MathsForFun

Draw $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v + w$ and $v - w$ in a single xy plane.

Vector Addition.

$\boxed{c_1 V + c_2 W}$ → Linear combination of vectors.

Vector Multiplication

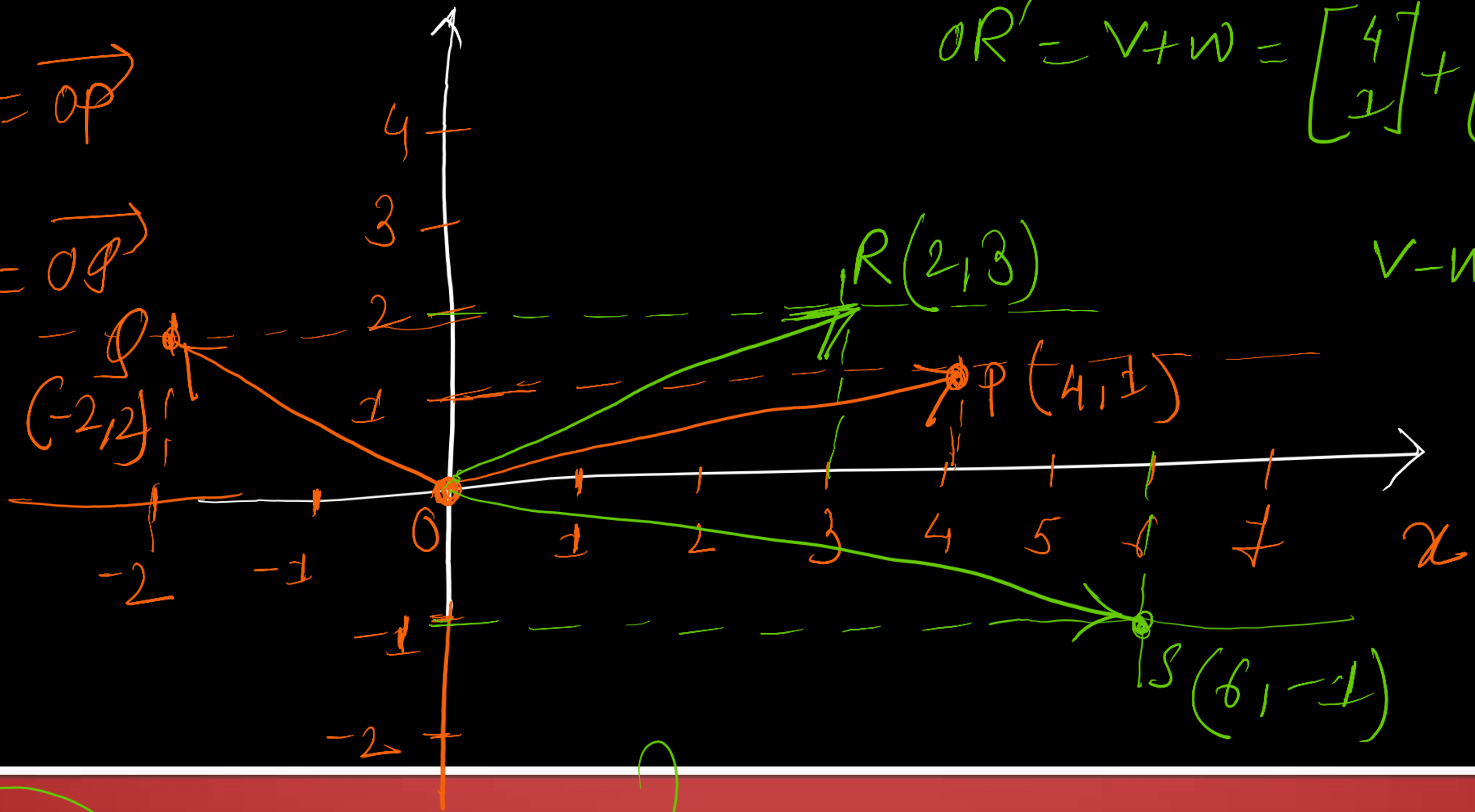
$V = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \in \mathbb{R}^2$, $W = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \in \mathbb{R}^2$

x

y

$V = \begin{bmatrix}4\\1\end{bmatrix} = \overrightarrow{OP}$

$W = \begin{bmatrix}-2\\2\end{bmatrix} = \overrightarrow{OQ}$



$\overrightarrow{OR} = V+W = \begin{bmatrix}4\\1\end{bmatrix} + \begin{bmatrix}-2\\2\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$

$V-W = \begin{bmatrix}4\\1\end{bmatrix} - \begin{bmatrix}-2\\2\end{bmatrix}$

$= \begin{bmatrix}6\\-1\end{bmatrix}$

$\therefore \overrightarrow{OS} = V-W$

$V+W \rightarrow c_1 = 1, \; c_2 = 1$

$V-W \rightarrow c_1 = 1, \; c_2 = -1$

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Find two equations for c and d so that **the linear combination** $cv + dw$ **equals** b:

$$v = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad w = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

$$\boxed{cv + dw = b}$$

$$c\begin{bmatrix} 2 \\ -1 \end{bmatrix} + d\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2c \\ -c \end{bmatrix} + \begin{bmatrix} -d \\ 2d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2c-d \\ -c+2d \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\left.\boxed{\begin{aligned} 2c-d &= 1 \\ -c+2d &= 0 \end{aligned}}\right\} \boxed{c,\ d??}$$

Every combination of $\boldsymbol{v} = (1, -2, 1)$ and $\boldsymbol{w} = (0, 1, -1)$ has components that add to ___0___. Find c and d so that $c\boldsymbol{v} + d\boldsymbol{w} = (3, 3, -6)$. [redacted]

$$c_1 v + c_2 w$$

$$= c_1 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ -2c_1 + c_2 \\ c_1 - c_2 \end{bmatrix}$$

$$\cancel{c_1} - 2\cancel{c_1} + \cancel{c_2} + \cancel{c_1} - \cancel{c_2} = 0$$

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$$cv + dw = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$\Rightarrow c\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c \\ -2c+d \\ c-d \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$\boxed{c = 3}$$

$$-2c + d = 3$$

$$\Rightarrow -2(3) + d = 3$$

$$\boxed{\therefore d = 9}$$