Linear Dependence of Vectors:

→ L.D is defined for a set of vectors.

→ If atleast one vector can be represented as a linear combination of the other vectors, then the set of vectors is L.D.

MathsForFun

$\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\} \in \mathbb{R}^2$ (LD)

$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = (2) \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ✓

OR

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ ✓

$c_1 v_1 + c_2 v_2 + c_3 v_3$

linear combination of v_1, v_2 & v_3.

XU

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \left(\tfrac{1}{2}\right)\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\# \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix} \right\} \in \mathbb{R}^2$$

(LD)

$$\begin{bmatrix} 4 \\ 8 \end{bmatrix} = (4)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + (0)\begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

(or)

$$\begin{bmatrix} 4 \\ 8 \end{bmatrix} = (1)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + (1)\begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

(or)

$$\begin{bmatrix} 3 \\ 6 \end{bmatrix} = (3)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + (0)\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \right\} \in \mathbb{R}^3$$

$$\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} = (1) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (1) \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \checkmark$$

(or)

$$\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} = (-1) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (1) \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} c_1 \\ 2c_1 \\ 3c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ 4c_2 \\ 7c_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} c_1 + 2c_2 \\ 2c_1 + 4c_2 \\ 3c_1 + 7c_2 \end{bmatrix}$$

$$c_1 + 2c_2 = 3$$
$$2c_1 + 4c_2 = 6$$
$$3c_1 + 7c_2 = 10$$

$$c_1 + 2c_2 = 3 \times (3)$$
$$3c_1 + 7c_2 = 10$$

$$\cancel{3c_1} + 6c_2 = 9$$
$$-\cancel{3c_1} + 7c_2 = 10$$
$$- \quad - \quad -$$

$$-c_2 = -1 \Rightarrow \boxed{c_2 = 1}$$

$$3c_1 + 7 = 10$$
$$\Rightarrow 3c_1 = 3$$
$$\boxed{\therefore c_1 = 1}$$

MathsForFun

Let $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $y = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $z = \begin{bmatrix} 0 \\ 4 \\ 8 \end{bmatrix}$. Is $\cancel{\{x_1, x_2, x_3\}}$ $\{x, y, z\}$ linearly dependent?

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = c_1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 4 \\ 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3c_1 + 0 \\ 2c_1 + 4c_2 \\ c_1 + 8c_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \left(\frac{1}{3}\right) \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \left(\frac{1}{3}\right) \begin{bmatrix} 0 \\ 4 \\ 8 \end{bmatrix}$$

$\{x, y, z\} \in \mathbb{R}^3$ is LD.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3c_1 \\ 2c_1 + 4c_2 \\ c_1 + 8c_2 \end{bmatrix}$$

$$3c_1 = 1$$

$$\boxed{\therefore c_1 = 1/3}$$

$$2c_1 + 4c_2 = 2$$

$$\Rightarrow \frac{2}{3} + 4c_2 = 2$$

$$\Rightarrow 4c_2 = 2 - \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\therefore c_2 = 1/3}$$