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| NonSmooth 10 W/osc.]
Stalting with x = - m +2/XI-X/XI+Asin Qt, fixed m, And(1), 2>>1
Take a multiple scales approach with "slow" and "fast" time:

T=t, T=\Omega t \Rightarrow X_T=\Omega (-X_P-\mu+2|X|-X|X|+AsinT)
Next, introduce the asymptotic expansion: X~ Xo+Il X,+Il Xz+...
-> O(1): X07=0-> X0=X0(N)
     O(12"); XIT = -Xop - M + 2/Xol - XolXol + AsinT = R, (P,T)
Using the Fledholm alternative: T->0 TUO RI(T, u) du=0 -> Xor = - M+2/Xol -XolXd
Choosing to follow along the lower stable branch (\mu>0, \chi<0), stable \chi_0=-\mu-2\chi_0+\chi_0^2, which has equilibrium solution \chi_0=1-\sqrt{1+\mu}
 => XIT = ASINT -> AXXI = -ACOST + V, (7)
    O(-2): X2T = -X,p + 2/X,1-Xo/X,1-X,1/Xo/
                                                           Myren Jiles
 Again, given the order of the terms, we still have XLO,
             X_{2T} = -X_{1}P + 2X_{1}(X_{0} - 1)
= -V_{1}P + 2V_{1}(X_{0} - 1) - 2ACOST(X_{0} - 1)
 Fredholm once more gives, Vin = ZVI(Xo-1) = -ZVI+piV,
Which has Stable equilibrium V, = 0.
Thus, \times \sim 1 - \sqrt{1 + \mu} + \Omega^{-1}(-A cosT) + O(\Omega^{-2}) Touter solution
 We see this solution fails when terms reorder from uno(2"),
 It then is natural to rescale the problem,
       M= SIM, X= DZ got NO noitular Jamit
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Nonsmooth 10 W/ OSC. |
This rescaling induces,
     Ω'y = -Ω'm+Ω 2/4/-Ω y/4/ + Asin St
A similar multiple scales approach gives,
      y_ = S2 (-y_-m + 2/y1) - S2 y/y/ + AsinT
Which Suggests the expansion, you yo + I y, + ...
-> O(1): YOT = ASINT -> Yo = -ACOST + VO(7)
   O(II'): YIT = -YOT - M+21401
                                         Choosing to follow along
Here we again impose the Fredholm alternative, IT

> Lim 1 ft -yor -m+21yoldu = 0 -> yor = -m+2 Lim 1 (-Acosu + Vol7)/du

T>00 T (00 - yor -m+21yoldu = 0 -> yor = -m+2 Lim 1 (-Acosu + Vol7)/du
Which gives, Vor = -m+2 Lim 1 [ ] -Acosu+ Vo(7) Idu
This equation must be considered in two cases, first a picture:
                       -unstable branch
               μ= 21Al μ20(Ω-1)
                             Fledholm OKCE ME girles - Alle
                                 which has Stable equilibrium
                  130+ (190)
                  We see this solution fails when tellows leoloss was
                  It then is natural to reseale the froten
          Inner Solution Outer Solution
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Non Snooth 10 w/ osc. Vor=-m+2 Lim I (1-Acosu+Vo(7) Idu Case I: |Vo| = 1A/ -> Integlal is simple: Vop = -m+2/16/ Searching for equilibrium, IVol = = > Vo= -m stable. => yn-m-AcosT+O(si-1)->Xn(-m-AcosTsi-1)+O(si-2) Where |Vo|Z|A| => m = Z|A| => m = Z|A| (agrees with where outer soln fails)

Case II: IVol LIAI -> Integral is nontrivial and has no analytic Solution.

Approach!

· USE m 4 ZIAI region.
· Obtain numeric results for integral. (How?)
- Lim I of I-Acosut Vol7)/du = 297 of I-Acosut Vol7)/du
T-700 To To I of I-Acosut Vol7)/du

Find equilibrium in this region

- The integral is bounded from above & below

-m+ 4/A/ = Var <-m+2/A/ -> m= 4/H/ gives pos. Var

· Determine when equilibrium becomes unstable.

· Should see this around when the oscillations sump above the unstable blanch.

Issue; The numeric integrand is tricky. I've interpolated to get: 1Alt which indicates the integral is parabolic in Vo. 15 parabolic in Vo. 1 IAI VoCT) IN June 21Al + IAI (1-2) VoCT) VoCT) (4) Non-Smooth 10 osc.

If we assume $\lim_{T\to\infty} \frac{1}{T} \int_0^T |-AcosutVolT)|du$ is quadratic in $V_0(T)$, (estain Values are known, $f(0) = \frac{2lAl}{T}$) f(1Al) = |Al = f(-1Al) $= f(V_0(T)) = \frac{2lAl}{T} + \frac{1}{|Al|} \left(1 - \frac{2}{T}\right) V_0(T)^2$

 $= V_{on} = -m + 2 \lim_{T \to \infty} + \int_{0}^{T} |-Acosu + V_{o}(T)| du \approx -m + \frac{4|A|}{T^{2}} + \frac{2}{|A|} (1 - \frac{2}{H^{2}}) V_{o}(T)^{2}$ This approximation has maximum error of of for $A \sim O(1)$ (.4 for A = 30)

Seasching for an equilibrium with this approximation,

$$V_{o}(r) \approx \sqrt{\frac{m - \frac{4|A|}{r}}{\frac{3}{|A|}(1 - \frac{2}{r})}} = K\sqrt{m - \frac{4|A|}{r}} \neq /K/M/$$

Which now tells us that tipping occurs m = 4141

$$\rightarrow \mu \rho = \frac{41A1}{17\Omega}$$

> indicates Cabolic in