"The Classic"

Suppose $a = b$. Then

$$ab = a^2 \Rightarrow ab - b^2 = a^2 - b^2$$

$$\Rightarrow \cancel{(a-b)}\, b = \cancel{(a-b)}(a+b)$$

$$\Rightarrow b = a + b$$

$$\Rightarrow b = 2b$$

$$\Rightarrow \boxed{1 = 2}$$

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"Crippled currency"

$$£1 = 100 \text{ pence}$$

Take the square root:

$$\Rightarrow £\sqrt{1} = \sqrt{100} \text{ pence}$$

$$\Rightarrow \boxed{£1 = 10 \text{ pence}}$$

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"Self-proving statement"

$$1 = 0 \; !$$

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"The real unit"

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

Let $a = b = -1$:

$$\underbrace{\sqrt{-1}}_{i} \cdot \underbrace{\sqrt{-1}}_{i} = \sqrt{(-1)^2} = \sqrt{1} = 1$$

$$\Rightarrow i^2 = 1$$

$$\Rightarrow \boxed{-1 = 1}$$

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"Squared away"

$$(n+1)^2 = n^2 + 2n + 1$$

$$\Rightarrow (n+1)^2 - (2n+1) = n^2$$

$$\Rightarrow (n+1)^2 - (n+1)(2n+1) = n^2 - n(2n+1)$$

$$\Rightarrow (n+1)^2 - (n+1)(2n+1) + \frac{1}{4}(2n+1)^2$$
$$= n^2 - n(2n+1) + \frac{1}{4}(2n+1)^2$$

$$\Rightarrow \left((n+1) - \frac{1}{2}(2n+1)\right)^2 = \left(n - \frac{1}{2}(2n+1)\right)^2$$

$$\Rightarrow (\cancel{n}+1) - \cancel{\frac{1}{2}(2n}+1) = \cancel{n} - \cancel{\frac{1}{2}(2n}+1)$$

$$\Rightarrow \boxed{1 = 0}$$

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"Relatively confusing"

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$$

Convergent for $x = 1$, so

$$\log(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \ldots$$

$$2\log(2) = 2 - 1 + \frac{2}{3} - \frac{1}{2} + \frac{2}{5} - \frac{1}{3} + \frac{2}{7} - \frac{1}{4} + \ldots$$

Group terms with same denominator:

$$2\log(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$$

$$= \log(2)$$

$$\Rightarrow \boxed{2 = 1}$$

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"Absolutely confusing"

- $\dfrac{2}{(n-1)(n+1)} = \dfrac{1}{n-1} - \dfrac{1}{n+1}$, so

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots = \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \dots\right)$$

$$= \frac{1}{2}$$

- $\dfrac{1}{(2n-1)(2n+1)} = \dfrac{n}{2n-1} - \dfrac{n+1}{2n+1}$, so

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots = 1 - \cancel{\frac{2}{3}} + \cancel{\frac{2}{3}} - \cancel{\frac{3}{5}} + \cancel{\frac{3}{5}} - \cancel{\frac{4}{7}} + \dots$$

$$= 1$$

Hence $\boxed{\frac{1}{2} = 1}$

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"1 is the only integer"

Take any $n \in \mathbb{N}$.

$$1+2+3+\dots+n = \frac{n(n+1)}{2} \qquad (1)$$

Replace n by $n-1$:

$$1+2+3+\dots+n-1 = \frac{(n-1)n}{2} \qquad (2)$$

Add one:

$$1+2+3+\dots+n = \frac{(n-1)n}{2} + 1 \qquad (3)$$

Compare (1) and (3):

$$\frac{n(n+1)}{2} = \frac{(n-1)n}{2} + 1 \implies \boxed{n=1}$$

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"Every natural number can be described in fewer than 14 words"

Proof by contradiction

Let n be the smallest natural number that cannot be described in fewer than 14 words.

Then n is the smallest natural number
1 2 3 4
that cannot be described in fewer than
5 6 7 8 9 10 11
fourteen words
12 13

Contradiction! No such n exists.

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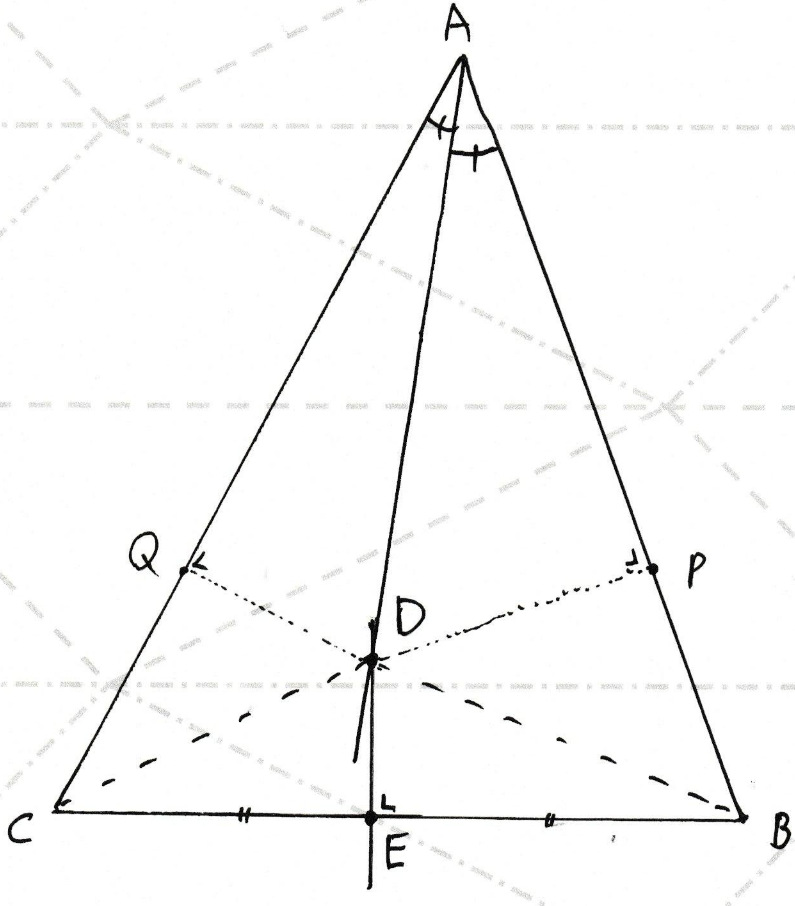
"Every triangle is isosceles"



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"Every triangle is isosceles"

Take any ΔABC.

- E: midpoint of BC.
- D: intersection of angle bisector at A and perpendicular bisector of BC.
- P, Q: base points of perpendiculars from D to AB and BC.

$\Delta BDE \cong \Delta CDE \Rightarrow |BD| = |CD|$

$\Delta APD \cong \Delta AQD \Rightarrow |PD| = |QD|$

$\Delta BPD \cong \Delta CQD \Rightarrow |BP| = |CQ|$

$\Delta APD \cong \Delta AQD \Rightarrow |AP| = |AQ|$

$\boxed{|AB| = |AC|}$

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Sources

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