

Let  $v_j$ 's be the vectors in  $\mathbb{R}^n$  for  $j = 1, 2, 3, 4$ .

Which of the following is/are CORRECT ?

~~X~~ A) If  $\{v_1, v_2, v_3, v_4\}$  is linearly dependent, so is  $\{v_1, v_2, v_3\}$ .

✓ B) If  $\{v_1, v_2, v_3\}$  is linearly dependent, so is  $\{v_1, v_2, v_3, v_4\}$ .

✓ ~~C~~)  $\{v_1, v_2, v_3, v_4\}$  is linearly dependent iff, atleast one vector from the given set is a linear combination of the other vectors, other than itself.

✓ ~~D~~) A set containing zero vector is always linearly dependent. (Always CORRECT)

$$\{v_1, v_2, v_3, v_4\} \in \mathbb{R}^n.$$

A)  $\{v_1, v_2, v_3, v_4\} \rightarrow$  Linearly dependent.

$\{v_1, v_2, v_3\}$   $\rightarrow$  L.D?? (NOT GUARANTEED)

$$\rightarrow v_4 = c_1 v_1 + c_2 v_2 + c_3 v_3$$

$$\begin{cases} v_1 \neq \text{L.C of } (v_2, v_3, v_4) \\ v_2 \neq \text{L.C of } (v_1, v_3, v_4) \\ v_3 \neq \text{L.C of } (v_1, v_2, v_4) \end{cases}$$

NOTE: If a set of vector is L.D, then its'  
Subset may or may not be L.D.

$$(A) \{v_1, v_2, v_3\} \rightarrow \text{L.D.}$$

$$v_2 = c_1 v_1 + c_3 v_3.$$

$$\{v_1, v_2, v_3, v_4\} : v_2 = \overbrace{c_1 v_1 + c_3 v_3} + (0) v_4$$

$\hookrightarrow \text{L.D (always)}$

NOTE: If a set of vectors is L.D, then its' Superset is always L.D.

$$c) \{v_1, v_2, v_3, v_4\} \rightarrow \text{L.D}$$

$$\left\{ \begin{array}{l} v_4 = c_1 v_1 + c_2 v_2 + c_3 v_3 \quad (\checkmark) \\ v_4 = c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 \quad (X) \end{array} \right.$$

$$v_4 = c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 \quad (X)$$