Artificial Neurogenesis in the Context Continual Learning

Master Masthematics, Vision et Apprentissage

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Abstract

Starting with a critical review of Elastic Weight Consolidation method (EWC), we argue that unconstrained memoryless modular strategies does not account for all dimensions of pollution which is an inherent limitation to solve continual learning. So why does the litterature seem to show positive results? The reason lies in the introduction of hyperparameters (HPs) that create trade-offs between the tasks. Therefore, we argue that EWC should not perform any better than any other approach that makes similar trade-offs. To verify this, we present naive baselines along with a more elaborated method that only make trade-offs and show that, with the same amount of hyperparameters, they achieve similar performance to EWC. Additionally, we study how performances are impacted by the amount of hyperparameters, from $O(1)$ to $O(n^2)$. However, with all these HPs, we are essentially overfitting on the benchmarks, which we demonstrate through the introduction of validation benchmarks.

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1 Introduction

1.1 Continual learning

We, humans are able to learn a wide variety of tasks throughout our lives. We can learn to play the piano, to speak a new language, to ride a bike, to cook a new recipe, etc. We can even learn to perform several tasks at the same time, such as playing the piano and singing at the same time. This ability to learn a sequence of tasks without forgetting the previously learned tasks is called continual learning. Continual learning is a fundamental aspect of human intelligence, and it is a key challenge in artificial intelligence. Indeed, when an artificial model using backpropagation is assigned

to learn a sequence of tasks, its final state after training on a task is percieved as an initial state to train on the next task, which will be overwritten during the training procedure on the new task, leading to catastrophic forgetting of the previous task. This is a major limitation of backpropagation on neural networks, and it is a key challenge in artificial intelligence. In this paper, we will discuss the problem of continual learning and we will introduce a new approach to continual learning that is based on the idea of growing neural networks.

In a very broad sense, one can define continual learning as

the problem of learning several tasks sequentially.

Although this assumption is sometimes relaxed in the literature [REFLA], the fact that tasks have to be learnt *sequentially* implies that once the training procedure on a task is done, its associated training data cannot be used again.

Continual learning comes in different flavors that vary depending on the working assumptions we make. Some of the most common frameworks include the following[REFLA].

In *Task Incremental Learning*[REFLA], the model is trained sequentially on a series of tasks, with each task involving a different set of data or objectives, such as sentiment analysis and then object classification within images. The model knows which task it is dealing with at any given time, and it often has task-specific components or outputs. This framework is very general and can be applied to a wide range of problems. Then comes *Domain Incremental Learning*[REFLA] framework, within which the model faces a sequence of tasks that involve the same objective but in different domains. During inference, the model is not informed which domain it is operating in and must generalize across these domains. Finally, *Class Incremental Learning*[REFLA] involves learning new classes sequentially, without task identifiers. The model is expected to classify data from any of the classes it has learned so far, even as new classes are introduced. This is often considered the most challenging form of continual learning, as the model must avoid catastrophic forgetting while continuously expanding its ability to recognize new classes.

Litterature also aknwoledges other frameworks such as *Lifelong Learning* [REFLA], and other classifications of these frameworks [REFLA].

1.2 Approaches to continual learning

The problem of continual learning has been heavily investigated in the past decade, leading to the development of various approaches to mitigate catastrophic forgetting. These approaches can be broadly categorized into regularization-based, replay-based and architectural.

Regularization-based approaches, which constrain the learning process through a regularization term embedded in the loss function, aim at preventing drastic changes to model parameters that are critical for previous tasks. Fundational methods to this category of approaches include Elastic Weight Consolidation (EWC) [REFLA], which estimates the importance of each parameter and penalizes changes accordingly, and Synaptic Intelligence (SI) [REFLA], which dynamically accumulates information about parameters importance throughout training of all the tasks.

Another important approach is *replay-based learning*, which involves storing or generating data from previous tasks to revisit during the training of new tasks. This helps maintain performance on older tasks by refreshing the model's memory of past data. Methods like Experience Replay [REFLA] store a small buffer of past samples which are forwarded to the model when training on new tasks, while Generative Replay [REFLA] uses a external generative model to recreate as many data from previous tasks as needed.

Finaly, *architectural approaches* modify the model's structure to accommodate new tasks while preserving knowledge from previous ones. Methods like Progressive Neural Networks (PNN) [REFLA] add new subnetworks for each task, allowing forward knowledge transfer from old tasks dedicated subnetworks, while keeping the new ones isolated. Conversely, Mixture of Expert Models [REFLA] leverage a gating system which is designed to identify and distribute tasks to expert subnetworks.

However, these categories do not constitute a proper partition of the litterature as they are not mutually exclusive. Approaches such as Architectural and Regularization 1 (AR1) [REFLA] fall both in regularization-based and architectural categories. Additionally, these categories do not encompass certain methods such as adversarial approaches [REFLA].

1.3 Our specific framework

Given the variety of frameworks and approaches, we need to narrow down and specify ours. In previous

subsections, we introduced frameworks and approaches to continual learning through categories. But beyond these categories, which provide a useful overview of the literature, what truly matters are the specific assumptions under which we operate.

First, we will focus on classification tasks involving the same set of classes. We assume that the model is notified when transitioning to the training of a new task. Each task must be handled strictly sequentially, meaning we do not permit any use of data from previous tasks. Additionally, we assume the total number of tasks is unknown and cannot be leveraged during the learning process.

As for the methods, our approach will be *modular*, meaning it will break down the learning process into subproblems and explicitly leverage dedicated modules to address these subproblems. These modules may consist of independent subnetworks, subnetworks that share some parameters, or groups of neurons distributed within the entirer network.

On the top of the modularity of our approach, we make to additional assumptions regarding the way we will process information. First, we will work with *unconstrained* architectures, meaning that we will not impose any architectural constraint on the model that could allow the processing or implicit remembering of the specificities of the past task. To this regard, convolutional or graph neural networks are out of our scope. We will also work with *memoryless* strategies, meaning that in addition to banning the use of data from previous tasks, we will not allow to leverage a replay buffer or generative rehearsal module.

2 Our approach : GroHess

2.1 Motivation

To address the problem of Continual Learning, one approach is to train independent specialized modules and a gating system that learns to recognize tasks, similar to Mixture of Experts Models [REFLA]. However, the tasks to be learned often have redundant features that we would like to leverage, so that only the new features of a task are learned without forgetting the old ones. The emergence of such features relies on constraints, starting with imposing small size of the model. Therefore, we would prefer to start training with a model of moderate size and expand it as more tasks are introduced.

Such an approach presents several challenges. First, it requires to determine which important features to retain. Then, we must figure out how to retain them. Finally, we need to determine how to use them to handle new tasks. To achieve this, we introduce GroHess, a new approach to architecture growth in the context of continual learning. GroHess is based on the idea of growing the model by adding new neurons to the model when the importance of a weight is high, as determined by the second derivative of the loss with respect to the weight. This is illustrated in 1. When the second derivative of the loss with respect to a weight is high - such as in w_1 - the weight is considered important as modifying it even a little bit would lead to a significant change in the loss. Conversely, when the second derivative is low - such as in w_2 - modifying the weight has little impact on the loss. This allows us to determine which weights are important and which are not, and to grow the model accordingly.

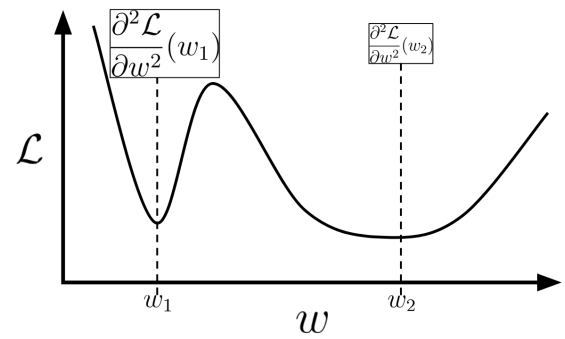


Figure 1: Illustration of our sense of importance of weights. When the second derivative of the loss with respect to a weight is high, the weight is considered important as modifying even a little bit would lead to a significant change in the loss. However, when the second derivative is low, modify the weight has little impact on the loss.

2.2 Description

More specifically, We control the growth of the model through two percentiles : the gradient percentile and the Hessian percentile. These percentiles allow us to define what is considered to be "high", relatively to the other weights first and second derivatives.

Once the training procedure on a task is done, we compute the second derivative of the loss with respect to each weight through all the data batches of the task. When the second derivative of a given weight with respect to the loss falls above the hessian percentile, comparatively to the second derivate of the other weights, it means that modifying this weight will strongly disturb what has been learnt on the task.

Thus, we compute a bianry mask recording which weights are important for the task according to the second derivative. Conversely, when training on a task, if the loss start to stagnate, we take a look at the last computed gradient with backpropagation. If the gradient of a given weight falls above the gradient percentile, comparatively to the other gradients, it means that according to backpropagation, this weight has to be strongly updated. As a consequence, when both the first and the second derivatives of a given weight are high, we have a tension that leads us into growing a new neuron, to accodmodate both backpropagation and the preservation of the knowledge of the task. The weight we grow from is then frozen, it wont be updated ever again, but we will keep backpropagating gradients through it.

Everytime one grows a neuron, this neuron has to be initialized, which means that 6 choices have to be made : the value of its bias, the value of the incoming weights, and the value of the outgoing weights and their respective gradients. The meaning of a growth is allow both the preservation of the knowledge of the task and the learning of the new task. As a consequence, we mostly copy the neuron we are growing from, and we only set the outgoing weights whose first and second derivatives were not both high to zero. This choice is motivated by the fact that we don't want the new neuron to disrupt the learning process.

2.3 Baselines

Our method is based on two ideas : a hessian-inspired mechanism to define weights importance and a growth mechanism to add new neurons to the model. To evaluate the relevance of these ideas, we introduce several baselines.

To evaluate the effectiveness of our method, which incorporates a Hessian-inspired mechanism for determining weight importance and a growth mechanism for adding new neurons to the model, we introduce four baselines.

In a first "vanilla" baseline, we define a model in the same initial configuration as the one used optimized with GroHess. During the training procedure through the tasks, no additional neurons are added, and no special mechanism is used to adjust or freeze weights according to their importance for the tasks. The model remains at its original size throughout the training process.

We also train a second "vanilla" baseline, but the initial size of its hidden layers is that of the average final hidden layer sizes when training with GroHess. So, from the beginning, this baseline is in the average configuration of a model trained with GroHess.

We also design a third baseline, where perform growth without GroHess, during the training procedure. This baseline involves starting the training process on the first task with a model of the same size as the one used with GroHess. However, everytime the model moves to training on a new task, it blindly adds new neurons to each layer, randomly initialezed, and without freezing any weight. This baseline helps evaluate the benefits of our approach of defining weights importance through Hessian coefficients.

Finally, our fourth and last baseline uses GroHess to freeze important weights without triggering the growing mechanism.This baseline is initialized in the same way as the second one, however, we leverage GroHess to freeze important weights during the training process, but without growing new neurons as GroHess would normaly do. This baseline is supposed to isolate the effectiveness of the growing mechanism to improve perfomances.

These baselines allow us to thoroughly analyze the contribution of the two main mechanisms of our approach, namely the Hessian-inspired weight importance mechanism and the growth mechanism, independently and in combination.

3 Main experiment

3.1 Permuted MNIST

A permuted MNIST (p-MNIST) dataset [REFLA] is a variant of the classic MNIST dataset where images are transformed through a fixed random permutation of pixel positions, creating a shuffled version of each original image. The label associated with each image remains the same, meaning the digits themselves are unchanged, but their visual structure is modified. Then, a p-MNIST task is define as solving the classification problem associated to a p-MNIST dataset. As each p-MNIST dataset is generated by a different permutation of MNIST dataset, there are 784! possible p-MNIST datasets and as many p-MNIST tasks. Finally, a p-MNIST benchmark is created by selecting a subset of these tasks. See 3 for illustration.

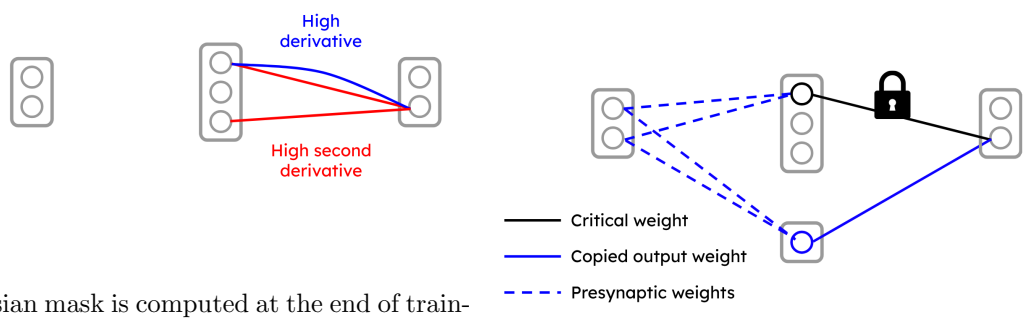
Permuted MNIST is particularly significant in the context of continual learning as it offers the opportunity to generate arbitrary long sequences of tasks with a fixed number of classes and a fixed number of sam-

Algorithm 1 Training GroHess on n tasks

Input:
- Model Σ_{ini}
- List of n tasks $L = [(X_1, Y_1), \ldots, (X_n, Y_n)]$
- Hessian percentile $\tau_{hessian}$
- Gradient percentile $\tau_{gradient}$

Output: Trained model Σ_{fin} solving each task

Step 1: Train Σ_{ini} on (X_1, Y_1)
Step 2: Compute $\nabla^2_{\Sigma_{ini}} \mathcal{L}_{(X_1,Y_1)}$ for each weight and define the binary mask $\mathcal{M}_{hessian}$ using $\tau_{hessian}$
for each reamining task (X_i, Y_i) **do**
  for each batch (x_i, y_i) in (X_i, Y_i) **do**
    Step 3: Perform training step on (x_i, y_i)
    if loss stagnates **then**
      Step 4: Compute $\nabla_{\Sigma_{ini}} \mathcal{L}_{(x_i,y_i)}$ and define the binary mask $\mathcal{M}_{gradient}$ using $\tau_{gradient}$
      Step 5: Compute the binary mask $\mathcal{M}_{overlap}$ as the element-wise product of $\mathcal{M}_{hessian}$ and $\mathcal{M}_{gradient}$
      Step 6: Grow as many neurons on Σ_{ini} as $\mathcal{M}_{grow}$ has column with at least a 1
      Step 7: Freeze the weights we just grew from
    end if
  end for
  Step 8: Compute $\nabla^2_{\Sigma_{ini}} \mathcal{L}_{(x_i,y_i)}$ and update $\mathcal{M}_{hessian}$ using $\tau_{hessian}$
end for



(a) Hessian mask is computed at the end of training on task $n-1$ and Gradient mask is computed during training on task n.

(b) The important weight is frozen and a new neuron is grown from it.

Figure 2: Illustration of the GroHess procedure.

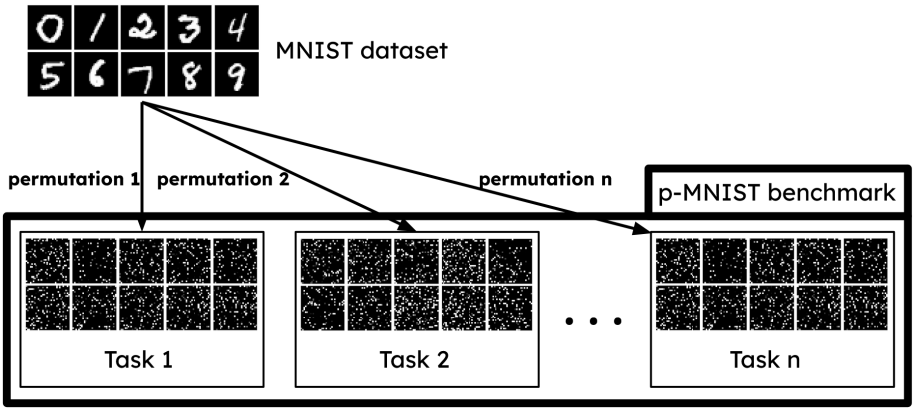


Figure 3: Illustration of the process of creating a p-MNIST benchmark.

ples per class. Additionally, users have a great control over the difficulty of a benchmark, which is directly associated with the similarity of the permutations used to generate the tasks of the benchmark. Finally, the p-MNIST benchmark is a well-established benchmark in the continual learning literature, which allows for comparison of different approaches.

On the other hand, litterature has come with a lot of criticism about the p-MNIST benchmarks. First, they are rather simple benchmarks, which do not reflect the complexity of real-world tasks. Second, the random permutations break the spatial structure of images we can find in the real world, which means that the p-MNIST benchmarks are not well-suited to evaluate method that require spatial reasoning on tasks such as object detection or segmentation. In particular, the permutations of pixels disrupt with the assumption of locality and translation invariance which are at the basis of convolutional neural networks. As a consequence, approaches leveraging convolutional neural networks are disadvantaged on p-MNIST benchmarks.

Despite this criticism, we have chosen to use p-MNIST benchmarks. They are relevant for our study of GroHess as their very simplicity allows to understand the fundamental principles of our exploratory method. Additionally, the ability to generate not only several tasks, but also several comparable benchmarks will allow us to motivate and introduce a validation paradigm in continual learning.

3.2 Training protocol

Our model consist in a multi layer perceptron with 2 hidden layers (or 3 layers in total). Input and output sizes are 784 and 10 respectively. Each hidden layer contains initialy contains 300 neurons. The growth of the model happens from the output layer, which means that when a weight has both a high first and second derivatives, we grow a neuron on the layer preceeding it (in the forward sense). We control the growth of the model through two percentiles : the gradient percentile and the Hessian percentile. When both first and second derivative of the loss with respect to a weight are above the gradient and Hessian percentiles, respectively, we grow a new neuron.

We train our models following GroHess algorithm on a sequence of 10 p-MNIST tasks, each with the usual 10 MNISTS classes. For each task 80% of the train set is used for training, 20% is used for validation and 100% of the test set is used to test the model. We train GroHess with the Adam optimizer and the cross-entropy loss. For each task, the number of epochs and the learning rate are left as optimizable hyperparameters. We use a batch size of 128. See Table 1 for the complete list of hyperparameters.

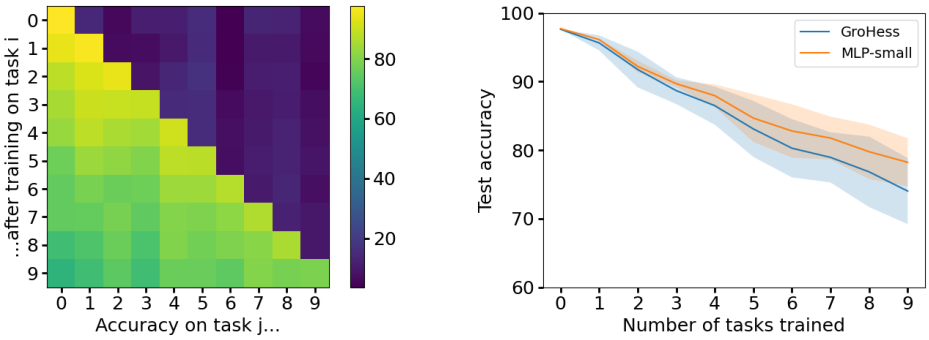
Hyperparameters (HPs)	
Fixed HPs	**Value**
Number of Layers	3
Initial hidden layers size	300
Batch Size	128
Optimizer	Adam
Loss	Cross-entropy
Growth happens from	Output
Gradient percentile	0.98
Hessian percentile	0.98
Optimized HPs	**Range**
Learning Rate	1e-5 - 2e-3
Number of epochs	2 - 10

Table 1: Hyperparameters value or range

As the training loop is called once for each task, it means that a learning rate and number of epoch have to be found through hyperparameter optimization (HPO) for each task. HPO is performed using the Optuna library [REFLA] with the Tree Parzen Estimator (TPE) algorithm. We use the validation set to evaluate the performance of the model and to optimize the hyperparameters. We use the average accuracy over all tasks as the metric to optimize. We perform HPO for each task sequentially, meaning that once the best hyperparameter values for a task are found, we retrain the model with them before moving to the next task. We repeat this process 10 times with random seeds 88 to 92 and we report the average test accuracy over all runs.

4 Results

In 4a, we show the test accuracy on each task as GroHess move forward through the tasks. This visualization provides a better insight than aggregated metrics as it allows us to inspect directly the performance of the model on each task during training. An interested feature of our method is that the coefficients below the diagonal of the matrix, on the same line are relatively equal, which means that our GroHess is able to retain knowledge of the previous tasks without completely overwritting them. Additionally, we observe that column-wise, the accuracy is decreasing, which is reasonnable : as the model learns tasks, it has to remember more past tasks, which makes the learning



(a) Test accuracy on each task as GroHess move forward through the tasks

(b) Average test accuracy on tasks trained so far

Figure 4: Evolution of the average test accuracy during training. The plots show the average test accuracy over all runs on seed 88 to 92. 4b also represents the standard deviation over runs.

process harder.

Additionnaly, 4b shows the average test accuracy on tasks trained so far. In other words, the value associated to abscisse i is the average of the coefficients below the diagonal of the ith line of the matrix in 4a. This plot is a more classical way to represent the performance of the model as it moves forward through the tasks, as it shows the ability of the approach to learn the first tasks of the benchmark. The last point of this plot represent the average test accuracy over all tasks, once the model has been trained on all of them.

Additionally, we present the results obtained with GroHess along with a comparison to litterature in 5a. We also compare our approach to the baselines we introduced in the previous section. Each dot in this plot represent the average accuracy over all tasks of a p-MNIST benchmark after training on all of them sequentially. For both our methods and the methods reproduced, the error bars represent the standard deviation over runs on seed 88 to 92. The permutations used to generate the p-MNIST tasks are the same for all the methods we ran, namely the first permutation as suggested by the random module of numpy when provide seeds 1 to 10.

Then, we group the results by type of approach in 5b. We identify detached groups. The methods relying on replay-based learning or rehearsal are performing much better than the other methods. The performances of the regularization methods are more spread, but they are generally better than our baselines and GroHess.

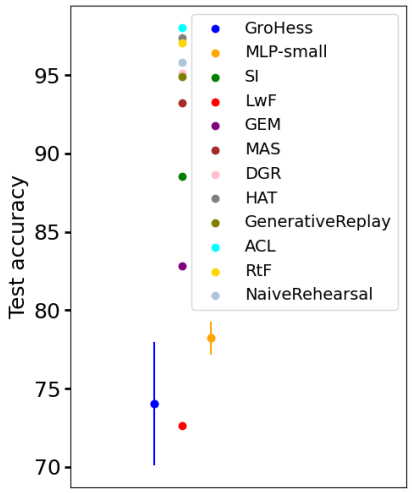
5 Paradigmatic discussion

5.1 Hyperparameters optimizations

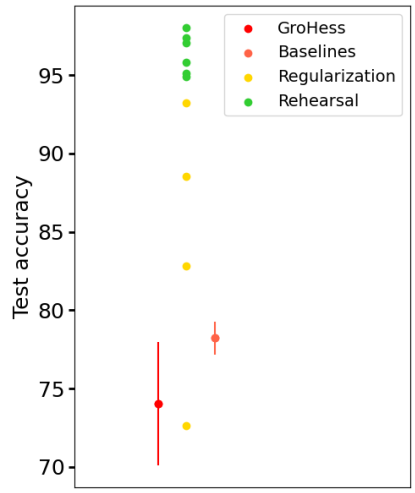
In the section in which the training protocol is introduced, we suggested to have only two hyperparameters, the number of epochs and the learning rate. However, as we mentionned, we performed one HPO for each task we train our model on. As consequence, despite having two hyperparameters, we end up with 10 values for each of them, which means that the whole training procedure on the benchmark carries 20 hyperparameters.

Continual learning enables several ways to manipulate hyperparameters. In this subsection, we will discuss various approaches to manipulate them.

Let's introduce a hyperparameter λ. One could set this hyperparameter to be the same for all the tasks, which would result in a total of 1 hyperparameter. To set the value of this hyperparameter, one could perform a single HPO on the first task, and keep using the same value for the forecoming task. However, when proceeding that way, the hyperparameter is not optimized for each task, only for the first one. Therefor, this way of performing HPO is not suited for our continual learning framework. In order to keep λ to only one value while optimizing it on all the tasks, one could perform a single HPO on the whole benchmark. That means that once a value for λ is suggested by the HPO algorithm, we would train through all the tasks using this value. At the end of the HPO, we would end up with a single value of λ that would be tuned for all the tasks. However, optimizing in such a way require to have access to all the tasks at the same time, which breaks the continual learning framework. Indeed the value of λ used when training on the first task as to



(a) Our results, along with results collected in the litterature on the 10 tasks p-MNIST benchmark [REFLA].



(b) Same results as in 5a, but grouped by type of approach.

Figure 5: Results

be decided before accessing data from the latter tasks. This way of performing HPO is highly problematic as it causes data leakage. That is why we refere to it as a cheated HPO with respect to continual learning framework. See [Table 2] top-left.

Alternatively, one could set this hyperparameter to a specific value λ_i for each task i, this would require to perform an HPO on each task and would result in a total of n hyperparameters. This way of performing HPO is suited for continual learning framework, as tasks are treated strictly sequentially. This is how we proceeding when training with GroHess and it is also the most common approach to HPO in the litterature. See [Table 2] bottom-center. As optimizing for the value of λ_{i+1} requires to have the value of λ_i, we refere to this approach as greedy HPO.

Remarkably, one could merge both of the previous approaches is a cheated HPO with n parameters. This would require to perform a single HPO on the whole benchmark, but instead of suggesting a single value of λ at each trial, the HPO algorithm would suggest values $\lambda_1, ..., \lambda_n$, one for each task i. This way of performing HPO is even more problematic than the first way as it combines data leakage and a rather high number of hyperparameters. See [Table 2] top-center.

Finally, some methods in the literature introduce a hyperparameter λ that is explicitly used to make tradeoffs between how much the model should learn from the new task versus how much it shoud retain from the previous tasks [REFLA]. Such a hyperparameter can be optimized as describded above, but it may also be used to perform tradeoffs directly between tasks. For instance, when training on task 2, one could have a $\lambda_{2,1}$ that would be used to make tradeoffs between how much the model should learn from task 2 and how much it should remember from task 1. When moving to task 3, one could have a $\lambda_{3,1}$, to tune how much the model should learn from task 3 and how much it should remember from task 1 along with a $\lambda_{3,2}$ for how much the model should learn from task 3 versus how much it should remember from task 2. More generally, one could even set a different value for this hyperparameter for each pair of tasks, meaning that one could introduce $\lambda_{i,j}$ for mitigating catastrophic forgetting of task $j < i$ when learning task $i > 1$. Ultimately, this would result in a total of $\frac{(n-1)n}{2}$ hyperparameters. Similarly to the HPO with n hyperparameters, this way of performing HPO could fit the continual learning framework, see [Table 2] bottom-right, but it could also be done is a cheated way, see [Table 2] top-right.

Despite hyperparameters manipulation being critical, it seems that some confusion remains in the litterature. The overwhelming majority of papers do not report their way of performing HPO or remain fuzzy about the management and values of certain hyperparameters [REFLA]. On the other hand, among the papers that do report their HPO procedure, some fall into the cheated HPO category, which calls into question the legitimacy of their results [REFLA].

For instance the original Elastic Weight Consolidation (EWC) paper [REFLA] seems to suffer from this is-

Number of HPs / Number of HPOs	$O(1)$	$O(n)$	$O(n^2)$
1 cheated HPO	$\boxed{T_1 \quad \cdots \quad T_m}$ $\boxed{\{\lambda\}}$ HPO	$\boxed{T_1 \quad \cdots \quad T_m}$ $\boxed{\{\lambda_1, ..., \lambda_n\}}$ HPO	$\boxed{T_1 \quad \cdots \quad T_m}$ $\boxed{\{\lambda_{i,j} \mid \forall 2 \leq j < i \leq n\}}$ HPO
n greedy HPOs		$\boxed{T_1}$ $\boxed{\{\lambda_1\}}$ HPO_1 $\longrightarrow \ldots \longrightarrow$ $\boxed{T_n}$ $\boxed{\{\lambda_n\}}$ HPO_n	$\boxed{T_2}$ $\boxed{\{\lambda_{2,1}\}}$ HPO_2 $\longrightarrow$ $\boxed{T_3}$ $\boxed{\{\lambda_{3,1}, \lambda_{3,2}\}}$ HPO_3 $\longrightarrow \ldots \longrightarrow$ $\boxed{T_n}$ $\boxed{\{\lambda_{n,1}, \ldots, \lambda_{n,n-1}\}}$ HPO_n

Table 2: HPOs in continual learning

sue. EWC is a regularisation method that attempts to tackle catastrophic forgetting by penalizing changes in important weights. The weights that are important for a task are the ones that updating would cause the model to move away from the solution of the task. Authors leverage the Fisher Information Matrix to define this importance and enforce the update of important weights to be performed in a direction that does not hurt the knowledge on the tasks already learnt. As introduced in the original paper, training on task 2 the loss used by EWC can be written as $\mathcal{L} = \mathcal{L}_2 + \lambda \mathcal{L}_{reg}$, where $\mathcal{L}_2$ accounts for maximizing the performance on task 2, λ is a positive scalar and $\mathcal{L}_{reg}$ accounts for the regularization between task 1 and 2.

However, in this paper, authors do not explain how this regularization term scales with the number of tasks. It could be the case that the same value of λ is used for all the tasks, which would lead to write the loss used while training on task m as

$$\mathcal{L} = \mathcal{L}_m + \lambda L_{reg,m} \tag{1}$$

or it could be the case that a different value of λ is used for each task, which would lead to write the loss used while training on task m as

$$\mathcal{L} = \mathcal{L}_m + \lambda_m L_{reg,m} \tag{2}$$

or the authors might want to make tradeoffs between each pair of tasks, introducing a loss of the following form when training on task m

$$\mathcal{L} = \mathcal{L}_m + \sum_{i=1}^{m-1} \lambda_{m,i} \mathcal{L}_{reg,m,i} \tag{3}$$

Each of these 3 possible ways of proceeding would result in a total of $O(1)$, $O(n)$ or $O(n^2)$ hyperparameters, respectively. However, the authors do not provide any information about the way they proceeded, which is highly problematic. Additionally, authors do not report the value of the hyperparameter they introduced nor the process of optimizing it and they do not provide their code. This lack of clarity is a major issue as

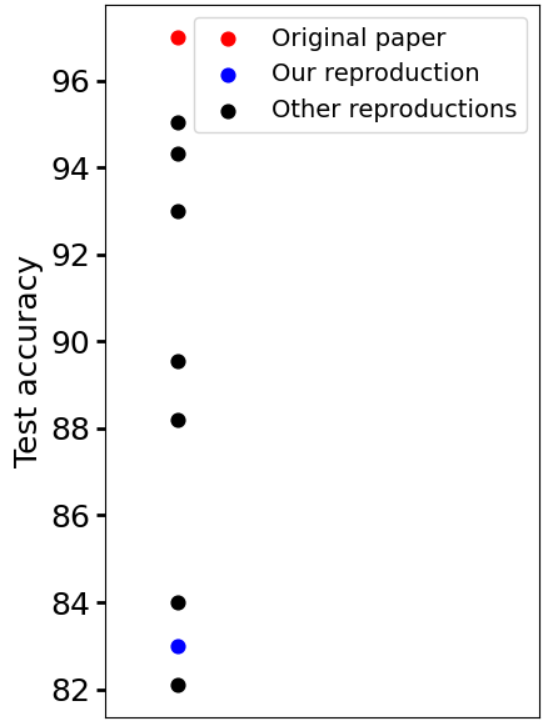


Figure 6: Summary of the results we identified in the litterature about EWC on 10 tasks p-MNIST benchmarks [REFLA].

it makes it impossible to reproduce the results of the paper. Despite the fact that the community reimplemented the method, the method itself does not include the manipulation of hyperparameters made by athors. As a consequence, attempts to reproduce these methods are very disparate and yield contrasted results. See [6] for a summary of the results we identified in the litterature.

Remarkably, all the papers reporting results above or equal to 90% accuracy on the 10 tasks p-MNIST benchmark are either performing a cheated HPO or not explaining their HPO procedure. In particular, the original paper mention that the size of the hidden layers is an optimizable hyperparameter. However, the size of the model does not change when training on the benchmark. This suggests that the HPO performed

by the authors of the original paper covers the entire benchmark, that is, the HPO is very likely cheated and include $O(n)$ hyperparameters. See Table 2 top-center.

5.2 Validation

Actually, what we refere to as "cheated HPO" where a single HPO is performed through all the tasks at the same time, could be just fine if one was performing the HPO on a benchmark, and then showing the performances of the approach on another benchmark with the same hyperparameters. Such an approach wouln't even break the hypothesis according to which we do not know the number of tasks in advance. On would just have to perform the HPO on a benchmark with a huge number of tasks, and then show the performances of the approach on another benchmark, with less tasks, using only the hyperparameters computed for the first tasks of the HPO benchmark.

Therefor, we introduce a validation paradigm on benchmarks to measure overfitting on benchmarks. We perform HPO on a benchmark and then measure the drop of performance on another benchmark. However, one could argue that performing HPO on a benchmark like p-MNIST and validating the approach using optimized hyperparameters on a benchmark derivated from CIFAR, for instance, would not be fair. And in order to avoid defining a brittle and handcrafted metric complexity or similarity between benchmarks, we will stick to p-MNIST benchmarks. Indeed, this validation paradigm we are introducing is one of the reasons why we chose to use p-MNIST benchmarks in the first place. Just as we did in 3, we generate 10 additional p-MNIST benchmarks, each with 10 tasks. As a consequence, we have benchmarks that sahre the same complexity and similarity from a statistical point of view. We do not perform any HPO on these validation benchmarks. Instead, we reuse the hyperparameter found on the previous benchmark, and simply retrain GroHess with them on the validation benchmarks. See 7 and 8 for the results of this validation paradigm.

For the HPO benchmark and each of the 10 validation benchmarks, 7 shows the test accuracy on each task once training with GroHess is complete on all the tasks. These results are run and averaged over seeds 88 to 92. At first glance, it seems that the performance on old tasks are better for the HPO benchmark, suggesting that it is harder for the model to mitigate catastrophic forgetting if the hyperparameters have not been specifically tuned on the benchmark. To confirm this, for each task, we compute the average test accuracy through the validation benchmarks (column wise average) and report the results in 8. In addition, this figure shows the standard deviation for performances on the validation benchmarks, along with the performances obtained on the HPO benchmark (left plot). The right plot shows the same results, averaged out over all tasks. We observe that the average test accuracy over all the tasks of the HPO benchmark are greater by an amount up to 9% than the averge one on the validation benchmarks. This figure confirms that, despite the fact all the benchmarks have the same complexity, performing an HPO on a benchmark is necessary to get the best performances. In other words, GroHess lack of versatility and is overfitting on the benchmark through the HPO process.

These results show that the choice of hyperparameters is at the very core of the performances of our method. It means that catastrophic forgetting is mit-

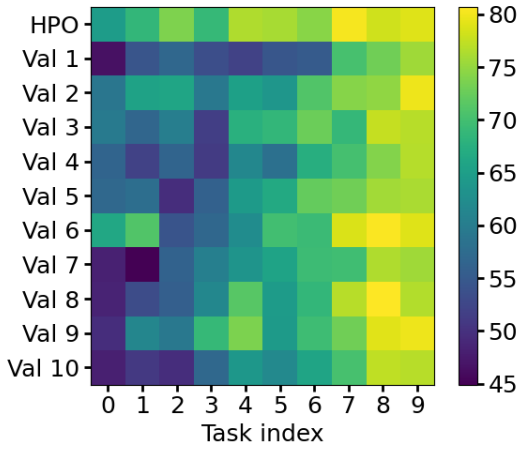


Figure 7: Test accuracy on each task once training with GroHess is complete on all the tasks.

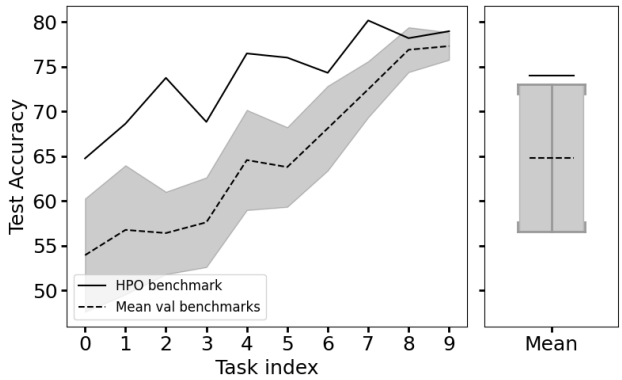


Figure 8: Results from 7 averaged over the valdiation benchmarks (left plot) and additionally averaged over all tasks (right plot).

igated through very specific arbitrations of how the model should learn from new tasks as they come, that is, it seems that our method relies on tradeoffs.

6 Critical comparison to litterature

In the following section, we will develop remarks about the methods fitting with our hypothesis. This will a opportunity to compare GroHess to other modular memoryless and unconstrained methods.

6.1 Tradeoffs in continual learning

In the last section, we aknwoledged that the hyperparameter optimization performed in the context of continual learning yield numerous hyperparameters. This can be an implicit way to perform trade-offs between how much the model should learn from the new task versus how much it should remember from the previous tasks. This lead has been confirmed through validation experiments, where we have seen that the very subtle differences between p-MNIST benchmarks are enough to impact the perfomances by an amount of 9%. As a consequence, we hypothesize that the HPO procedure is performing tradeoffs between the tasks.

An interesting way to investigate this hypothesis is to have a look at the specific hyperparameters retained by the HPO. Figure 9 shows the best values of the hyperparameters found by the HPOs ran on each task of the HPO benchmark. We observe that the learning rate is decreasing by over an order of magnitude as the model moves forward through the tasks. Similarly, we note that the best number of epochs used to train on a task was the highest allowed for the first tasks, and then quickly decreased for the latter tasks. As a consequence, the model model is learning less and less of each tasks. Now we interpret this behavior as a attempt to keep track on previously acquired knowledge while learning on the new task. That is, our approach is mitigating catastrophic through tradeoffs between the specificities of the tasks. As the defenrences between the specifities of the tasks rely on the permutations used to generate the tasks, our interpretation provides an explanation why the model is overfitting on the HPO benchmark. Indeed, the tradeoffs necessary to perform well on a given validation benchmark would require other hyperparameters than the ones found on the HPO benchmark.

Furthermore, in addition to making tradeoffs between learning and memory for each pair of tasks, methods like GroHess and those based on regularization,

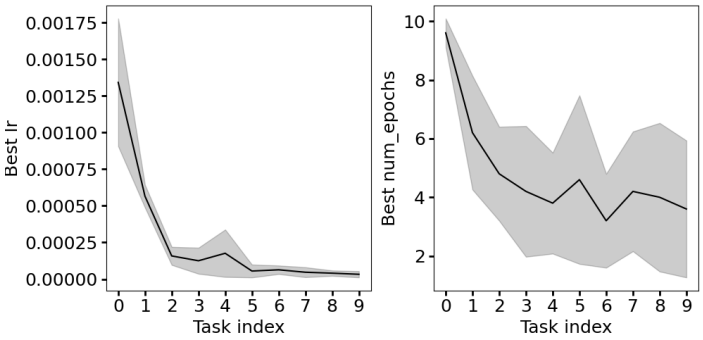


Figure 9: Best values of the hyperparameters (learning rate on the left and number of epochs on the right) found by the HPOs ran on each task of the HPO benchmark.

such as EWC, SI, LwF or AFEC [REFLA], incorporate trade-offs within the optimization process. Indeed, the regularization term used in these methods is intended to perform tradeoffs between the importance of the new task and the importance of the old tasks. They are controlled by a scalar, which is a hyperparameter is optimized through the HPO procedure. Similarly, through the computation of the Hessian and Gradient masks of importance, GroHess is trading how much to update the weights that are the most crucial for the past tasks versus how much to update these weights for the benefit of the new task.

Even though all these methods are performing tradeoffs, all the tradeoffs are not the same. From a method to another, the hyperparameters are not the same, which means that the tradeoffs that can be performed through HPO are not the same. This explain the disparities between the results of regularization methods gathered in 5b. Additionally, the tradeoffs perfomed by regularization methods are continuous, in the sense that they affect all the weights of the model more or less intensively. On the other hand, GroHess performs discrete tradeoffs, only impacting a few weights, leaving the other ones completely unchanged. Finally, to explain results obtained with GroHess that happen to be even poorer than the ones yielded by our baselines, we note that our growing procedure is not optimal. Indeed, it is not embedded in an optimization procedure, as it is the case for the baselines and regularization methods who are optimized through backpropagation only. Thus, it is likely that our growing procedure is simply disturbing the learning process done through backpropagation.

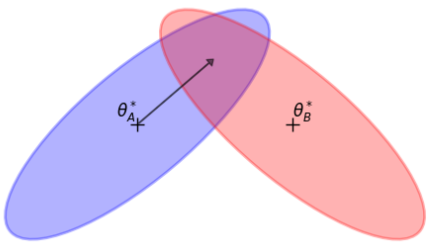


Figure 10: Representation of the parameters space as depicted in some regularization papers [REFLA]

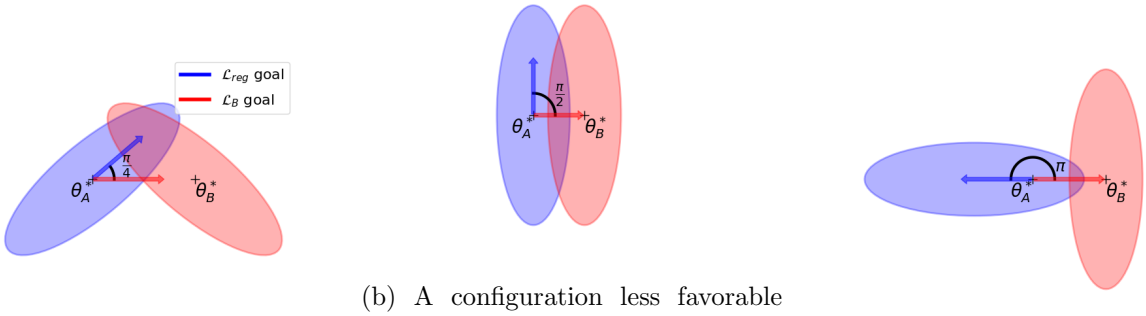
6.2 Geometrical argument

In this subsection, we will discuss the geometrical motivation behind several regularization methods such as EWC [REFLA]. Very often, the method is motivated by the representation of a 2-dimensional parameter space, where two ellipses represent regions of low loss when provided data from task A or task B. See 10. Once training on task A is done, one assumes that the model is in a state close to the argument of minimum of the loss for data associated to task A, θ_A*. Then, the intuition is that the regularization term introduced in the method will prevent the parameters from converging directly toward the argument of the minimum of the loss for task 2, θ_B*, when learning on task B. Instead, the parameters should converge to an overlapping region where both loss on task A and loss on task B are small. However, this representation is misleading, does not reflect the complexity of the problem and, as a consequence, does not hold in practice.

A first objection is that with a regularization method, the direction of parameters modifications does not aim for a minimum of the model trained on the combined data from tasks A and B, but rather towards the local minimum of the model on task B's data that is closest to the local minimum reached by the model trained solely on task A's data.

Second, the Fisher Information Matrix introduced in EWC is leveraged to approximate a metric that indicates what is considered to be the best path to move toward low loss on task B, compromising with increase of loss on task A. However, loss on task A is not directly leveraged, as when training on task B, samples from task A are not accessible anymore. As a consequence, authors says that in order to stay close to the region of low loss on task A, it is reasonable to try to update the parameters in a direction of high curvature of the ellipse associated to task A. Indeed, the part of the ellipse that has the highest curvature is also the one that is the furthest from θ_A*, which makes it a direction in which updates of the parameters can be carried without increasing the loss on task A too much. However, contrary to the representaion in 10, this is not a guarantee that the model will converge to the center of the overlapping region. Indeed, the overlapping region might not be aligned with the axis passing through θ_A* and the point of ellipse associated with task A with the highest curvature. See **??**. In these representation, the red arrow represent the update direction according to $\mathcal{L}_B$ and the blue arrow represent the update direction enforced by $\mathcal{L}_{reg}$. The actual update direction is then decided by the value of λ and falls inside the cone generated by the two arrows, without covering the whole cone as the linear combination is done through a single scalar. Thus, what matters in the situations depicted in 10 is the angle between the two arrows along with the orientation of the blue arrow relatively to the red ellipse. The orientation of the red ellipse relatively to the blue arrow accounts for the succes of the update : is the update going to land in a region where both loss on task A and on task B is low ? On the other hand, the angle between the red and the blue arrows accounts for how much $\mathcal{L}_B$ and $\mathcal{L}_{reg}$ are acting jointly. In the case depicted in 11a, no matter what the value of λ is, the update will be perfomed toward the overlapping region. This means that the configuration depicted in EWC papers is a really convenient scenario. On the other hand, looking at the configuration in 11b, we see that the update will be perfomed toward the overlapping region only if the value of λ is low enough, because $\mathcal{L}_{reg}$ is pulling the parameters away from the overlapping region. Ultimately, 11c depicts an extreme scenario, where $\mathcal{L}_{reg}$ is actively acting against $\mathcal{L}_B$. In this configuration, any none zero value of λ would make the update worse both with respect to task A and B.

Furthermore, these illustrations represent a 2-dimensional parameters space, however, in the experiments conducted in the litterature, the parameter space is $\mathbb{R}^n$ with $n \sim 10^5$ or even orders of magnitudes bigger. Additionally, as we already mentionned earlier, the number of tasks in a continual learning benchmark is generally above or equal to 10, and the research community ultimately expects to be able to tackle as many as hundreds of tasks. As a consequence of these



(a) Configuration depicted in EWC papers

(b) A configuration less favorable where both the angle and the orientation of the red ellipse are not convenient

(c) Extreme configuration where $\mathcal{L}_{reg}$ is actively acting against $\mathcal{L}_B$

Figure 11: Several possible configurations in the parameters space

two remarks,the area (or volume) of the overlapping region tends doubly towards 0. On one hand, this representation accounts for two tasks, but in real life scenarios, there are many more. And as the number of tasks increases, the area of the overlapping region decreases, has 0 zero as a lower bound, so converges. But it has no reason to converge toward a non-zero value as the relative position of the low loss ellipses is a priori independant between the tasks, or at least, nobody has provided a better lower bound than 0, to the best of our knowledge. On the other hand, the previous remark implies that there exist a number of tasks for which the overlapping region of interest is small enough to fit within an Euclidean sphere whose radius is strictly smaller than 1, which volume, therefore, converges toward 0 as the dimension of the parameters space increases.

So, an increase in the number of tasks makes it less likely to find a region where the loss is low for all the tasks, while an increase in the number of parameters of the model somewhat pulls the low loss regions appart from each other. These two remarks do not constitute a strict argument against the regularization methods but rather bring to light the fact the motivation behind them is somewhat flawed. Additionally, the explain why litterature report very poor performances of EWC on longer and harder benchmarks than p-MNIST [REFLA]. Overall, our remarks mostly point out the absence of guarantees. The absence of guaranty is not dramatic though, as deep learning tends to be able to perform well in practice without strong theoretical results. But what is more problematic is that authors of modular, memoryless and unconstrained methods generally do not even have control over the object of our remarks, such as the relative position of the ellipses.

Finally, the worse inherent limitations of modular, memoryless and unconstrained methods is beautifully captured on the 10 :

Improving on task B means deteriorating on task A, and reciprocally.

It does not mean that the more tasks there are to solve, the harder it gets to solve them all, which is true. Our statement is stronger, it means that tasks are inhrently put in a competition setting with each other. This implies that performing continual learning with modular, memoryless and unconstrained methods inherently boils down to perform tradeoffs, which confirms our prior analysis of tradeoffs through HPOs. Ultimately, it imlpies that modular, memoryless and unconstrained methods are inherently limited when it comes to perform continual learning on numerous tasks. By contrast, we do not find this inherent limitation with in Mixture of Experts models [REFLA], where modules dedicated to each tasks are in parallel, and improving one particular module does not hurt another module as long as the gating system remains the same. And this gating system can be implemented using replay or rehearsal methods, or leveraging feature extractor such as CNN masks. But here we break one of our two assumptions: either we leverage some form of memory or we employ architectural constraints.

6.3 Combinatorial argument

Let's assume we are given I tasks indexed in increasing order by the set $[\![1, I]\!]$. We define a neural network as the set of its parameters

$$\phi = \{\{w_{l,m,n} \in \mathbb{R} | m, n \in [\![1, N_{l-1}]\!] \times [\![1, N_l]\!]\} \, | l \in [\![1, L+1]\!]\} \tag{4}$$

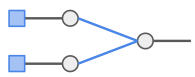
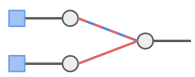
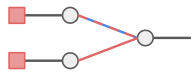
Tested \ Trained	Taks 1	Task 2
Task 1		
Task 2		

Table 3: After training on task 1, there is no pollution (top left). After training on task 2, there is $P^{1\to 2}$, pollution from task-2-specific weights when forwarding task 1 samples (top right) and $P^{2\to 1}$, pollution from task-1-specific weights when forwarding task 2 samples (bottom right).

where L is the number of hidden layers and N_l is the number of neurons on layer l.

Let's define $\mathbb{W}^i_{l,n}$ to be the set of protected weights for task i with respect to neuron n of layer l, for any $i \in [\![1, I]\!]$, $l \in [\![1, L+1]\!]$ and $n \in [\![1, N_l]\!]$:

$$\mathbb{W}^i_{l,n} = \{w_{l,m,n}, m \in [\![1, N_{l-1}]\!] | w_{l,m,n} \text{ protected}\} \quad (5)$$

which allows to define the pollution of task i by protected weights for task j on neuron n of layer l as

$$p^{i\to j}_{l,n} = \mathbb{E}_{x\sim D_i}\left[\frac{\sum_{w\in\mathbb{W}^j_{l,n}} w(x)}{\sum_{w\in\mathbb{W}^i_{l,n}} w(x)}\right] \quad (6)$$

Finally, we can define the pollution of task i by protected weights for task j on the whole network as

$$P^{i\to j} = \frac{1}{L+1}\sum_{l=1}^{L+1}\frac{1}{N_l}\sum_{n=1}^{N_l} p^{i\to j}_{l,n} \quad (7)$$

The notion of pollution $P^{i\to j}$ encapsulates how much neurons specialized for a task j interfere whith neurons specialized for a task i when provided a sample from task i, after training on the first $\max(i, j)$ tasks at least. See Table 3 for a simple illustration. Once the model is trained on task 1, we can test it on data from task 1 (top left cell) but we can't expect it to perform well on data from the next tasks (bottom left cell), so no pollution is occuring. Once the model is trained on task 2, we expect it to perform well on task 1 and 2, so the pollutions that can occur are $P^{2\to 1}$ (top right cell) and $P^{1\to 2}$ (bottom right cell). $P^{2\to 1}$ is generally handled by regularization approaches that keep certain weights of the model close to the weights of the model after training on task 1. The training process on task 2 take into consideration these dedicated weights, and the model learns to forward samples from task 2 through all the weights. However, $P^{1\to 2}$ is not handled by these approaches, as the model can't learn how to forward samples from task 1 through the undedicated weights retrained on task 2.

More boradly, the challenge is to deal with $P^{i\to j}$ for $j \neq i$, which requires to learn data from task i without being polluted by data from task j. Regularization approaches such as EWC claim to solve pollution of task j over task i ($P^{i\to j} = 0$) for $j < i$. While they can't really overcome it, they can at least mitigate it by penalizing the distance between the weights of the model after training on task i and the weights of the model after training on task j. However, they do not tackle the minimization of $P^{i\to j}$ for $j > i$. But they have a good reason no to tackle it : this is a fundamental limitation of this familly approaches, as it is inherently not possible to deal with it. Indeed, without contraints or some form of memory of the previous tasks, when provided a sample from task i, the model has no discrimination power of weights updated for task j from the ones the approach identified as specific for task i when training on task j, for $j > i$. So it can't know how to react to a sample from task i after training on task $j > i$.

Conversely, allowing some form of memorization of previous data distribution may enable to discriminate between samples features. This is has been attempted through replay memory approaches [19], [18] and rehearsal methods [17]. Alternatively, it can also be achieved through gating mechanisms such as mixture of expert models [6], [3]. These approaches are not memoryless. However, in memoryless approaches we have no direct control over the way the neurons specialized to task j will react when fed with samples from task $i < j$. So the very last hopes without memory is that the weights are such that they implicitly encode information about the task's specificities. This can be performed with constrained models, such as CNN, through which one could enforce convolution kernels associated to one task to be orthogonal to the convolution kernels associated to another task. And with additional contraints, this would prevent the model from using the same features for two different tasks while selecting for the most relevant features for each task. But, we considere unconstrained approaches, so the model has no reason to encourage such behavior.

7 Additional remarks and opening

8 Conclusion

In the end, this efforts put in writting this paper beautifully illustrate Brandolini's law :

> The amount of energy needed to refute bullshit is an order of magnitude bigger than that needed to produce it

We corrected the work subsequent to two small mistakes, namely, that could have been avoided by a more careful preliminary analysis of the problem. The fact that a method relies on an attempt to balance how much we hurt the learning process on each task naturally introduces trade-offs. These trade-offs are performed through hyperparameters in the loss that set the relative importance of the task, which can easily be mismanipulated or lead to overfitting on the benchmarks at hand.

Finally, it appears that manipulation of hyperparameters in the context of continual learning is subtle, and we encourage more care in their management.

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