

The due date for this assignment is Tuesday November 24-th. The total length of your assignment should be at most four pages (including figures). There is no need to hand in any computer code.

1. Size distortions: To illustrate the size distortions of the 2SLS t -statistic, we simulate data from the following model:

$$\begin{aligned} Y &= X\beta + \varepsilon \\ X &= Z\Pi + V \end{aligned}$$

where Y and X are $n \times 1$ vectors which contain the endogenous variables and Z is a $n \times k$ matrix of instruments. ε and v are $n \times 1$ vectors that contain the disturbances. The different rows of $(\varepsilon : V)$, $(\varepsilon_i : V_i)'$, $i = 1, \dots, n$, are independently normal distributed: $(\varepsilon_i : V_i) \sim N(0, \Sigma)$, $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. We use $n = 100$, $k = 10$, $\Pi = a \times e_{10}$ with e_{10} a 10×1 vector whose top element is one and all remaining elements are equal to zero. All elements of Z are independently standard normal distributed. We only simulate them once and keep them fixed throughout the simulation experiment.

We use seven values of a : (0.3 0.25 0.2 0.15 0.1 0.05 0) and ten different values of ρ : (0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.9 0.95).

For each value of a make a figure of the rejection frequency as a function of ρ when testing $H_0 : \beta = 0$ with 95% significance using the 2SLS t -statistic (so five figures which show the rejection frequency as a function of a). Use 5000 simulations from the above model.

What do you conclude?

2. Compute and make a figure of the 95% critical value function of the LR statistic as a function of $r(\beta_0)$ for $k = 10$. What can you say about the critical value when $r(\beta_0) = 0$ or infinite?
3. Repeat the exercise in 1 for the AR, score and LR statistics. What do you conclude?
4. Compute and make a figure of the 95% critical value function of the LR statistic as a function of $r(\beta_0)$ for $k = 4$.

Question 5 is on the other side.

5. Card (1993)¹ analyzes the return on education. He uses different proximity to college variables as instruments. The file `assignmentweakinstruments.mat` contains that part of the Card data which we use for this assignment. The different variables in the file are: `nearc2`: if near a 2 year college, `nearc4`: if near a 4 year college, `nearc4a`: if near a 4 year community college, `nearc4b`: if near a 4 year private college, `ed`: years of education, `wage`: log-earnings, `age`: age in years, `age2`: squared age, `exper`: experience, `exper2`: experience squared, `south`: lives in the South, `smsa`: lives in a metropolitan area, `race`: racial indicator.

The variables `wage` and `ed` constitute the endogenous variables (y and x), `nearc2`, `nearc4`, `nearc4a`, `nearc4b` are instruments (z) and `exper`, `exper2`, `south`, `smsa`, `race` and the constant term are the included exogenous variables (w) (We do not use `age` and `age2`).

- (a) Using only `nearc2` as an instrument, construct the 95% confidence set for the return on education using the 2SLS t -statistic and the AR statistic.
- (b) Is there a difference between these confidence sets and if so can you explain why this difference occurs?
- (c) What is the value of the first stage F-statistic and what does the value of the AR statistic look like when the tested parameter is large.
- (d) We did not use the LM and LR statistics in a or did we?
- (e) Using `nearc4`, `nearc2`, `nearc4a` and `nearc4b` as instruments, construct the 95% confidence set for the return on education using the 2SLS t -statistic, AR, LM and LR statistics.
- (f) Is there a difference between these confidence sets and if so can you explain why this difference occurs?

¹Card, D. Using geographic variation in college proximity to estimate the return to schooling. In L.N. Christofides, E.K. Grant and R. Swidinsky, editor, *Aspects of Labour Market Behaviour: essays in honor of John Vanderkamp*, pages 201—222. University of Toronto Press, Toronto, Canada, 1995. (NBER Working Paper 4483 (1993)).