Weekly Puzzle

TW-C

25/11/2024 - 1/12/2024

What you need to know:

Euler's formula
Hyperbolic functions
Trigonometric angle sum formulae
Logarithmic identities

Questions:

1. (a) What is $\cosh(i)$ in terms of trigonometric functions?

   (b) What is $\cosh(ix)$ in terms of trigonometric functions?

   (c) Hence, by making a suitable substitution, what is $\cos(i)$?

   (d) What is $\sin(i)$?

   (e) Express $\cos(z)$, where z is a complex number in the form $a + bi$, in terms of trigonometric and hyperbolic functions of real numbers.

2. (a) By using the definition of the natural logarithm of real numbers in relation to exponents, come up with a definition for the logarithm of complex numbers.

   (b) Why is this not a function? (Hint: why do we restrict the domains of trigonometric functions when defining their inverses)

   We shall use the notation $Log(z)$ to represent the principal branch of the complex logarithm, meaning the argument of z will be restricted to lie in the interval $(-\pi, \pi]$.

   (c) By using the exponential form of complex numbers, what is $Log(z)$?