MA4413

Question 1:

Sample Space $S=\left\{\begin{matrix}(R,1),(R,2),(R,3),\\(B,1),(B,2),(B,3),\\(G,1),(G,2),(G,3)\end{matrix}\right\}$

G: Green R:Red B: Blue

Question 2:

$$P(C)=80/200=0.40$$

$$P(F\cap B)=50/200=0.25$$

$$P(B|F)=\frac{P(F\cap B)}{P(B)}=50/100=0.50$$

$$P(F|B)=\frac{P(F\cap B)}{P(B)}=50/70=0.7142$$

Question 3

part a

The sum of the probabilities is equal to 1.

i.e. $0.30+0.20+k+0.10+0.20 = 1$

Necessarily k = 0.20

part b

$$E(X)=\sum x_i P(x_i)$$

$$E(X)=(1\times 0.30)+(2\times 0.20)+(3\times 0.20)+(4\times 0.10)+(5\times 0.20)$$

$$E(X)=2.7$$

part c

We were given $E(X^2)$ as 9.5, but lets see how it was computed.

$$E(X^2)=\sum x_i^2 P(x_i)$$

$$E(X^2)=(1^2\times 0.30)+(2^2\times 0.20)+(3^2\times 0.20)+(4^2\times 0.10)+(5^2\times 0.20)$$

$$E(X^2)=9.5$$

$V(X) = E(X^2) - E(X)^2$

$V(X) = 9.5 - (2.7)^2 = 2.21$

Question 4

part a

$^{11}C_2 = \frac{11!}{9! \times 2!} = \frac{11 \times 10 \times 9!}{9! \times 2!} = \frac{110}{2} = 55$

part b

Given parameters $n = 11$ $p = 0.48$

$P(X=2) = {}^{11}C_2 (0.48)^2 (1-0.48)^9$ (Remark: Full marks were given once this was presented)

$P(X=2) = 0.035227$

part a

For a one hour period, the Poisson mean is $m = 4$.

So for a 30 minute period, the Poisson mean is $m = 2$.

part b

$P(X=1) = \frac{2^1 \times e{-2}}{1!} = 2 \times e^{-2} = 0.2707$

Question 6
X is a normally distributed random variable with mean $\mu = 500$ and $\sigma = 24$

part a

$P(X \geq 518)$

Compute Z value $Z_{518} = \frac{518 - 500}{24} = 0.75$

From tables $P(Z \geq 0.75) = 0.2266$

Therefore $P(X \geq 518) = 0.2266$

part b

$P(X \leq 482)$

Compute Z value $Z_{482} = \frac{482 - 500}{24} = -0.75$

Using symmetry rule $P(Z \leq -0.75) = P(Z \geq 0.75) = 0.2266$

Therefore $P(X \leq 482) = 0.2266$

Remark: No need for complement rule in this question.

part c

$P(482 \leq X \leq 518)$

Use "Too Low/Too High" approach.

Too Low: $P(X \leq 482) = 0.2266$

Too High: $P(X \geq 518) = 0.2266$

Inside Interval = 1 -(Too Low + Too High) = 1 - (0.2266 + 0.2266) = 0.5468

$P(482 \leq X \leq 518) = 0.5468$