INSA Toulouse

Software-Defined Radio - Report

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1. Introduction

During these lab, the objective is to learn how to use a Universal Software Radio Peripheral (USRP). USRP is a tool for radio communication. This communication is wireless and therefore works through electromagnetic waves. So we can work on three parameters : frequency (which is fixed here), amplitude and phase.

As we are working with phase and amplitude on this report, most of the calculation will implies complex numbers in the following form :

$$re^{i\phi} = r(cos(\phi) + i\,sin(\phi))$$

Data is modulated in carrier signals, we can express it the following way :

$$s_{RF}(t) = A(t)cos(2\pi f_0 t + \phi(t)$$

$$= A(t)cos(2\pi f_0 t) * cos(\phi(t) - A\,sin(2\pi f_0 t) * sin(\phi(t))$$

$$s_R(t) = A(t)cos(\phi(t))$$

$$s_I(t) = A(t)cos(\phi(t))$$

so :

$$s_{RF}(t) = s_R(t) * cos(2\pi * f_0 t) - s_I * sin((2\pi f_0 t)(2)$$

2. Part 1 : Presentation of the acquisition device - USRP

2.1 Description

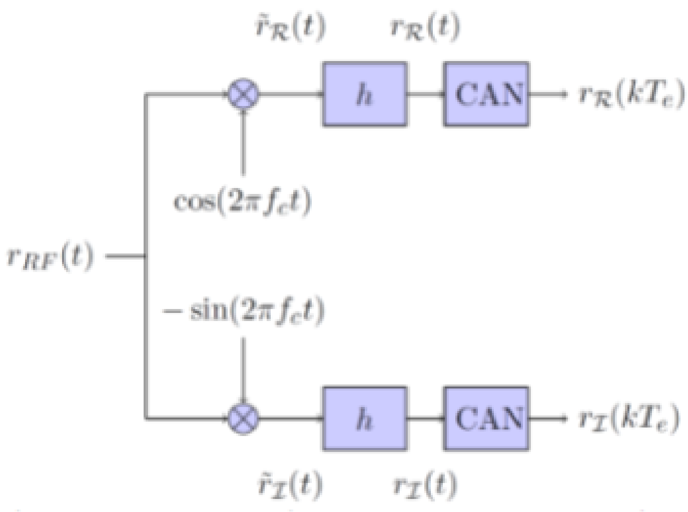

Figure 1: Block schema

As we can see in this figure and the previous calculation, the first objective of this lab will be to segregate the real and imaginary figure so that we can treat the whole signal easily. Then, we will filter the high frequencies and convert the signal to a digital form to be able to treat it with a computer.

2.2 Theory

We assume that received and transmitted signals are similar, we can note :

$$r_{RF}(t) = s_{RF}(t)$$

So, we have :

$$\tilde{r}_R(t) = [s_R * cos(2\pi f_0 t) - s_I(t) * sin(2\pi f_0 t)]cos((2\pi f_0 t)$$
$$\tilde{r}_I(t) = -[s_R * cos(2\pi f_0 t) - s_I(t) * sin(2\pi f_0 t)]sin((2\pi f_0 t)$$

$$\tilde{r}_R(t) = \frac{s_R(t)}{2}[cos(2\pi(f_0 + f_c)t) + cos(2\pi(f_0 - f_c t)] - \frac{s_I(t)}{2}[sin(2\pi(f_0 + f_c)t) + sin(2\pi(f_0 - f_c)t)]$$
$$\tilde{r}_I(t) = \frac{s_I(t)}{2}[cos(2\pi(f_0 - f_c)t) - cos(2\pi(f_0 + f_c t)] - \frac{s_R(t)}{2}[sin(2\pi(f_0 + f_c)t) - sin(2\pi(f_0 - f_c)t)]$$

We consider that $f_c = f_0$ to bring back the signal to the base.

We then have,

$$\tilde{r}_R(t) = \frac{s_R(t)}{2}[cos(4\pi f_0 t) + 1] - \frac{s_I(t)}{2}sin(4\pi f_0 t)$$
$$\tilde{r}_I(t) = \frac{s_I(t)}{2}[1 - cos(4\pi f_0 t] - \frac{s_R(t)}{2}[sin(4\pi f_0 t)]$$

After Fourier transform :

$$\tilde{R}_R(f) = S_R * \frac{1}{2}\delta(f) * [\delta(f) + \frac{1}{2}[\delta(f + 2f_0) + \delta(f - 2f_0)]] - S_I(f) * \frac{1}{2}\delta(f) * [\frac{j}{2}[\delta(f + 2f_0) - \delta(f - 2f_0)]]$$

$$\tilde{R}_I(f) = S_I * \frac{1}{2}\delta(f) * [\delta(f) - \frac{1}{2}[\delta(f + 2f_0) - \delta(f - 2f_0)]] - S_R(f) * \frac{1}{2}\delta(f) * [\frac{j}{2}[\delta(f + 2f_0) - \delta(f - 2f_0)]]$$

$$\tilde{R}_R(f) = \frac{1}{4}[2S_R(f) + S_R(f - 2f_0) + S_R(f + 2f_0) + jS_I(f - 2f_0) - jS_I(f + 2f_0)$$
$$\tilde{R}_I(f) = \frac{1}{4}[2S_I(f) - S_I(f - 2f_0) - S_I(f + 2f_0) + jS_R(f - 2f_0) - jS_R(f + 2f_0)$$

The characteristics of h filter with this hypothesis are then :
Gain : $|H(f)| = 2$
Bandwidth : $\frac{BW}{2} \leq f_{cut} \leq 2f_0 - \frac{BW}{2}$

The gain has to be 2 because with this manipulation the amplitude of the signal is divided by 2. The choice of this bandwidth allows us to ignore the signal which was present around f_0 before bringing it back to base.

Since the beginning, we work with the hypothesis of Narrow Band : $f_0 \leq \frac{BW}{2}$
Because if band is too wide, ($f_0 \leq \frac{BW}{2}$), there will be overlap. It is therefore impossible to design a correct filter because there are overlays in frequencies. Numerically it is possible, but with a non-causal filter, that we will not design in this lab.

In order to recover the real signal from the sampled signal, the sampling period has to follow Shannon - Nyquist theorem :

$$F_e \geq 2f_{max}$$
$$F_e \geq 2 * \frac{BW}{2}$$
$$F_e \geq BW$$

So,

$$T_e \geq \frac{1}{BW}$$

It is interesting to wonder if we can interchange the stages of frequency transposition and analog to digital conversion. But we can easily realise that if we put the transposition after the ADC it will be more expensive because you need to convert high frequencies which are not necessary. In fact, ADC are very expensive for GHz (radio). It can also lead to possible errors after the conversion, due to the accuracy and the number of levels for digital conversion. That's why we choose the place the ADC after the transposition.

The positive analytic signal associated to the real narrow-band signal needs to have an amplitude that is twice as important so that it conserve all its power. The complex envelope is then obtained after re-centring the analytic signal. So from the expression of the real narrow-band signal we can determine the expressions of complex envelope and positive analytic signals.
We consider that the expression of the real-narrow band is :

$$S_{RF}(t) = s_R(t)cos(2\pi f_0 t) - s_I(t)sin(2\pi f_0 t), \forall t \in \mathbb{R}$$

Applying Fourier transform, we have:

$$S_{RF}(t) = \frac{S_R(f)}{2}[\delta(f - f_0) + \delta(f + f_0)] - \frac{S_I(f)}{2j}[...])$$

$$S_{RF}(f) = \frac{1}{2}[S_R(f - f_0) + S_R(f + f_0) + jS_I(f - f_0) - jS_I(f + f_0)]$$

The analytic signal is:

$$S_a(f) = \begin{cases} 2S_{RF}(f), \forall f \geq 0, \\ 0, \forall f < 0 \end{cases}$$

$$\iff S_a(f) = S_R(f - f_0) + jS_I(f - f_0) = [S_R(f) + jS_I(f)] * \delta(f - f_0), \forall f > 0$$
$$s_A(t) = (s_R(t) + j\, S_I(t))\, e^{j2\pi f_0 t}$$

$$S(f) = S_a(f + f_0) = S_R + jS_I(f)$$
$$s(t) = s_R(t) + j\, s_I(t)$$

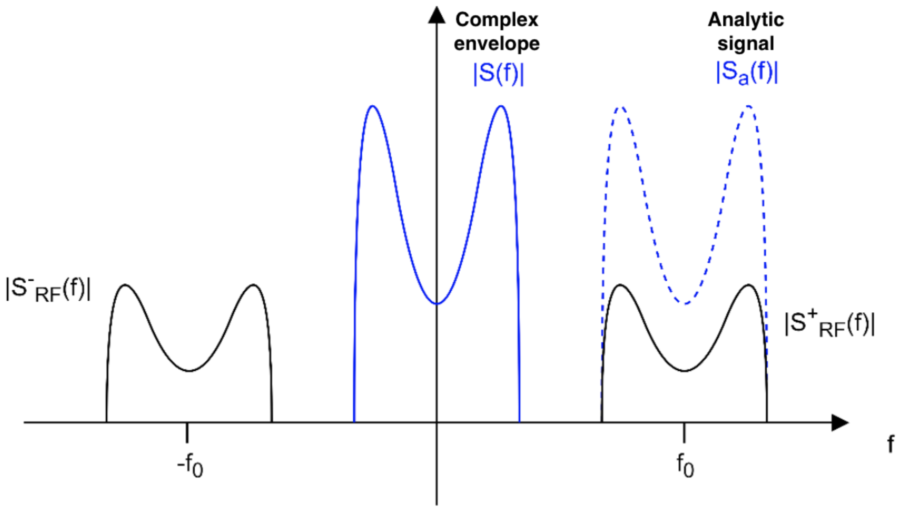

Figure 2: Complex signal analysis

3. Part 2 : Reception of frequency modulation (FM) broadcasting

In the first part, we introduced all the theoretical and mathematical aspects. Now, in this second part, we are going to implement receivers thanks to GNURadio SDR software. This software allows us to receive or read signals and analyse their frequency characteristics precisely.

Firstly, we will analysed a recording, then we will proceed a channel extraction and we will start a frequency demodulation and restitution. Finally, we didn't had time to try a real time implementation, but we have seen it through other students experimentation.

3.1 Analysis of the recording

Firstly, we analysed a recording thanks to the chain presented in figure below.

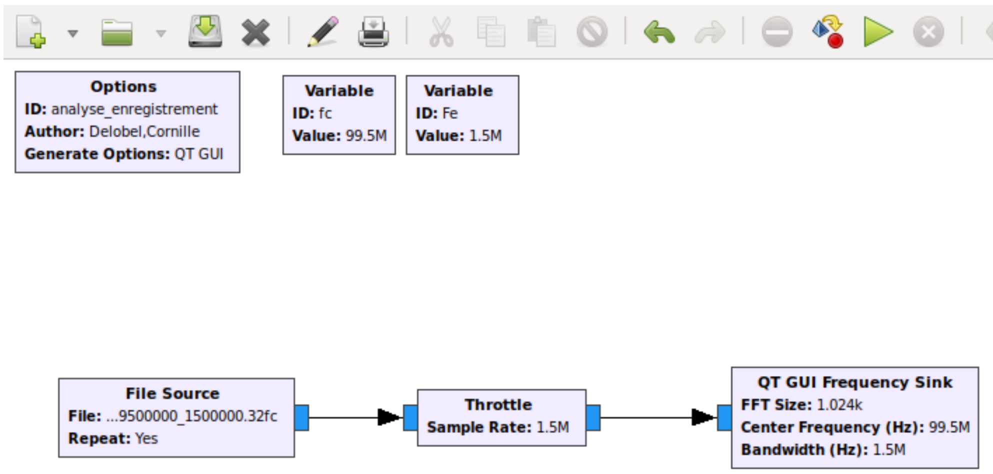

Figure 3: GNURadio Processing Chain

The "File Source" block is used to read the recording from a file. The "Throttle" block allows us to specify the sample reading speed, we throttle the flow of samples such that the average rate does not exceed the number of samples per second we want. The "QT Gui Frequency Sink" block is used to display the signal in frequency domain.

The sample we used was recorded at Toulouse in 2015, using a center frequency of $f_c = 99.5MHz$ and a sampling frequency of $F_e = 1.5MHz$. We used these values for the variable of our chain. We defined a sample rate similar to F_e for the throttle characteristics. We centered the display around f_c and chose to use a bandwidth of 1.5M so that we can visualise all the interesting part of the signal.

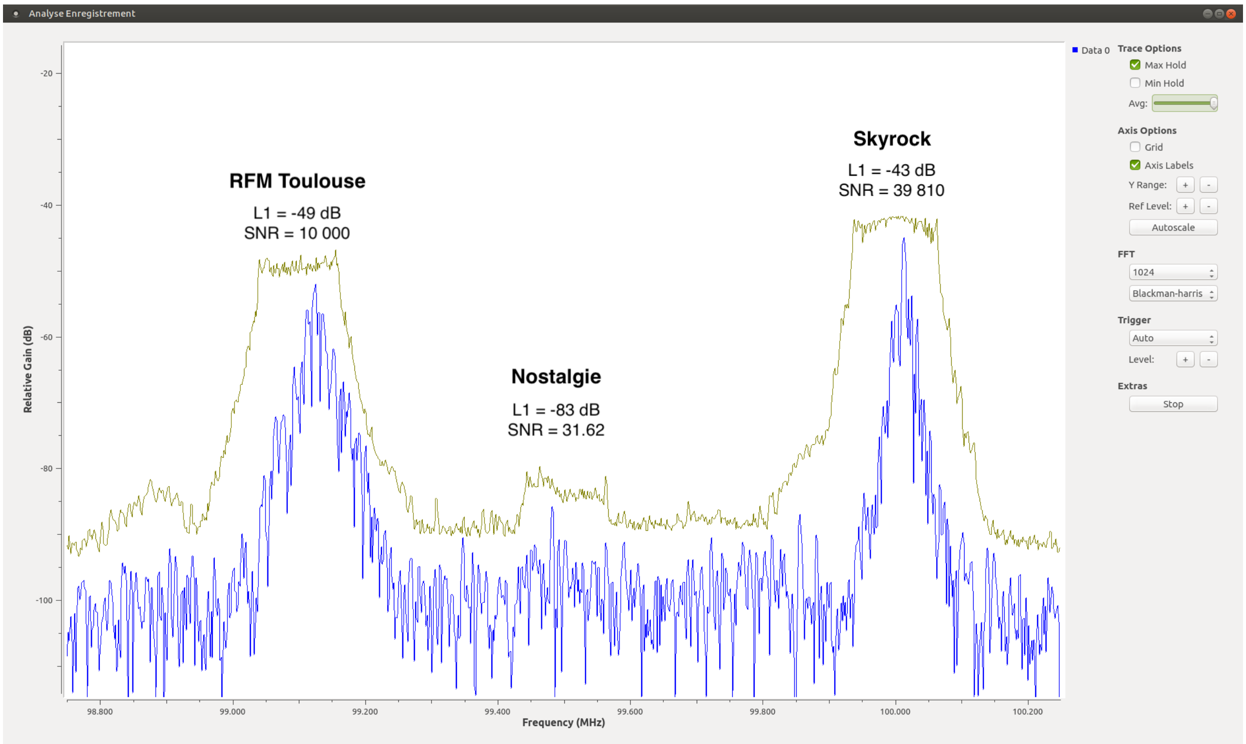

Figure 4: Frequency response of radio recording

Doing the analysis we observed 3 main intensity peaks.

- L_1 : 99.1 MHz : RFM Toulouse
- L_2 : 99.5 MHz : Nostalgie (low ratio)
- L_3 : 100.0 MHz : Skyrock

We found the corresponding radio station thanks to a website referencing the allocation of frequencies in the FM band near Toulouse.

We measured the signal-to-noise ratio of these 3 stations on our recording. We did it with the maximum mean because we found it more representative of the signals.

As the display is in dB we used the following formula :

$$P_{dB} = 10\,log_{10}(P) \iff P = 10^{P_{dB}/10}$$

We evaluate the noise to $-89dB$

The respective maximum mean for each of the 3 stations are :
$L_1 = -49dB$

$L_2 = -83dB$
$L_3 = -43dB$

So we calculate the signal-to-noise ratio of the 3 channels :
$\Delta_1 = 10^{89-49/10} = 10\,000$
$\Delta_2 = 10^{89-83/10} = 31.62$
$\Delta_3 = 10^{89-43/10} = 39\,810$

In order to calculate the bandwidth of a channel we decided to measurate the width of the signal around -3dB.

We have :
$BW_1 = [99.038, 99.159] = 0.121$
$BW_2 = [99.434, 99.566] = 0.132$
$BW_3 = [99.936, 100.066] = 0.130$

So the approximate bandwidth of a channel can be defined as the mean of these three bandwidth, which is : $BW_m = 0.128MHz = 128KHz$ which is greater than 100 KHz, the minimum compulsory value imposed by regulation. We realise that the radio assignation is quite optimistic with only 100KHz for each channel. That is why sometimes interference occur.

3.2 Channel extraction

Now that we have identified several broadcasting stations, we want to receive each one separately. To do so we will add some block to our GNURadio chain. This separation is done through two main steps : a frequency transposition in order to center the useful signal, and a low-pass filter in order to attenuate the out-of-band noise.

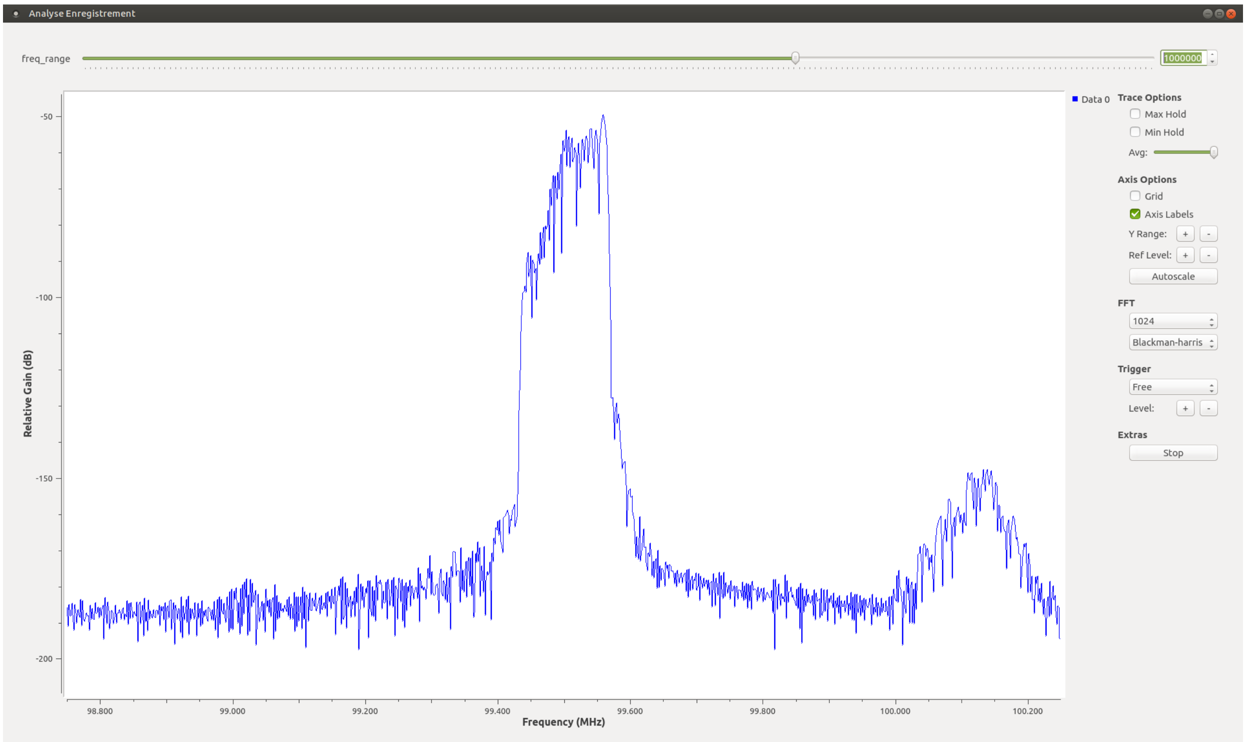

Figure 5: Channel filtered and centered

To center the channel we want to extract, we need an offset of $f_{centrale} - f_{channel-porteuse}$.

So respectively the offsets are :
$L_{1offset} = 99.5 - 99.1 = 400.6KHz$
$L_{2offset} = 0$ (this channel is already centered)
$L_{3offset} = 99.5 - 100 = -500KHz$

We apply this offset thank to the multiplication a source signal (which is a cosine) with our signal. The block named "QT Gui Range" allows us to modify the source signal in order to vary the offset.
If the frequency offset A is higher than the sampling frequency F_e, it is the same as a frequency offset of $A - F_e$. It means that in fact, the frequency offset is set modulo the sampling frequency.

Now that the channel is centered in term of frequency we need to filter the noise to have a satisfying sound. To do so, we used a "Low Pass Filter" block in GNURadio. In the filtering, we can introduce a decimation factor in order to lighten the computational load. After that, the sample rate will be divided by this same factor so we need to divide it by the decimation factor (we used 6) so that the points we kept are not read too fast and the signal is not deteriorated.

3.3 Demodulation

In order to restore the content of each radio broadcasting station and listen to it, we tried to understand the modulation method used. The signal is in fact composed of two stereophonic channels which are centered in frequency and have a maximum frequency of 15kHz. To ensure compatibility between monophonic receivers and stereophonic ones, these two channels are multiplexed. We obtain a message m(t) which is frequencially represented by the following figure.

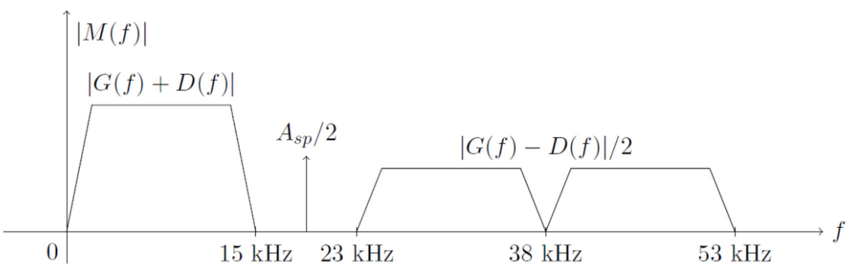

Figure 6: Frequencial stucture of radio broadcasting

The Carson rule defined the theoretical bandwidth by $B_{FM} = 2(\Delta_f + f_m)$ with f_m the maximum frequency of the composite message m(t). We have $\Delta_f = 75KHz$, so the bandwith we measured in the precedent part verify this theory.

Mathematically, it gives :

$$s_{RF} = A\,cos(2\pi f_0 t + \frac{\Delta f}{max(m(t))} \int_{-\infty}^{t} m(u)\mathrm{d}u) \quad (10)$$

By identification we have:

$$s_{RF} = A(t)cos(2\pi f_0 t + \phi(t))$$

$$and : s_R(t) = A(t)cos(\phi(t))$$

$$s_I(t) = A(t)sin(\phi(t))$$

We know :

$$s_{RF} = A(t)cos(\phi(t))cos(2\pi f_0 t) - A(t)sin(\phi(t))sin(2\pi f_0 t)$$

$$and : s(t) = s_R(t) + j\, s_I(t)$$

So :

$$s(t) = A(t)cos(\phi(t)) + jA(t)sin(\phi(t))$$

$$\iff s(t) = A(t)e^{j\phi(t)}$$

$$\iff s(t) = A(t)e^{j\frac{\Delta_f}{max(|m(t)|)}\int_{-\infty}^{t} m(u)\mathrm{d}u}$$

So, after sampling :

$$s[k] = A[\frac{k}{Fe}]e^{j\frac{\Delta_f}{max(|m(t)|)}\sum_{i=0}^{k} m(i)} + b[k] \; with \; A[\frac{k}{Fe}]$$

We used a block to plot the spectrum of the demodulated channel. We can identify the different elements seen of the following figure. We can see a peak for the monophonic modulation until 15KHz, for the stereophonic modulation until 53KHz and then the RDF transmissions that have a low SNR.

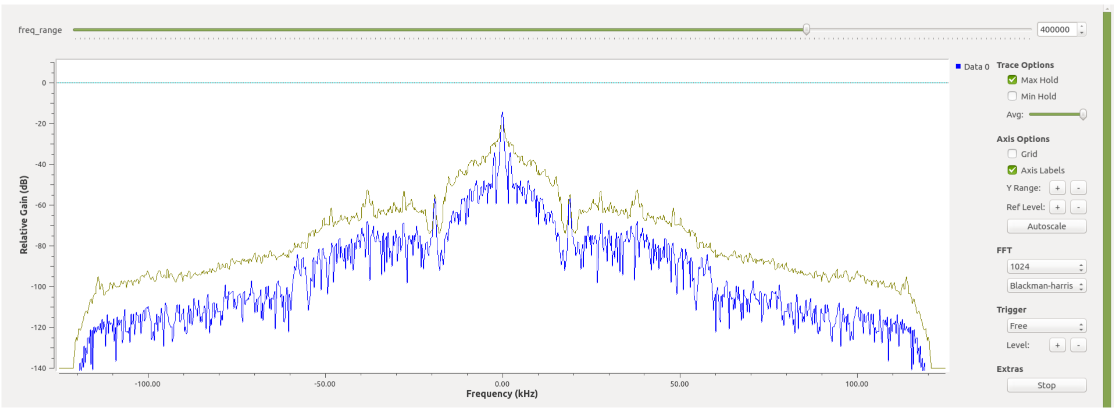

Figure 7: Frequency structure of the modulation

After this operation we were able to listen to the radio using a audio sink block and we heard on Skyrock that Jordy won Sam Smith album. Counting Stars was playing on RFM and YMCA on Nostlagie.

4. Part 3 : Reception of VOLMET messages in AM-SSB

We then worked with a VOLMET recording. In fact, the frequency sub-band between 11.175MHz and 11.4MHz is now reserved to the international aeronautic communications and in particular to the VOL METEO service (VOLMET). This is a periodic broadcasting of meteorological information, using a single side band amplitude modulation.

We used a sample that was recorded at Toulouse in 2015, using a center frequency of $f_c = 11.2965MHz$ and a sampling frequency of $F_e = 250kHz$. There is a single station in this record where the transmitter is located in the Royal Air Force air-base of St-Eval, United Kingdom.

First, we did a frequency analysis of this record. We observed a peak at 11.253 and we verified the location on VOLMET website. It was well located in United Kingdom ("GBR military one").

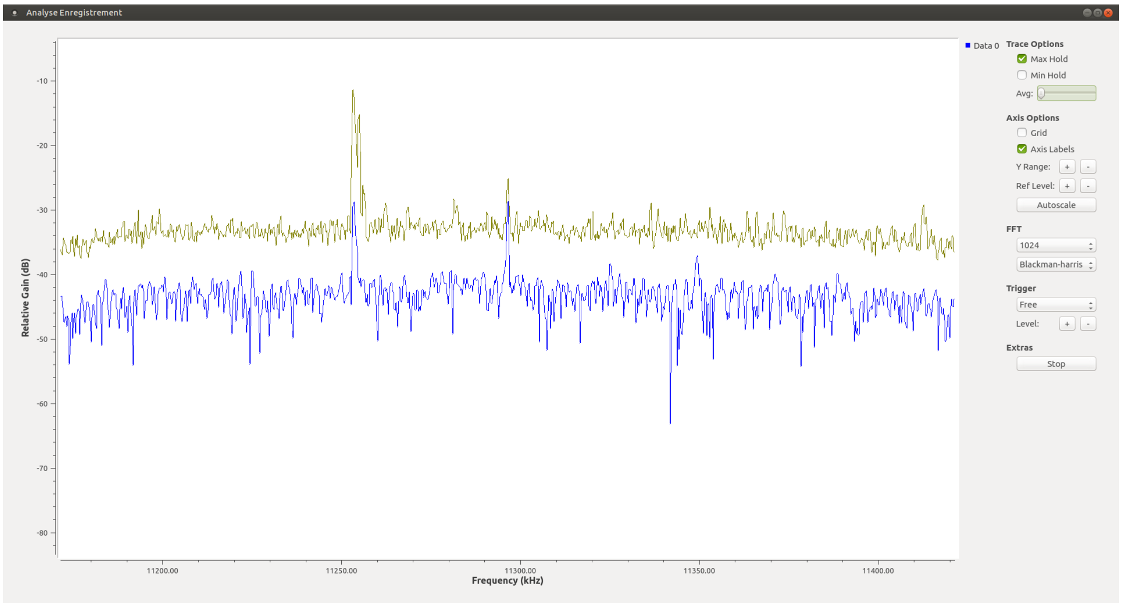

Figure 8: Frequency structure of the VOLMET recording

As we want to listen the record we need to center the signal. We used the same block as in the question We calculate the needed offset : f_1 =11.2965 - 11.253 = 43 500Hz.

Following expressions represent the recorded signal :

$$s(t) = m(t) \pm j\mathcal{H}\{m(t)\}$$

$$s_{RF}(t) = \Re\{m(t) \pm j\mathcal{H}\{m(t)\}e^{j2\pi f_0 t}\}$$

We know :

$$S_a(f) = S_{RF}(f) + j\mathcal{H}\{S_{RF}(f)\}$$

$$\iff = S_{RF}(f) + sgn(f)\, S_{RF}(f)$$

So :

$$S_a(f) = [m(f) + sgn(f)m(f)]\, \delta(f - f_0)$$

$$\iff = m(f - f_0) + sgn(f - f_0)m(f - f_0)$$

And :

$$S_{RF}(f) = \frac{1}{2}(S_a(f) + S_a^*(-f))$$

$$\iff = \frac{1}{2}[m(f - f_0) + sgn(f - f_0)\, m(f - f_0 + m(-f - f_0) + sgn(-f - f_0\, m(-f - f_0)]$$

Experimentally, We observed that the signal is between 11.253 and 11.256, we can conclude that it represents the right (upper) part of the signal (+3dB around 11.253).

Then, we design a filter using the tool "Filter Design Tool". The objective is a complex bandpass filter with finite impulse response that have lower and upper cut-off frequencies at 0Hz and B/2, in-band and out-band gain of 0dB and -30dB.

The following figures show the modulus and phase of frequency response of the filter.

Figure 9: Step response of the filter

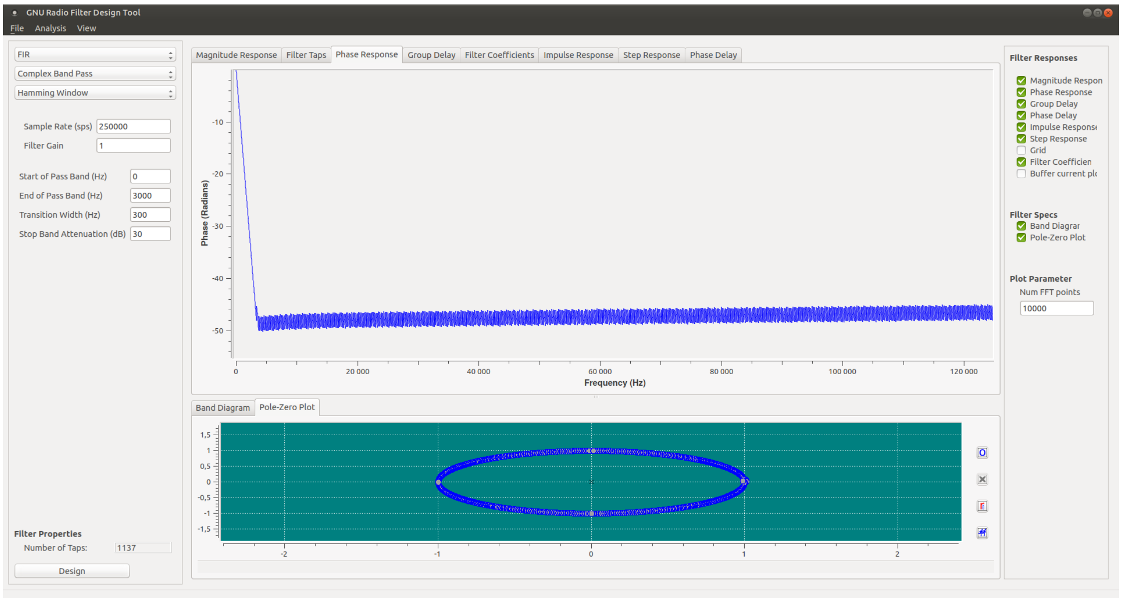

Figure 10: Phase response of the filter

Then, we used this specific filter by pasting the generated filter coefficient in a FFT Filter block. When applied to the VOLMET recording, it gives the following figure where we can see that the communication peak has a high SNR.

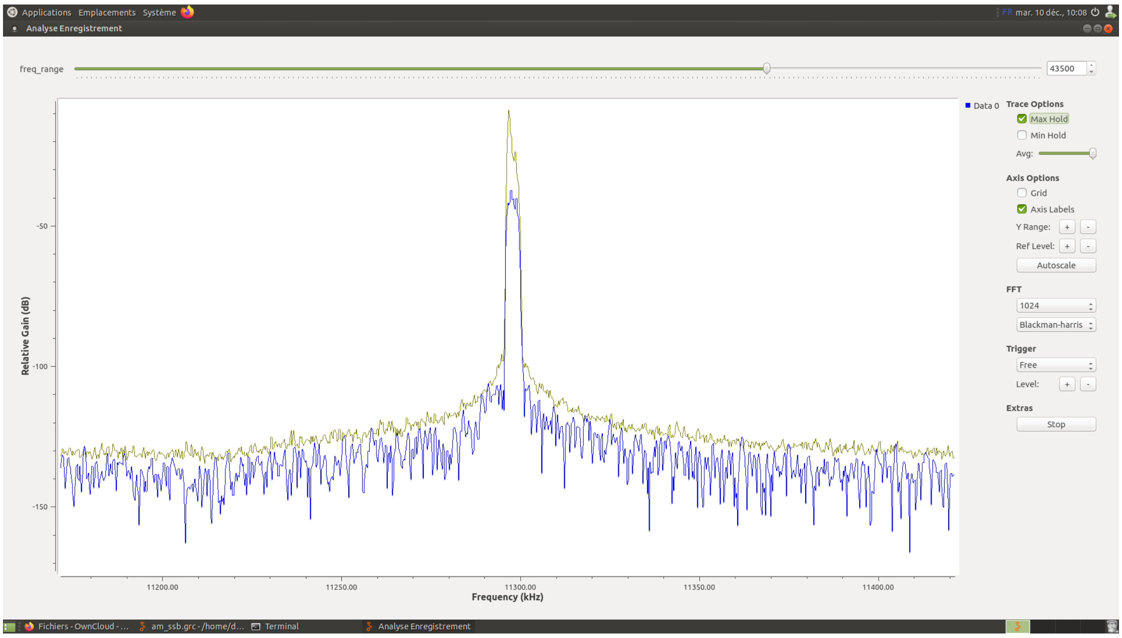

Figure 11: Filtered VOLMET signal

5. Conclusion

During these three labs, we learnt the basis of USRP operation. With both theoretical and experimental approaches, we discovered the physics of this tool and how to use it. With the use of the GNU-Radio software, we were able to treat and apply filter to retrieve audio (therefore, useful information such as weather situation) from wide band recordings.