MA4413 Weeks 10 and 11 Tutorials

Question 1 (Paired t-test)

The weight of 10 students was observed before commencement of their studies and after graduation (in kgs). By calculating the realisation of the appropriate test statistic, test the hypothesis that the mean weight of students increases during their studies at a significance level of 5%.

Student	1	2	3	4	5	6	7	8	9	10
Weight before	68	74	59	65	82	67	57	90	74	77
Weight after	71	73	61	67	85	66	61	89	77	83

[**Recall Descriptive Statistics**]
You may be required to carry out these calculations in the exam.

- Case-wise differences are

$$d = \{3, -1, 2, 2, 3, -1, 4, -1, 3, 6\}$$

- The sum of case-wises differences and squared case-wise differences are $\sum d_i = 20$ and $\sum d_i^2 = 90$ respectively.

- Mean of case-wise differences $\bar{d} = 2.00$.

$$\bar{d} = \frac{3 + (-1) + 2 + \ldots + 6}{10} = \frac{20}{10}$$

- Standard deviation of casewise differences $s_d = 2.36$
(Modified version of standard deviation formula)

$$s_d = \sqrt{\frac{\sum(d_i^2) - \frac{(\sum d_i)^2}{n}}{n-1}}$$

$$s_d = \sqrt{\frac{90 - \frac{(20)^2}{10}}{9}} = \sqrt{\frac{50}{9}} = 2.36$$

- Standard Error

$$S.E.(d) = \frac{s_d}{\sqrt{n}} = \frac{2.36}{\sqrt{10}} = 0.745$$

- From Murdoch Barnes, the CV is 1.812 (small sample,df = 9,one-tailed procedure)

Writing the Hypotheses

H_0 $\mu_d \leq 0$
mean of case-wise differences not a positive number. (i.e. no increase in weight)

H_1 $\mu_d > 0$
mean of case-wise differences is a positive number. (i.e. increase in weight)

Question 1 Part B

Calculate a 95% confidence interval for the amount of weight that students put on during their studies. Using this confidence interval, test the hypotheses that on average students put on **3 kilos** during their studies

Question 2 (Two Sample Means - One Tailed)

A pharmaceutical company wants to test, a new medication for blood pressure. Tests for such products often include a '*treatment group*' of people who use the drug and a '*control group*'of people who did not use the drug. 50 people with high blood pressure are given the new drug and 100 others, also with high blood pressure, are not given the drug.

The systolic blood pressure is measured for each subject, and the sample statistics are given below. Using a 0.05 level of significance, test the claim that the new drug **reduces** blood pressure.

Treatment	Control
$n_1 = 50$	$n_2 = 100$
$x_1 = 189.4$	$x_1 = 203.4$
$s_1 = 39.0$	$s_1 = 39.4$

Standard Error Formula

$$S.E.(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Question 3 (Two Sample Means - One Tailed)

The average mass of a sample of 64 Irish teenagers (Let say - 18 year old males) was 73.5kg with a variance of 100kg^2. The average mass of an equivalent sample of 81 Japanese teenagers was 68.5kg with a variance of 81 kg^2.

(i) Test the hypothesis that Irish students are larger (in terms of mass) than Japanese teenagers.

Standard Error Formula

$$S.E.(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$S.E.(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{10^2}{64} + \frac{9^2}{81}} = \sqrt{2.56} = 1.6$$

Question 4 (Two Sample Means, small samples, one tailed)

The working lifetimes of 100 of both of two different types of batteries were observed. The mean lifetime for the sample of type 1 batteries was 25 hrs with a standard deviation of 4hrs. The mean lifetime for the sample of type 1 batteries was 23 hrs with a standard deviation of 3hrs.

Type 1	Type 2
$n_1 = 100$	$n_2 = 100$
$x_1 = 25$ hours	$x_1 = 23$ hours
$s_1 = 4$ hours	$s_1 = 3$ hours

(i) Test the hypothesis that the mean working lifetimes of these batteries do not differ at a significance level of 5% .

(ii) Calculate a 95% confidence interval for the difference between the average working lifetimes of these batteries.

(iii) Using this confidence interval, test the hypothesis that battery 1 on average works for 3 hours longer than battery 2.

Question 5 (Two Sample proportions, one tailed)

A simple random sample of front-seat occupants involved in car crashes were taken. The first sample was on cars with airbags available and it was found that there were 29 occupant fatalities out of a total of 1110 occupants. The second sample was on cars with no airbags available and there were 62 fatalities out of a total 1553 occupants.

(i) Using a 5% significance level, determine whether or not there is a difference in the proportion of fatality rates of occupants in cars with airbags and cars without airbags.

(ii) Calculate a 95% confidence interval for the difference between the two proportions of fatality rates.

Standard Error Formula

Confidence Intervals

$$S.E.(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 \times (100 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \times (100 - \hat{p}_2)}{n_2}}$$

Hypothesis testing

$$S.E.(\pi_1 - \pi_2) = \sqrt{\bar{p} \times (100 - \bar{p}) \times \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Aggregate Sample Proportion

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Confidence Intervals (in terms of percentages)

95% confidence interval

$$(\hat{p}_1 - \hat{p}_2) \times (1.96 \times S.E.(\hat{p}_1 - \hat{p}_2))$$

$$1.4 \times (1.96 \times 0.683) = (1.27, 1.53)$$

Question 6 (Two Sample proportions, one tailed)

- The government wishes to increase the proportion of people taking government training courses who obtain a job in the following 3 months.
- Before they introduced the new schemes this figure was 58%, according to sample of 400 people, with 232 successes.
- A survey of 300 people who took the new courses indicated that 188 of them gained a job. A government official stated that this indicates that the new courses have been more successful.
- Is this statement reasonable at a significance level of 5%?

Some Calculations

- Aggregate proportion
$$\bar{p} = \frac{232 + 188}{400 + 300} = \frac{420}{700} = 60\%$$
- Standard Error for Hypothesis Test
$$S.E$$
$$S.E.(\pi_1 - \pi_2) = \sqrt{60 \times 40) \times \left(\frac{1}{400} + \frac{1}{300}\right)} = 3.74$$

Question 7 - Two Sample Means (Small Samples)

A new process has been developed to reduce the level of corrosion of car bodies.

- Experiments were carried out on 11 cars using the new process and 11 cars using the old process.
- The average level of corrosion using the new process was 3.4 with a standard deviation of 0.5.
- The average level of corrosion using the old process was 4.2 with a standard deviation of 0.8.

(i) Test the hypothesis that the variance of the level of corrosion does not depend on the process used.

(ii) Is there any evidence that the new process is better at a significance level of 5%?

(iii) Calculate a 95% confidence interval for the difference between the mean levels of corrosion under the two processes. Can it be stated that the mean level of corrosion is reduced by 1.5 at a significance level of 5%?

Question 8 - Two Sample Means

Deltatech software has 350 programmers divided into two groups with 200 in Group A and 150 in Group B. In order to compare the efficiencies of the two groups, the programmers are observed for 1 day.

- The 200 programmers in Group A averaged 45.2 lines of code with a standard deviation of 8.4.
- The 150 programmers in Group B averaged 42.7 lines of code with a standard deviation of 5.2.

Let $\bar{x}_A$ denote the average number of lines of code per day produced by programmers in Group A and let $\bar{x}_A$ be the corresponding statistic for Group B. Provide an estimate of $\mu_A \mu_B$ and calculate an approximate 95% confidence interval for

Test the claim that Group A are more efficient than Group B by

(i) Interpreting the 95% confidence interval.

(ii) Computing the appropriate test statistic.

Question 9 - Two Sample Proportions

In a recent British election 40.12% of the voters voted for the Labour party. A survey of 98 people indicated that 49 of them wish to vote for the Labour party.

(i) Does this figure indicate that support for the Labour party has changed at a significance level of 5% (calculate the realisation of the appropriate test statistic)?

(ii) Calculate a 95% confidence interval for the present support of the Labour party. Comment on your result taking your conclusion from part i) into account.

Question 10 - Testing Equality of Variances

Interpret the output from the following tests of equality of variances. State your conclusion both by referencing the $p-$value and the confidence interval. You may assume the significance level is 5%.

(Remark : This procedure is a one-tailed procedure. However, we will base our conclusion on whether or not we arbitrarily decide the p-value is large or small)

```
> var.test(X,Y)

        F test to compare two variances

data:  X and Y
F = , num df = 13, denom df = 13, p-value = 0.02725
alternative hypothesis:
   true ratio of variances is not equal to 1
95 percent confidence interval:
  1.164437 11.299050
sample estimates:
ratio of variances
```

```
> var.test(X,Z)

        F test to compare two variances

data:  X and Z
F = , num df = 13, denom df = 11, p-value = 0.7813
alternative hypothesis:
   true ratio of variances is not equal to 1
95 percent confidence interval:
 0.2526643 2.7401535
sample estimates:
ratio of variances
```

Question 11 - Shapiro-Wilk Test

Interpret the output from the three Shapiro-Wilk tests. What is the null and alternative hypotheses? State your conclusion for each of the three tests.

```
> shapiro.test(X)

        Shapiro-Wilk normality test

data:  X
W = 0.9001, p-value = 0.113
>
```

```
> shapiro.test(Y)

        Shapiro-Wilk normality test

data:  Y
W = 0.8073, p-value = 0.006145
>
```

```
> shapiro.test(Z)

        Shapiro-Wilk normality test

data:  Z
W = 0.9292, p-value = 0.372
```

Question 12 - Classification Metrics

For each of the following classification tables, calculate the following appraisal metrics.

- accuracy
- precision
- recall
- F-measure

	Predict Negative	Predict Positive
Observed Negative	9500	85
Observed Positive	115	300

	Predict Negative	Predict Positive
Observed Negative	9700	140
Observed Positive	60	100

	Predict Negative	Predict Positive
Observed Negative	9530	10
Observed Positive	300	160