MA4413 Weeks 12 and 13 Tutorials

Question 1a

Consider a source X that produces five symbols with probabilities 1/2, 1/4, 1/8, 1/16 and 1/16. Determine the source entropy $H(x)$.

Question 1b

A input source is a random variable X with a four letter alphabet $\{A, B, C, D\}$. There are four different probability distributions presented below. Compute the entropy for each case.

	X_i	A	B	C	D
Case 1	$p(X_i)$	0.25	0.25	0.25	0.25
Case 2	$p(X_i)$	0.25	0.5	0.125	0.125
Case 3	$p(X_i)$	0.7	0.1	0.1	0.1
Case 4	$p(X_i)$	0.97	0.01	0.01	0.01

Question 1c

Consider a source X that produces 8 symbols with equal probabilities for each symbol. Determine the source entropy $H(x)$.

Question 2

The input source to a noisy communication channel is a random variable X over three symbols $\{a, b, c\}$. The output from this channel is a random variable Y over the same three symbols. The joint distribution of the these two random variables is as follows:

	x=a	x=b	x=c
y=a	0.25	0	0.125
y=b	0	0.125	0
y=c	0.125	0.25	0.125

- Write down the marginal distributions for X and Y.
- Compute the marginal entropies $H(X)$ and $H(Y)$
- Compute the joint entropy $H(X, Y)$ of the two random variables.

Question 3a

A four letter alphabet is encoded into binary form according to

Case 1	A: 10	C: 110	G: 111	T: 0
Case 2	A: 00	C: 01	G: 10	T: 11

Using the code presented in case 1, decode the following sequence:

`11110001011010`

Encode this message using the code from case 2. Compare the length of messages in both cases.

Question 3b

Given that the alphabet has the following distribution

x_i	A	C	G	T
$p(x_i)$	0.25	0.125	0.125	0. 5

Compute the average symbol length for both cases.

Question 4

A DMS X has live symbols $\{x_l, x_2, x_3, x_4, x_5\}$ with $P(x_1) = 0.4$, $P(x_2) = 0.19$, $P(x_3) = 0.16$, $P(x_4) = 0.15$, and $P(x_5) = 0.1$.

(a) Construct the Shannon-Fano code for X, and calculate the efficiency of the code.

(b) Repeat for the Huffman code and compare the results.

Question 5

A discrete memoryless source has a five symbol alphabet $\{x_1, x_2, x_3, x_4, x_5\}$ with the following probabilities 0.2,0.15,0.05,0.10 and 0.5.

(i) Construct a Shannon-Fano code for X, and calculate the code efficiency.

(ii) Construct a Huffman code for X, and calculate the code efficiency.

Question 6

The input source to a noisy communication channel is a random variable X over three symbols a,b,c. The output from this channel is a random variable Y over the same three symbols. The joint distribution of these two random variables is as follows:

	x=a	x=b	x=c
y=a	0.25	0	0.125
y=b	0	0.125	0
y=c	0.125	0.25	0.125

- Write down the marginal distributions for X and Y.
- Compute the marginal entropies $H(X)$ and $H(Y)$
- Compute the joint entropy $H(X,Y)$ of the two random variables.
- Compute the mutual information $I(X;Y)$.
- Compute the conditional entropies $H(X|Y)$ and $H(Y|X)$.
- From Formulae:

$$I(X,Y) = H(X)H(X|Y)$$

Question 7

The frequency of 0 as an input to a binary channel is 0.6. If 0 is the input, then 0 is the output with probability 0.8. If l is the input, then l is the output with probability 0.9.

Write out the channel transition matrix

- Calculate the output probabilities [P(Y)]
- Compute the joint probabilities [P(X,Y)]
- Calculate the probability that the input is 0 given that the output is O.
- Calculate the probability that the input is l given that the output is 1,
- Calculate the probability that the input is l given that the output is O.
- Calculate the probability that the input is 0 given that the output is l.

Question 8

Consider a DMS X with symbols $\{x_l, x_2, x_3, x_4\}$. The table below lists four possible binary codes.

x_i	Code *A*	Code *B*	Code *C*	Code *D*
x_1	00	0	0	0
x_2	01	10	11	100
x_3	10	11	100	110
x_4	11	110	110	111

Figure 1:

(i) Show that all codes except B satisfy the Kraft inequality (formula below) m

(ii) Show that codesA and D are uniquely decodable but code B and C are not uniquely demdable.

$$K = \sum_{i=1}^{m} 2^{-n_i} \leqslant 1$$

Figure 2:

Question 9

A DMS X has live symbols $\{x_l, x_2, x_3, x_4, x_5\}$ with $P(x_1) = 0.2$, $P(x_2) = 0.15$, $P(x_3) = 0.05$, $P(x_4) = 0.10$, and $P(x_5) = 0.5$.

(a) Construct a Shann0nFano code for X, and calculate the efficiency of the code.

(b) Repeat for the Huffman code and compare the results.