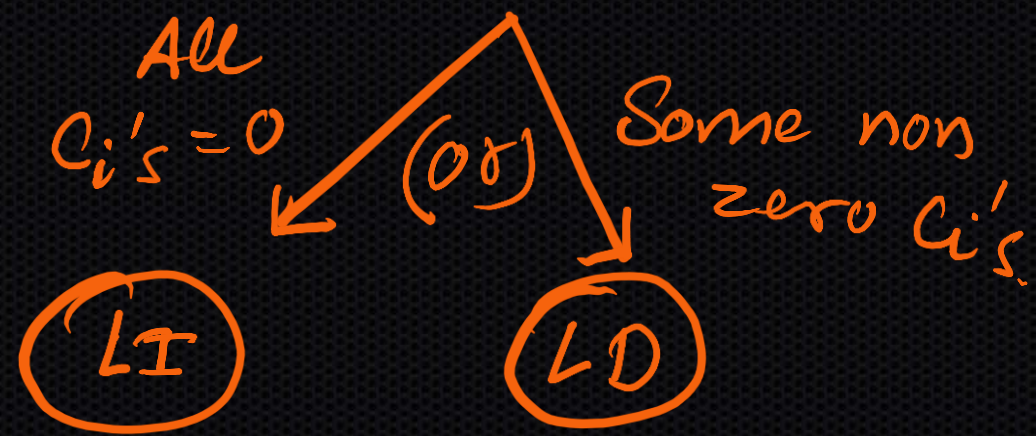


Let  $C_1, C_2, C_3, \dots, C_n$  be scalars, not all zero, such that  $\sum C_i a_i = 0 (\forall i=1, \dots, n)$ , where  $a_i$  are column vectors in  $\mathbb{R}^n$ , then the column vectors are

- A. Linearly Dependent  $\{a_1, a_2, a_3, \dots, a_n\} \in \mathbb{R}^n$
- B. Linearly Independent  $C_1 a_1 + C_2 a_2 + C_3 a_3 + \dots + C_n a_n = 0$   
(vector eqn.)
- C. None of the above

$$c_1 a_1 + c_2 a_2 + c_3 a_3 + \dots + c_n a_n = 0$$



$$\{a_1, a_2, a_3, \dots, a_n\} \in \mathbb{R}^n \rightarrow \text{LD}$$



Let  $\{u, v, w, x\}$  be linearly independent in a vector space  $V$ . Does it imply that  $\{\underline{u+v}, \underline{v+w}, \underline{w+x}, \underline{x+u}\}$  is linearly independent in  $V$ ?



$\{u, w, w, x\} \in V$

$\hookrightarrow$  (Linearly Independent)

$$\{u+v, v+w, w+x, x+u\}$$

Vector equation:

$$c_1(u+v) + c_2(v+w) + c_3(w+x) + c_4(x+u) = 0$$

$$\Rightarrow c_1u + c_1v + c_2v + c_2w + c_3w + c_3x + c_4x + c_4u = 0$$

$$\Rightarrow (c_1+c_4)u + (c_1+c_2)v + (c_2+c_3)w + (c_3+c_4)x = 0$$

Given:  $\{u, v, w, x\} \rightarrow$  Linearly Independent

$$c_1 + c_4 = 0, \quad c_1 + c_2 = 0$$

$$c_2 + c_3 = 0, \quad c_3 + c_4 = 0$$

$$c_1 = c_2 = c_3 = c_4 = 0$$

$\{u+v, v+w, w+x, x+u\} \rightarrow$  LINEARLY  
INDEPENDENT.

Suppose  $S$  is a set of vectors and some  $v \in S$  is not a linear combination of other vectors in  $S$ . Does it follow that  $S$  is linearly independent?

(NO)

$$S = \{v_1, v_2, v_3, v_4, v_5\}$$

$$\underline{v_4 \neq c_1 v_1 + c_2 v_2 + c_3 v_3 + c_5 v_5}$$

## TRUE / FALSE

1. All supersets of a linearly dependent set are linearly dependent. (T)

2. All subsets of a linearly independent set are linearly independent. (T)

$$\textcircled{1} \quad S = \{v_1, v_2, v_3\} \text{ (LD)} \rightarrow v_3 = c_1 v_1 + c_2 v_2$$

$$T = \{v_1, v_2, v_3, v_4, v_5\}$$

✓ ( $S \subseteq T$ )

$$T = \{v_1, v_2, v_3, v_4, v_5\} \quad (S \subseteq T)$$

$$v_3 = c_1 v_1 + c_2 v_2 + (0) v_4 + (0) v_5 \quad (\checkmark)$$

(or)

$$v_3 = c_1 v_1 + c_2 v_2 + c_4 v_4 + c_5 v_5 \quad (\checkmark)$$

$$\textcircled{2} \quad S = \{ \underline{v_1}, \underline{v_2}, v_3 \} \quad (L \cdot T)$$

$$\hookrightarrow T = \{ \underline{v_1}, \underline{v_2} \} \quad (T \subseteq S)$$

$\hookrightarrow (L \perp)$