1

Consider a binary decision problem with the following conditional PDFs

$$f(x \mid H_0) = \frac{1}{3} e^{-|x|}$$

$$f(x \mid H_1) = e^{-2|x|}$$

$$C_{00} = C_{11} = 0 \qquad C_{01} = 3 \qquad C_{10} = 1$$

Determine the Bayes test if $P(H_0) = 3/4$.

Compute the associated Bayes risk

## Likelihood Ratio Test

$$\Lambda(x) = \frac{f(x \mid H_1)}{f(x \mid H_0)} = 3 e^{-|x|}$$

$$\eta = \frac{(1-0)\frac{3}{4}}{(3-0)^1/_4} = \frac{3/4}{3/4} = 1.$$

$$\therefore \quad e^{-|x|} \underset{H_0}{\overset{H_1}{\gtrless}} +1.$$

$$\boxed{e^{-1} = 0.3678}$$

$$|x| \underset{H_0}{\overset{H_1}{\lessgtr}} 0.3678$$

2)

The Decision Regions are therefore

- $R_0 = \{ x : |x| > 0.3678 \}$
- $R_1 = \{ x : |x| < 0.3678 \}$

ANS

Compute $P(D_1 | H_0)$ and $P(D_0 | H_1)$

- $$P(D_1 | H_0) = \int_{-0.3678}^{0.3678} \frac{1}{3} e^{-|x|}\, dx$$

$$= 2 \int_0^{0.3678} \frac{1}{3} e^{-x}\, dx$$

$$= \frac{2}{3}\left[ -e^{-0.3678} + 1 \right]$$

$$= \frac{2}{3}\left[ 0.3077 \right] = 0.2051$$

- $$P(D_0 | H_1) = 2 \int_{0.3678}^{\infty} e^{-2x}\, dx$$

$$= \cancel{2}^{1}\left[ \cancel{\frac{e^{-\infty}}{2}}^{0} + \frac{e^{-(2\times 0.3678)}}{\cancel{2}^{1}} \right] = 0.4792$$

3)

**Bayes cost**

$$\bar{C} = 3 \times 0.4792 + 1 \times 0.2051$$

$$= \underline{1.6427}.$$