

Formal Proofs for Mobile Robot Swarms

Lionel Rieg

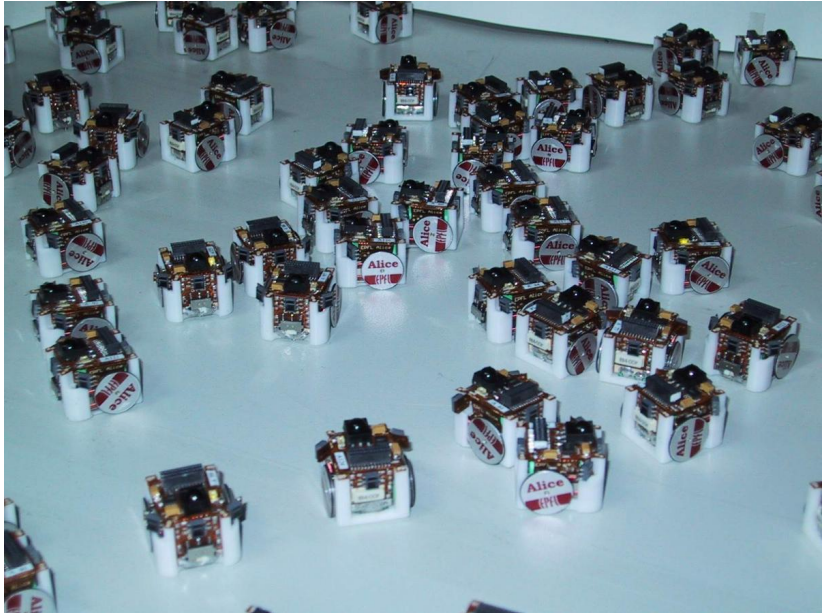
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Réunion Estate/Descartes, Roscoff, 2 avril 2019

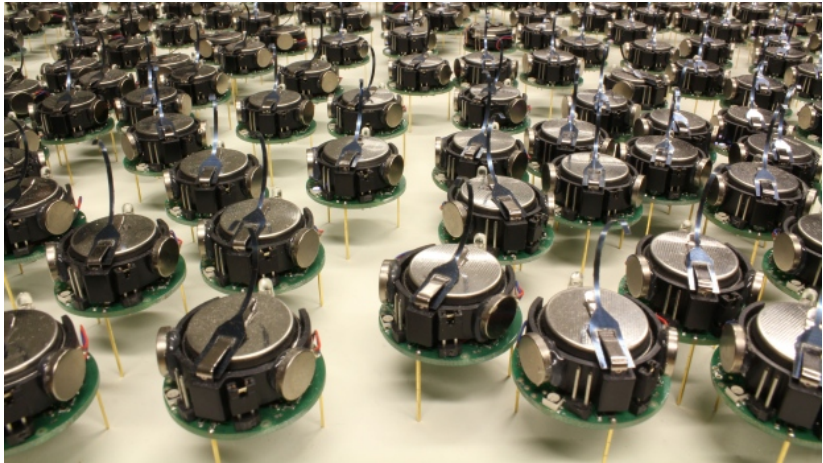
Inspiration: Swarms of Mobile Robots

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 - ▶ entertainment
 - ▶ rescue
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 - ▶ lots of (small) identical robots
- ▶ Where?
 - ▶ entertainment
 - ▶ rescue
 - ▶ exploration
 - ▶ ...
- ▶ Opportunities?
 - ▶ cooperative behavior (swarm intelligence)
 - ▶ resilience
- ▶ Main challenge?
 - ▶ Understand what happens!

Why Formal Methods for Mobile Robots Swarms?

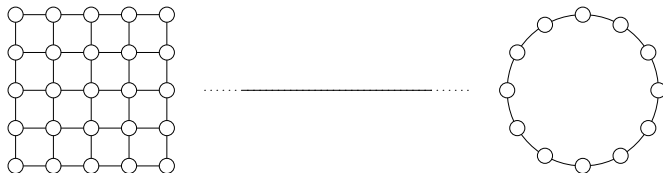
Many mobile robots swarm models:

- ▶ Space discrete/continuous, bounded/unbounded, topology, ...
- ▶ Sensors multiplicity, range, accuracy, orientation, ...
- ▶ Faults none, crash, Byzantine, ...
- ▶ Execution synchronous/asynchronous, fairness, interruption, ...

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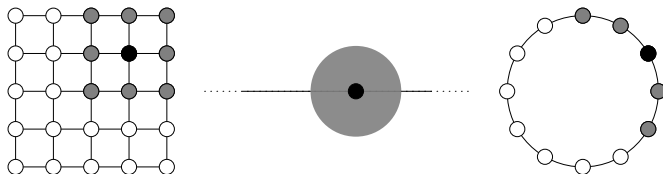
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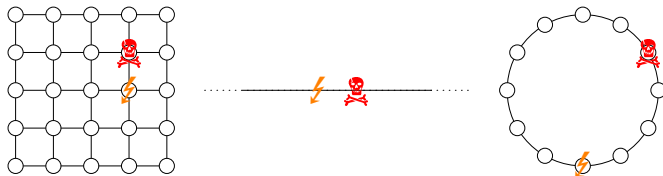
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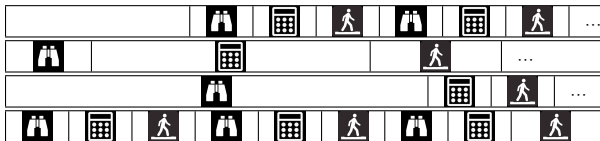
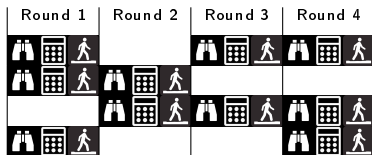
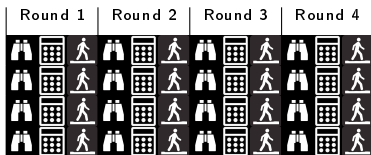
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▶ Subtle differences in models \rightsquigarrow Very error-prone

▶ Careful of mismatch spec/proof!

▶ Lots of proof cases

\Rightarrow Formal methods can help

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Which model? (process algebra, TLA, ...)

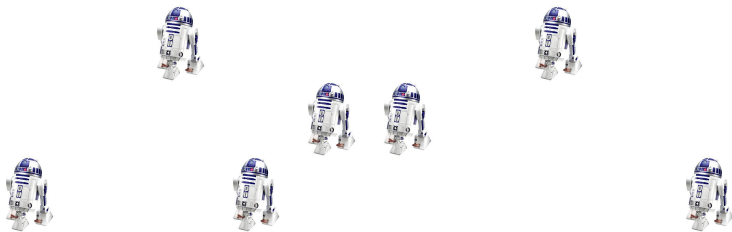
Which tool? (model checking, proof assistant, ...)

We need a suitable framework

What are the requirements?

Robot Model

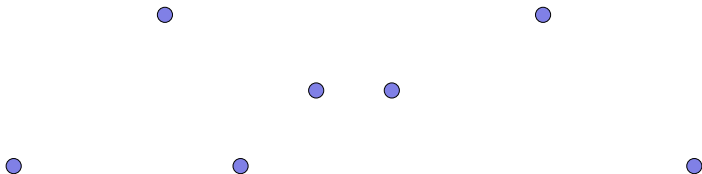
[Suzuki, Yamashita 99]



Robot Model

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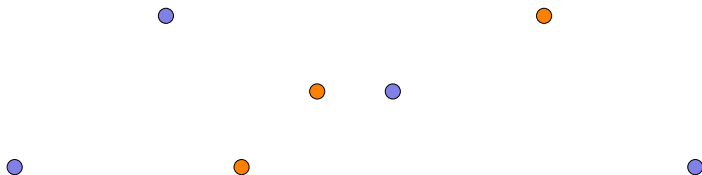
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Robot Model

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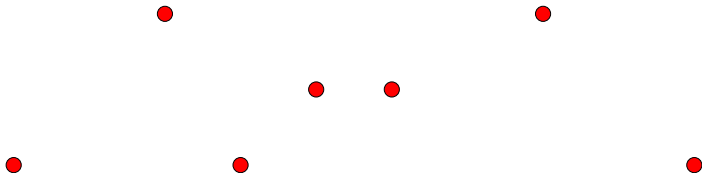
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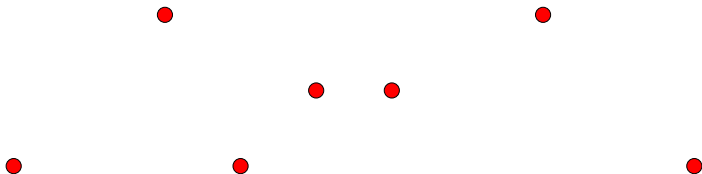
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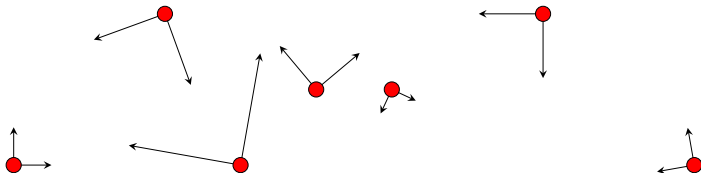
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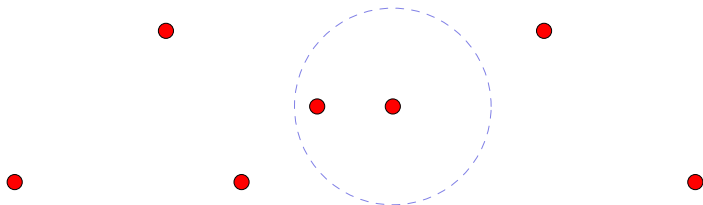
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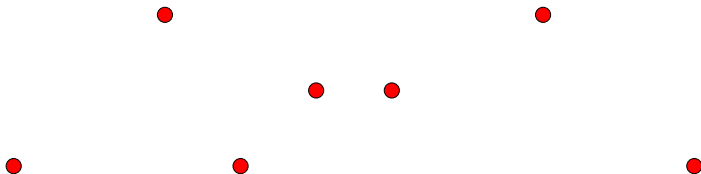
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- ▶ Limited/unlimited vision? multiplicity?
- ▶ Same (deterministic) program everywhere



3 phases for each robot:

1.  **Look**: observe its surrounding
 - ▶ indirect communication
 - ▶ depends on sensor capabilities
2.  **Compute**: choose what to do
 - ▶ choose an objective
 - ▶ depends on observation, program
3.  **Move**: do it (or try to)
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and repeat

Execution Models (2): Scheduling

[Suzuki, Yamashita 99]

Scheduling of robots is

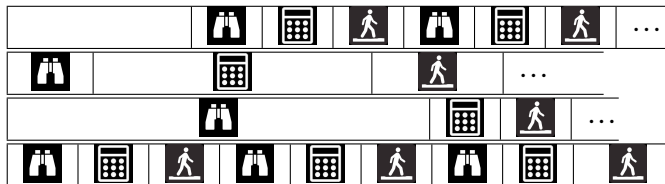
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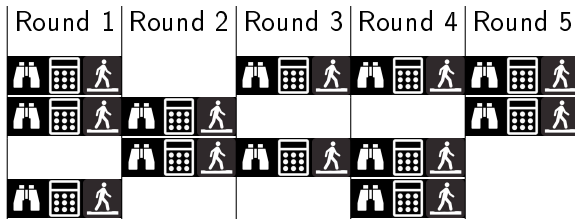
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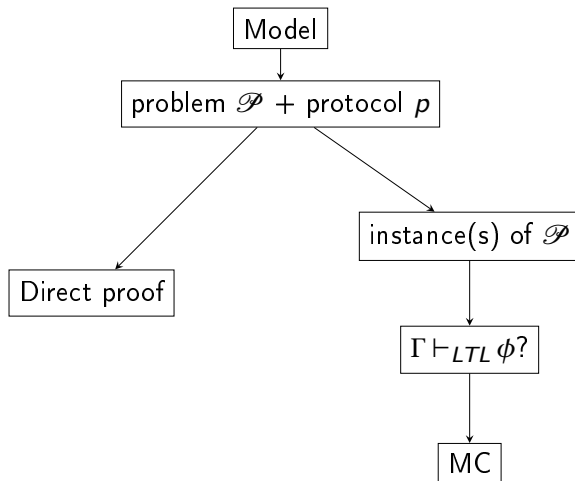
or Same phase for all active robots

- ▶ Time split into rounds
- ▶ **FSync**: **all** robots are activated each round
- ▶ **SSync**: only a **subset** is activated

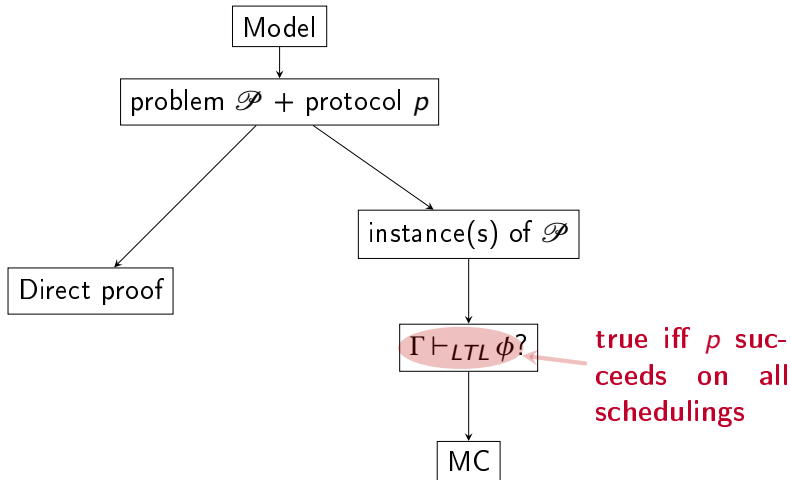
↪ **Fairness** assumptions on the scheduling (demon)



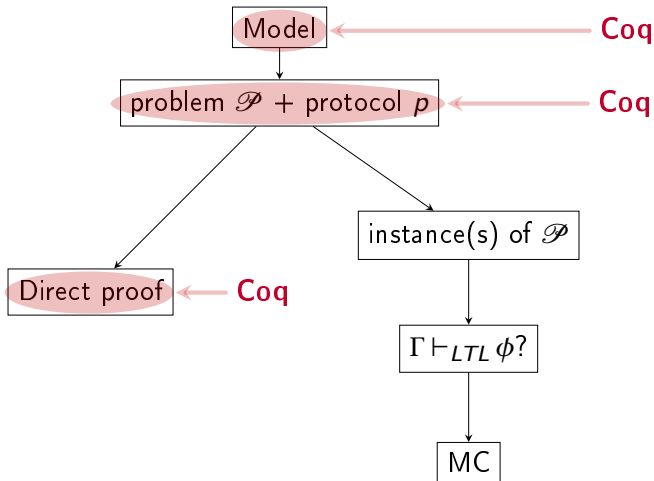
Proving Distributed Protocol



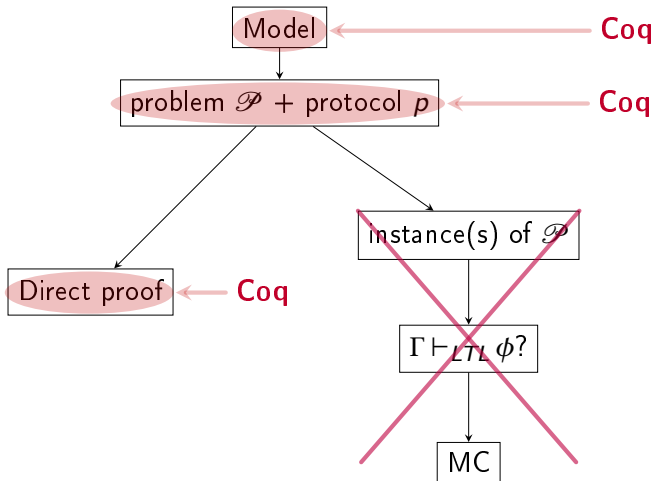
Proving Distributed Protocol



Pactole



Pactole



Pactole: a Coq Framework for Mobile Robots

Very Parametric (but still useful):

- ▶ Space
- ▶ State of Robots (**location**, memory, battery level, etc.)
- ▶ Sensors
- ▶ How states are updated during the move phase

Key Ingredients:

- ▶ Configuration function ident \rightarrow state
- ▶ Spectrum info on config from sensors
- ▶ Robogram function spectrum \rightarrow location
- ▶ Demon (scheduler) adversarial environment
- ▶ Round one step of execution

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Robogram: what happens for a robot?

Demon: what does the environment decide?

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4. Select the new frame of reference for non Byzantine ones
5. Decide how to update them depending on their destination

The Core of Pactole: the Round Function

Coq

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Definition round (r:robogram) (da:demonic_action) cfg
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fun id  $\Rightarrow$       (* for a given robot, we compute the new state *)
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fun id ⇒      (for a given robot, we compute the new state)
if da.(activate) id (see whether the robot is activated)
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  match id with
  | Byz b ⇒ da.(relocate_byz) cfg b          (byzantine *)
  | Good g ⇒
    (change the frame of reference)
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    let local_pos := get_location (local_cfg (Good g)) in
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    (apply r on spectrum)
    let local_traj := r spect in
    (return to the global frame of reference)
    let global_traj := lift_path (new_frame-1) local_traj in
    (the demon chooses how to perform the state update)
    let choice := da.(choose_update) cfg g global_traj in
    (actual state update performed by the update function)
    update cfg g global_traj choice
  end
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Proving in Pactole

Correctness proof:

1. Formalize your problem

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Correctness proof:

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3. Express your algorithm in the global frame of reference

Two different points of view

Global View (demon + proof)	Local View (robots)
absolute location	local frame of reference
robots: Byzantine or not (B / G)	indistinguishable robots
identifiers $\text{ident} = G + B$	
configuration = $\text{ident} \rightarrow \text{position}$	local configuration
	spectrum
	robogram
	: $\text{spectrum} \rightarrow \text{location}$
$\text{round } r \text{ d} : \text{config} \rightarrow \text{config}$	
$\text{exec} = \text{stream of configurations}$	

spectrum = degraded local view of a configuration

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Impossibility proof:

1. Formalize your problem
2. Assume given a robogram (a variable) + its properties
3. Prove that the algorithm does not solve the problem
(`round_simplify` not always needed)

Some results (✓ verified in Pactole)

- ▶ Convergence \mathbb{R}^2 , rigid, $\geq 1/3$ Byzantine, SSYNC
Impossibility [Bouzid et al. TCS'10] [SSS'13] ✓
- ▶ Gathering \mathbb{R}^2 , rigid, no Byzantine, SSYNC
Impossibility ($2 \times n$ robots) [Suzuki, Yamashita SICOMP'99]
[IPL'15] ✓
- ▶ Gathering \mathbb{R}^2 , rigid, no Byzantine, SSYNC, start not bivalent
Effective (universal) algorithm [DISC'16] ✓
- ▶ Gathering \mathbb{R}^2 , non rigid, no Byzantine, FSYNC, no multiplicity
Effective (universal) algorithm [SSS'16] ✓
- ▶ Exploration $\mathbb{Z}/n\mathbb{Z}$, discrete, k robots, no bizantine, SSYNC
Impossibility (k divides n) [ICDCN'17] ✓
- ▶ Discrete/continuous graphs, non rigid, ASYNC
Model equivalence [SSS'18] ✓
- ▶ Connection \mathbb{R}^2 , rigid, no bizantine, SSYNC
Effective (non optimal) algorithm [SSS'21] ✓

Case Study: Gathering

Objective

Have all (non byzantine) robots reach in finite time the same location (unknown ahead of time) and then stay there.

Let's formalize it!

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Coq

```
(* All good robots are at the same location [pt] (exactly). *)
```

```
Definition gathered_at (pt : location) (cfg : configuration) :=  
  ∀ g, get_location (cfg (Good g)) ≡ pt.
```

```
(* At all rounds of [e], robots are gathered at [pt]. *)
```

```
Definition Gather (pt : location) (e : execution) : Prop :=  
  Stream.forever (Stream.instant (gathered_at pt)) e.
```

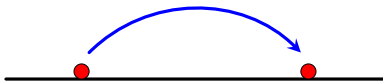
```
(* The infinite execution [e] is *eventually* [Gather]ed. *)
```

```
Definition WillGather (pt : location) (e : execution) : Prop :=  
  Stream.eventually (Gather pt) e.
```



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Generalizations:

- ▶ Even number of robots
- ▶ Type of line (\mathbb{Q} ou \mathbb{R}), higher dimension

Universal Algorithm

Key Ideas:

- ▶ Center of the smallest enclosing circle
- ▶ Move in stages (not all robots at once)
- ▶ Never create the configuration of the impossibility proof

Hard part of the proof: **termination** \rightsquigarrow **termination order**

- ▶ #robots on the circle decreases
- ▶ Stages
- ▶ Avoid looping with triangles

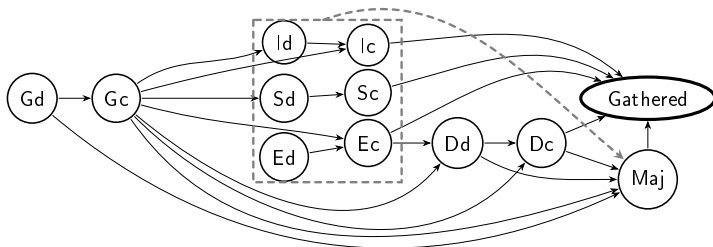
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Termination Order in Coq

Coq

```
Definition m_clean s := nG-s[target s].
Definition m_dirty s := nG-SECT_cardinal s.
Function measure (s : observation): nat * nat :=
  match support (max s) with
  | nil  $\Rightarrow$  (0, 0) (* no robot *)
  | pt :: nil  $\Rightarrow$  (0, nG - s[pt]) (* majority *)
  | _ :: _ :: _  $\Rightarrow$ 
    match on_SEC (support s) with
    | nil | _::nil  $\Rightarrow$  (0,0) (* impossible cases *)
    | pt1::pt2::nil  $\Rightarrow$  (* diameter case *)
      if clean s then (1,m_clean s) else (2,m_dirty s)
    | pt1::pt2::pt3::nil  $\Rightarrow$  (* triangle case *)
      if clean s then (3,m_clean s) else (4,m_dirty s)
    | _  $\Rightarrow$  (* general case *)
      if clean s then (5,m_clean s) else (6,m_dirty s)
  end
end.

Definition lt_config x y :=
  Lexprod.lexprod lt lt (measure (!! x)) (measure (!! y)).
```

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- ⊕ Ease of use for specification
- ⊕ Broadly applicable

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- ▶ Highly parametric space, sensors, execution model, ...
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- ▶ Common base of definition (no more mismatches)

⇒ A junction point for several formal results?

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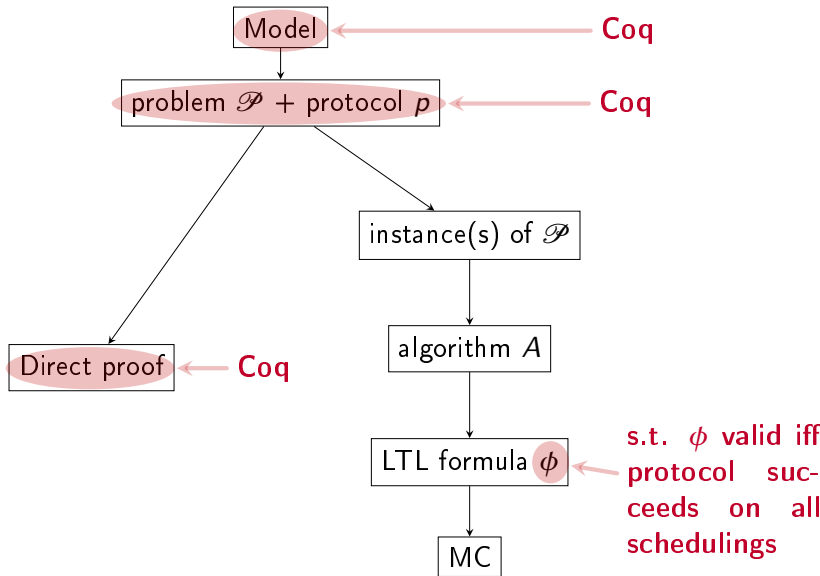
⊖ Caveat

- ▶ No fully automated procedure (yet)
- ▶ Building proofs is a lot of work

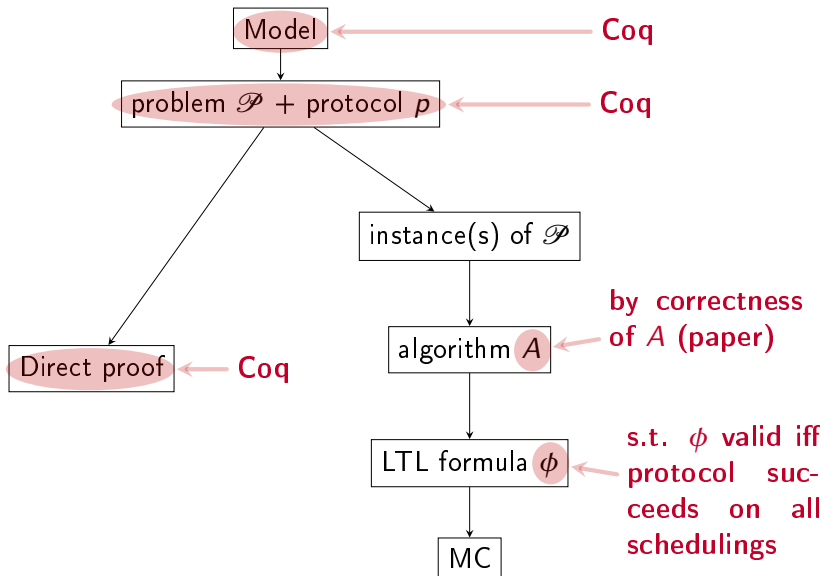
Future

- ▶ ANR SAPPORO (2019-2023):
 - ▶ Randomized protocols
 - ▶ Automated proofs (graph rewriting)
 - ▶ Non euclidean geometry (non-perfect sensors)
 - ▶ Luminous robots
- ▶ Link with model checkers

Appendix: Link with model checking



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