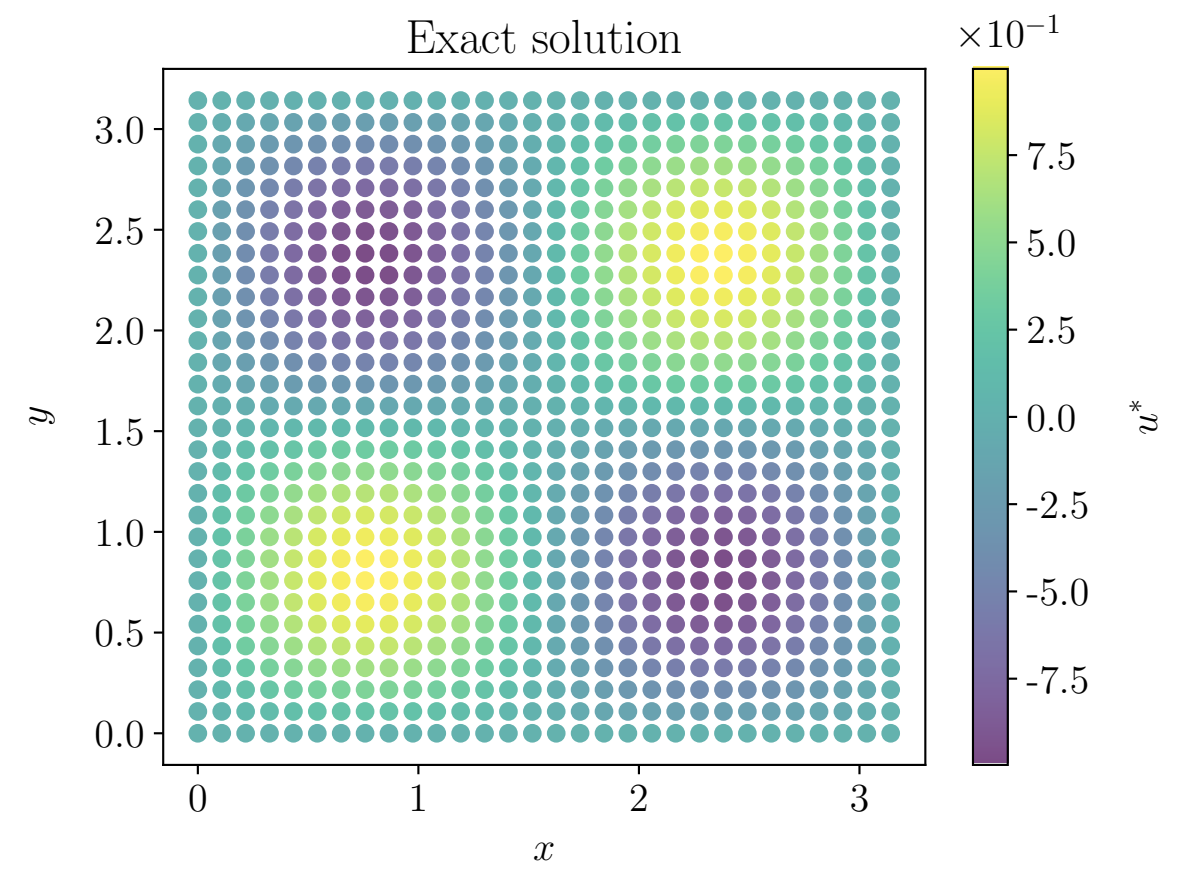
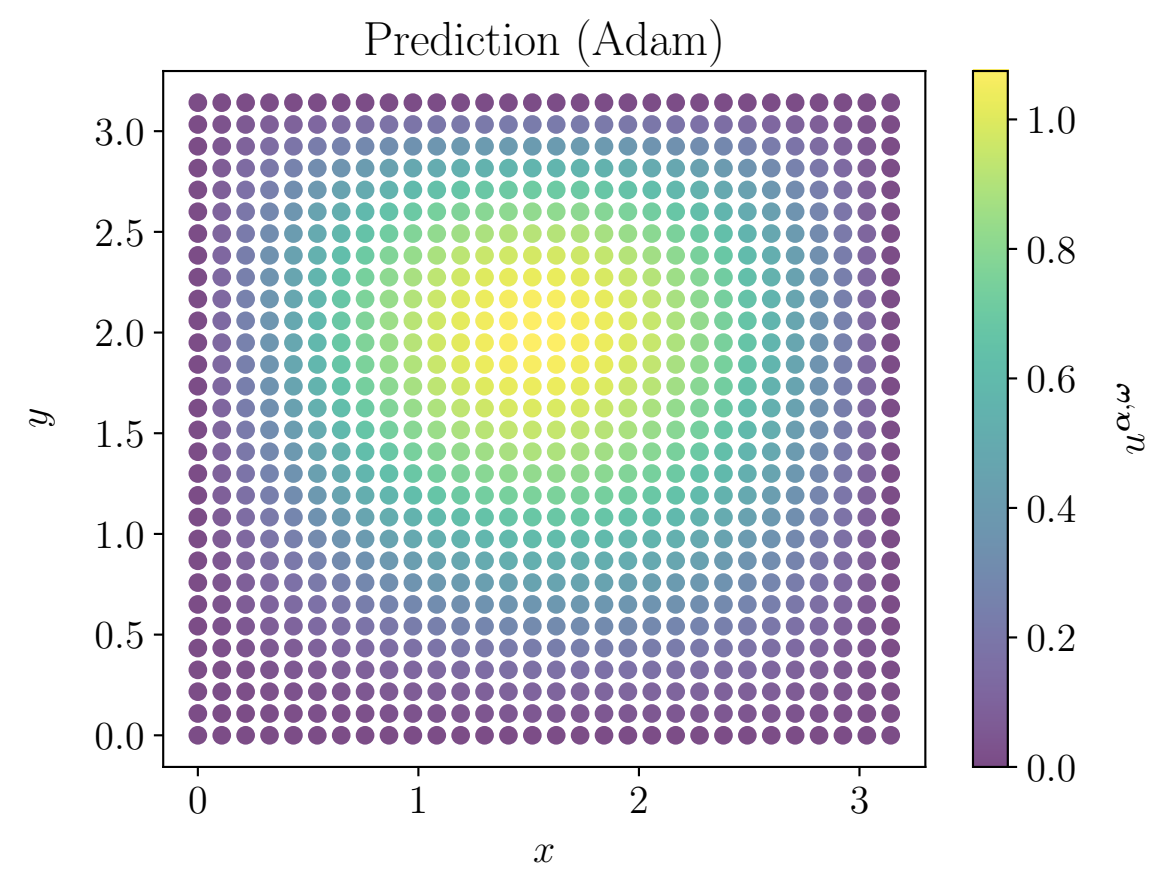
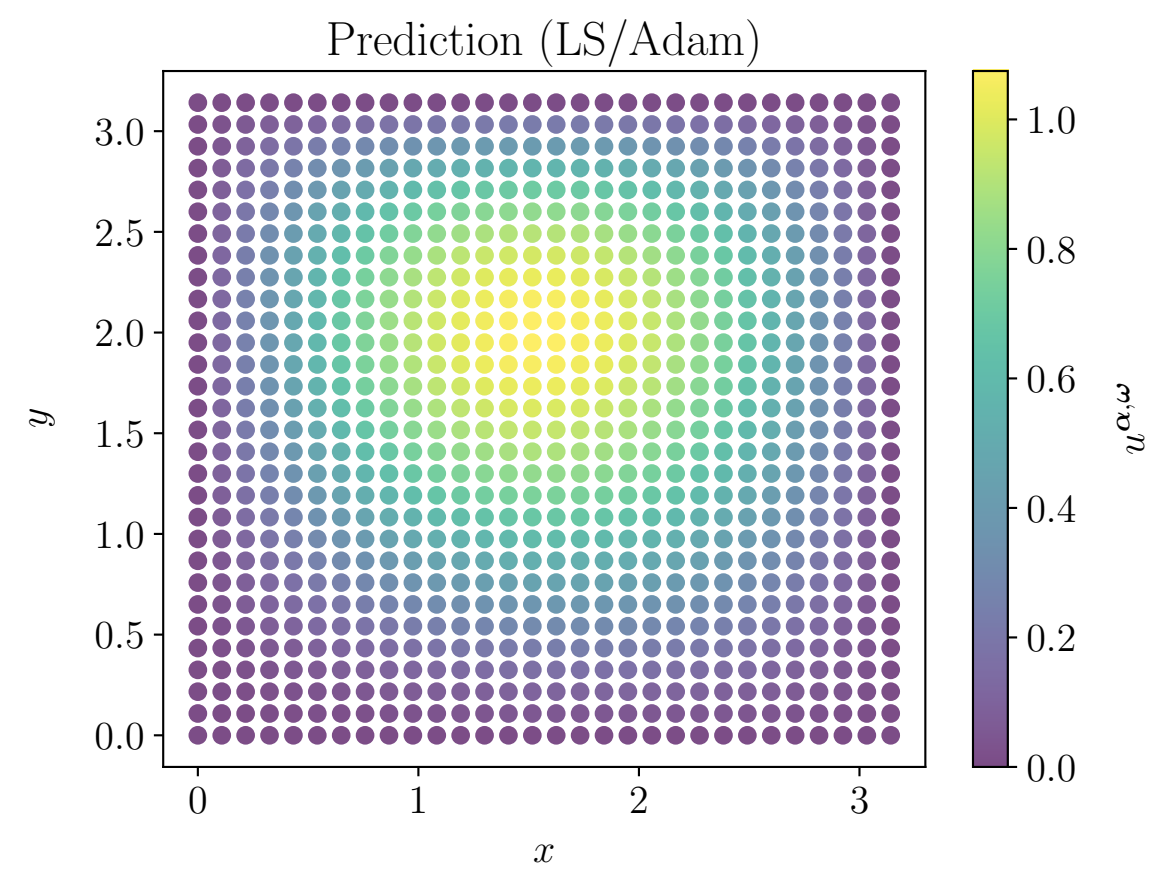
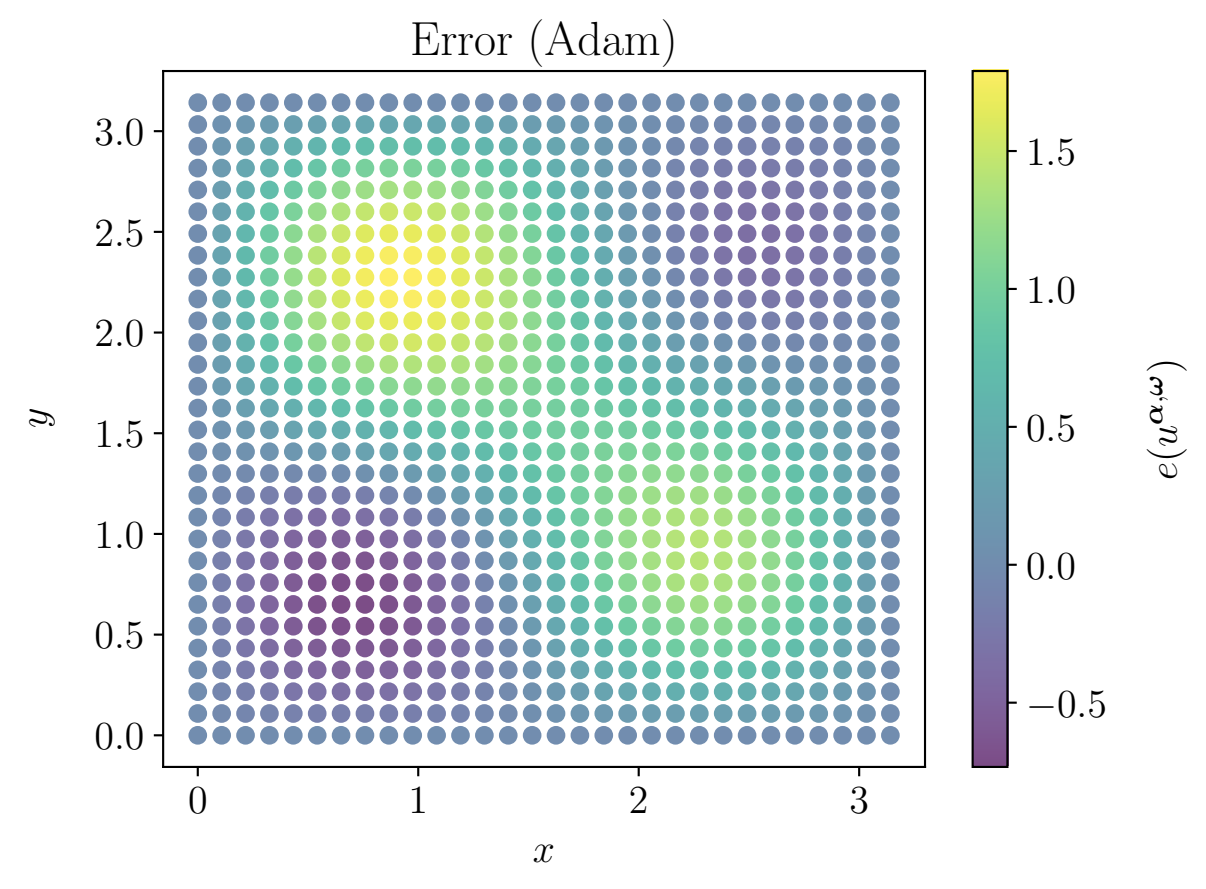
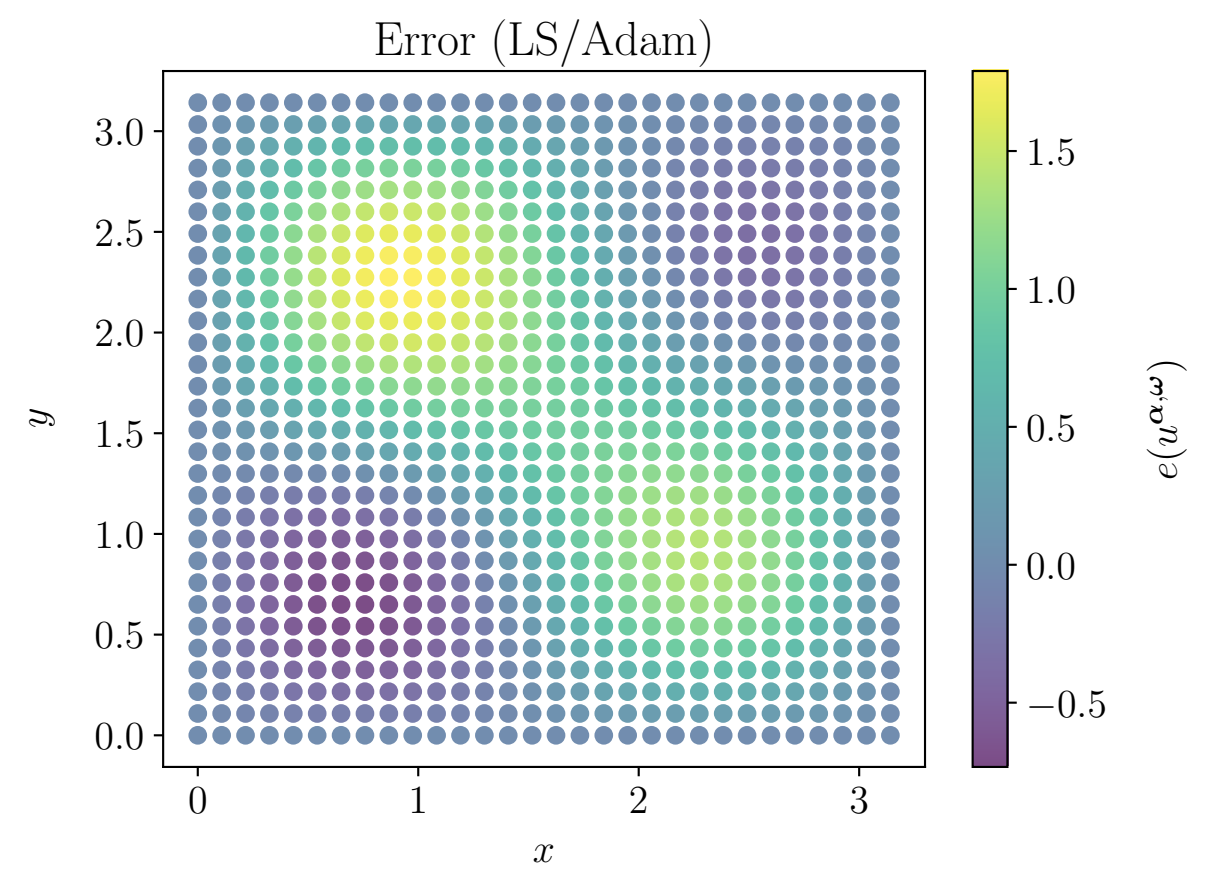
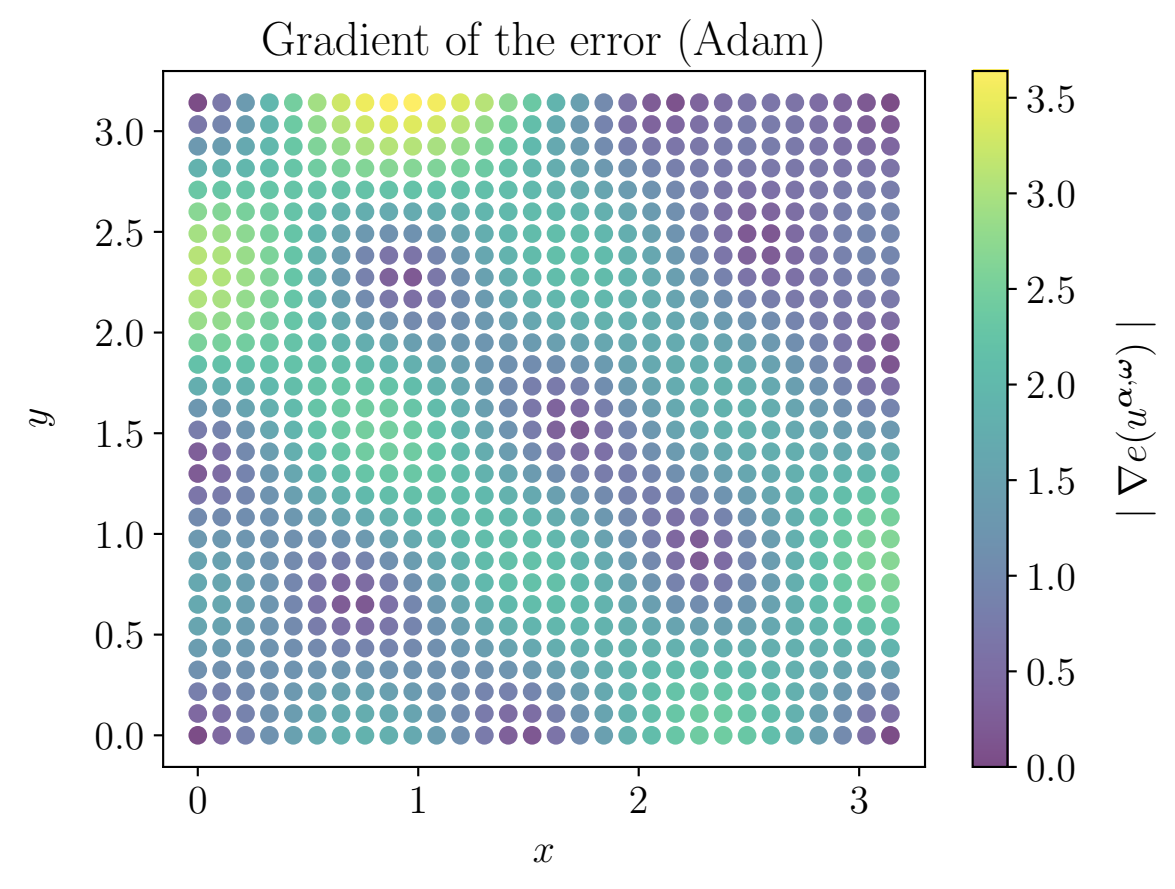

Exact solution
$\times 10^{-1}$
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
u^*
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x

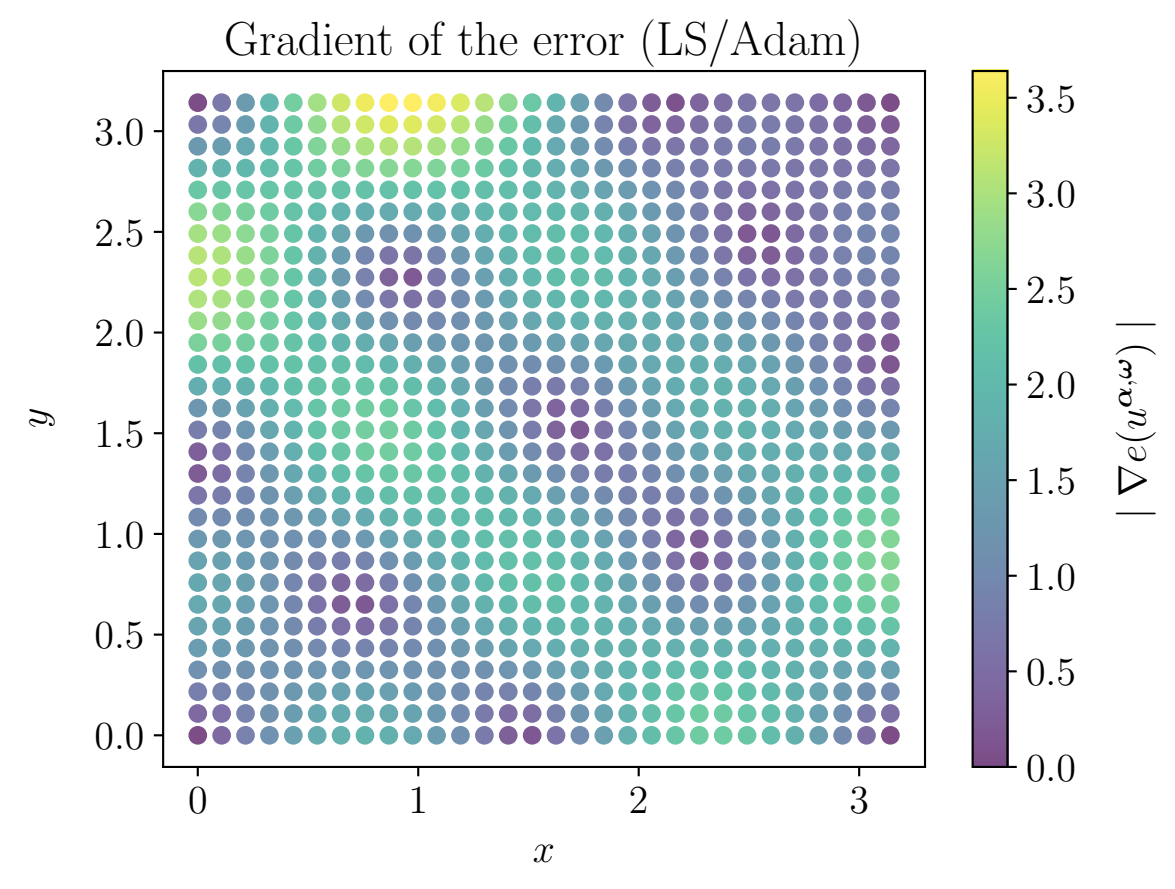

Prediction (Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.0
0.8
0.6
0.4
0.2
0.0
$u^{\alpha,\omega}$

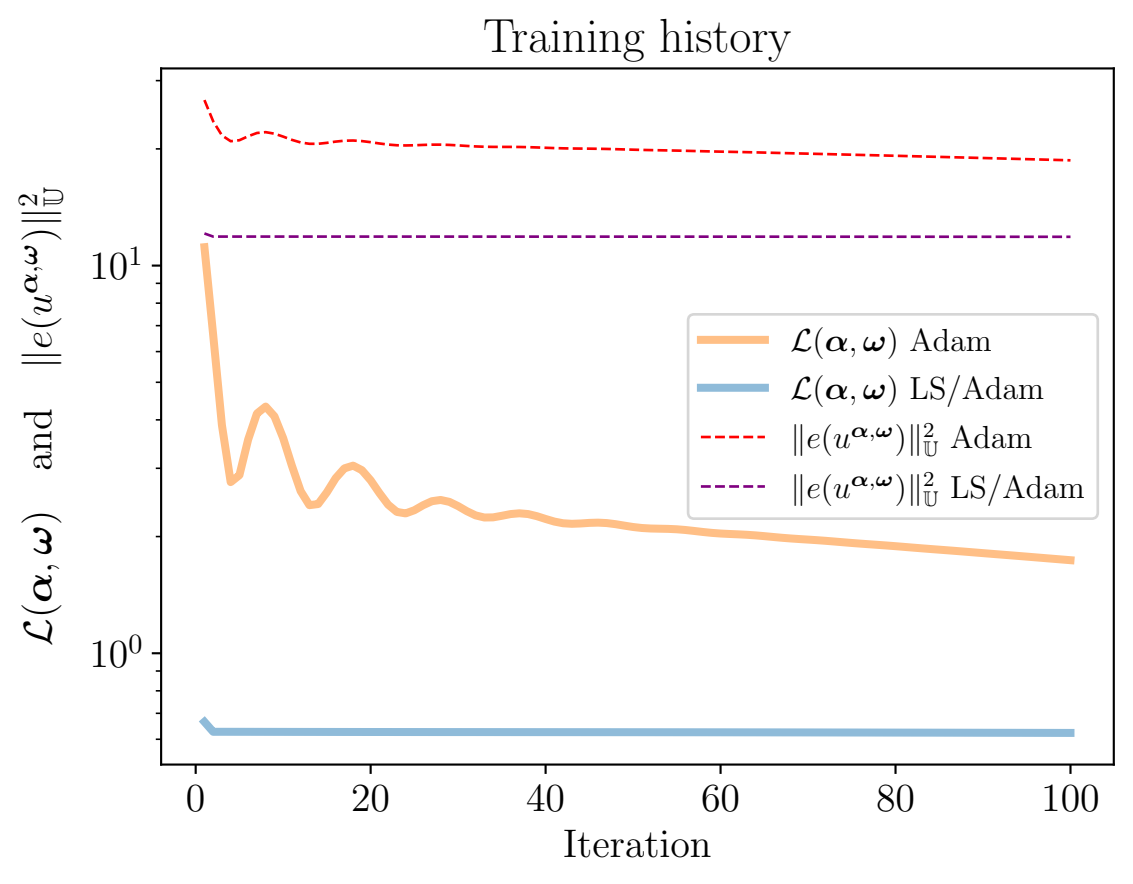

Prediction (LS/Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.0
0.8
0.6
0.4
0.2
0.0
$u^{\alpha,\omega}$

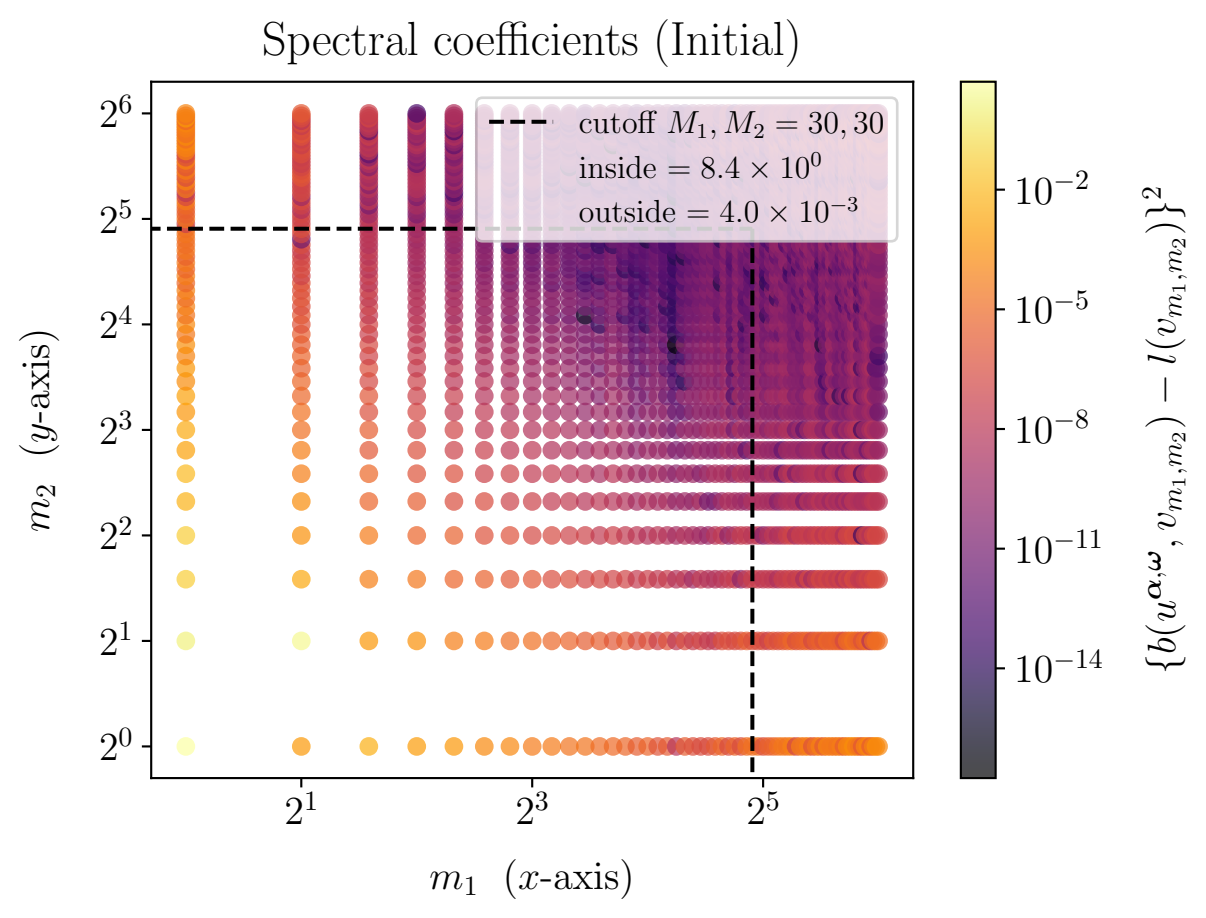

Error (Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.5
1.0
0.5
0.0
−0.5
$e(u^{\alpha,\omega})$

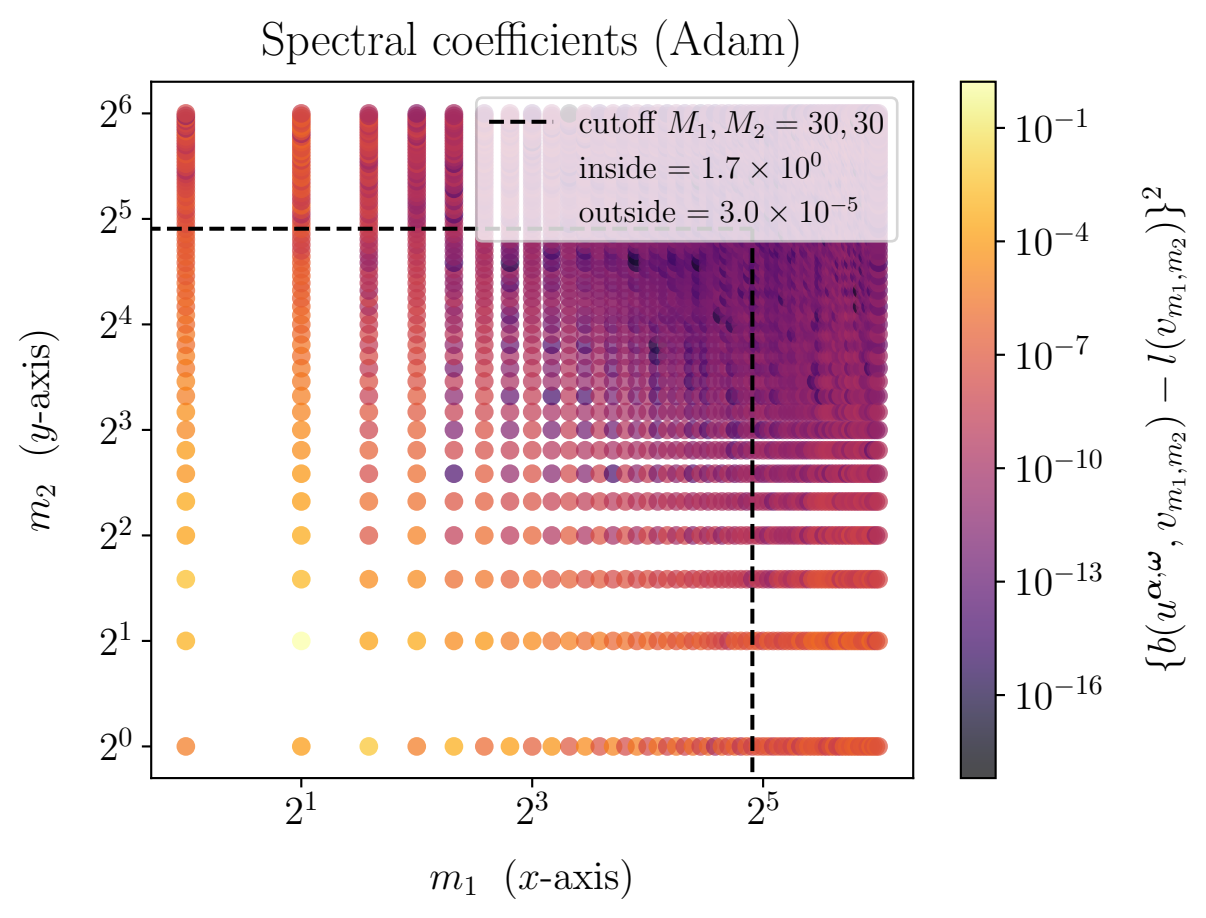

Error (LS/Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.5
1.0
0.5
0.0
−0.5
$e(u^{\alpha,\omega})$

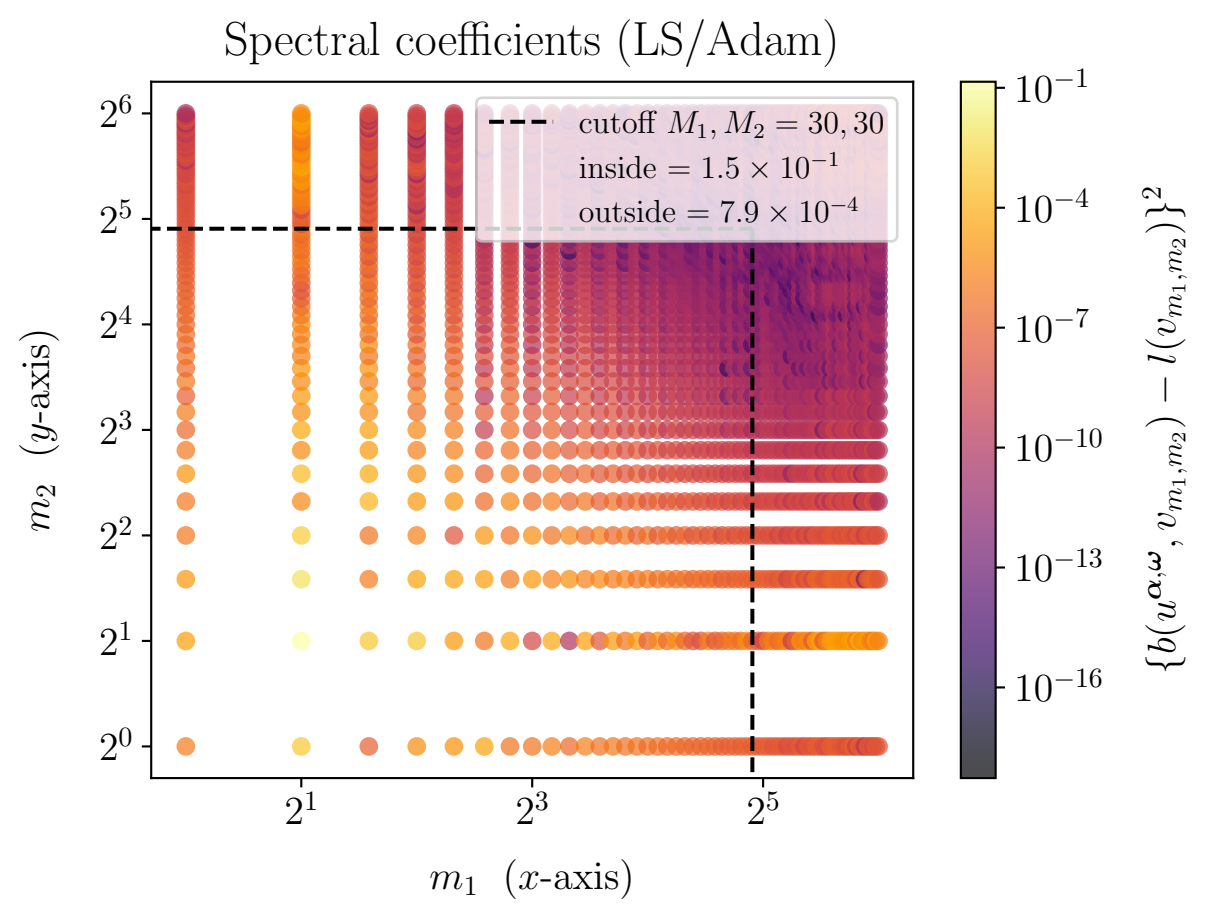

Gradient of the error (Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$|\nabla e(u^{\alpha,\omega})|$

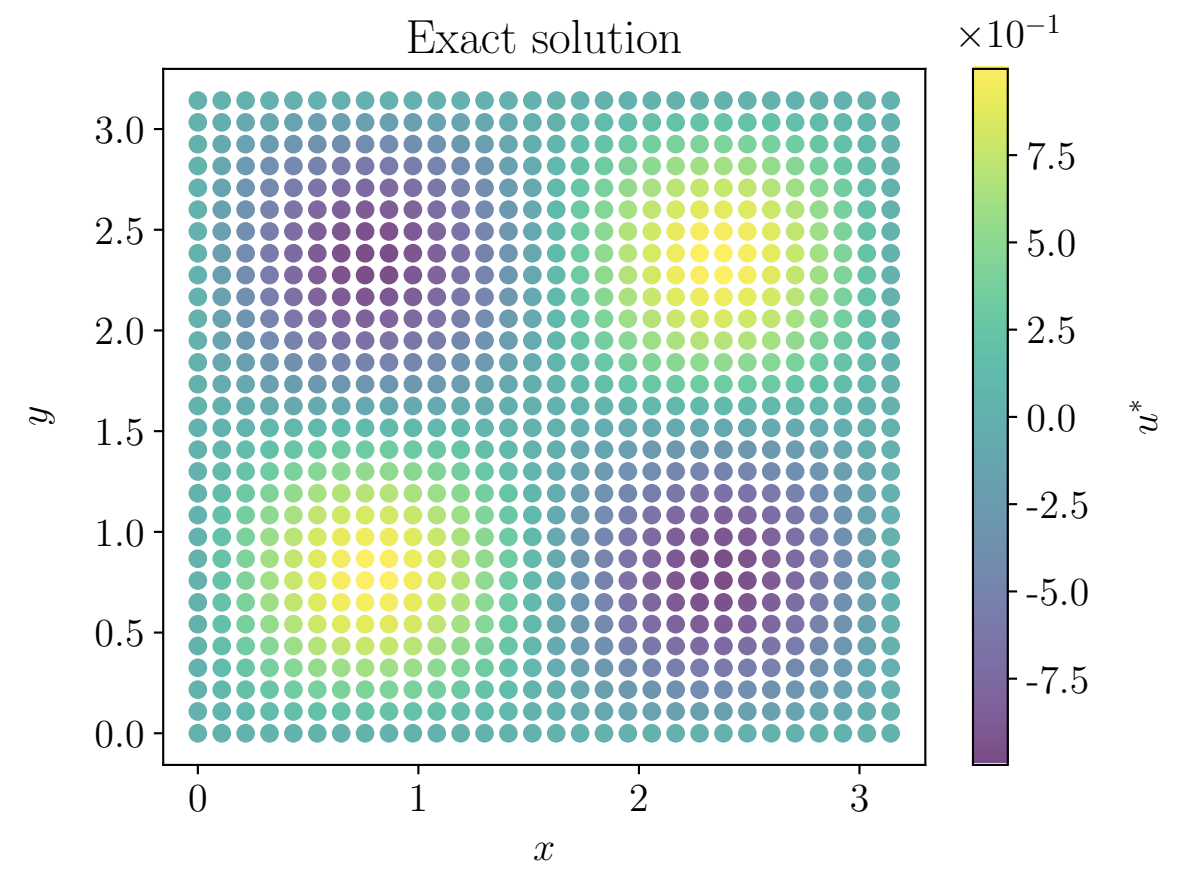

Gradient of the error (LS/Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$|\nabla e(u^{\alpha,\omega})|$

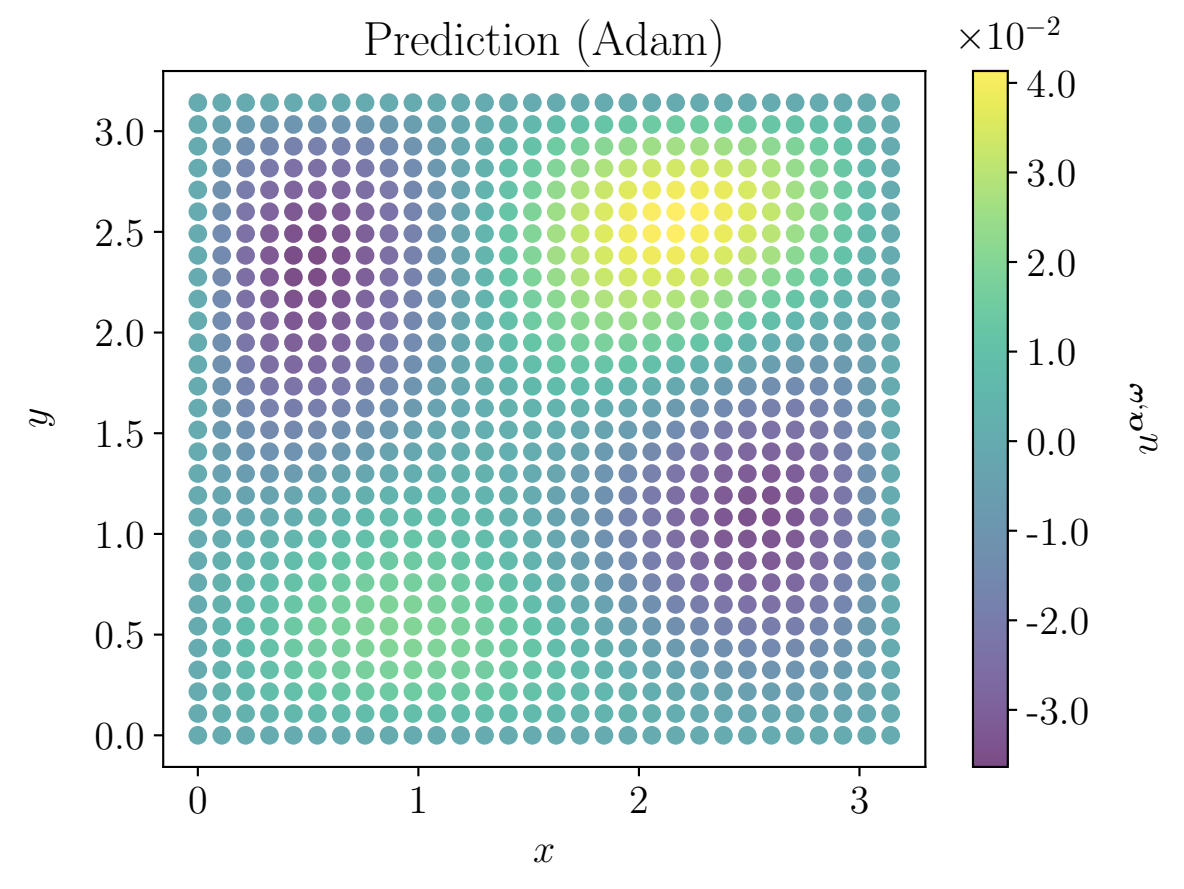

Training history
$\mathcal{L}(\boldsymbol{\alpha},\boldsymbol{\omega})$ and $\|e(u^{\boldsymbol{\alpha},\boldsymbol{\omega}})\|^2_{\mathbb{U}}$
10^1
10^0
0
20
40
60
80
100
Iteration
$\mathcal{L}(\boldsymbol{\alpha},\boldsymbol{\omega})$ Adam
$\mathcal{L}(\boldsymbol{\alpha},\boldsymbol{\omega})$ LS/Adam
$\|e(u^{\boldsymbol{\alpha},\boldsymbol{\omega}})\|^2_{\mathbb{U}}$ Adam
$\|e(u^{\boldsymbol{\alpha},\boldsymbol{\omega}})\|^2_{\mathbb{U}}$ LS/Adam

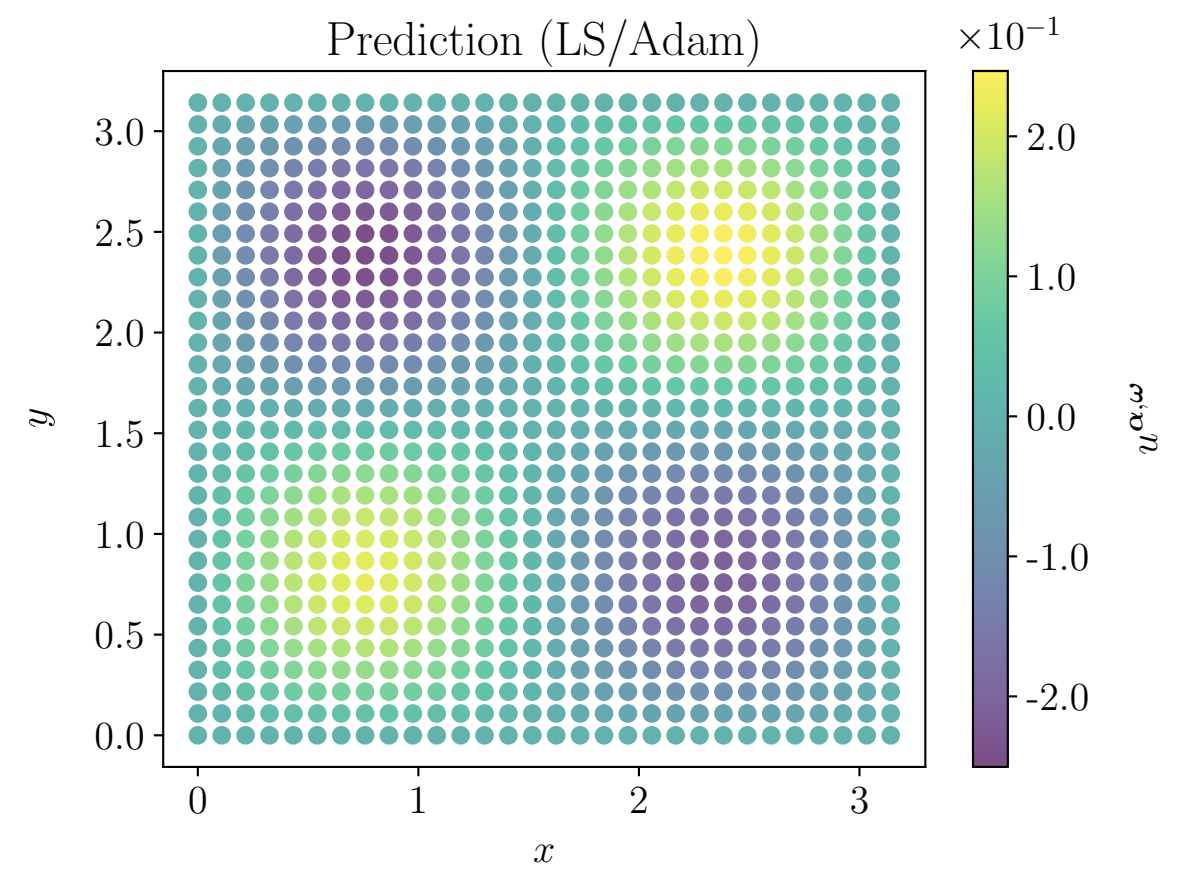

Spectral coefficients (Initial)
cutoff $M_1, M_2 = 30, 30$
inside $= 8.4 \times 10^{0}$
outside $= 4.0 \times 10^{-3}$
2^6
2^5
2^4
2^3
2^2
2^1
2^0
m_2 (y-axis)
2^1
2^3
2^5
m_1 (x-axis)
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
$\{b(u^{\boldsymbol{\alpha},\boldsymbol{\omega}}, v_{m_1,m_2}) - l(v_{m_1,m_2})\}^2$

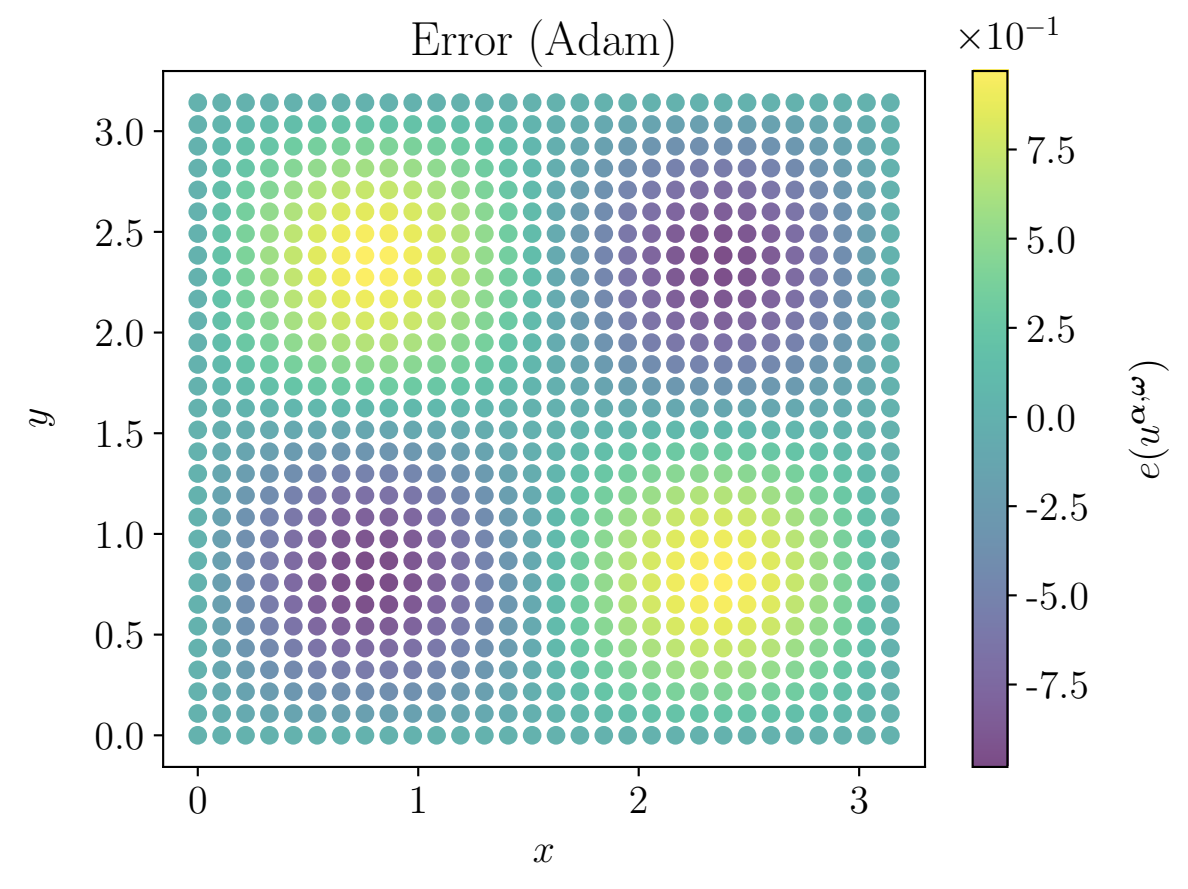

Spectral coefficients (Adam)
cutoff $M_1, M_2 = 30, 30$
inside $= 1.7 \times 10^{0}$
outside $= 3.0 \times 10^{-5}$
2^6
2^5
2^4
2^3
2^2
2^1
2^0
m_2 (y-axis)
2^1
2^3
2^5
m_1 (x-axis)
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
$\{b(u^{\boldsymbol{\alpha},\boldsymbol{\omega}}, v_{m_1,m_2}) - l(v_{m_1,m_2})\}^2$

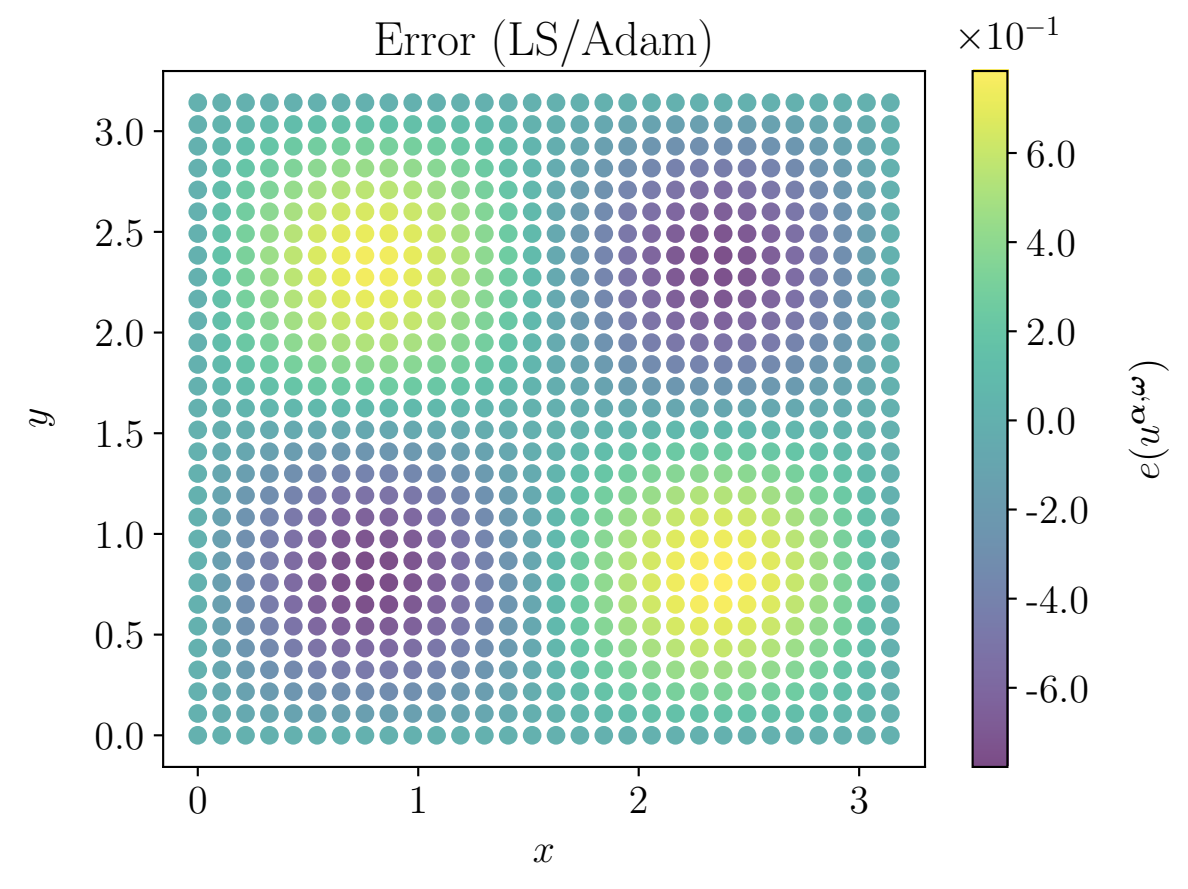

Spectral coefficients (LS/Adam)
cutoff $M_1, M_2 = 30, 30$
inside $= 1.5 \times 10^{-1}$
outside $= 7.9 \times 10^{-4}$
2^6
2^5
2^4
2^3
2^2
2^1
2^0
m_2 (y-axis)
2^1
2^3
2^5
m_1 (x-axis)
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
$\{b(u^{\boldsymbol{\alpha},\boldsymbol{\omega}}, v_{m_1,m_2}) - l(v_{m_1,m_2})\}^2$

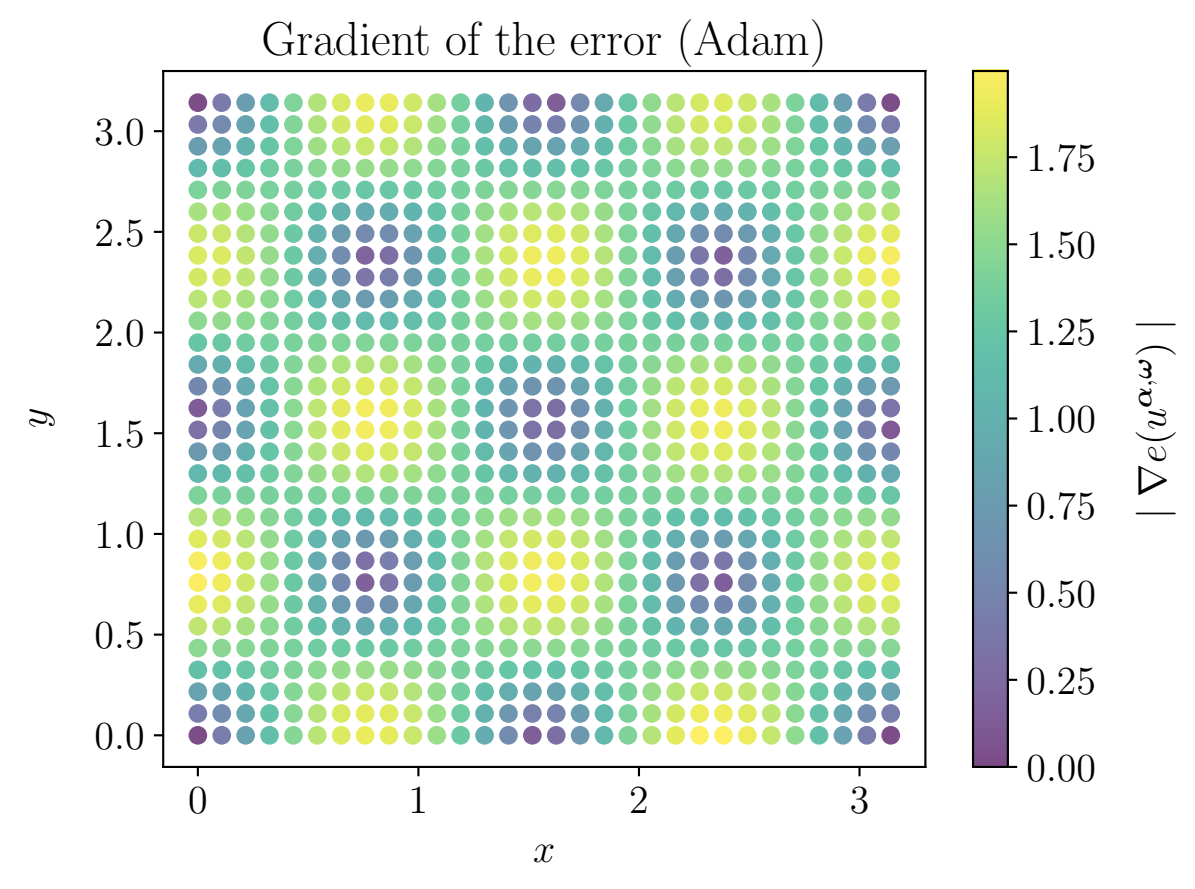

Exact solution
$\times 10^{-1}$
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
u^*
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x

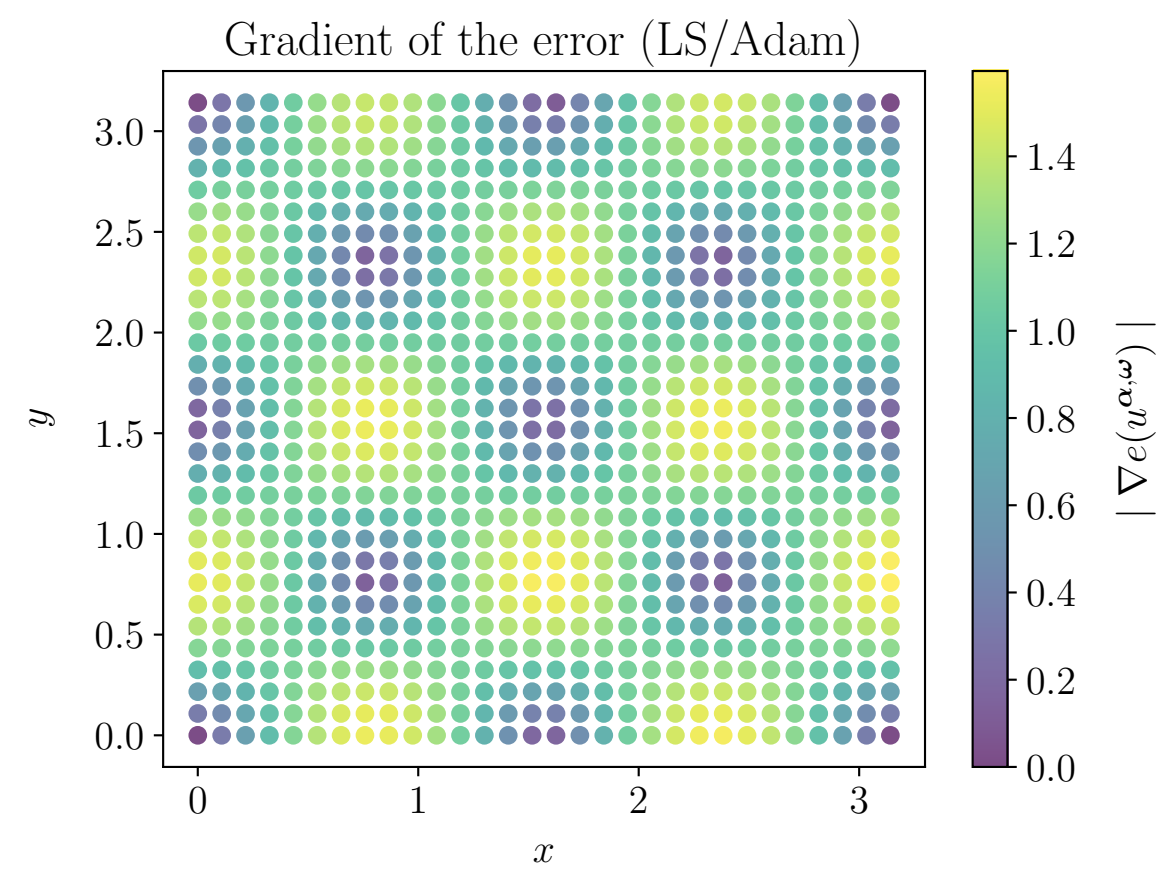

Prediction (Adam)
$\times 10^{-2}$
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
$u^{\alpha,\omega}$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x


Prediction (LS/Adam)
$\times 10^{-1}$
2.0
1.0
0.0
-1.0
-2.0
$u^{\alpha,\omega}$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x


Error (Adam)
$\times 10^{-1}$
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
$e(u^{\alpha,\omega})$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x


Error (LS/Adam)
$\times 10^{-1}$
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
$e(u^{\alpha,\omega})$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x


Gradient of the error (Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
$|\nabla e(u^{\alpha,\omega})|$


Gradient of the error (LS/Adam)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
x
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$|\nabla e(u^{\alpha,\omega})|$