ANALYTICAL - 1...

1). Solve the following using recurrence relation:

a). $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

$x(1)=0$

substitute $n=2$:- $x(2) = x(2-1)+5 \Rightarrow x(1)+5$

$= 0+5$

$x(2) = 5$

$n=3$ $\quad x(3) = x(3-1)+5 = x(2)+5$

$= 5+5$

$x(3) = 10$

$n=4$ $\quad x(4) = x(4-1)+5 = x(3)+5 = 10+5$

$x(4) = 15$

For the recurrence relation, each term is 5 more than previous term.

$\boxed{\text{So, } x(n) = 5n-5 \text{ for } n>1}$

2). $x(n) = 3x(n-1)$ for $n>1$ $\boxed{x(1)=4}$

$x(1)=4$

$n=2$ $\quad x(2) = 3x(2-1) = 3x(1) = 3(4)$

$x(2) = 12$

$n=3$ $\quad x(3) = 3x(3-1) = 3x(2) = 3(12)$

$x(3) = 36$

$n=4$ $\quad x(4) = 3x(4-1) = 3(36)$

$x(4) = 108$

So, $\boxed{x(n) = 4 * 3^{n-1} \text{ for } n \geq 1}$

$x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ solve for n

$\underline{n=2}$ $x(2) = x(2/2) + 2 = x(1) + 2$

$n = 2^1 = 2$ $x(2) = 3$

$n = 4$ $x(4) = x(4/2) + 4 = x(2) + 4$

$n = 2^2 = 4$ $= 3 + 4$

$x(4) = 7$

$n = 8$

$n = 2^3 = 8$ $x(8) = x(8/2) + 8 = x(4) + 8$

$= 7 + 8$

$x(8) = 15$

for this the reccurrence relation, $2^{n-1} = 2^k - 1$

for $\boxed{n = 2^k}$, $\boxed{x(2k) = 2^{2k} - 1}$

$x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$

$n=3$ $x(3) = x(3/3) + 1 = x(1) + 1$

$x(3) = 2$

$n=9$ $x(9) = x(9/3) + 1 = x(3) + 1 = 2 + 1$

$x(9) = 3$

$n=27$

$x(27) = x(27/3) + 1 = x(9) + 1$

$= 3 + 1$

$x(27) = 4$

for this the reccurrence relation is

$x(n) = \log_3 n$

$x(k) = \log_3 3^k$

$n = 3$

$n = 3^1$

$n = 3^2$

$n = 3$

... following recurrences completely

(i) $T(n) = T(n/2) + 1$ where $n = 2k$ for all $k \geq 0$

Given, $T(n) = T(n/2) + 1$ $\quad n = 2^k$

Sub $n = 2^k$ $\quad T(2^k) = T(2^k/2) + 1 = T(2^{k-1}) + 1$

Now, $T(2^{k-1}) = T\left(\frac{2^{k-1}}{2}\right) + 1 = T(2^{k-2}) + 1$

Again $T(2^{k-2}) = T(2^{k-2}/2) + 1 = T(2^{k-3}) + 1$

$T(2^1) = T(2^0) + 1$

now, $n = 2^k \Rightarrow k = \log_2 n$

$T(2^k) = T(2^{k-1}) + 1 = T(2^{k-2}) + 1 + 1 \ldots T(2^0) + k$

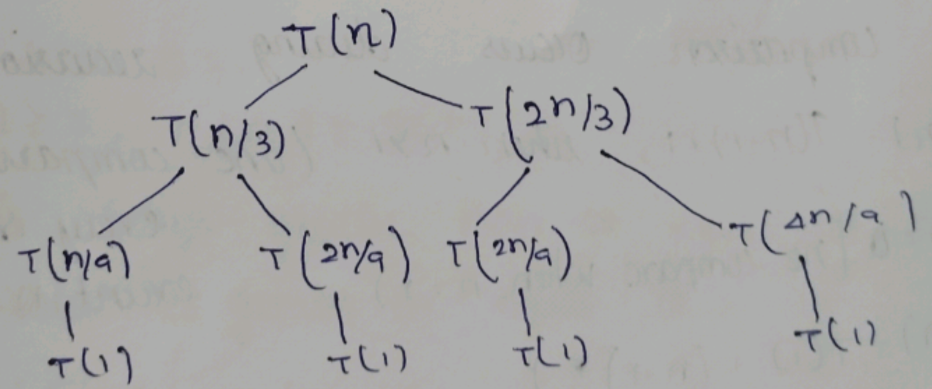
Since, $2^0 = 1$, $T(2^0) = T(1)$ $\quad T(1) = 1$

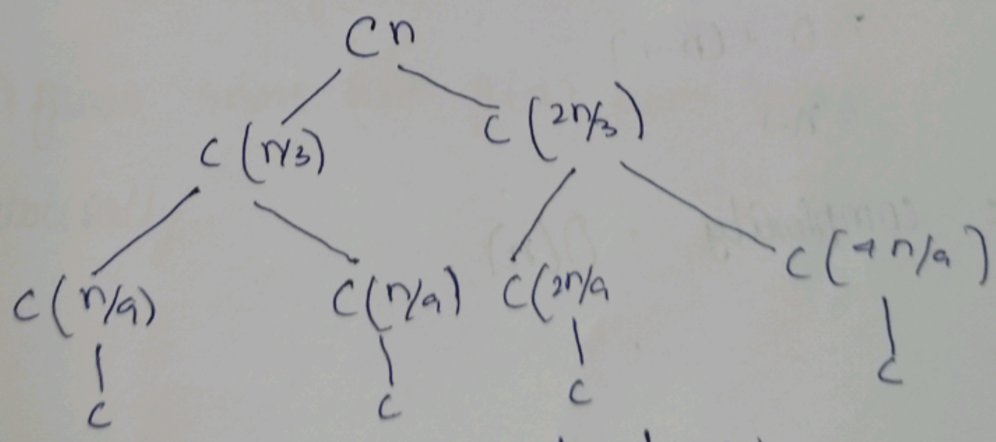
$T(2^k) = 1 + k$

$\boxed{T(n) = 1 + \log_2 n}$ $\quad \boxed{O(\log n)}$

(ii) $T(n) = T(n/3) + T(2n/3) + cn$

$T(n) = T(n/3) + T(2n/3) + cn$





length = $\log_3 n$ (divided by 3)

$T(n) = cn \log_3 n \Rightarrow \omega(n \log n)$

3) consider following Algorithm:

min (A [0.... n-1]) —— 1

if $n = 1$ return A [0]

else temp = min 1 [A [0.... n-2]]

If temp $\leq$ A [n-1] return temp

else return A [n-1] – n-1

a). what does it compute?

b) Setup a recurrence relation for algorithm & solve it

a). It computes minimum element in an array A of size n.

If $i < n$, A [i] is smaller than all elements, then A[j], $j = i+1$ to $n-1$, then it returns A[i]. It also returns the leftmost minimal element.

b). mainly comparison occurs during recursion

So, $T(n) = T(n-1)+1$, when $n > 1$ (one comparison at every step except $n=1$)

$T(1) = 0$ (no compare when $n = 1$)

$T(n) = T(1) + (n-1) * 1$

$= 0 + (n-1)$

$= n-1$

$\therefore$ Time complexity $= O(n)$

4) Analyze order of growth

(i) $F(n) = 2n^2+5$ and $g(n) = 7n$ use $\Omega(g(n))$ notation

Given: $F(n) = 2n^2+5$

$c \cdot g(n) = 7n$

$n=1$

$F(1) = 2(1)^2+5 = 7$

$g(1) = 7$

$n=1, \quad 7 = 7$

$n=2, \quad F(2) = 2(2)^2+5$

$= 8+5 = 13$

$g(2) = 7 \times 2 = 14$

$n=2, \quad 13 = 14$

$n=3$

$F(3) = 2(3)^2+5$

$= 18+5$

$= 23$

$g(3) = 21$

$n=3, \quad 23 = 21$

$n \geq 3, \quad F(n) \geq g(n) \cdot c$

F(n) is always greater than or equal to $c * g(n)$ when, n Value is greater or equal to 3.

$\therefore F(n) = \Omega(g(n))$.

F(n) grows more than g(n) from below asymptotically.

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Design and Analysis Of Algorithm for Polynomial Problems...

ASSIGNMENT- 2

~ Mathimozhi.E.R.
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