True / False (TRUE)

Let $\{V_1, V_2, \ldots, V_k\}$ be a set of of atleast two vectors in a vector space V. Then $\{V_1, V_2, \ldots, V_k\}$ is linearly dependent if and only if of at least one of the vectors in the set can be expressed as a linear combination of the others.

$\{v_1, v_2\} \rightarrow LD$

$v_1 = c_2 v_2$

$\{v_1, v_2, v_3\} \rightarrow LD$

$v_3 = c_1 v_1 + c_2 v_2$

www.math.purdue.edu/files/academic/courses/2010spring/MA26200

If $V_1 = (1, 2, -1)$ $V_2 = (2, -1, 1)$ and $V_3 = (8, 1, 1)$, show that $\{V_1, V_2, V_3\}$ is linearly dependent in R^3, and determine the linear dependency relationship.

$\{v_1, v_2, v_3\}$

$v_2 = c_1 v_1 + c_3 v_3, \quad (c_1, c_3) \in \mathbb{R}.$

MathsForFun

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ 2c_1 \\ -c_1 \end{bmatrix} + \begin{bmatrix} 8c_3 \\ c_3 \\ c_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 + 8c_3 \\ 2c_1 + c_3 \\ -c_1 + c_3 \end{bmatrix}$$

MathsForFun

$$c_1 + 8c_3 = 2 \quad \text{—(i)}$$

$$\begin{cases} 2c_1 + c_3 = -1 \quad \text{—(ii)} \\ -c_1 + c_3 = 1 \quad \text{—(iii)} \end{cases}$$

$$+ \qquad - \qquad -$$

$$3c_1 = -2$$

$$\boxed{\therefore c_1 = -\frac{2}{3}}$$

$$\frac{2}{3} + c_3 = 1$$

$$\Rightarrow c_3 = 1 - \frac{2}{3}$$

$$\boxed{\therefore c_3 = \frac{1}{3}}$$

$$\underset{(v_2)}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}} = \underline{\left(-2/3\right)} \underset{(v_1)}{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}} + \underline{\left(1/3\right)} \underset{(v_3)}{\begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}}$$

$$\boxed{\{v_1, v_2, v_3\} \rightarrow LD}$$