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Short Note

The accuracy of estimating *Q* from seismic data

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INTRODUCTION

A major aim of seismic interpretation is the inference of petrophysical properties of reservoir rocks. Because the inversion from seismic to petrophysical characteristics is far from unique, this task requires a range of seismic parameters, prominent among which are seismic velocity, impedance, and Poisson's ratio. The inclusion of seismic absorption in this list could add desirable complementary information. For example, absorption may be more sensitive to clay content than seismic velocity (Klimento and McCann, 1990). However seismic absorption is difficult to measure, particularly over depth intervals as short as most reservoir intervals.

This note presents equations for the standard errors in estimating the absorption parameter Q from surface seismic and downhole seismic data. The equations provide a useful guide to the accuracy that can be expected in estimating Q, while at the same time illustrating the inherent trade-offs. They are based on simplifying assumptions and are not intended as anything more than a guide; an actual analysis should as a matter of course determine error estimates from the data in hand.

Whether constant-Q absorption is a valid way of describing seismic absorption is not considered. What this note demonstrates is that, if absorption is to be estimated from seismic data, there is little prospect of achieving useful accuracy in the estimates if the absorption model is *overparameterized*. The equations presented here set fundamental limits on the accuracy of estimates of Q. These limits show that it is essential that the estimation model is parameterized according to the principle of parsimony (Box and Jenkins, 1976).

The origin of the estimation errors depends on the methods of estimation of Q. Three methods are considered:

1) estimation from the power spectral ratio of the reflections from two time intervals in surface seismic data;
2) estimation from the amplitude spectra of wavelets extracted by matching seismic data to well-log synthetic seismograms over two time intervals; and
3) estimation from waveforms recorded at two levels downhole from a common seismic source.

Whatever the method, it should be possible to set out the amount and quality of data required to achieve a specified accuracy in its parameter estimates.

In method 1 the errors in Q arise principally from the sampling fluctuations inherent in power spectral estimates from any finite segment of data. Whatever the true spectral coloring of the reflection signal in a segment of data, estimates of that coloring exhibit sampling fluctuations whose magnitude increases as the duration of the segment decreases. The other two methods aim to remove the effects of errors from sampling the reflection signal. Their estimation errors come principally from noise. Additive noise also causes estimation errors to vary inversely with the duration of the data segment and inversely with the signal-to-noise ratio of the data. When Q is estimated from matching synthetic seismograms, modeling errors in the well-log synthetic are likely to be a major source of noise, whereas, provided that downgoing waveforms are isolated from any significant upgoing or other coherent interfering events in estimating Q from downhole recordings, the accuracy of the third method depends on the level of random noise in the data. The accuracy of all three methods depends strongly on the frequency bandwidth of the data. The third method is the most accurate of the three.

The equations below relate only to the way in which sampling fluctuations and noise propagate into estimates of Q. Problems of considerable practical concern to the estimation of Q, such as the reduction of noise in the data and the separation of upgoing and downgoing waves in VSPs or the separation of primary and multiple reflections in surface seismic data, are a separate issue. Raikes and White (1984)

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describe, with examples, the application of the three methods to seismic data.

CONSTANT-Q ATTENUATION

When constant-Q absorption operates, the amplitude spectrum of a seismic pulse at frequency f obeys:

$$|A_t(f)| = |A_0(f)| \exp(-\pi f t/Q), \tag{1}$$

where t is the traveltime of the pulse from time zero and $|A_0(f)|$ is the unattenuated amplitude spectrum of the pulse. Q can be estimated from the amplitude spectra at traveltimes t_1 and t_2 using

$$\ln |A_2(f)/A_1(f)| = -\pi f(t_2 - t_1)Q = -\mu f, \tag{2}$$

where

$$\mu = \pi(t_2 - t_1)/Q. \tag{3}$$

Because the traveltimes and frequency can be specified with negligible error during the course of analysis, the error in estimating Q can be attributed to the errors in estimating $|A_2(f)/A_1(f)|$. The next section sets out the variances in spectral amplitude ratios from the three methods.

SPECTRAL ESTIMATION ERRORS

Estimation from power spectral ratios

This method estimates Q from the power spectral ratios of the reflection signals in two time intervals of surface seismic data:

$$\begin{aligned}\ln (P_2(f)/P_1(f)) &= 2 \ln |A_2(f)/A_1(f)| \\ &= -2\pi f(t_2 - t_1)/Q \\ &= -2\mu f.\end{aligned} \tag{4}$$

This equation follows from the relation:

$$P_s(f) = |A(f)|^2 P_r(f) \tag{5}$$

between the power spectrum $P_s(f)$ of a seismic reflection signal $s_t = a_t * r_t$ and the power transfer function $|A(f)|^2$ of the seismic wavelet a_t and the power spectrum $P_r(f)$ of the reflectivity sequence r_t from which the signal derives. It is assumed that the spectral coloring of the reflectivity either does not change between the two intervals or is compensated and that no other frequency-dependent effects are operating.

The power spectra can be estimated in various ways. One way would be to use multiple coherence analysis (White, 1973). This separates the signal and noise spectra on the basis of the short-range, trace-to-trace coherence of their spectral components. Alternatively, the random noise in the data could be attenuated by applying a spatial filter such as a Karhunen-Loeve filter or a signal-preserving f-x filter (Harris and White, 1991) to the aligned reflection signal. The alignment of signals is very important because timing jitter would cause a loss of coherent spectral power at high frequencies.

The variance of signal power spectral estimates from multiple coherence analysis using q traces at a time is (White, 1973):

$$\frac{\operatorname{var}\{\hat{P}_s(f)\}}{P_s^2(f)} \simeq \frac{1}{bT}\left[1 + \frac{2}{q\rho}\right], \tag{6}$$

where ρ is the harmonic mean of the signal-to-noise ratios of the traces at frequency f, b is the bandwidth of the spectral window employed in the analysis, and T is the duration of the data segment. Throughout this note a circumflex denotes an estimate.

On the assumption that they come from separate time gates, the power spectral estimates $\hat{P}_1(f)$ and $\hat{P}_2(f)$ are independent. Also, within the seismic bandwidth, $q\rho$ is much greater than 1 normally, so that over the frequency range from which estimates are made the variance of the estimated amplitude ratio is

$$\frac{\operatorname{var}\{|\hat{A}_2(f)/\hat{A}_1(f)|\}}{|A_2(f)/A_1(f)|^2} = \operatorname{var}\{\ln |\hat{A}_2(f)/\hat{A}_1(f)|\} = \frac{1}{2bT}. \tag{7}$$

This equation is also valid for spectral ratios estimated after applying a noise-reducing spatial filter.

Estimation by matching seismic data and synthetic seismograms

Instead of correcting power spectral estimates over two time intervals for reflectivity coloring using well logs, it may be possible to estimate the seismic wavelets in the two intervals by matching the seismic data to well-log synthetic seismograms. The matching technique and its associated spectral estimation errors are described by White (1980) and Walden and White (1984). The relative variance in the amplitude spectrum of a matching wavelet is given by:

$$\frac{\operatorname{var}\{|\hat{A}(f)|\}}{|A(f)|^2} = \operatorname{var}\{\ln |\hat{A}(f)|\} = \frac{\gamma^{-2} - 1}{2bT}, \tag{8}$$

where $\gamma^2 = \gamma^2(f)$ is the spectral coherence between the data and synthetic seismogram at frequency f, and the dependence on frequency has been dropped for convenience in later equations.

Q can then be estimated from the ratio of the amplitude spectra and, on the assumption that the estimates come from separate time gates and are therefore independent,

$$\operatorname{var}\{\ln |\hat{A}_2(f)/\hat{A}_1(f)|\} = \frac{1}{2bT}[\gamma_1^{-2} + \gamma_2^{-2} - 2]. \tag{9}$$

Estimation from two downhole waveforms

In this method, Q can be estimated from the amplitude spectra $|\hat{A}_{12}(f)|$ and $|\hat{A}_{21}(f)|$ of the filters that best match the deeper to the shallower and the shallower to the deeper of two downhole recordings. Both filters are biased estimates of the absorption response because each contains a noise suppression filter (Raikes and White, 1984). Bias and random errors are both small at frequencies where the coherence is high. Random error and bias are reduced in the ratio

$$\frac{|\hat{A}_{12}(f)|}{|\hat{A}_{21}(f)|} = \frac{P_2(f)}{P_1(f)}, \tag{10}$$

which is also the spectral ratio of the two trace segments. This ratio is unbiased if the signal-to-noise ratios on two recordings are the same. Note that coherence analysis is essential to find the reliable frequency range and to monitor the estimation errors.

The variance of the power spectral ratio is related to the variances and covariance of the two power spectral estimates by means of the propagation of errors formula:

$$\frac{\text{var}\,\{\hat{P}_2(f)/\hat{P}_1(f)\}}{[P_2(f)/P_1(f)]^2} = \frac{\text{var}\,\{\hat{P}_1(f)\}}{P_1^2(f)} + \frac{\text{var}\,\{\hat{P}_2(f)\}}{P_2^2(f)} - 2\,\frac{\text{cov}\,\{\hat{P}_1(f),\, \hat{P}_2(f)\}}{P_1(f)P_2(f)}. \quad (11)$$

The fractional variances and covariances in power spectral estimates are given by [Goodman, 1957, section 4.4; Jenkins, 1963, equation (42)]:

$$\frac{\text{var}\,\{\hat{P}_j(f)\}}{P_j^2(f)} = \frac{1}{bT} \quad (12)$$

and

$$\frac{\text{cov}\,\{\hat{P}_1(f),\, \hat{P}_2(f)\}}{P_1(f)P_2(f)} = \frac{\gamma^2}{bT}, \quad (13)$$

where $\gamma^2 = \gamma^2(f)$ is the spectral coherence between the two recordings. Substituting these values into equation (11) gives:

$$\text{var}\,\{\ln|\hat{A}_2(f)/\hat{A}_1(f)|\} = \frac{\text{var}\,\{\sqrt{\hat{P}_2(f)/\hat{P}_1(f)}\}}{P_2(f)/P_1(f)} = \frac{1-\gamma^2}{2bT}. \quad (14)$$

This variance is smaller than the relative variance in amplitude frequency response given by equation (8) above.

Comment on variances

The equations for variance in this section are large-sample expressions and they may underestimate actual variances slightly. Provided the spectral estimates strike a sensible balance between smoothing and sampling errors (White, 1984), the difference is of no consequence, particularly when the purpose of the expressions is simply to provide guides to accuracy. Also any inflation of the variances due to the leptokurtic distribution of seismic reflection coefficients (Walden and Hosken, 1986; White, 1988) is very weak.

All the expressions are based on the theory of stationary stochastic processes. However only the noise need be assumed to be a stationary stochastic sequence in methods 2 and 3.

ERRORS IN ESTIMATING Q

Suppose that n independent estimates of $\ln|A_2(f)/A_1(f)|$ have been obtained. For independence, the spacing of these estimates cannot be less than b, the analysis bandwidth. There is no point in using a wider spacing because that would be throwing away information. If $f_\ell = \ell b$ is the frequency at which the first of these estimates is centered and $f_h = hb$ is the frequency corresponding to the last estimate, then $n = h - \ell + 1$. To avoid serious errors, n must be restricted to the frequency band over which the spectral coherence is reasonably high. This defines a bandwidth B given by:

$$B = f_h + (b/2) - (f_\ell - (b/2)) = nb \quad (15)$$

over which useful measurements can be made.

It is also supposed that the estimates of $\ln|A_2(f)/A_1(f)|$ have a nearly constant variance σ^2:

$$\sigma^2 = \text{var}\,\{\ln|\hat{A}_2(f)/\hat{A}_1(f)|\}. \quad (16)$$

Inspection of the examples in Raikes and White (1984) shows that this is a reasonable assumption; it implies that the coherence is constant over the seismic bandwidth. The slope $\hat{\mu}$ of the attenuation response over the frequency range f_ℓ to f_h can then be found by linear unweighted regression, and its estimate has a variance:

$$\text{var}\,\{\hat{\mu}\} = \sigma^2 \Big/ \sum_{i=\ell}^{h} (f_i - \bar{f})^2. \quad (17)$$

The f_i are equispaced and, with the help of series numbers 19 and 32 in Jolley (1961), a little algebra gives

$$\text{var}\,\{\hat{\mu}\} = 12\sigma^2/n(B^2 - b^2) \simeq 12\sigma^2/nB^2. \quad (18)$$

The variance in the estimate of Q can be related to the variance in estimated slope $\hat{\mu}$ through equation (3):

$$\frac{\text{var}\,\{\hat{Q}\}}{Q^2} = \frac{\text{var}\,\{\hat{\mu}\}}{\mu^2} = \frac{12Q^2\sigma^2}{\pi^2(t_2 - t_1)^2 n(B^2 - b^2)}. \quad (19)$$

This result can be specialized by substituting the values of σ^2, the variance of the logarithmic amplitude ratio, from the three methods considered in the previous section.

If estimates of $\ln|\hat{A}_2(f)/\hat{A}_1(f)|$ come from multiple coherence analysis of surface seismic data then, using equations (7) and (15),

$$\frac{\text{var}\,\{\hat{Q}\}}{Q^2} = \frac{6Q^2}{\pi^2(t_2 - t_1)^2(B^2 - b^2)BT} \simeq \frac{6Q^2}{\pi^2(t_2 - t_1)^2 B^3 T}. \quad (20)$$

If the estimates come from matching surface seismic data to a well-log synthetic over two separate time intervals, then the relative variance in $\hat{Q}$ becomes [see equation (9)]:

$$\frac{\text{var}\,\{\hat{Q}\}}{Q^2} = \frac{6Q^2(\gamma_1^{-2} + \gamma_2^{-2} - 2)}{\pi^2(t_2 - t_1)^2(B^2 - b^2)BT} \simeq \frac{6Q^2(\gamma_1^{-2} + \gamma_2^{-2} - 2)}{\pi^2(t_2 - t_1)^2 B^3 T}. \quad (21)$$

The factor $(\gamma_1^{-2} + \gamma_2^{-2} - 2)$ will reduce the variance compared with equation (20) as long as $(\gamma_1^{-2} + \gamma_2^{-2}) < 3$. In this case, the reduction in variance comes from removing the effect of sampling two different segments of data.

If the estimates come from the spectral ratio of two downhole recordings [equation (14)], then the relative variance in $\hat{Q}$ becomes:

$$\frac{\operatorname{var}\{\hat{Q}\}}{Q^2} = \frac{6Q^2(1-\gamma^2)}{\pi^2(t_2-t_1)^2(B^2-b^2)BT} \approx \frac{6Q^2(1-\gamma^2)}{\pi^2(t_2-t_1)^2B^3T}. \tag{22}$$

For good downhole recordings the factor $(1 - \gamma^2)$ can be small.

The relative standard error in $\hat{Q}$ is simply the square root of the relative variance.

DISCUSSION

Let us put some representative numerical values into the expressions of the previous section. Suppose that Q is 100 and data segments 0.5 s long and separated by 0.5 s in two-way time are analyzed. In methods 1 and 2, the time separation $(t_2 - t_1)$ must be at least 0.5 s for estimates from the two intervals to be independent. The seismic bandwidth is taken to be 54 Hz (e.g., 8 to 62 Hz). Then for estimation from surface seismic data (method 1) the relative standard error in $\hat{Q}$ is 0.56, which is enormous. Doubling the duration of the analysis intervals, and correspondingly their time separation, reduces the relative standard error in $\hat{Q}$, which now represents an average over a considerable depth interval, to a tolerable 0.20. Doubling the bandwidth brings the same reduction in standard error and is a much more desirable way of improving accuracy if it can be achieved.

In tying surface data to synthetic seismograms, good matches have a mean spectral coherence of around 0.7. If both coherences γ_1^2 and γ_2^2 are 0.7 over the seismic bandwidth, the relative standard error in $\hat{Q}$ is 0.51. This method removes bias from reflectivity coloring, but its random errors are comparable with those of method 1.

The coherence of downhole waveforms is generally much higher than that between surface seismic data and synthetic seismograms. Values above 0.9 are not uncommon [e.g., Figures 12 to 14 of Raikes and White (1984)]. Also the bandwidth may be higher than that of surface data. On the other hand, an 0.5 s difference in two-way time must be halved in method 3 to investigate Q over the same depth range. Thus, using a coherence of 0.9 with $Q = 100$, $B = 54$ Hz, and $T = 0.5$ s, the relative standard error in $\hat{Q}$ drops to 0.35, which is still a considerable error. For a not-unrealistic coherence of 0.95 the relative standard error in $\hat{Q}$ is 0.25 s.

The lesson of these calculations is that, of the three methods considered, only the use of downhole recordings offers a reasonable expectation of reliable estimates of Q without averaging over an unduly long depth interval. This method could run into difficulty if the depth interval were a short one. However the equations show that, in the case of most interest in seismic exploration, where the absorption is strong (Q is low), better accuracy can be attained. This makes the task of identifying zones of strong absorption less demanding.

The errors in estimating Q discussed here all relate to random errors. The analysis was assumed to be bias-free. The causes of the random fluctuations in the data segments analyzed were discussed in the introduction. While it is well recognized that downhole recordings offer more scope for better resolved and more accurate estimates of absorption, equation (22) provides a useful and hitherto unpublished guide to the potential estimation accuracy.

The accuracy of all three methods depends strongly on the frequency bandwidth of the data. There is also a strong dependence on the value of Q itself and the time interval over which it is measured. The expressions confirm the physical intuition that Q is most easily estimated when the effect of absorption is large. The remaining factor $1/BT$ in the expressions is the usual averaging term found in any error measure and denotes the inverse relationship between variance and the effective number of independent samples in the data segment.

There is no reason to suggest that methods employing measures such as instantaneous frequency or rise time, which do not make full use of the waveform information available, would be anything but less accurate than the methods analyzed here. For example, without averaging or amplitude weighting, instantaneous frequency exhibits a very large variance (White, 1991) which must propagate into any estimate of absorption made from it.

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