Weekly Puzzle
Geometry

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What you need to know:

Cyclic quadrilaterals

Questions:

Cyclic quadrilaterals:

1. (a) What is a cyclic quadrilateral?

   (b) What fact is there about opposite angles in a cyclic quadrilateral?

   (c) Hence, or otherwise, prove Thale's Theorem, which says that if a triangle ABC is constructed in a circle such that A, B and C are distinct and lie on the circumference of a circle, and AC is the diameter of that circle, then angle ABC is $90°$.

   (d) Prove Ptolemy's Theorem, which states the product of the lengths of the diagonals in a cyclic quadrilateral is equal to the sum of the products of the lengths of the pairs of opposite sides.

Area and Perimeter:

2. (a) Derive a formula for the area of a regular polygon with n sides, with side length l.

   (b) Derive Heron's semiperimeter formula, which says that $A = \sqrt{s(s-a)(s-b)(s-c)}$, where A is the area, s is the semiperimeter (half the perimeter) and a, b and c are the side lengths of the triangle.

   (c) Derive the trapezoid formula for areas, which says that the area of a simple polygon, which has an anticlockwise sequence of vertices $P_i = (x_i, y_i)$, where $i = 1, \ldots, n$, is $\frac{1}{2}\sum_{i=1}^{n}(y_i + y_{i+1})(x_i - x_{i+1})$. You may use the convention $P_{n+1} = P_1$. (Hint: Consider trapeziums enclosed by the points P_i, P_{i+1}, and the x-axis, by translating the polygon so all vertices lie in the first quadrant.) (Hint 2: Consider whether the area has to be subtracted or added depending on if there is an increase or decrease in x.)

   (d) Show your answer to part c is equivalent to $\frac{1}{2}\sum_{i=1}^{n}(x_i y_{i+1} - x_{i+1} y_i)$.

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