Operations Research - Tutorial Set Question 1 to 30

1. Find a solution to each game, i.e, an optimum strategy p^o for R, an optimum strategy q^o for C, and the value of the game.

(i)

3	−3
2	1

(ii)

3	2
1	2

(iii)

1	2
1	3

- *Identify the saddle point and value of a Strictly Determined Game (Week 1)*

2. Which of the following games are strictly determined ? For the strictly determined games, find the value v of the game and find an optimum strategy p^o for the row player R and an optimum strategy q^o for the column player C.

(i)

6	−4	−2
1	−1	3
−8	−3	7

(ii)

3	−1	4	−2
0	3	−1	6
3	−3	5	1

(iii)

1	2	0	3
2	3	2	5
3	0	1	4

3. Find the solution to each of the following 2 x 2 games.

(i)

2	−1
0	2

(ii)

1	−1
3	−2

(iii)

1	−1
−3	5

(iv)

−3	5
2	−3

4. Two players R and C simultaneously show 2 or 3 fingers. If the sum of the fingers shown is even, then R wins the sum from C; if the sum is odd, then R loses the sum to C. Find optimum strategies for the players and to whom the game is favorable.

5. Find the solution to the following game

1	−3	−2
0	−4	2
−5	2	3

6. Identify optimal strategy profiles and value of a non-strictly determined game.
    - *Similar to Question 3*
7. Statement of a game matrix, given a scenario.
    - *See Week 5 handout 2 - Amy and Beth Example*
8. Questions on Iterative Elimination of Dominated Strategies
    - *Zero Sum Games - expect to reduce to a "2 by 2" or "2 by 3"*
    - *Show your workings. Expect one or two such matrices*
9. Short definition of Nash Equilibrium
10. ***Worked Example of Nash Equilibrium***
    Determine the Nash Equilibrium of the following game.

	L	R
T	(1,1)	(6,-2)
M	(-2,6)	(3,3)

11. Essay Question about Prisoner's Dilemma
    - *Include a relevant Pay-off matrix*
    - *Discuss Nash Equilibrium*
12. Describe a graphical method for presenting extensive form games.
    - *Kuhn Trees - from Week 5 handout 1*
13. Depict the following matrix game as an extensive form game. You may assume that Player 1 moves first, and Player 2 reacts.

	L	C	R
T	6	-7	-3
B	-6	5	4

    - *See Week 5 handout 1*
14. Design a binary search tree for an ordered list of 17,21 and 30 Records. For each case, what is the maximum number of comparisons that the computer would have to make to match any existing record?
15. A mail order company has 5,000,000 records on its database. Calculate the maximum number of comparisons that would need to be made to match a target with any record in the database.
    - *For last two questions - see Week 3 handout*

16. Consider the following game. Player 1 moves first and can take action A or B. Player 2 observes the action of Player 1 and independently of the action of Player 1 can take action A or B. If both play A, then Player 1 obtains a payoff of 4 and Player 2 obtains a payoff of 5. If Player 1 plays A and Player 2 plays B, then Player 1 obtains a payoff of 2 and Player 2 obtains a payoff of 7. If Player 1 plays B and Player 2 plays A, then Player 1 obtains a payoff of 6 and Player 2 obtains a payoff of 3. If both play B, then Player 1 obtains a payoff of 0 and Player 2 obtains a payoff of 1.

    (a) Draw the tree depicting the extensive form of the game.

    (b) Solve the game using recursion.

    (c) Give the matrix form of the game.

    - *See Week 5 handout 2*

17. Consider the following matrix game. Find the minimax solution of this game.

	A	B
A	(4,5)	(2,7)
B	(6,3)	(0,1)

18. By removing all strategies which are dominated by pure or mixed strategies, derive the reduced version of the following matrix game.

	D	E	F	G
A	(5,2)	(2,4)	(3,3)	(4,4)
B	(3,3)	(5,2)	(3,5)	(2,3)
C	(4,4)	(4,6)	(4,3)	(5,4)

    (a) Derive the minimax solution of this game.

19. Reduce the following bimatrix by

    (a) Iterative Elimination of Strongly Dominated Strategies (IESDS)

    (b) Iterative Elimination of Weakly Dominated Strategies (IEWDS)

	L	C	R
T	(3,2)	(3,2)	(2,3)
M	(3,2)	(2,3)	(1.1)
B	(2,3)	(1,1)	(1,1)

20. State the weakly dominated strategy equilibrium for both games below

		Player 2 (Steven)			
		Split		Steal	
Player 1 (Sarah)	Split	3	3	2	4
	Steal	4	2	2	2

		Player 2 (Steven)			
		Split		Steal	
Player 1 (Sarah)	Split	4	3	2	4
	Steal	3	2	1	2

21. (a) Provide a short description of the Dynamic Programing Paradigm.
    (b) What is a Greedy Algorithm? Support your answer with a simple example, and discuss the advantages and disadvantages of using Greedy Algorithms.
    (c) In the context of the design of algorithms, describe the Divide and Conquer paradigm.

22. Minimax Question
    - Short Definition
    - Worked Example - See Handouts and Question 29

23. ***Short Game Theory Questions*** : Provide Short definitions of the following.
    - Perfect Information
    - Assumption of Rationality
    - Pareto Optimality

24. Linear Programming (LP) Solution
    - *This will be part of IP Branch and Bound Question later in course.*

25. Explain the difference between Linear Programming (LP) and Integer Programming (IP). Explain what is meant by LP Relaxation.
    - *This will be part of IP Branch and Bound Question later in course.*

26. Apply the IDSDS procedure to the following game. Is there a strict iterated dominant-strategy equilibrium?

		Player 2		
		d	*e*	*f*
Player 1	*a*	8 , 6	0 , 9	3 , 8
	b	3 , 2	2 , 1	4 , 3
	c	2 , 8	1 , 5	3 , 1

    - *See Question 19.*

27. Consider the following game:

		Player 2					
		D		*E*		*F*	
Player 1	*a*	2	3	2	2	3	1
	b	2	0	3	1	1	0
	c	1	4	2	0	0	4

(a) Apply the IDSDS procedure to it. Is there an iterated strict dominant-strategy equilibrium?

(b) Apply the IDWDS procedure to it. Is there an iterated weak dominant-strategy equilibrium?

- *See Question 19 and 26.*

28. *Statement of a Simplex Tableau to solve a Game Theory Problem*

29. By removing all strategies which are dominated by pure or mixed strategies, derive the reduced version of the following matrix game. Then derive the minimax solution of this game.

	L	C	R
T	3	-2	2
M	-1	0	4
B	-4	-3	1

30. Express the following Kuhn Tree in matrix form. Use Backward Induction to state an equilibrium solution.