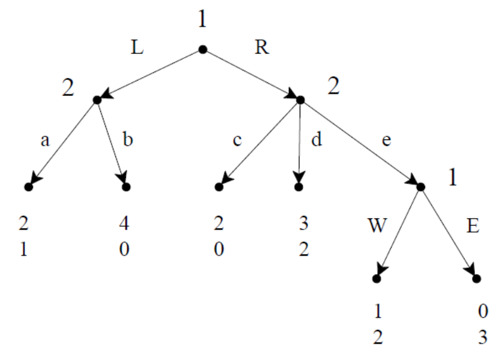
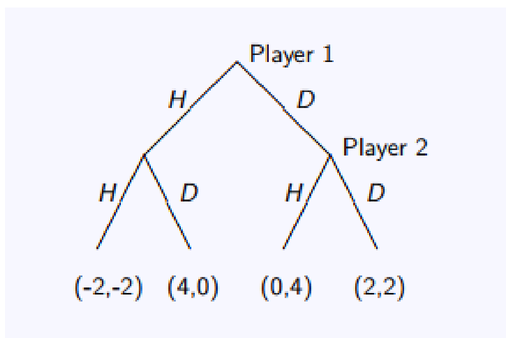
Operations Research - Tutorial Set Question 31 to 60

31. Find the backward-induction solutions of the following extensive form game.



32. Solve the following game using Backward Induction. Express the game as a matrix game.



33. Consider the following game. Player 1 moves first and can take action A or B. Player 2 observes the action of Player 1 and independently of the action of Player 1 can take action A or B. Once the players have chosen their actions a die is thrown. If the result of the die roll an odd number the payoffs obtained by the players are given by

	A	B
A	(2,1)	(1,3)
B	(0,4)	(3,0)

If the result of the die roll an even number the payoffs obtained by the players are given by

	A	B
A	(0,8)	(6,0)
B	(5,2)	(2,6)

(a) Draw the tree depicting the extensive form of the game.

(b) Solve the component games using backward induction.

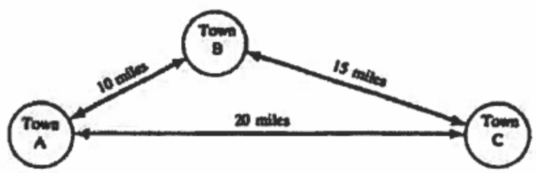
(c) Give the matrix form of the game.

34. Find the Nash equilibria of the following strategic game.

	L	R
T	(2,2)	(0,0)
B	(0,0)	(1,1)

35. **Construct a payoff matrix far the following game.**

- Each of two supermarket chains proposes to build a store in a rural region that is served by three towns. The distances between towns are shown in the figure below.



- Approximately 43 percent of the. regiions population live near town A, 33 percent live near town B. and 70 percent live near town C.
- Because Chain 1 is larger and has developed a better reputation than chain 2. chain 1 will control a majority of the business whenever their situations are comparable.
- Both chains are aware of the other's interest in the region and both have completed marketing surveys that give identical projections.
- If both chains locate in the same town or equidistant from a town, chain 1 will control 65 percent of the business in that town.
- If chain I 1 closer to a town than chain 2. chain I will control 90 mem of that towns business.
- If chain 1 is farther from a town than chain 2, it will still draw 40 percent of that town's business, The remaining business under all circumstances will go to chain 2.
- Furthermore, both chains know that it is the policy of chain I not to locate in towns that are too small, and town C falls into this category.

36. **Construct a payoff matrix far the following game.**

A barrel contains equal number of red and green marbles, Player I randomly selects one marble and inspects it (or color without showing it to player II).

- ***Player I***
  - If the marble is red, player I says, "I have a red marble," and demands $1 from player II.

  - If the marble is green, either player I says, "The marble is green," and pays player II.
  - Alternatively player I can bluff by saying, "The marble is red," and demands demands $1 from player II.

- ***Player II***
  - Whenever player I demands $1, player II either can pay or can challenge player I's claim that the selected marble is red.
  - Once challenged, player I must show the marble to player II.
  - If it is indeed red. player II pays player I $2.
  - if it is not red, player I pays player 11 $2.
- *Use a game tree to help solve this problem.*
- *(Hawk-Dove Game)*

37. ***MS4315 Spring 2012 Q5 (MB / JK)***

    (a) Define what is meant by a minimax strategy in a 2-player game.

    (b) For the following matrix game

	A	B
A	(5,3)	(4,4)
B	(2,6)	(4,4)

    Find the minimax strategies and payoffs for each player. If both players play their minimax strategies, what is the value of the game ?

    (c) In the context of a 2-person game, define what a Nash equilibrium is.

    (d) Does the game of part (b) have Nash equilibria ? Justify your answer.

38. ***MS4315 Autumn 2014 Q5 (MB / JK)***

    (a) In the context of a 2-person game, define what a Nash equilibrium is.

    (b) What is a pure strategy ? What is a mixed strategy ?

    (c) For strategies define strict dominance and weak dominance.

    (d) By removing all strategies which are dominated by strict pure or mixed strategies, derive the reduced version of the following 2-player matrix game:

	D	E	F
A	(3,5)	(5,1)	(1,2)
B	(1,1)	(6,9)	(6,4)
C	(2,6)	(4,7)	(0,8)

    (e) Derive the Nash equilibria and values of this game.

39. ***MS4315 Autumn 2014 Q5 (MB / JK)***

    (a) In the context of a 2-person game, define what a Nash equilibrium is.

    (b) What is a pure strategy ? What is a mixed strategy ?

(c) For strategies define strict dominance and weak dominance.

(d) By removing all strategies which are dominated by strict pure or mixed strategies, derive the reduced version of the following 2-player matrix game:

	D	E	F
A	(4,-2)	(3,0)	(-3,-1)
B	(-1,1)	(2,2)	(2,3)
C	(2,1)	(-1,-1)	(0,4)

(e) Derive the Nash equilibria and values of this game. 8

40. ***MS4315 Autumn 2015 Q5 (MB / JK)***

(a) In terms of a strategic (matrix) game, what is a dominated strategy? Describe the technique of iterated elimination of dominated strategies. What is the technique used for ?

(b) By removing all strategies which are dominated by strict pure or mixed strategies, derive the reduced version of the following 2-player zerosum matrix game:

	D	E	F
A	(5,-5)	(1,-1)	(2,-2)
B	(1,-1)	(0,0)	(3,-3)
C	(2,-2)	(3,-3)	(6,-6)

(c) In terms of a 2-player zero-sum game, what is a minimax strategy?

(d) Derive the minimax strategies and value of the above game.

41. **Monopoly - Introduction to Duopolies**
Let the inverse demand function and the cost function be given by $P = 502Q$ and $C = 10+2q$ respectively, where Q is total industry output and q is the firms output. Cosnider the case of a monopoly ($Q = q$). Determine the optimal output level, market price, and profit for the firm.

42. **Cournot Equilibrium - Introduction to Duopolies**

Determine the Cournot-Nash Equilibrium of the following Duopoly Model.

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43. Cournot 2

Determine the Cournot-Nash Equilibrium of the following Duopoly Model.

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44. Cournot 3

Determine the Cournot-Nash Equilibrium of the following Duopoly Model.

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45. **Bertrand Duopoly 1 (Identical Goods)**

    Determine the Bertrand Equilibrium of the following Duopoly Model.

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    -

46. **Bertrand Duopoly 2 (Different Goods)**

    Determine the Bertrand Equilibrium of the following Duopoly Model.

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47. **Stackleberg Quantity Leadership Problem**

    Determine the Stackleberg Equilibrium of the following Duopoly Model.

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48. **Stackleberg : 2 Firm with Identical Goods** Determine the Stackleberg Equilibrium of the following Duopoly Model.

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49. **Cartel Question**

    Consider an industry with two firms. Firms are identical and produce an homogenous product. Firms have to select outputs (capacity) in order to maximize profits. Each firm knows its own total cost of production, the total cost of production of the competitor and the industry demand.
    The following data are known by both firms and describe the industry situation:

    - P = 140 - (Q1+Q2) *(industry demand)*
    - TC1 = 20Q1 *(total cost of firm 1),*
    - TC2 = 20Q2 *(total cost of firm 2).*

    Suppose that both firms agree to form a cartel. The goal of the cartel is to set the industry output at a level that maximizes industry profits. A rule governing the cartel behavior specifies how the industry output and profits must be shared among the cartel members.

50. **MS4315 Autumn 2013 Combined Duopoly Question**
Consider the asymmetric duopoly game: Firm i, i = 1, 2 produces xi items at a cost of

$$C(x_i) = \frac{1}{i}x_i + 20.$$

The items sell at a price of

$$p(x1, x2) = 5\frac{x_1 + x_2}{500}$$

each.

(a) Find the equilibrium of the game if it is played as a Cournot game, and prove that it is a Nash equilibrium.

(b) Find the equilibrium if it played as a Stackelberg game with Firm 1 as leader.

(c) Contrast and comment on the two solutions.

51. **(*From a Previous Exam Paper*)**
In a game show, contestants Máire and Séamus start the last round with €500 and €400 respectively. Each must decide to pass or play. If a player passes, they keep their money but if they opt to play they win or lose €200 each with probability 1/2. These outcomes are independent of each other. The player with the most money at the end of the round gets a bonus of €300.

(a) If Máire goes first and Séamus sees her move, draw the game tree.

(b) Show that the strategic form of the game is

	Pass	Play
Pass	(8,4)	$(\frac{13}{2}, \frac{11}{2})$
Play	$(\frac{13}{2}, \frac{11}{2})$	$(\frac{29}{4}, \frac{19}{4})$

where payoffs are expected values in 00's.

(c) Solve the game using Backward Induction.

52. **(*From a Previous Exam Paper*)**
In a game show, contestants Síle and Séan start the last round with €500 and €400 respectively. Each must decide to pass or play. If a player passes, they keep their money but if they opt to play they win or lose €200 each with probability 1/2. These outcomes are independent of each other. The player with the most money at the end of the round gets a bonus of €200.

(a) Suppose Síle goes first, with Séan then taking his turn, after seeing her move. Draw the game tree.

(b) Show that the strategic form of the game is

	Pass	Play
Pass	(7,4)	(6, 5)
Play	(6, 5)	$(\frac{9}{2}, 5)$

where payoffs are expected values in 00's.

(c) Solve the game using Backward Induction.

53. **Binary Classification Question (Decision Theory)**
Consider the following confusion matrix.(The total number of experiments is 10,000)

	Predict Negative	Predict Positive
Observed Negative	9700	80
Observed Positive	100	120

Compute the following measurements

(a) Accuracy
(b) Recall
(c) Precision
(d) The F-measure

54. Explain the class imbalance problem in binary classification procedures. Explain how this would advserely affect some performance measures for binary classification procedures.

55. For following binary classification outcome table (i.e. confusion matrices), calculate the following appraisal metrics.

(a) Accuracy;
(b) Recall
(c) Precision;
(d) F-measure.

Confusion Matrix 1

	Predict Negative	Predict Positive
Observed Negative	9560	100
Observed Positive	270	70

Confusion Matrix 2

	Predict Negative	Predict Positive
Observed Negative	9500	320
Observed Positive	20	160

56. Decision Theory (Bayes Test)

57. Binary Classification

58. **Monopoly Question (MS4315 Mark Burke)**
The costs incurred by a firm in a production period are $c = 100 + 2x$ where x is the number of items produced in that period. The items each sell at a price of

$$P(x) = 10 - \frac{x}{50}.$$

Find the level of production that maximises the firms profits when the firm has a monopoly.

59. (a) Big O-notation is used to classify algorithms according to their relative complexity. Compare the complexity of algorithms of order $O(\log n)$, $O(n)$, $O(n \log n)$, $O(2^n)$and $O(n!)$. Illustrate your answer with a sketch.
    (b) Classify the Binary Search Tree algorithm using Big O-notation. Justify your answer.
    (c) Compare and contract Big O-Notation, Big Omega Notation and Big Theta Notation.
    (d) What is meant by Combinatorial Explosion? Why is it relevant for Binary Integer Problems?

60. **Algorithm Definition Questions**

- Knapsack Problem
- Provide illustrations / Examples