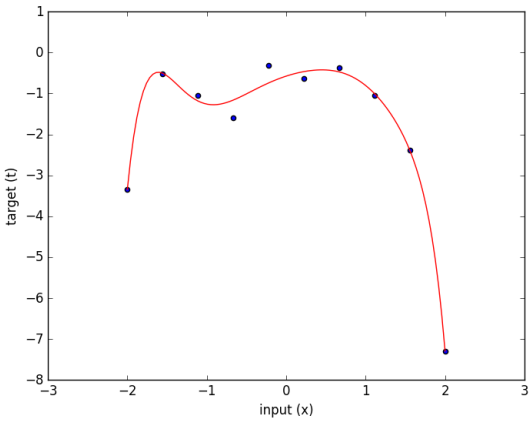


Figure 2: The prediction plot for the polynomial function with order = 9 and $\lambda = 0.1$