MathsForFun

Let $C_1, C_2, C_3, \ldots, C_n$ be scalars, not all zero, such that $\Sigma C_i a_i = 0 (\forall i=1,..,n)$, where a_i are column vectors in $\mathbb{R}^n$, then the column vectors are

A. Linearly Dependent

B. Linearly Independent

C. None of the above

$\{a_1, a_2, a_3, ---, a_n\} \in \mathbb{R}^n$

$c_1 a_1 + c_2 a_2 + c_3 a_3 + --- + c_n a_n = 0$

(Vector eqn.)

$$c_1 a_1 + c_2 a_2 + c_3 a_3 + \cdots + c_n a_n = 0$$

All c_i's $= 0$ (or) Some non zero c_i's

LI LD

$$\{a_1, a_2, a_3, \ldots, a_n\} \in \mathbb{R}^n \longrightarrow \text{LD}$$

Let $\{u, v, w, x\}$ be linearly independent in a vector space V. Does it imply that $\{u+v, v+w, w+x, x+u\}$ is linearly independent in V ?

V

$\{v, w, w, x\} \in V$

↳ (Linearly Independent)

https://home.iitm.ac.in/mtnair/LAE-Assign-Prob.pdf

$\{u+v,\ v+w,\ w+x,\ x+u\}$

Vector equation:

$$c_1(u+v) + c_2(v+w) + c_3(w+x) + c_4(x+u) = 0$$

$$\Rightarrow c_1u + c_1v + c_2v + c_2w + c_3w + c_3x + c_4x + c_4v = 0$$

$$\Rightarrow (c_1+c_4)u + (c_1+c_2)v + (c_2+c_3)w + (c_3+c_4)x = 0$$

Given: $\{u, v, w, x\} \rightarrow$ Linearly Independent

$c_1 + c_4 = 0, \quad c_1 + c_2 = 0$

$c_2 + c_3 = 0, \quad c_3 + c_4 = 0$

$$\boxed{c_1 = c_2 = c_3 = c_4 = 0}$$

$\{u+v,\ v+w,\ w+x,\ x+u\} \rightarrow$ LINEARLY INDEPENDENT.

Suppose S is a set of vectors and some v ε S is not a linear combination of other vectors in S. Does it follow that S is linearly independent? (NO)

$S = \{v_1, v_2, v_3, v_4, v_5\}$

$v_4 \neq c_1 v_1 + c_2 v_2 + c_3 v_3 + c_3 v_5$

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TRUE / FALSE

1. All supersets of a linearly dependent set are linearly dependent. (T)

2. All subsets of a linearly independent set are linearly independent.

(T)

1. $S = \{v_1, v_2, v_3\}$ (LD) $\rightarrow$ $v_3 = c_1 v_1 + c_2 v_2$

$T = \{v_1, v_2, v_3, v_4, v_5\}$

$(S \subseteq T)$

$T = \{v_1, v_2, v_3, v_4, v_5\}$ $(S \subseteq T)$

$v_3 = c_1 v_1 + c_2 v_2 + (0) v_4 + (0) v_5$. $(\checkmark)$

(or)

$v_3 = c_1 v_1 + c_2 v_2 + c_4 v_4 + c_5 v_5$ $(\checkmark)$

② $S = \{\underline{v_1, v_2}, v_3\}$ (L.I)

$T = \{\underline{v_1, v_2}\}$ $(T \subseteq S)$

$\rightarrow$ (L.I).