Inner eq. for 1D slowly osc.

Let $\Omega = \varepsilon^{-\lambda}$

$$\dot{x} = -\mu + 2|x| - x|x| + A\sin\Omega t \quad (1)$$
$$\dot{\mu} = -\varepsilon$$

Inner Scaling: $\mu \sim O(\varepsilon) \longrightarrow x \sim O(\varepsilon)$

Let $\mu = \varepsilon m$, $x = \varepsilon y - \varepsilon^{\lambda} A\cos\Omega t$ ⇐ term that shows up in outer

$$\rightarrow \dot{y} = -m + 2|y - \varepsilon^{\lambda-1}A\cos\Omega t| - \varepsilon(y - \varepsilon^{\lambda-1}A\cos\Omega t)|y - A\cos\Omega t| \quad (2)$$
$$\dot{m} = -1$$

Multiple Scales: $\tau = t$, $T = \Omega t$, $y = y(\tau, T)$, $m = m(\tau)$

$$\rightarrow y_T + \varepsilon^{\lambda} y_\tau = \varepsilon^{\lambda}(-m + 2|y - \varepsilon^{\lambda-1}A\cos T| - \varepsilon(y - \varepsilon^{\lambda-1}A\cos T)|y - \varepsilon^{\lambda-1}A\cos T|) \quad (3)$$
$$m_\tau = -1$$

Try expansion $y \sim y_0 + \varepsilon^{\lambda} y_1 + \dots$

$O(1)$: $Y_{0T} = 0$

$O(\varepsilon^{\lambda})$: $Y_{1T} + Y_{0\tau} = -m + 2|y_0 - \varepsilon^{\lambda-1}A\cos T| - \varepsilon(y_0 - \varepsilon^{\lambda-1}A\cos T)|y_0 - \varepsilon^{\lambda-1}A\cos T|$

Fredholm

$$\Rightarrow Y_{0\tau} = -m + (2 - \varepsilon y_0)\lim_{T\to\infty}\frac{1}{T}\int_0^T |y_0 - \varepsilon^{\lambda-1}A\cos u|\,du + \varepsilon^{\lambda}\lim_{T\to\infty}\int_0^T A\cos u\,|y_0 - \varepsilon^{\lambda-1}A\cos u|\,du$$

Change Vars,

$$y_{0m} = m + (\varepsilon y_0 - 2)\left[\frac{1}{2\pi}\int_0^{2\pi}|y_0 - \varepsilon^{\lambda-1}A\cos T|\,dT\right] - \varepsilon^{\lambda}\left[\frac{1}{2\pi}\int_0^{2\pi} A\cos T\,|y_0 - \varepsilon^{\lambda-1}A\cos T|\,dT\right] \quad (4)$$

Case I: $|y_0| \geq \varepsilon^{\lambda-1}|A|$

Turns solution into an airy and is a different order.

$$(5)\quad y_{0m} = m - 2|y_0| + \varepsilon y_0|y_0| + \varepsilon^{2\lambda-1}\frac{A|A|}{2}$$

In this case the slowly varying is dominant, leading order equil. is:

$$|y_0| = \frac{m}{2} + \varepsilon^{2\lambda-1}\frac{A|A|}{4}$$

The condition $|Y_0| \geq \varepsilon^{\lambda-1}|A| \Rightarrow m \geq 2\varepsilon^{-1}\frac{|A|}{\Omega} - \varepsilon^{-1}\frac{A|A|}{2\Omega^2}$

-or-

$$\mu \geq \frac{2|A|}{\Omega} - \frac{A|A|}{2\Omega^2}$$

Case II: $|y_0| < \varepsilon^{\lambda-1}|A|$

$$y_{0m} = m - \dot{2}\left[\frac{1}{2\pi}\int_0^{2\pi} |y_0 - \varepsilon^{\lambda-1} A\cos T|\,dT\right] + \varepsilon^{\lambda}\left[\frac{1}{2\pi}\int_0^{2\pi} A\cos T\,|y_0 - \varepsilon^{\lambda-1} A\cos T|\,dT\right]$$

I'm <u>looking for when this equation is</u> <u>Last</u> <u>negative,</u>
<u>use find to detelmine this and declare this the tipping?</u>

Issues: I have no explicit form for tipping and I cannot show the relation between λ and tipping or between the delayed tip and osc. bif.