Dialetheism in Arabic Logic: Determining a Deviant Tradition?

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Contents

All Arabic names in this paper have been transcribed into Latin script without distinguishing between emphatic and normal consonants or long and short vowels. Exceptions are Averroes and Avicenna, whose Latinised names are used, according to convention.

1 Introduction

One need not be a logical pluralist to admit that here is no logic 'as such'. Even in introductory courses at university one soon finds out that there is in fact a wide, and theoretically infinite, range of systems that can be studied as logics. Yet, what is less evident is that the canonical logical systems taught at most universities are actually the product of an intensely occidental development, which tends to overshadow traditions that have existed elsewhere. One such alternative tradition is that of the logic developed in the Arab world during the Middle Ages: Arabic logic.

It is the deviant tradition of Arabic logic that will be studied in this paper. Unfortunately, limited research has been conducted on this theme, which tends to be underappreciated at Western universities, and which may appear exotic or even irrelevant to those used to operating within an occidental framework. This paper will depart from an intuition on a possible explanation of the unwillingness and discomfort that characterise much of the Western attitude towards Arabic logic: it will aim to evaluate the extent to which the tradition is based on a conception of logical truth fundamentally different from common occidental views. More particularly, it will set out to analyse whether the tradition of Arabic logic could be considered *dialetheist*: could the acceptance of a non-exclusive relation between truth and falsity have been a determining aspect of the tradition, and thus account for its divergence with respect to the predominantly bivalent logic of occidental discourse?

Answering this question demands that a clear understanding of the general development of Arabic logic be established first. This is why the paper's first chapter will be dedicated to a historical overview of the tradition, where its key moments and turning points will be identified. Thus, the instances representative of the tradition's evolvement will be clarified, so that an accurate analysis in terms of dialetheist truth conceptions is made possible. The second chapter will explain the central concept of dialetheism, explicitly contrasting it with the Principle of Non-Contradiction that has been at the basis of most Western logic. Next, the third and final chapter will analyse the perspectives of the central thinkers, schools and movements identified in the first chapter, and assess the extent to which their truth conceptions could indeed be regarded as dialetheist. A concluding section will reflect on the results thus obtained.

By pursuing the programme outlined above, I wish to establish whether the pretheoretical impression of Arabic logic as structured by dialetheist notions is justified.

In doing so, I also hope to contribute to a more active debate on this tradition, which is a field that still contains many research gaps. Moreover, I believe there is a lot to be learnt about the dominant practices and systems of Western logic by juxtaposing them to alternative discourses, such as the Arabic one. Especially when it comes to such a sensitive topic as dialetheism, the rejection of which has long been an indubitable cornerstone of occidental philosophy, studying a deviant tradition may hopefully facilitate an enhanced self-awareness of the exclusively bivalent conceptions structuring the Western tradition at the very core.

2 Arabic logic: a concise historical overview

This first chapter will provide a bird's-eye perspective on the historical development and constitution of what has come to be known as Arabic logic. After a brief consideration of the grounds underlying the coherent distinction of such a thing as 'Arabic logic', a rough overview will be given of those movements, schools and thinkers that have articulated its main products. This historical overview is an important preliminary to the paper, as it will facilitate the distinction of some of the tradition's most significant and influential views and theories, which will later be studied in more technical detail. The main objective will be to identify those moments that have been representative of the tradition in its totality, so that the later analysis in terms of dialetheist truth conceptions will also apply to the development as a whole.

2.1 What is Arabic logic?

Before sketching its historical development, it is necessary to create a clear impression of what is meant by the term 'Arabic logic'. As pointed out by Street (Street 2001), Arabic logic is by no means restricted to the work of logicians who wrote in Arabic. A great number of sources were originally written in other languages, such as Syriac, Hebrew, Persian and Urdu, which are nonetheless best characterised as Arabic logic. At the same time, it should be noted that not all works on logic written in Arabic are to be regarded as samples of Arabic logic in the sense maintained in this paper, as there are examples of logical works that were written in Arabic, although content-wise, they are best contextualised in the framework of occidental or 'Latin' logic. This is the case with certain translations, or with Arabic scholars who spent time at European universities and introduced notions or ideas thus acquired in their respective countries of origin.

Given these caveats, we would probably do best not to attempt any rigid demarcation of what qualifies as Arabic logic and what does not, but to treat the term as a general classification of those logical views and theories developed in the Arab world without immediately and completely corresponding to ideas advanced in the European tradition. Of course, such a description will always remain somewhat vague, but what matters here is that we will focus on the logics developed by scholars at universities and schools in the Arab world since the 8th century CE. This is also the understanding on which most of the literature on Arabic logic is based. (Rescher 1964, Street 2001)

The Arab world is taken to encompass regions that once were part of the caliphates, emirates or kingdoms that arose after the great territorial expansions of the Muslim world following the death of Muhammad in the 7th century CE. The fact that Arabic logic is thus spatio-temporally related to Islamic religion and politics does not, however, imply that it is itself a synonym for Islamic logic. As Bergstra points out (Bergstra 2011), we should take Islamic logic to be the deductive system formalising the inferences that Islam allows or requires its followers to draw, whereas Arabic logic should be regarded as the abstract and general study of reasoning patterns and truth ascriptions on a theoretical level that is not necessarily phrased in religious terms.

2.2 Early period

With the stabilisation of the Abbasid Caliphate in the 8th century, the Arabic interest in philosophy and science increased, and a blooming intellectual climate came to exist. Especially the Levant and the city of Alexandria were locations where scholars started paying much attention to subjects such as rhetoric, grammar, and also to logic. This period can be regarded as the beginning of the logical tradition in the Arab world. However, it is hard to reach a theoretical understanding of this early stage and its main exponents, because much of the debate on logic was centred around the construction of a generally acceptable vocabulary, in addition to the translation of available works by Aristotle. It is perhaps too early to speak of a distinctly Arabic school of logic at this point; rather, we are faced with what seem to be the preparations paving the way for later developments. (Street 2001, Rahman, Street, and Tahiri 2008)

This is not to say that these early signs of an Arabic interest in logic should be neglected altogether. For instance, it is relevant to note that at this stage, only a limited fraction of Aristotle's work on logic was available to Arab scholars. Those writings that were known were usually truncated or corrupted by a series of preceding translations, but among the (partly) available works were significant parts of the *Categories*, *On Interpretation*, fragments of *Prior* and *Posterior Analytics*, the *Topics* and the *Rhetoric*. Most of these titles were eventually translated into Arabic by Syriac Christians such as Kindi, Hunayn and Ishaq. Translations of other works, such as the Metaphysics, are not known to have circulated at this point in time. (Nasr 2006, Black 1990)

2.3 The Baghdad School

Following the early days of logic in the Arab world, some major developments took place in Baghdad from the 10th century onwards. New theories and perspectives were formulated and discussed by a group of polymaths that could be called the 'Baghdad School'. Prominent members of this movement were Al-Farabi and Al-Baghdadi, who both took a particular interest in logic, far beyond the mere translation or interpretation of Aristotelian texts. (Alwishah and Sanson 2009)

Especially Al-Farabi has been a widely influential figure, with regard to his own contemporaries as well as later generations of Arabic philosophers and logicians. Although Al-Farabi's logical writings have sometimes been misclassified as simple commentaries on available fragments of Aristotle's *Organon*, his project amounted to much more than that. What he aimed to achieve was so-called 'Lesser Harmony', that is, the distillation of one consistent doctrine from Aristotle's works, in order to overcome the ambiguities that the latter's writings were often thought to contain. In doing so, however, he went beyond the interpretation of individual Aristotelian treatises, and clearly came to construct a novel theoretical framework, in which a special position was accorded to logic in general, and syllogisms in particular. Furthermore, Al-Farabi stressed the importance of logic in the context of the educational system that was developed in his time, and thus contributed to the sustained interest in the topic that was maintained by later generations. (Lameer 1994)

Another important thinker of the Baghdad School was Al-Baghdadi, a philosopher who particularly focused on the study of grammar and logic. Al-Baghdadi is especially well-known for his treatment of some logical paradoxes, and the perspectives that he advanced on these issues had a considerable influence on his contemporaries. He advanced a theory of truth and falsehood to which many other Arabic logicians subscribed, and his name features often as a reference in later writings. (Alwishah and Sanson 2009, Street 2001)

2.4 Avicenna

After the accomplishments of the Baghdad School, the next major turning point in the development of Arabic logic is marked by the work of the famous Khorasmian philosopher Avicenna, in the 11th century. His critical insights had a particularly great impact on the prevailing attitude towards Aristotle. Whereas Al-Farabi and others associated with the Baghdad School still attempted to interpret Aristotelian treatises in such a way that they

could be reconciled with their own theories, Avicenna's works imply a definite departure from Aristotelian doctrines. This is not to say that Avicenna was not influenced by Aristotle, but that his philosophical project was not primarily aimed at a consistent reading of the latter's writings in order to attain such a thing as Lesser Harmony. Rather, Avicenna was led by what he himself called 'Intuition', which enabled him to set up a new theoretical framework. (Ahmed 2010, Thom 2008)

Avicenna's main logical achievement is probably the notion of modality that he developed. He set up a syllogistic system that dealt with modal propositions in a way radically different than any reading of Aristotle would allow. In fact, Avicenna promoted a modalised interpretation of many kinds of propositions that his predecessors would have taken to be non-modal, which also influenced his truth conception. (Thom 2008, Street 2002) Avicenna's notion of modality and the accompanying logical framework have had a major influence on subsequent developments in Arabic logic. Although Al-Farabi and his followers advocated the importance of logic as part of the educational curriculum, Avicenna's theory soon came to dominate their views. Avicenna's logic was integrated in the academic programme at most Arabic universities, and apparently, such an authority was accorded to his work as to justify later Arabic scholars to refer to it as 'the Qur'an of the philosophers'.(Wisnovsky 2013)

2.5 Averroes

The next philosopher whose work on logic should be considered is Averroes, who was active in Cordoba in the 12th century. Averroes reacted against Avicenna by attempting to restore some of Aristotle's authority, and was sympathetic to Al-Farabi's objective of Lesser Harmony, which he wished to establish more rigorously. In his logical treatises, Averroes paid much attention to the syllogistic structures that he believed had been misinterpreted by Avicenna. (Wohlman 2008, Leaman 2013)

In a wish to continue the programme initiated by Al-Farabi and the Baghdad School, Averroes hoped to realise a return to Aristotelian conceptions of syllogistic inference. Primarily, however, he is known for his theory of double truth, which aimed to reconcile the reasoning of Aristotelian logic and syllogistic inference with the doctrines of Islam. Although he enjoyed quite some appreciation and support during his career, mainstream Arabic logic was never as heavily influenced by Averroes as it was by Avicenna. In fact, when Averroes wrote his philosophical works, Avicennan logic had already been institu-

tionalised in the so-called 'madrasa', the educational system at place in most of the Arab world. (Street 2001, Wohlman 2008)

There are two reasons why Averroes must still be taken into consideration in a broad historical overview of Arabic logic. First, he could be regarded as one of the last Arabic logicians who were determined to maintain a consistent relationship with Aristotle, and in this capacity Averroes represents the end of an important strand in the tradition of Arabic logic. Secondly, Averroes' works provoked critical responses from a variety of angles. On the one hand, there were the Avicennan logicians of the Arabic madrasa who believed Averroes' logical viewpoints to be reactionary or plainly incorrect. On the other hand, his writings were translated and made available at Western universities rather soon. His theory of double truth was applied to Christianity and became popular among so-called 'Latin Averroists', whose position was heavily criticised and condemned by the Catholic Church. (Leaman 2013)

2.6 The Maragha School

The last major moment in the development of Arabic logic is usually taken to be the so-called 'Maragha School', the members of which wrote some important treatises in the 13th century. The Maragha School consisted of scholars who had originally joined forces in order to set up an astronomical observatory, but later gained popularity and influence for their philosophical works as well. (Street 2001)

Important logicians belonging to the School were Abhari and Tusi. Both are known for their work on logical paradoxes and conceptions of truth and falsehood, which they approached with greater technical rigour than earlier logicians in the Arabic tradition. Abhari was the first Maragha School member to emphasise the relevance of these logical issues. The theory that he presented was commented upon by Tusi, who promoted a slightly modified form of Avicennan logic. Moreover, in a response to Abhari, Tusi proposed a highly sophisticated solution to the Liar Paradox. (Alwishah and Sanson 2009)

Apart from the technical sophistication of the formulated views, an important reason for taking the Maragha School into account is the impact that Abhari's and Tusi's writings have had on the solidification of the logical and philosophical curriculum of the madrasa system, which remained the dominant educational structure in the Arab world for several centuries, and in some regions up until today. (Nasr 2006) The School has therefore had a decisive influence on the tradition of Arabic logic.

In fact, as noted above, the Maragha School qualifies as the final turning point in the history of Arabic logic. This is not to say that any interest in logic disappeared after the 13th century, but that works stemming from this later period are not known to have posed serious challenges to the quickly dominant paradigm developed by the Maragha School. Some treatises proposing novel perspectives have been discovered, but the influence of such manuscripts seems to have been minimal and strictly regional. (Street 2001, Rescher 1964)

Hence, this historical overview is concluded here. We have identified the major moments and the most influential thinkers in the history of Arabic logic, and thus created a clear basis on which to evaluate what role dialetheist conceptions have played throughout its development.

3 Contradictions and dialetheism

Before investigating the extent to which the tradition of Arabic logic could be considered a dialetheist tradition, this chapter will set out to establish a clear understanding of what dialetheism actually is. This will be done by explicitly contrasting the position with the Principle of Non-Contradiction that has been so dominant in the Western tradition.

3.1 The Principle of Non-Contradiction

The Principle of Non-Contradiction (PNC) has been an essential cornerstone of Western logic and philosophy. It has been formulated in varying terms, but the version that will be most relevant to the course of this paper is the semantic one, with reference to truth and falsity. One of the first and most influential statements of the principle in such terms is usually thought to occur in Aristotle's *Metaphysics*: 'The most certain of all basic principles is that contradictory propositions are not true simultaneously.' (IV.6, 1011b13-14). PNC thus declares that contradictory statements cannot both be true at the same time: the truth of a statement and that of its negation cannot occur simultaneously. (Wedin 2004)

Aristotle called PNC 'the firmest principle of all', as he ultimately considered it to be an undeniable axiom on which all other truths depend. It is not possible to prove it by demonstration, as its acceptance is a prior requirement for engaging in any kind of rational argument. Hence, it serves as the most fundamental postulate for any logical or epistemic enterprise, and subscribing to the principle 'is necessary for anyone $(\cdots)$ who knows any of the things that are' (Lawson-Tancred n.d., IV.3, 1005b15). It is clear how, for Aristotle, rejection of PNC is equivalent to a dismissal of rational discourse, which would make it impossible to acquire any kind of knowledge. (Crivelli 2004) Many later philosophers have adhered to this conception by refusing to engage in debate on the principle.

Although there has been some discussion about the appropriate formalisation of PNC (Brady et al. 2004), is usually taken to be the syntactical formulation that most accurately captures the corresponding semantic principle in classical logic: the negation of a contradiction, i.e. a sentence of the form . Should a contradiction occur in a proof, or feature as an assumption, then any other proposition classically follows by the following inference:

1.	$\varphi \wedge \neg\varphi$	assumption
2.	φ	$1, \wedge_E$
3.	$\neg\varphi$	$1, \wedge_E$
4.	$\varphi \vee \psi$	$2, \vee_I$
5.	ψ	3, 4, disjunctive syllogism

The inference rule that follows from the above proof is known as Ex Falso Sequitur Quodlibet (EFSQ): $(\varphi \wedge \neg\varphi) \vdash \psi$. Thus, in classical logic, anything follows from a contradiction. This result implies an *explosion*, or *trivialisation*, of the system: everything can be proven. It is for this reason that inconsistency should at all times be avoided in systems of classical logic. Under no circumstances can a sentence be true and false at the same time: truth and falsity cannot coincide, so that true and false statements form mutually exclusive sets.

Its intuitive appeal, the influence of Aristotele's *Metaphysics* and the threat of EFSQ seem to explain PNC's dominance in the development of Western logic. It has been preserved and respected by the scholastic philosophers of the Middle Ages, and has not been subjected to explicit criticism by any influential logicians before the second half of the 20th century. Some have interpreted the writings of philosophers such as Hegel, Meinong and Marx as implying an implicit rejection of the principle, but it was not until Priest's advancement of a dialetheist logic in the 1970s that challenges to PNC gained a more pronounced relevance. (Martin 1993, Priest 2006)

3.2 Dialetheism

Rejection of PNC entails the recognition of sentences that may be true along with their negation. Such sentences are called 'dialetheias', and dialetheism is the view that accepts and asserts their existence. The fact that sentences φ may be true together with their negation $\neg\varphi$ opens the possibility of true contradictions: given that φ and $\neg\varphi$ are both true, the inconsistent conjunction $\varphi \wedge \neg\varphi$ is also true, which is classically impossible due to the fundamental status of . Maintaining that falsity is negated truth, dialetheism thus also implies that sentences can be true and false at the same time, and that truth and falsity are no longer mutually exclusive. (Priest 1979, Priest and Routley 1989)

Dialetheism has found an important contemporary advocate in Priest, whose Logic of Paradox (LP) stems from the conviction that certain contradictory, paradoxical statements

are best taken to be true. Therefore, the semantics of LP is constructed in such a way as to accommodate true contradictions. Because the existence of true contradictions is equivalent to the possibility of simultaneous truth and falsity, the truth values of LP do not only include 0 (falsity) and 1 (truth), but also the third value both, which is a truth glut assigned to sentences that are both true and false. The inclusion and particular ('glutty') interpretation of the third truth value both in the LP semantics render the system dialetheist. (Priest 1979)

The main motivation for Priest's dialetheism is the existence of paradoxes such as the notorious Liar Sentence, which is most popularly stated as 'This sentence is not true'. Paradoxes of this kind have been addressed by innumerable philosophers throughout history, but according to Priest, a satisfactory resolution was never found. He believes that the unquestionable adherence to PNC is an important obstacle blocking the appropriate treatment of paradoxes. According to Priest, the countless failed attempts to overcome paradoxes indicate that something may be wrong with the underlying truth conception: 'Suppose we stop banging our heads against a brick wall trying to find a solution, and accept the paradoxes as brute facts.' (Priest 1979, 220) Hence, Priest believes, the best approach to paradoxes is not to make them disappear, but to simply accept the fact that certain sentences, such as the Liar, are true and false at the same time. Dialetheism thus facilitates a philosophy that is tolerant towards paradoxes.

An important threat to any dialetheist logic, however, concerns EFSQ. Obviously, no dialetheist would wish to see the logic trivialise, which is a risk as soon as one accepts that sentences and their negations may be true simultaneously, by the inference shown in the previous section. What is needed is a logic that tolerates contradictions, without risking explosion. Such logics are called 'paraconsistent', and block EFSQ as an inference rule. Returning to LP, this logic is paraconsistent because it takes the designated truth values to be 1 *and* both. As true contradictions take the value both, EFSQ no longer holds, and the system does not explode. Take, as a counterexample, the valuation in which the contradiction $\varphi \wedge \neg\varphi$ is both true and false, so that φ and $\neg\varphi$ are both assigned designated truth value both. Then let another sentence ψ be false and it follows that $(\varphi \wedge \neg\varphi) \models \psi$ does not hold for LP: the premises have a designated truth value, but the conclusion does not. (Priest 1979, Priest and Routley 1989)

Although Priest's paradox-based justification of dialetheism is a famous one, the position as such does not require any specific grounds. In order for someone to be a dialetheist,

the only necessary condition is that one's semantics include the possibility of true contradicitons, or equivalently, simultaneous truth and falsity, and the assignment of such a status to at least one actual sentence.

4 Dialetheism in Arabic logic

In the first chapter, a historical overview was provided in order to establish the major hinges and turning points in the development of Arabic logic. The second chapter clarified the notion of dialetheism, so that it is now possible to present a grounded analysis of the role this concept has played in the Arabic logical tradition. This will be the main objective of the third chapter. Before starting the actual analysis, however, some notes have to be made in order to resolve issues that may arise with respect to the translatability of dialetheism, and its applicability to the non-occidental discourse of Arabic logic.

4.1 Translation

Despite its controversial nature, what must be appreciated about dialetheism as understood in the above is that it was itself developed in a theoretical framework that is in fact occidental. This raises the question if an application of the term to a deviant, non-Western tradition is justified, and if so, if there are any particular conditions that must be satisfied.

First, there are the general notions of truth and falsity whose translatability must be assessed. Although there may be some initial worries about possible semantic incompatibility of different logical discourses, truth and falsity generally do not appear to constitute major problems when it comes to their application in non-occidental contexts. For the Arabic tradition in particular, a division between truth and falsity is at place that displays a sufficient parallel with the relevant Western counterparts in order for the notions' application to be defensible. In fact, as described in the first chapter, it may not even be appropriate to regard Arabic and Western logic as definitely autonomous in this respect, as logicians in the Arab world were greatly indebted to Aristotle in the construction of their logical vocabulary, which included the elementary notions of truth and falsity. Obviously, differences may exist when it comes to the kind of entities that are considered truth-apt, but such debates on truth-bearers also exist within traditions and should therefore entail no complications here. (Street 2001)

Secondly, with regard to dialetheism specifically, one must be careful not to confuse

the view's technical aspects with the controversial status that it is commonly assigned in the context of Western logic. Even though dialetheist positions have been rare and generally unsuccessful in the history of occidental logic, the conception need not be that revolutionary in other intellectual traditions. In other words, connotation and reception should not block the way towards an accurate analysis of the view's position in other traditions, such as the Arabic one. Without anticipating any of the results of the upcoming sections, it is important to bear in mind that nothing prevents dialetheism from being an acceptable, or even a basic semantic scheme in contexts that have not been exposed to Western (pre)judgments. Here, it is worth referring to some research done by Priest on traditional Indian logic. The semantics of this logic, as it turns out, is primarily based on the so-called *Catuskoti principle*, which states that sentences may be true, false, neither or both, and all of these four possible truth values are actually assigned to non-empty sets of sentences. (Priest 2010) Thus, Indian logic contains a dialetheist component (acceptance and application of a truth glut) by default. This illustrates that dialetheism need not be an exotic variation on existing semantic set-ups, and that a translation of the notion should therefore not be biased by its provocative reputation in Western philosophy.

Now that these methodological cautions have been taken into account, the actual analysis of Arabic logic in terms of dialetheist truth conceptions can start. The approach will be as follows. The key moments identified in the first chapter will be revisited, and studied in terms of their perspective on logical truth. Whenever possible, particular attention will be paid to the treatment of paradoxes. Thus, by addressing representative instances in the development of Arabic logic, it will be possible to assess the extent to which dialetheism has determined the tradition as a whole.

4.2 Early period

As described in the first chapter, the period in which the Arabic interest in logic started to take shape primarily comprised the translation of Aristotelian texts and the formation of a basic, shared vocabulary. Although it would not yet be appropriate to speak of distinct systems of Arabic logic at this stage, there seem to be two hypothetical statements to make about this preliminary phase and the influence it may have had on the truth conceptions of later generations.

First, an important issue concerns the partial availability of the Aristotelian corpus. A significant number of texts was known and translated into Arabic, but some writings

of great logical importance were probably not known to Arabic scholars of this early generation. Most importantly, the *Metaphysics* seems to have been unavailable, which may have set the tone for a tradition resting on other principles than the ones Aristotle so famously articulated in this work. (Wedin 2004, Black 1990) Of course, what is most relevant to this paper is the fact that PNC has found its most influential formulation in the Metaphysics, as explained in the preceding chapter. As the early generations of Arabic logicians mainly relied on Aristotle in the construction of a basic logical framework, the fact that they probably did not have access to this text may justify the belief that PNC did not enjoy the firmly established status that Aristotle claims it should have.[1] The foundational texts that Arabic scholars knew and translated simply did not include any explicit statement of PNC as emphatic and definite as the one in the *Metaphysics*, which may, speculatively, have led to a less resistant attitude towards true contradictions.

A second consideration concerns religion. Although in the beginning of the first chapter, it was stressed that Islamic and Arabic logic are not the same thing, it must not be neglected that all Arabic logicians were in fact Islamic, too. Actually, the relative centralisation, linguistic unity and perhaps even the identification of such a thing as the 'Arab world' are all dependent on Islamic religion to a considerable degree. It was after decades of religious wars that the Abbasid Caliphate came into existence in the 8th century, and the intellectual climate started flourishing. In turn, this intellectual development was also coloured by religious interests, and mainly took place at universities whose function was primarily theological. (Rescher 1964, Street 2001, Nasr 2006) This is a sociological fact that should not be lost sight of. Recognising that Arabic logicians participated in this intensely religious society, and that they probably knew the Qur'an by heart before starting a career, it is not hard to imagine how this may have impacted their intuitions concerning truth and falsity. The Qur'an, as most other religious revelations, contains contradictory statements, e.g.: 'Ye have not, beside Him, a protecting friend or mediator' (32:4) versus 'The Believers, men and women, are protectors one of another' (9:71). Given some reasonable assumptions, these two fragments are in contradiction, yet, as Islam teaches that the Qur'an contains only truths, the conjunction cannot be evaluated as false. Appreciating that Arabic scholars operated within an intellectual framework that assigned first priority to the Qur'an, it thus makes sense to assume that true contradictions were not

[1] One may contend that this assumes the stability of PNC in the Western tradition to be, above all, an Aristotelian legacy. Yet, also without making such far-reaching suppositions, it seems worthwhile to stress that the Arabs did not have access to a clear statement of the principle in the available texts of the philosopher who enjoyed most authority in this early period, namely Aristotle.

regarded as very objectionable at all. Obviously, this condition should not be regarded as a sufficient explanation of possible later occurrences of dialetheism, but it may be a background consideration to bear in mind.

4.3 The Baghdad School

After the preliminary stage, the next key moment identified in the first chapter was the Baghdad School. Focusing on logic, Al-Farabi and Al-Baghdadi were the two main exponents of this 10th century intellectual movement. Both logicians took a stance on the non-exclusive relation between truth and falsity, thus influencing later generations.

When it comes to Al-Farabi, it is interesting to first pay attention to the role contradictions tend to play in his argumentations. Reading Al-Farabi, one is struck by what seem to be inconsistencies in his texts - between works, between fragments or sometimes even within sentences. Scholars have spoken of Al-Farabi's 'brazen commission of self-contradiction' (Mahdi 1970a), and attempted to justify this predicament as a rhetorical device: 'Al-Farabi enables himself simultaneously to embrace and to deny what is 'respectable'; or he can make himself appear to 'change his mind' back and forth; or he can take the sting out of the shocking by denying it, before or after having affirmed it.' (Idem) Projects aiming at a fully rhetorical interpretation of Al-Farabi's self-contradictions, however, do not seem to have an explanatory power covering the multitude and frequency of all such instances. The rate at which Al-Farabi engages in self-contradiction, in addition to his generally unpretentious, scientific tone of voice, suggests that he may have been more serious in his subsequent expression of contradictory statements than PNC would allow. There is an obvious sense in which Western readers may be tempted to reduce Al-Farabi's self-contradictions to tools of rhetoric, but an interpretation that takes a possible, underlying dialetheism seriously is not completely implausible.

Next, Al-Farabi generally seems to be more interested in the notion of oppositions than in that of contradictions. (Chatti 2012) Logical oppositions are pairs of statements that are semantically conflicting in some sense. The three classical, Aristotelian kinds of oppositions are contraries, subcontraries and contradictions. Contraries are of the form 'All S are P' versus 'All S are not-P', subcontraries have the structure 'Some S are P' versus 'Some S are not-P', and contradictions are as explained in the second chapter. For Al-Farabi, not only contradictions, but also contraries and subcontraries are instances of negation of a sentence. (Abed 1991) Uncontroversially, Al-Farabi believed that contraries

can be false at the same time, and subcontraries can be true at the same time. Thus, for the general class of logical oppositions, it is not possible to state that either one of the two opposing sentences must be true, and the other false. For Al-Farabi, this also implies that it is not necessarily the case that a sentence is true when its negation is false, as all oppositions are treated as kinds of negations. Thus, $j\neg(\varphi \wedge \neg\varphi)$ fails, which could be regarded as an implicitly dialetheist corollary.

Another dialetheist aspect in Al-Farabi's thinking becomes clear when his views on future contingency are investigated. The correct interpretation of the relevant passages has been subject to debate, largely because of the following fragment, which appears to argue that the truth value of future contingency statements is an indefinite superposition of truth and falsity:

> [I]n the case of the equally possible [the truth] is completely indefinite, and truth and falsehood are by chance. But in the case of the possible that happens more often, the truth of one of the antitheses regarding it is more appropriate than its falsehood. But in the case of what happens less often, its falsehood is more appropriate than [its] truth. Likewise, opposed beliefs about the types of existing things are in the same state regarding truth and falsehood. (Commentary 97.7-15, trans. in Michael E Marmura and Zimmermann 1983, 68)

Al-Farabi speaks of 'appropriate' truth and falsity, and 'opposed beliefs about the types of existing things', including the things existent in future, which 'are in *the same state* regarding truth and falsehood'. Appropriate truth depends on an event's likelihood, which is always paired with an extent of appropriate falsity. According to some scholars, Al-Farabi is actually aiming at a conception of truth in degrees here (Adamson 2006, Lameer 1994), but what matters for this paper's argument is that sentences on the future contingent are stated to be in an identical position regarding both truth and falsity. It seems that, for future contingency statements, truth and falsity are in a superposition that collapses into one definite truth value once the relevant future moment has been reached. Hence, future contingency is semantically characterised by a truth distribution, which, according to probability, assigns both truth and falsity to single statements. Obviously, this reading does not admit the assignment of full truth as well as full falsity to single sentences, but it is clearly the case that *partial* truth and falsity can occur simultaneously,

which qualifies as a form of *dialetheism*.[2]

Whereas Al-Farabi's dialetheism is still somewhat implicit, Al-Baghdadi articulates the position very clearly in his *Principles of Religion*, where he discusses the Liar Paradox. He is the only Baghdad School member who is known to have studied the paradox, and given the intellectual coherence of the movement, it is not unlikely that his proposed solution enjoyed the other members' support as well. In fact, Al-Baghdadi is one of the first Arabic logicians to approach the paradox in a form that is strongly reminiscent of its typical Western formulation, which justifies that we refer to it as the Liar Paradox. Remarkably, there is no evidence of any contact between Arabic and Western philosophers about the Liar at this stage, so that the issue is commonly supposed to have arisen in both traditions independently: '[e]ach tradition reads as though it invented the paradox anew' (Alwishah and Sanson 2009, 99). Although Al-Baghdadi is not the one who 'invented' the paradox in the Arabic tradition, he is probably the first to suggest a technical and systematic resolution, the crux of which is quoted below:

> There is no declarative sentence that is both true and false together, except one: namely, the declaration by he who has not lied at all, about himself, that he is a liar, and this declarative sentence, from him, is false. And a liar who declares that he is a liar says the truth. And therefore this one declarative sentence is true and false, and it has one subject. (trans. in Alwishah and Sanson 2009, 101)

For Al-Baghdadi, declarative sentences are generally either true or false. The Liar is presented as an exception to this bivalence, for it resists valuation as exclusively true or false. The paradox in Al-Baghdadi's words, 'the declaration by he who has not lied at all, about himself, that he is a liar', is first argued to be false, but as falsity of the declaration renders its utterer a liar, it is consequently evaluated as a true sentence as well. Therefore, exclusive bivalence must be rejected in favour of dialetheism, which grants simultaneous truth and falsity to the Liar Paradox. In other words, Al-Baghdadi's view on logical truth is that declarative sentences generally obey exclusive bivalence, but that the convoluted semantics of a Liar sentence necessitate that this scheme be applied more leniently in such extremely paradoxical cases. Thus, truth and falsity are not mutually exclusive -

[2] This view is very similar to the notion of *subvaluation* in the context of Priest's LP, according to which statements involving vague predicates are assigned simultaneous truth and falsity by lack of a precisification that allows the truth value to collapse into exclusive truth or falsity.

their intersection is non-empty, as this is the domain of the Liar. This approach to the paradox has been much cited by later logicians in the Arab world. It is intriguing to see that Al-Baghdadi's 10th century adoption of a dialetheist truth conception is motivated by exactly those reflections on logical paradoxes that induced Priest's LP a millennium afterwards.

4.4 Avicenna

As explained before, no research engaging with the development of Arabic logic could possibly ignore the works of Avicenna. Reacting against the Baghdad School, which still aspired to be prevailingly Aristotelian in spirit, Avicenna introduced a novel and sophisticated modalisation of existent logical structures, with implications that reach far beyond the scope of this paper. What matters here are the consequences that Avicenna's heavily modal perspective on logic has had for his notion of truth. In order to comprehend these consequences, it is key to first clarify the basic understanding of modality that is in place. Citing Avicenna:

> That which is existent as of now does not deny the non-existent in a second moment, let alone that which necessitates neither existence nor non-existence. It is not the case that if a thing is moving as of now that it is impossible that it not move in the future, let alone that it be non-necessary for it to move and not move at every moment in the future. (trans. in Inati 1981, 277)

In this fragment, Avicenna underlines the modal qualities of entities that change, become, move and disappear. The world and the objects of which we speak are recognised as intensely modal, and it is this intuition that guides Avicenna's later constraints on the logical structures that are capable of adequately expressing such modality. (Lagerlund 2009) This is also the reason for Avicenna to maintain a vital distinction between *absolute* and *modal* propositions. Absolute propositions, on the one hand, have not been modally quantified in terms of time or necessity/possibility. Modal propositions, on the other hand, have been specified with respect to the relevant modalities. The relevant Avicennan modalities are alethic (with respect to necessity and possibility), and temporal (with respect to moments in time). (Ahmed 2010, Thom 2008)

For Avicenna, a syllogistic system containing only absolute propositions would be very unattractive. Alethic and temporal quantification are the formal components that provide

a suitable structure to sentences intended to convey non-trivial information, which explains their central position in Avicennna's logical treatises.[3] Indeed, appropriate modalisation of absolute propositions appears to be a necessary condition for the validity of certain logical principles, such as exclusive bivalence:

> The situations of the contradictory [propositions] in their division of truth and falsehood among themselves should not be the same in every case. For the truth of the quantified [propositions] is determined in virtue of the essence of the proposition and of the nature of the actual state of affairs. Similarly, for the singular temporal propositions which concern the past and the present, the time which obtained has of necessity made one of the two things [i.e. truth and falsehood] corresponding to the actual state of affairs. (trans. in M. Marmura 2005, 70)

Hence, according to Avicenna, 'division of truth and falsehood' between contradictory propositions depends on the nature of the propositions in question. Quantified propositions, according to alethic 'essence' or temporal state, are true or false in virtue of actual states of affairs. This is possible because their logical structure is sufficiently modalised in order for a correspondence to worldly states of affairs to be possible at all. For absolute propositions, this is generally not the case. An example that returns throughout the history of Arabic logic, and that is also mentioned by Avicenna, is the sentence 'zaydun jaalisun' ('Zayd [is] sitting'). In its current form, it is an absolute proposition, for the Arabic sentence contains no temporally specified copula. It is true when Zayd is indeed sitting, but when he stands up after some time it is false. Yet, as the sentence itself contains no temporal component, its valuation cannot be time-specific either.[4] Only 'the time which obtained has of necessity made one of the two things [i.e. truth and falsehood]

[3] Possibly, what could have contributed to Avicenna's emphasis on modalisation with respect to time is the fact that the Arabic language, in which Avicenna wrote his works, is a so-called 'zero copula' language: it does not require subject and predicate terms to be explicitly connected by a form of the verb 'to be'. As it is usually by means of verb tenses that the time-context of a sentence is inferred, Avicenna may have been unhappy with the temporally indeterminate status of certain Arabic sentences, which thus prompted him to require such modalisation. Yet, as it is grammatically and pragmatically legitimate for Arabic sentences not to be specified in this way, a lack of modalisation is not identical to a lack of contextualisation.

[4] Of course, one could resort to a Fregean approach according to which the (state of the) world must be included in the evaluation of a sentence, so that a lack of temporal specification could be resolved by equating the time of evaluation with that of expression. Avicenna, however, does not seem to have taken such options into account. Absolute propositions really are devoid of temporal aspects, and even if their context is not, Avicenna does not suggest that this should add any implicit time-specification to the evaluation of the sentence.

corresponding to the actual state of affairs', but while this time has not been made explicit in the sentence, both things obtain: truth as well as falsehood. (Street 2000, Street 2002)

Thus, for Avicenna, bivalence requires appropriate modalisation. Absolute propositions may be true and false in different alethic or temporal respects, but as they contain no reference to such modalities themselves, their valuation necessarily conjoins truth and falsity.[5] This is where Avicenna's dialetheism arises: it is modal specification that makes a proposition's truth value collapse from a semantic superposition into either truth or falsity, but in its absolute form, a proposition may be true along with its negation.

4.5 Averroes

The first chapter already presented Averroes as the philosopher who introduced the notorious doctrine of double truth, which did not have as much impact on the development of Arabic logic as the views discussed so far, but which caused such controversy that discussing it is still significant. Not only among Arabic intellectuals, but also in the Latin West were Averroes' thoughts the subject of a storm of criticism, commotion and religious concern. Yet, whereas Arabic scholars tended to deem Averroes' philosophy as old-fashioned in its refusal to accept Avicennan logic, the critical Western philosophers and theologians generally regarded it as a provocative and exotic recency. (Wohlman 2008)

Essentially, Averroes' doctrine of double truth is a simple one. It states that there are two equally legitimate ways to attain knowledge. There is religious revelation, which Averroes himself saw epitomised by the Qur'an, and there is philosophical reasoning, which rational human beings can pursue independently. Both methods will lead to truth, yet, the resulting truths need not be in agreement. Given a truth attained by philosophical reflection, Averroes argues:

> (· · ·) either the revealed text has nothing to say about the subject in question, or it makes a statement. In the first case, no contradiction is possible; in the second, here are two possible results: that the plain sense of the Qur'anic statement agrees with the result of demonstration, or they are contradictory. (trans. in Van Den Bergh 2008, 31)

Apparently, Averroes is not troubled by the conclusion that independently justified truths may be contradictory. This is simply one out of two options once the Qur'an turns

[5] Note that this is somewhat reminiscent of Al-Farabi's treatment of future contingency statements.

out to make a statement about 'the subject in question'. The equal epistemic reliability of both human reason and religious revelation appears to weigh more heavily than the risk of inconsistency: contradictory statements may arise, but as there is no reason to doubt either one, the conjunction must also be accepted as a truth. Clearly, Averroes is a dialetheist. Another striking passage that makes this even more evident:

> (· · ·) God's knowledge cannot be divided into the opposites of true and false (· · ·) both propositions, that He knows what He knows and that He does not know it, are true (· · ·) (trans. in Van Den Bergh 2008, 33-34)

Not only when humans combine epistemic resources may contradictions arise, but also within the knowledge body of God. This serves to explain that certain truths of reasoning may contradict Qur'anic truths, for Averroes' God tolerates contradictions. Once this view had made its way to Western universities in a Christianised form, it became known as 'Latin Averroism'. Although it initially enjoyed some popularity, it was soon condemned on multiple occasions towards the end of the 13th century. The Catholic Church listed Averroism as a heretical position, and some of its exponents actually faced the death penalty.(Leaman 2013) Speculatively, one may wonder to what extent their deaths could be regarded as a result of conflicting truth conceptions: the Western PNC versus Averroist dialetheism.

Although it is clear at this point that Averroes' dialetheism is not exceptional in the context of Arabic logic, what renders his position somewhat unique is its justification. So far, all occurrences of dialetheism investigated in this paper were motivated by logical, and usually strictly semantic considerations. With Averroes, however, we encounter a dialetheism that is driven by epistemological and religious arguments.

4.6 The Maragha School

The final focus of this chapter will be on the 13th century Maragha School. Abhari and Tusi were mentioned as the most famous logicians participating in this intellectual movement. Their treatises contain some noteworthy logical arguments, the most interesting of which expand on the Liar Paradox.

Starting with Abhari, what perhaps first deserves attention is his new formulation of the Liar: 'All that I say at this moment is false'. For the sentence to be paradoxical, it must be assumed that there is someone expressing, or more precisely, saying it. Below,

the passage is quoted in which Abhari struggles with the paradox, and eventually presents his solution:

> One of the difficult fallacies is the conjunction of the two contradictories when someone says, 'All that I say at this moment is false'. This sentence is either true or false. If it is true, then it must be true and false. And if it is not true, then it is necessary that one of his sentences at this moment is true, as long as he utters something. But, he says nothing at this moment other than this sentence. Thus, this sentence is necessarily true and false. (Alwishah and Sanson 2009, 125)

As seen before with Al-Baghdadi, the argument starts with the bivalent premise that, generally, sentences are 'either true or false'. This supposition then turns out to be challenged by the Liar, which generates a sentence to which exclusively bivalent semantics do not seem to apply. Assuming that 'this moment' in Abhari's Liar only encompasses the moment in which the sentence is uttered, as Abhari does himself, the paradox effectively reduces to the classical formulation 'This sentence is false'. Abhari first derives simultaneous truth and falsity from the assumption that the sentence is true, and afterwards reasons towards the same result upon assuming its falsity. Either way, the paradox can only be evaluated as true and false at the same time, which is, again, a dialetheist conclusion.

Tusi, then, draws the same dialetheist conclusion on the paradox as Abhari, but adds an extra layer of sophistication to his analysis. He is the first logician in the Arabic tradition to recognise that the Liar is in fact a paradox of self-reference. To articulate this insight, Tusi distinguishes between declarative sentences and objects of declaration, and states that the Liar emerges because it makes the declaration coincide with its object: it is a sentence declaring something about itself. How this Tarskian understanding subsequently leads Tusi to a dialetheist perspective becomes clear in the following citation:

> Thus, the following paradox can be generated: The first declarative sentence ['This sentence is false'], which is a declaration about itself, namely that it is false, is either false or true. If it is true, then it must be false, because it declares itself to be false. If it is false, then it must be true, because if it is said falsely, then it will become true $(\cdots)$ [I]f it is false, then what it declares about itself, namely that it is false, is true at the same time. (trans. in Alwishah and Sanson 2009, 125-26)

By discriminating the hierarchy of the declaration and its object, Tusi identifies the instability caused by a self-referential sentence such as the Liar Paradox. He does not, however, proceed by imposing any constraints on the admissible objects of declarative sentences. Hence, Tusi seems to recognise that self-reference is a phenomenon of natural language, which must not be avoided in order to prevent cumbersome logical paradoxes, but which comes with certain implications that one cannot but accept. The most important such implication is that some (self-referential) sentences are both true and false: true contradictions exist.

5 Conclusion

Before answering the main question motivating this paper, let us briefly recall the findings of the preceding chapters. The initial chapter aimed to identify the most representative moments in the development of Arabic logic, which turned out to be, in chronological order, an early, preliminary stage, the Baghdad School, Avicenna, Averroes and finally the Maragha School. These are the movements and thinkers whose thoughts on logic have proven most influential and constitutive to the Arabic discourse. Consequently, the second chapter explained dialetheism as the belief that sentences can be true and false simultaneously, which is not possible if one adheres to PNC, a principle that has been dominant in Western logic ever since Aristotle.

In a sense, the first two chapters were only preliminary to the work done in the third, where the major moments in the history of Arabic logic were analysed in terms of the applied conceptions of logical truth. More specifically, this chapter investigated how pervasive dialetheism has actually been in the tradition as a whole. As it turned out, the theories of all studied logicians contained more or less explicitly dialetheist ideas. Starting with the early period of Arabic logic, it was noted that the unavailability of Aristotle's *Metaphysics*, where PNC is influentially pronounced, in addition to religious concerns may speculatively have created a condition that was rather tolerant towards dialetheism. When the Baghdad School was considered, it became clear that Al-Farabi's argumentative style, as well as his views on logical oppositions and future contingency, may best be interpreted from a dialetheist perspective. Al-Baghdadi even explicitly admitted that the Liar must be true and false at the same time. Next, Avicenna turned out to postulate simultaneous truth and falsity for insufficiently modalised, absolute propositions, and few arguments were required for demonstrating the epistemologically grounded dialetheism of Averroes' doctrine of double truth. Finally, Maragha School members Abhari and Tusi both endorsed overtly dialetheist solutions to the Liar Paradox.

This analysis seems to justify the conclusion that dialetheism is indeed a structuring and determining aspect in the tradition of Arabic logic. Of course, one may argue that the examination of a limited number of cases does not legitimise a conclusion of such generality, but given the careful historical preparations made in the first chapter, it should at least be admitted that dialetheism has been an important thread throughout the tradition, with unequivocal manifestations in the works of those logicians who have been most influential. Perhaps it deserves more emphasis that the Maragha School members effectively shaped

the curriculum of the madrasa system, which remained in place for centuries. Considering that Abhari and Tusi presented an explicit dialetheism, the position is likely to have been textbook material in the Arab world for a long time.

Although it is tempting to extrapolate the paper's conclusions, and state that the identified strand of dialetheism explains the divergence between Arabic and Western logic, it also seems that more research has to be done first. A more in-depth, comparative study into the prevalence and importance of PNC in Western logic versus the acceptance of simultaneous truth and falsity by the Arabs may be necessary in order to assess if this really is the one main aspect in which the deviance between both discourses originates. Moreover, an obvious way to continue this paper's research would be to evaluate the implications of the observed dialetheist truth conceptions for Arabic logics more generally. In the second chapter, the risk of trivialisation was underlined, so it would be worthwhile to investigate if Arabic logicians took this into account, and whether they took measures to assure paraconsistency. Hopefully, this paper will prove useful as a starting point for future research addressing these and similar issues.

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