Statistics for Computing

MA4413 Lecture 4B and 5B

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Current Status

- Mid Term Examination next Monday (Week 5) at 4pm
- Currently covering Continuous Probability Distributions
- Lecture notes are out of synch with published class notes (we are about half a lecture ahead)
- The Exponential distribution will be examinable. (I will confirm that at the end of this lecture)
- Next Wednesday We will start looking at the Normal Distribution.

Continuous Distributions

- (The Continuous Uniform Distribution, Not examinable)
- The Exponential Distribution (Examinable for midterm)
- The Normal Distribution
- The Standard Normal (Z) Distribution.
- Applications of Normal Distribution

Exponential Distribution

The Exponential Distribution may be used to answer the following questions:

- How much time will elapse before an earthquake occurs in a given region?
- How long do we need to wait before a customer enters our shop?
- How long will it take before a call center receives the next phone call?
- How long will a piece of machinery work without breaking down?

Exponential Distribution

- All these questions concern the time we need to wait before a given event occurs. If this waiting time is unknown, it is often appropriate to think of it as a random variable having an exponential distribution.
- Roughly speaking, the time X we need to wait before an event occurs has an exponential distribution if the probability that the event occurs during a certain time interval is proportional to the length of that time interval.

Probability density function

The probability density function (PDF) of an exponential distribution is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

The parameter λ is called ***rate*** parameter.

Cumulative density function

The cumulative distribution function (CDF) of an exponential distribution is

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Expected Value and Variance

The expected value of an exponential random variable X is:

$$E[X] = \frac{1}{\lambda}$$

The variance of an exponential random variable X is:

$$V[X] = \frac{1}{\lambda^2}$$

Exponential Distribution: Example

Assume that the length of a phone call in minutes is an exponential random variable X with parameter $\lambda = 1/10$. If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait

(a) less than 5 minutes,

(b) between 5 and 10 minutes.

Exponential Distribution: Example

As it is CDF values that we are interested in, we use the output from the `pexp()` commands.

(a) $P(X \leq 5) = 0.39346934$

(b) $P(5 \leq X \leq 10)$
$= P(X \leq 10) - P(X \leq 5)$
$= 0.63212056 - 0.39346934$
$= 0.2386512$
$= 23.84\ \%$

Exponential Distribution

- The Exponential Rate
- Related to the Poisson mean (m)
- If we expect 12 occurrences per hour - what is the rate?
- We would expected to wait 5 minutes between occurrences.