Stability analysis: 1D osc.

$$\dot{V}_0 = -m + 2\left[\frac{1}{2\pi}\int_0^{2\pi} |V_0 - A\cos T|\,dT\right]$$

Case 1: $|V_0| \geq |A|$, $V_0 < 0$ region

$$\dot{V}_0 = -m - V_0 \Rightarrow V_0 = \frac{-m}{2} = X^0 \text{ equil.}$$

Linearizing, $V_0 = X^0 + u$, $\|u\| \ll 1$

$$\Rightarrow \dot{u} = f(X^0) + f_{V_0}(X^0)u + O(\|u\|)$$

$$\dot{u} = -u$$

where this is a hyperbolic system with $\lambda = -1$, thus our equilibrium, X^0, is asymptotically stable.

Case 2: $|V_0| < |A|$, $V_0 = \cancel{X}^0$ is the numeric equilibrium

Subcase: Interpolation: $\dot{V}_0 \approx -m + \frac{4|A|}{\pi} + \frac{2}{|A|}\left(1 - \frac{2}{\pi}\right)V_0^2$

$$V_0 = \pm C\sqrt{m - \frac{4|A|}{\pi}} = \underbrace{-C\sqrt{m - \frac{4|A|}{\pi}}}_{X^0} \text{ (stable)}$$

$$\Rightarrow V_0 = X^0 + u, \ \|u\| \ll 1$$

$$\Rightarrow \dot{u} = f(X^0) + f_{V_0}(X^0)u + O(\|u\|)$$

$$\dot{u} = \frac{4}{|A|}\left(1 - \frac{2}{\pi}\right)X^0 \cdot u, \text{ but } X^0 < 0$$

$\hookrightarrow$ hyp. stable for $m > \frac{4|A|}{\pi}$

Nonhyp at $m = \frac{4|A|}{\pi}$

$\dot{u} = f(X^0) + f_{V_0}(X^0)u + O(\|u\|)$, X^0 is negative given the region

$$f_{V_0}(X^0) = \frac{1}{\pi}\int_0^{2\pi} \mathrm{Sgn}(X^0 - A\cos T)\,dT, \quad f_{V_0}(X^0) < 0 \text{ iff } X^0 < 0$$

$\Rightarrow \dot{u} = f_{V_0}(X^0)u$ is hyperbolic and asymptotically stable, this holds for $m > \frac{4|A|}{\pi}$.

$\dot{u} = f_{V_0}(X^0)u$ is nonhyperbolic at $\cancel{m} = \frac{4|A|}{\pi} \Rightarrow$