Slowly varying... (24)-(25)

$$\dot{x} = a - x^2 + A\sin(\Omega t), \quad \dot{a} = -\mu, \quad \mu \ll 1, \quad \Omega \gg 1, \quad A \sim O(\Omega)$$

Our goal is to incorporate the amplitude into the dynamics of a and x as it is their relationship we care for.

Thus, $x = \sqrt{A}\, z$, $\cancel{h = \frac{a}{A}}$

$$\rightarrow \sqrt{A}\,\dot{z} = a - A z^2 + A\sin(\Omega t)$$

Rescaling time, $t = s/\sqrt{A} \rightarrow \dot{z} = \frac{dz}{ds}\cdot\sqrt{A}$, $\dot{a} = \sqrt{A}\,\frac{da}{ds}$

$$\rightarrow A\frac{dz}{ds} = a - A z^2 + A\sin\left(\frac{\Omega}{\sqrt{A}} s\right), \quad \sqrt{A}\,\frac{da}{ds} = -\mu$$

Lastly, Let $h = \frac{a}{A}$, $w = \frac{\Omega}{\sqrt{A}}$, $M = A^{3/2}\mu$

$$\rightarrow \frac{dz}{ds} = h - z^2 + \sin(ws), \quad \frac{da}{ds} = -A^{-3/2}\mu = -M$$

where $z(0) = z_0 = \frac{x_0}{\sqrt{A}}$, $h(0) = h_0 = \frac{a_0}{A}$ (24)

Now, to search for ways to reuse former asymp. series,

Let ① $w = \frac{\Omega}{\sqrt{A}}$, ② $M = A^{-3/2}\mu$, ③ $\{A = \Omega^P, P \geq 1\}$, ④ $w = M^{\xi}$

① & ④ $\frac{\Omega}{\sqrt{A}} = M^{\xi(P,\lambda)} \xrightarrow{③} \Omega^{1-P/2} = M^{\xi(P,\lambda)}$

Thus, if $P = 2 \rightarrow 1 = M^{\xi(2,\lambda)} \Rightarrow \xi(P,\lambda) = (P-2)\, f(P,\lambda)$

Recall ⑤ $\Omega = \mu^{-\lambda}$ ~~used~~ in our High frequency calculations.

②+③+⑤:

$$M = \Omega^{-3/2 P} \mu \longrightarrow M = \mu^{1+\frac{3}{2}P\lambda} \longrightarrow M = 1 \text{ if } 3P + \frac{2}{\lambda} = 0$$

But since $\omega = M^{\xi}$, we need to restrict $\omega \sim O(1)$,

so we choose $f(P,\lambda) = \frac{1}{3P + 2/\lambda}$

$$\Rightarrow \xi(P,\lambda) = \frac{P-2}{3P + 2/\lambda} \quad (25)$$

To use our High frequency calculations,

$$\xi < -\frac{1}{6} \longrightarrow P < \frac{4}{3} - \frac{2}{9}\lambda \quad (\text{Here } \lambda > 0)$$

$$\Rightarrow P < \frac{4}{3} \longrightarrow 1 \leq P < \frac{4}{3}$$

$$\text{choose } P = 1 \longrightarrow \frac{-1}{3 + 2/\lambda} < -\frac{1}{6} \longrightarrow \lambda > \frac{2}{3}.$$

Thus, $\xi < -\frac{1}{6}$, $1 \leq P < \frac{4}{3}$, and $\lambda > \frac{2}{3}$ to use our high frequency approach.