Advanced Micro Econometrics

Computer assignment

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The jupyter Python notebook used for this assignment is available on Github[1].

Exercise 1

Figure 1 shows the results from the simulation study. For each value of a, the rejection frequency is shown as a function of ρ when testing $\mathrm{H}_0 : \beta = 0$ with 95% significance using the 2SLS t-statistic. The rejection frequency increases in the correlation ρ, that is the endogeneity of X, for all values of a. Hence, the 2SLS t-statistic becomes more size distorted when the endogeneity increases.

When comparing the graphs for the different values of a, we find that the rejection frequency given ρ becomes larger for smaller values a. Hence, the weaker the instruments, the more size distorted the 2SLS t-statistic becomes.

We conclude that strong endogeneity and weak instruments both lead to consistent size distortions of the 2SLS t-statistics making it unviable in such cases.

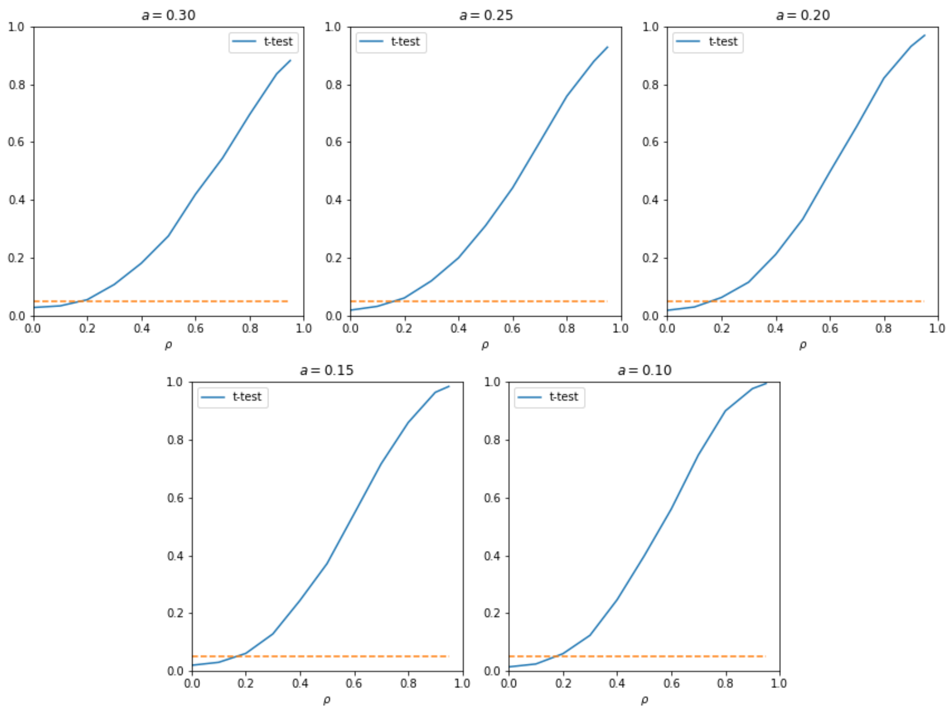


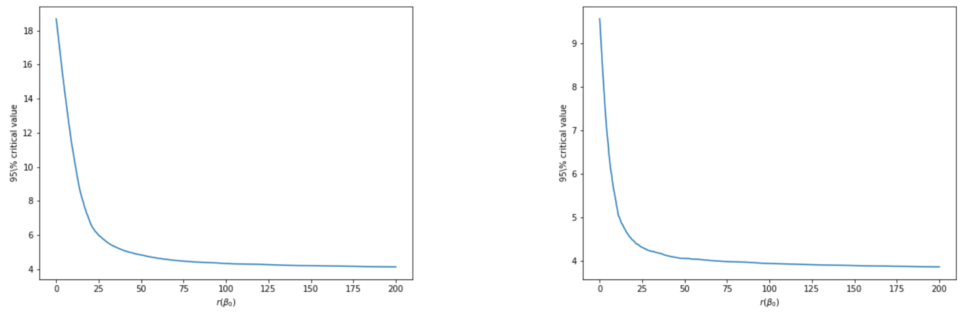
Figure 1. Rejection frequency as a function of ρ when testing $\mathrm{H}_0 : \beta = 0$ with 95% significance using the 2SLS t-statistic for different values of a and $\rho \in \{0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95\}$.

Exercise 2

The simulated 95% critical value function of the LR statistic as a function of $r(\beta_0)$ for $k = 10$ and $k = 4$ are shown in Figure 2. When $r(\beta_0) = 0$, the 95% critical value of the LR statistic equals the critical value of the $\chi^2(k)$ distribution, that is, 18.307 for $k = 10$ and 9.488 for $k = 4$. In this case the LR statistic is equal to k times the Anderson-Rubin statistic.

When $r(\beta_0) \to \infty$, the 95% critical value converges to 3.841, that is, the 95% critical value of the $\chi^2(1)$ distribution. In such cases, the LR statistic converges to the score statistic.

[1] Jupyter notebook: https://github.com/Mathijs995/Advanced-Micro-Econometrics/blob/master/TI152%20-%20Computer%20Assignment%20-%20Mathijs%20de%20Jong.ipynb



(A) $k = 10$ (B) $k = 4$

FIGURE 2. The simulated 95% critical value function of the LR statistic as a function of $r(\beta_0)$ using 10000 simulations and sampling β between 0 and 200 with 1000 steps.

Exercise 3

Figure 3 shows the results from the simulation study for the AR, score and LR statistics. For each value of a, the rejection frequency is shown as a function of ρ when testing $\mathrm{H}_0 : \beta = 0$ with 95% significance using the AR, score and LR statistics. We find that for all values of a and ρ, the tests are stable. The rejection frequencies are approximately 0.065 which is above the desired 5% significance level. The score statistic has the lowest rejection frequency and the AR statistic has the largest rejection frequency.

The problems that occurred for strong endogeneity and weak instruments when using the 2SLS t-statistics are no longer present, and hence we conclude that the test AR, score and LR test statistics are robust to strong endogeneity and weak instruments making them generally applicable.

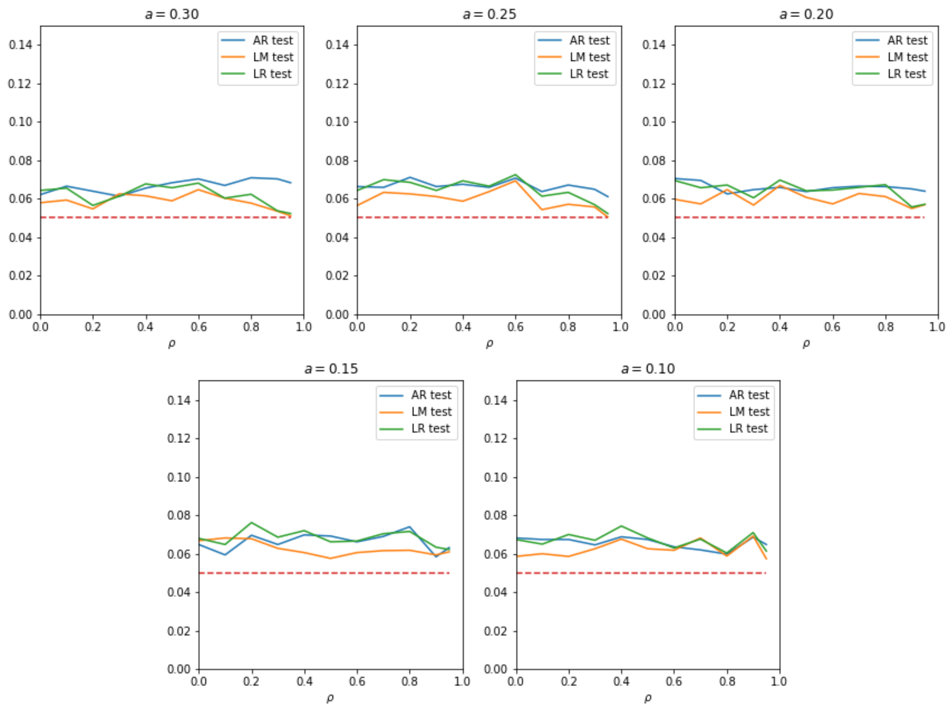


FIGURE 3. Rejection frequency as a function of ρ when testing $\mathrm{H}_0 : \beta = 0$ with 95% significance using the AR, score and LR statistics for different values of a and $\rho \in \{0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95\}$.

Exercise 4

See the answer of Exercise 2.

Exercise 5

(a) The 95% confidence set for the return on education based on the 2SLS t-statistic is given by $(-0.043, 0.743)$. The 95% confidence set for the return on education based on the the AR statistic is given by $(-\infty, -1.474) \cup (0.117, \infty)$.

(b) The confidence sets of the 2SLS t-statistic and the AR statistic differ considerable. The confidence set of the 2SLS t-statistic is bounded while the confidence set of the AR statistic consists of two disjoint sets and is unbounded towards both positive and negative numbers which indicates weak instruments. As we saw in question 1, the 2SLS t-statistic is not robust in this case.

(c) The first-stage F-statistic is equal to 2.810. For 2SLS t-statistics to be reliable, the rule of thumb is to require a first-stage F-statistic of at least ten. The AR(β_0) statistic that tests for an infinite value of β_0 is equal to the first-stage F-statistic for the test $\mathbf{\Pi} = \mathbf{0}$. Therefore, when the tested parameter is large, the AR(β_0) statistic converges to the first-stage F-statistic.

(d) Since the AR, score and LR statistics are equivalent when there is only one instrument, that is, when $k = 1$, we used all three statistics in question 5a.

(e) The 95% confidence sets for the return on education, using `nearc4`, `nearc2`, `nearc4a` and `nearc4b` as instruments are shown in the Table 1. Note that the interval $(-0.390, -0.232)$ was not included for the score statistic. For a justification, see the answer in 5f.

	2SLS	AR	Score	LR
LB	0.094	0.086	0.104	0.104
UB	0.237	0.315	0.273	0.274

TABLE 1. The 95% confidence sets for the return on education using `nearc4`, `nearc2`, `nearc4a` and `nearc4b` as instruments.

A graphical representation of the confidence intervals for the AR, score and LR statistics is shown in Figure 4

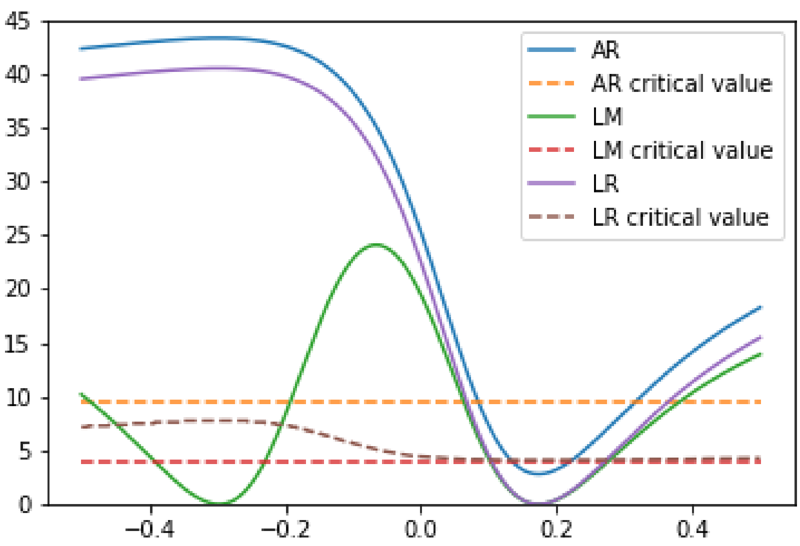


FIGURE 4. The 95% confidence sets for the return on education using `nearc4`, `nearc2`, `nearc4a` and `nearc4b` as instruments for the AR, score and LR statistics.

(f) The confidence sets of the 2SLS t-statistic, AR, score and LR statistics show no clear differences besides that the score statistic has the additional confidence region $(-0.390, -0.232)$. The additional interval is a result from using the concentrated likelihood function and can be seen as a weakness of the method. We can ignore this interval for the analysis.

The reason that there are no further clear differences is because we have strong instruments which is supported by the first-stage F-statistic that is equal to 34.672.