Mathis Bouverot-Dupuis

CNAM – ??

12 juillet 2022

Model

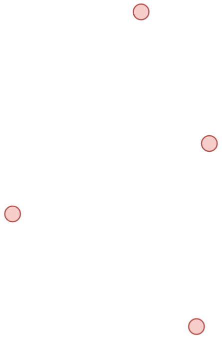
Weber Point

Alignment

Suzuki & Yamashita's Model

Very simple (dumb) robots :

- Points in R^2 (can overlap)



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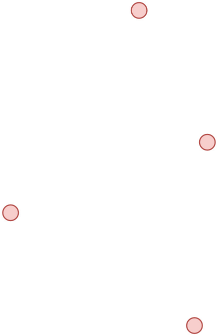

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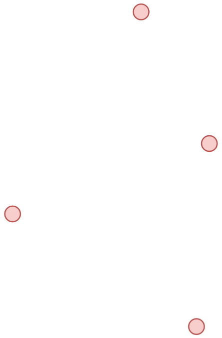

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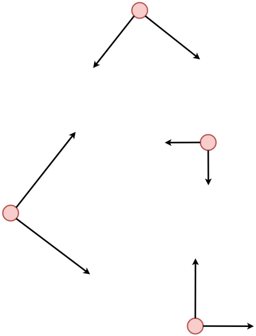

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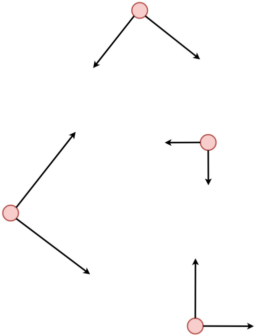

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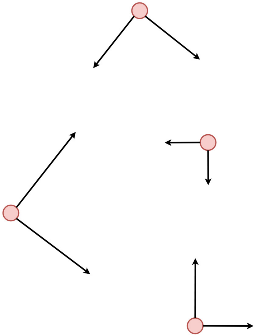

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- Similar to barycenter (sum of distances v.s. sum of distances squared).

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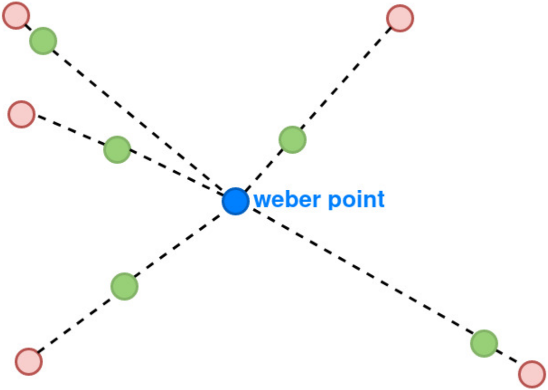
Why use the weber point ?

Contraction Lemma

- Let X, Y : multiset of points (cf. figure).



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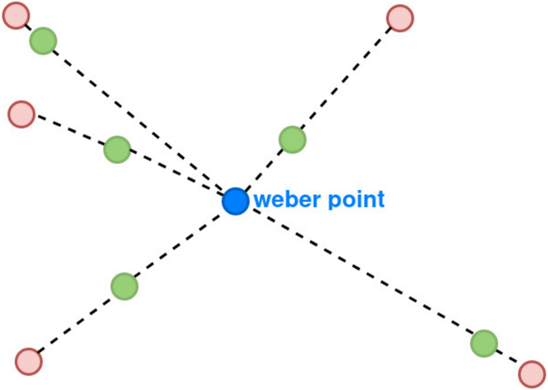

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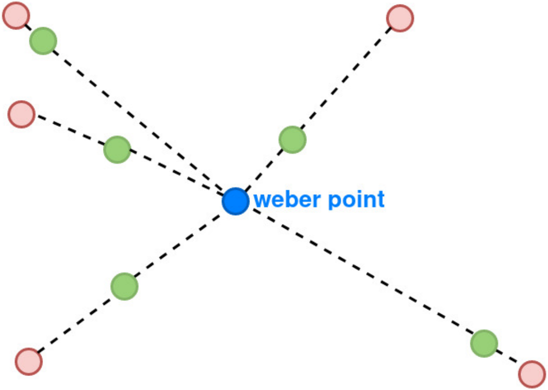

Contraction Lemma (proof idea)

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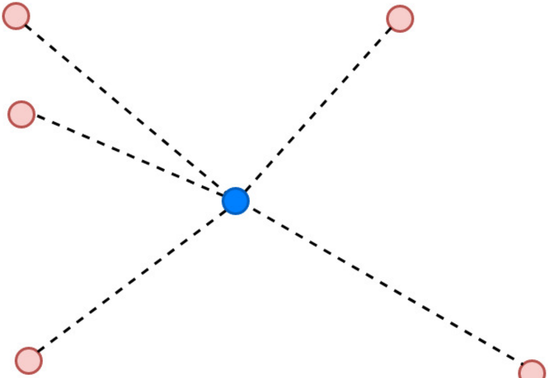

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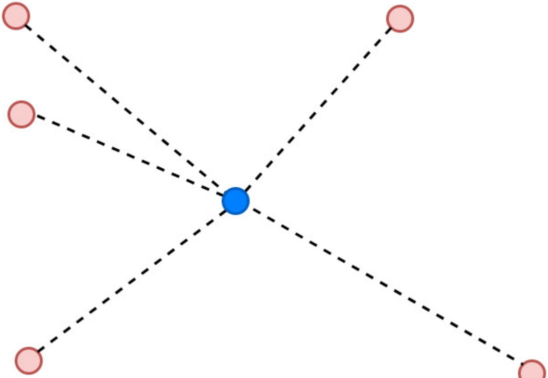

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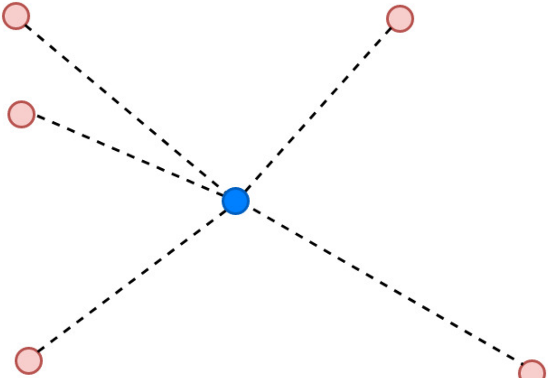

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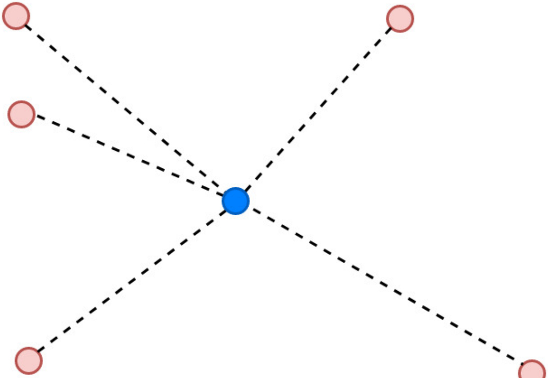

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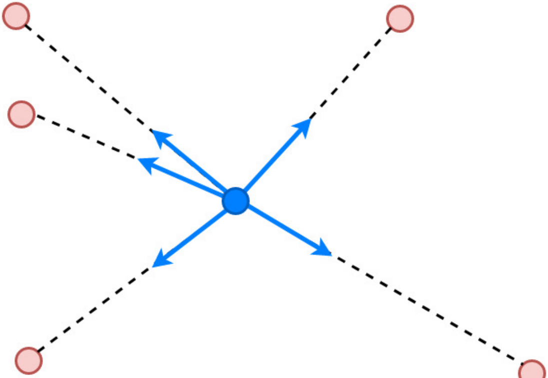
- Gradient : $\nabla D_X(p) = \sum_{x \in X} \frac{p-x}{\|p-x\|}$



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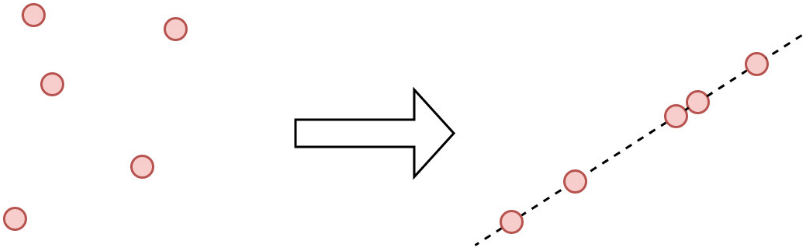
Goal : move robots to a common line, and make them stay on the line.



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Theorem alignment_correct :=
  ∀ (c:configuration) (d:demon),
    (* Hypotheses *) →
    eventually_aligned c d align_robogram.
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Definition observation := multiset R2.
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- ASYNC (first time in Pactole) & flexible.

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new_start <- straight_path start dest ratio
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- Other robots :

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new_ratio <- ratio + demon_ratio
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Flexible :

```
Coq
Definition flex_da_prop (da:demonic_action) (δ:R) :=
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    da.(activate) id = true →
    get_location (c id) ≡ get_destination (c id) ∨
    δ ≤ dist (get_start (c id)) (get_location (c id)).
```

Decreasing measure

We define a measure that decreases each time a robot is activated.