

1. Consider a system of linear equations $Ax = b$ where $A \in \mathbb{R}^{m \times n}$, $x = (x_1, x_2, \dots, x_n)^t$ and $b \in \mathbb{R}^m$.

What among the following should be the rank R of $[A|b]$ such that there exists infinitely many solutions ?

- a) $m = n$, provided $(m > R)$
- b) $m < n$, provided $(m = R)$
- c) $m > n$, provided $(m < R)$
- d) $m \geq n$, provided $(m \% R)$

2. For what values of a does the following system of equations have a solution ?

$$3x_1 - x_2 + ax_3 = 1$$

$$3x_1 - x_2 + x_3 = 5$$

- a) $a \in \mathbb{R}$
- b) $a \in \mathbb{R} - \{2\}$
- c) $a \in \mathbb{R} - \{1, 2\}$
- d) $a \in \mathbb{R} - \{1\}$

3. STATEMENT-1 : If there exists a system of m homogeneous linear equations such that $n > m$, then there are infinitely many solutions.

STATEMENT-2 : A subset of linearly dependent vectors is always linearly dependent.

Which of the following option is CORRECT?

- a) STATEMENT-1 is true and STATEMENT-2 is false.
- b) STATEMENT-1 is true and STATEMENT-2 is true.
- c) STATEMENT-1 is false and STATEMENT-2 is true.
- d) None of the above

4. Consider four column vectors $([1 \ 1 \ 1]^T, [1 \ 0 \ 0]^T, [1 \ 1 \ 1]^T, [1 \ 1 \ 0]^T)$ such that a matrix A is composed of the above given vectors in the same order as mentioned. Consider a vector b as $[6 \ 4 \ 2]^T$ such that there exists a system $AX=b$.

Which of the following options are CORRECT?

- a) $\text{Rank}[A|b] = \text{Rank}[A]$
- b) $\text{Rank}[A|b] \neq \text{Rank}[A]$
- c) Number of linearly independent solution is 1
- d) Nullity of A is 2.

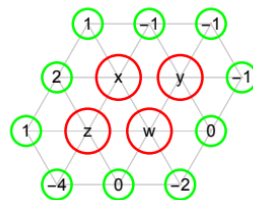
5. Consider a matrix A of (m x n) size.

Let $\text{rank}(A) = m = n$

Which of the following is TRUE about the solutions of $AX=b$?

- a) There are infinitely many solutions.
- b) There is exactly one solution.
- c) There is zero or one solution.
- d) There is zero or infinite solution

6. Shyam has a herb garden, where the soil has the property that at any given point the humidity is the sum of the neighbouring humidities. Samples of the soil are taken and plotted on a hexagonal grid as follows:



The humidity at four locations x , y , z and w are known to be unknown.

Given relation among the humidities are as follows:

$$\begin{cases} x = y+z+w+2 \\ y = x+w-3 \\ z = x+w-1 \\ w = x+y+z-2 \end{cases}$$

Which of the following option is correct?

- a) $x = 1$, $y = 2$, $z = 3$, $w = 4$
- b) $x = -1$, $y = 2$, $z = 0$, $w = -3$
- c) $x = 2$, $y = -1$, $z = 1$, $w = 0$
- d) $x = 2$, $y = -1$, $z = -1$, $w = 1$

7. Rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ is?

- a) 1
- b) 2
- c) 0
- d) 3

8. Given to you is a system of linear equation:

$$a^3x + (a+1)^3y + (a+2)^3z = 0$$

$$ax + (a+1)y + (a+2)z = 0$$

$$x + y + z = 0$$

For what value of “a” will there exist infinitely many non-trivial solutions?

[NOTE : $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

- a) 0
- b) -1
- c) 2
- d) -2

9. Let there be a matrix A whose dimension is (40 x 30). Consider performing row operation on A such that you obtain another matrix D whose specifications are such that it contains 25 non-zero rows. Carefully observe the below given match:

A

B

- | | |
|--|--------|
| a) Maximum rank of B is | i) 0 |
| b) Minimum rank of B is | ii) 30 |
| c) If w is a vector such that $Aw=0$, then value of Bw is | iii) 1 |

Which of the following option is CORRECT?

- a) a-i , b-ii , c-iii
- b) a-iii , b-ii , c-i
- c) a-ii , b-iii , c-i
- d) a-i , b-iii , c-ii

10. The entries of a matrix A and a vector v are a_{ij} and v_j . The first component of the product Av is given by $\sum a_{1j}v_j = a_{11}v_1 + a_{12}v_2 + a_{13}v_3 + \dots + a_{1n}v_n$. The operation R_{ij} is defined as subtraction of row j from row i. Which of the following options are CORRECT?

- a) The third component of Av is : $\sum a_{3j}v_j$
- b) The (3,2) entry of $R_{31}A$ is : $(a_{32}-a_{12})$
- c) The (2,1) entry of $R_{21}(R_{21}A)$ is : $(a_{21}-2a_{11})$
- d) The rank of A is 2.

11. Consider a matrix A be given as $([2 \ 2 \ 4 \ 4], [2 \ 2 \ 2 \ 6], [x \ y \ y \ 2])$. The rank of A is 2. What should the possible value of a?

- a) -1 b) 2 c) 2/3 d) 3

12. Let A be a matrix consisting of the column vectors $([1 \ -1 \ k]^T, [0 \ 1 \ 2]^T)$ and b be a vector $([1 \ 0 \ 1]^T)$. For what value of k, the system $Ax=b$ is consistent and have unique solution?

- a) $k=0$
- b) $k=1$
- c) $k=-1$
- d) No real value of k.

13 The nullity of a matrix A given as $([2 \ 1 \ 9]^T, [3 \ 1 \ 2]^T, [7 \ 9 \ x]^T)$ is equal to 1. Which one of the following option correctly determines the value of x?

- a) 200
- b) 158
- c) 160
- d) 199

14 Consider a linear regression problem $y=wx+b$. The least-squares solution, i.e., the values of w and b are 3 and 2 respectively. Let A be any real matrix consisting of the elements $([0 \ 0 \ 9]^T, [0 \ 2 \ 4]^T, [-3 \ 1 \ -1]^T)$. It is also known that the rank of A is k. Given, to you is an expression:

$$6(w+b) / (w-1)k = \alpha .$$

Which one among the following options correctly determine the value of α ?

- a) 2
- b) -1
- c) 0
- d) 5

15 Consider a 4×5 matrix $A = \begin{bmatrix} 1 & 7 & -1 & -2 & -1 \\ 3 & 21 & 0 & 9 & 0 \\ 2 & 14 & 0 & 6 & 1 \\ 6 & 42 & -1 & 13 & 0 \end{bmatrix}$

The sum of the elements of the first row of the row-reduced echelon form of A is k.

Which one of the following option correctly determines the value of k?

- a) 10
- b) 11
- c) 12
- d) 13