So, if $\underbrace{c_1 = c_2 = c_3 = \text{---} = c_K = 0}$

then, $\{v_1, v_2, v_3, \text{---}, v_K\}$

→ LINEARY INDEPENDENT.

NOTE: When all the co-efficients are zero,

i.e., $c_1 = c_2 = c_3 = \text{---} = c_K = 0$, they are

Known as "TRIVIAL SOLUTION".

→ LD or LID, both are defined for a set of vectors.

→ $c_1 v_1 + c_2 v_2 + c_3 v_3 + \cdots + c_k v_k = 0$

When all the co-efficients are equal to zero, i.e., $(c_1 = c_2 = c_3 = \cdots = c_k = 0)$, then the set of vector is "LINEARLY INDEPENDENT."

$$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 + \cdots + c_kv_k = 0$$

Case-2: When not all co-efficients are zero.

$$c_4v_4 = -c_1v_1 - c_2v_2 - c_3v_3 - \cdots - c_kv_k$$

$$\Rightarrow v_4 = \left(\frac{-c_1}{c_4}\right)v_1 + \left(\frac{-c_2}{c_4}\right)v_2 + \left(\frac{-c_3}{v_4}\right)v_3 + \cdots + \left(\frac{-c_k}{c_4}\right)v_k.$$

Let us assume: $(c_1, c_2, c_4 \neq 0)$

(and remaining $= 0$)

$$v_4 = \left(\frac{-c_1}{c_4}\right)v_1 + \left(\frac{-c_2}{c_4}\right)v_2 + \left(\frac{-c_3}{c_4}\right)v_3 + \cdots + \left(\frac{-c_r}{c_4}\right)v_4$$

$$\Rightarrow v_4 = \left(\frac{-c_1}{c_4}\right)v_1 + \left(\frac{-c_2}{c_4}\right)v_2 + (0)v_3 + \cdots + (0)v_4$$

$$v_4 = k_1 v_1 + k_2 v_2 + (0) v_3 + (0) v_4 + \cdots + (0) v_k$$

$$\{ v_1, v_2, v_3, \ldots, v_k \}$$

(LINEARLY DEPENDENT)

In other words, $\{v_1, v_2, \ldots, v_k\}$ is linearly dependent if there exist numbers $x_1, x_2, \ldots, x_k$, not all equal to zero, such that

$$x_1 v_1 + x_2 v_2 + \cdots + x_k v_k = 0.$$

This is called a *linear dependence relation* or *equation of linear dependence*.

textbooks.math.gatech.edu/ila/linear-independence.html