MA4413 Weeks 12 and 13 Tutorials Solutions

Question 1b Solutions

	Xi	A	B	C	D	Entropy
Case 1	$p(x_i)$	0.2500	0.2500	0.2500	0.2500	
	$I(x_i)$	2.0000	2.0000	2.0000	2.0000	
	$p(x_i) \times I(x_i)$	0.5000	0.5000	0.5000	0.5000	2.0000
Case 2	$p(x_i)$	0.2500	0.1250	0.1250	0.1250	
	$I(x_i)$	2.0000	3.0000	3.0000	3.0000	
	$p(x_i) \times I(x_i)$	0.5000	0.3750	0.3750	0.3750	1.6250
Case 3	$p(x_i)$	0.7000	0.1000	0.1000	0.1000	
	$I(x_i)$	0.5146	3.3219	3.3219	3.3219	
	$p(x_i) \times I(x_i)$	0.3602	0.3322	0.3322	0.3322	1.3568
Case 4	$p(x_i)$	0.9700	0.0100	0.0100	0.0100	
	$I(x_i)$	0.0439	6.6439	6.6439	6.6439	
	$p(x_i) \times I(x_i)$	0.0426	0.0664	0.0664	0.0664	0.2419

Question 1c Solutions

For source alphabet of size m where each symbol has an equal probability of occurring

$$H(X) = \log_2(m)$$

Here there are 8 equally probable symbols.

$$H(X) = \log_2(8) = 3\text{b/sym}$$

Question 2 Solutions

Marginal probabilities

	y=a	y=b	y=c	
x=a	0.25		0.125	**0.375**
x=b		0.125	0	**0.125**
x=c	0.125	0.25	0.125	**0.5**
	0.375	**0.375**	**0.25**	1

Marginal Entropies : $H(X)$ and $H(Y)$

$P(x_i)$	$I(x_i)$	$P(x_i) \times I(x_i)$		$P(y_i)$	$I(y_i)$	$P(y_i) \times I(y_i)$
0.3750	1.4150	0.5306		0.3750	1.4150	0.5306
0.1250	3.0000	0.3750		0.3750	1.4150	0.5306
0.5000	1.0000	0.5000		0.2500	2.0000	0.5000
	$H(X)$	1.4056 b/sym			$H(Y)$	1.5613 b/sym

Joint Entropy : $H(X, Y)$

$P(x_i)$	Freq(x_i)	$I(x_i)$	$p(x_i) \times I(x_i) \times \text{Freq}(x_i)$
0	3	...	0
0.125	4	3	1.5
0.25	2	2	1
		H(X,Y)	2.5