

1. Which of the following option correctly gives the determinant of the given matrix?

$$A = \begin{bmatrix} x & y & 0 & 0 & 0 \\ 0 & x & y & 0 & 0 \\ 0 & 0 & x & y & 0 \\ 0 & 0 & 0 & x & y \\ y & 0 & 0 & 0 & x \end{bmatrix}$$

[MCQ]  
[POINTS: 2]

- A)  $x^3 + y^5$   
B)  $x^4 - y^4$   
C)  $x^5 + y^5$   
D)  $x^5 - y^5$

2. Let T be the collection of all column vectors  $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  such that  $(b_1, b_2, b_3) \in \mathbb{R}$

and the system of equations

$$3x + 5y - 4z = b_1$$

$$-3x - 2y + 4z = b_2$$

$$6x + y - 8z = b_3$$

has atleast one solution. Then which of the following systems have minimum one

solution for each  $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in T$ ?

[MSQ]

[POINTS: 2]

- A.  $x - 3y + z = b_1, -x + 2y - 5z = b_2, 5x - 13y + 13z = b_3$   
B.  $x + 2y + 3z = b_1, 4y + 5z = b_2, x + 2y + 6z = b_3$   
C.  $x + 2y + 5z = b_1, 2x + 3z = b_2, x + 4y - 5z = b_3$   
D.  $x + 3y + 5z = b_1, 2x - y - 3z = b_2, 4x + 5y - z = b_3$

3. If a matrix A has row 1 + row 2 = row 3, then which of the following are CORRECT?

- A. Matrix A is invertible.  
B. Matrix A is not invertible.  
C. A is a non-singular matrix.  
D. Determinant of  $A^T$  is zero.

[MSQ]

[POINTS: 2]

4. If A is a  $m \times 100$  matrix and  $Ax = 0$  has a unique solution, then which of the following option is CORRECT?

[MCQ]

[POINTS: 2]

- A.  $m \geq 5$
- B.  $m \geq 100$
- C.  $m \leq 100$
- D.  $m$  can be any positive integer.

5. If the numerator of a fraction is increased by 2 and the denominator is decreased by 1, then it becomes  $\frac{2}{3}$ . If the numerator is increased by 1 and the denominator is increased by 2, then it becomes  $\frac{1}{3}$ . Which of the following options CORRECTLY determines the fraction?

[MCQ]

[POINTS: 2]

- A.  $\frac{1}{5}$
- B.  $\frac{1}{6}$
- C.  $\frac{2}{9}$
- D.  $\frac{2}{7}$

6. Let C be a  $4 \times 4$  matrix with its determinant being any non-zero integer. Then which of the following options is/are CORRECT?

[MSQ]

[POINTS: 2]

- A. For all vectors  $d$ , the system  $Cx = d$  have atleast one solution.
- B. For some vector  $d$ , the system  $Cx = d$  have infinitely many solutions.
- C. For some vector  $d$ , the system  $Cx = d$  have exactly one solution.
- D. For any vector  $d$ , the system  $Cx = d$  have no solution.

7. Consider the system of equations:

$$x + y - z = a$$

$$x - y + 2z = b$$

Which of the following options are CORRECT?

[MSQ]

[POINTS: 2]

- A. The rank of the Co-efficient matrix for the above system is 2.
- B. The above given system has exactly one solution.
- C. There exists only one linearly independent solution.
- D. For the system if  $a = 1$  and  $b = 2$ , then solution set is  $\{1,1,1\}$ .

8. Let  $K_n$  be some  $n \times n$  matrix with all unit entries. For what values of  $\alpha, \beta \in \mathbb{R}$  is the matrix  $\alpha I_n + \beta K_n$  invertible?

[MCQ]

[POINTS: 2]

- A.  $\alpha = 0, \beta = 0$
- B.  $\alpha \in \mathbb{R}, \beta \in \mathbb{R}$
- C.  $\alpha \in \mathbb{R} - \{0\}, \beta \in \mathbb{R}$
- D.  $\alpha \in \mathbb{R}, \beta \in \mathbb{R} - \{0\}$

9. Assume a  $(m \times n)$  matrix  $A$ , where  $(m > n)$ . Given below are the following statements.

**STATEMENT-1** : It is possible that  $Ax=b$  has infinite solutions for some  $b$  and no solution for other  $b$ .

**STATEMENT-2** : It is possible that  $Ax=b$  has unique solution for some  $b$  and no solution for other  $b$ .

**STATEMENT-3** : It is possible that  $Ax=b$  has infinite solutions for some  $b$  and unique solution for other  $b$ .

Which **ONE** of the following options is **CORRECT**?

[MCQ]

[POINTS: 2]

- A. Both **STATEMENT-1** and **STATEMENT-2** are true.
- B. Both **STATEMENT-2** and **STATEMENT-3** are true.
- C. All the statements are true.
- D. None of the statements are true.

10. Let  $B$  be a matrix such that  $B = [b_{ij}]$ ,  $1 \leq i, j \leq n$ , with  $n \geq 3$  and  $b_{ij} = i \cdot j$ . What could possibly be the rank of  $B$ ?

[MCQ]

[POINTS: 2]

- A.  $n-2$
- B.  $0$
- C.  $1$
- D.  $n-1$

11. If  $A$  is a  $2 \times 2$  matrix such that  $\text{trace } A = \det A = 25$ , then what is the trace of  $A^{-1}$ ?

[MCQ]

[POINTS: 2]

- A.  $2$
- B.  $25$
- C.  $(1/25)$
- D.  $1$

12. Suppose the rank of the matrix  $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \\ a & b & b & 1 \end{bmatrix}$

is 2 for some real numbers a and b. Then b equals

- A. 2
- B. 1/3
- C. 1/2
- D. 1

[MCQ]

[POINTS: 2]

13. Find a number t such that (3,1,4), (2,-3,5), (5,9,t) is not linearly independent in  $\mathbb{R}^3$ ?

[SUBJECTIVE]

[POINTS: 1.5]

14. The value of the determinant  $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$  is

[SUBJECTIVE]

[POINTS: 2.5]

15. Find the missing number in the given series:

0,1,1,\_\_,3,5,8,13,.....

- A. 1.5
- B. 1
- C. 2
- D. 2.5

[MCQ]

[POINTS: 2]