Weekly Puzzle
Mechanics

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What you need to know:

Velocity is the time derivative of displacement, and acceleration is the time derivative of velocity
Small angle approximations
Polar coordinates

Questions:

Newtonian Mechanics:

1. (a) An elastic collision is a collision in which both momentum and kinetic energy are conserved. Given that there are two particles in one dimension with known masses and initial velocities, derive formulae for the velocities of each particle after the collision. (Hint: Change the reference frame so that the initial velocity of one of the particles is 0.)

   (b) By considering a particle moving in circular motion, determine the magnitude of the acceleration of the particle in terms of the magnitude of the velocity and the radius of motion. (Hint: Express x and y in terms of polar coordinates, a constant radius and changing angle.)

   (c) For a small sphere falling slowly through a fluid, there are 3 main forces acting upon it, the weight of the sphere, the upthrust (which is equal in magnitude to the weight of the displaced fluid), and Stoke's drag force (with magnitude $6\pi\eta rv$, where η is the viscosity of the fluid, r is the radius of the sphere, and v is the speed). By rearranging, what is the maximum speed that the sphere could reach when falling through a fluid?

   (d) Friction is often given by the formula $F_f \leq \mu F_n$, where F_f is the frictional force, μ is the coefficient of friction and F_n is the normal force exerted by each surface. By considering friction at its maximum and weight, what is the acceleration of a particle of mass m moving down a slope with incline θ?

   (e) By considering the forces in a pendulum (a particle attached to a light inextensible string attached to a fixed point), derive an equation relating the angle with the vertical with its double time derivative.