IMPORTANT NOTE :

Note that linear dependence and linear independence are notions that apply to a *collection of vectors*. It does not make sense to say things like "this vector is linearly dependent on these other vectors," or "this matrix is linearly independent."

Collection of vectors : Set of vectors.

textbooks.math.gatech.edu/ila/linear-independence.html

WHAT IS THE ACTUAL DEFINITION OF LINEAR INDEPENDENCE ?? 🤔

(GEORGIA INSTITUTE OF TECHNOLOGY)

⚠ **Definition.** A set of vectors $\{v_1, v_2, \ldots, v_k\}$ is *linearly independent* if the vector equation

$$x_1v_1 + x_2v_2 + \cdots + x_kv_k = 0$$

has only the trivial solution $x_1 = x_2 = \cdots = x_k = 0$. The set $\{v_1, v_2, \ldots, v_k\}$ is *linearly dependent* otherwise.

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LET'S TRY TO UNDERSTAND

$\{v_1, v_2, v_3, ----, v_k\} \in \mathbb{R}^n$.

Equation of vectors:

$$c_1 v_1 + c_2 v_2 + c_3 v_3 + --- + c_k v_k = 0$$

Case-1: If $c_1 = c_2 = c_3 = ---- = c_k = 0$

$$(0) v_1 + (0) v_2 + (0) v_3 + ---- + (0) v_k = 0$$

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$$c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots + c_k v_k = 0$$

$$\Rightarrow c_2 v_2 = -c_1 v_1 - c_3 v_3 - c_4 v_4 - \cdots - c_k v_k$$

$$\Rightarrow v_2 = \left(-\frac{c_1}{c_2}\right) v_1 + \left(-\frac{c_3}{c_2}\right) v_3 + \left(-\frac{c_4}{c_2}\right) v_4 + \cdots + \left(-\frac{c_k}{c_2}\right) v_k$$

$$\Rightarrow v_2 = \left(-\frac{0}{0}\right) v_1 + \left(-\frac{0}{0}\right) v_3 + \cdots + \left(-\frac{0}{0}\right) v_k$$

(NOT DEFINED)

So, if $\underbrace{c_1 = c_2 = c_3 = \text{---} = c_K = 0}$

then, $\{v_1, v_2, v_3, \text{---}, v_K\}$

$\longrightarrow$ LINEARY INDEPENDENT.

NOTE: When all the co-efficients are zero, i.e., $c_1 = c_2 = c_3 = \text{---} = c_K = 0$, they are known as "TRIVIAL SOLUTION".