Two Sample Procedures

- So far we have looked at single sample procedures.
- We can generalise our methodologies for comparing two samples.
- Our point estimates are typically differences in sample statistics. i.e.

$$\bar{X}_1 - \bar{X}_2$$

$$\hat{p}_1 - \hat{p}_2$$

- We can use these point estimates to make inference on differences for populations.

$$\mu_1 - \mu_2$$

$$\pi_1 - \pi_2$$

Two Sample Procedures - Confidence Intervals

- When computing confidence intervals, all that is required is the calculation of the appropriate standard error value.
- The two-sample standard error calculations contain statistical information from both samples.
- See the formula sheet. (Practice with these calculaions will take place in next week's tutorials.)
- There are some minor issues that will arise with each type of procedure. These will be explained in relevant examples.

Two Sample Procedures - small samples and degrees of freedom

- Let n_1 and n_2 be the sample sizes of two samples.
- When deciding whether to use the large sample approach or the small sample appproach, we will use the following rule of thumb:
  Small sample : $n_1 + n_2 \leq 30$
  Large sample : otherwise

- For small samples the appropriate degrees of freedom is $(n_1 - 1) + (n_2 - 1)$ i.e. $n_1 + n_2 - 2$

Two Sample Procedures - small samples and degrees of freedom

- When performing hypothesis tests, we are usually interested in determining whether or not the population parameters can be considered equal for both populations.
- Another way of expressing this is that the difference in population parameters is 0.
- The following two hypotheses are directly equivalent.

$$H_o : \mu_1 = \mu_2$$

$$H_o : \mu_1 - \mu_2 = 0$$

- Equivalently, for proportions:

$$H_o : \pi_1 = \pi_2$$

$$H_o : \pi_1 - \pi_2 = 0$$

0.1 Hypothesis test for the means of Two independent samples

The procedure associated with testing a hypothesis concerning the difference between two population means is similar to that for testing a hypothesis concerning the value of one population mean. The procedure differs only in that the standard error of the difference between the means is used to determine the test statistic associated with the sample result. For two tailed tests, the null hypothesis states that the population means are the same, with the alternative stating that the population means are not equal.

- Two samples are referred to as independent if the observations in one sample are not in any way related to the observations in the other.
- This is also used in cases where one randomly assign subjects to two groups, i.e. in give first group treatment A and the second group treatment B and compare the two groups.
- Often we are interested in the difference between the mean value of some parameter for both groups.

The approach for computing a confidence interval for the difference of the means of two independent samples, described shortly, is valid whenever the following conditions are met:

- Both samples are simple random samples.
- The samples are independent.
- Each population is at least 10 times larger than its respective sample. (Otherwise a different approach is required).
- The sampling distribution of the difference between means is approximately normally distributed

0.2 Confidence Intervals for Difference Of Two Means

In order to construct a confidence interval, we are going to make three assumptions:

- The two populations have the same variance. This assumption is called the assumption of homogeneity of variance.
- For the time being, we will use this assumption. Later on in the course, we will discuss the validity of this assumption for two given samples.
- The populations are normally distributed.
- Each value is sampled independently from each other value.
- If the combined sample size of X and Y is greater than 30, even if the individual sample sizes are less than 30, then we consider it to be a large sample.
- The quantile is calculated according to the procedure we met in the previous class.
- Assume that the mean (μ) and the variance (σ) of the distribution of people taking the drug are 50 and 25 respectively and that the mean (μ) and the variance (σ) of the distribution of people not taking the drug are 40 and 24 respectively.

$$(\bar{X} - \bar{Y}) \pm \left[\text{Quantile } \times S.E(\bar{X} - \bar{Y})\right]$$

0.3 Computing the Confidence Interval

- As always the first step is to compute the point estimate. For the difference of means for groups X and Y, the point estimate is simply the difference between the two means i.e. $\bar{x} - \bar{y}$.
- As we have seen previously, sample size has a bearing in computing both the quantile and the standard error. For two groups, we will use the aggregate sample size $(n_x + n_y)$ to compute the quantile. (For the time being we will assume, the aggregate sample size is large $(n_x + n_y) > 30$.)
- Lastly we must compute the standard error $S.E.(\bar{x} - \bar{y})$. The formula for computing standard error for the difference of two means, depends on whether or not the aggregate sample size is large or not. For the case that the sample size is large, we use the following formula (next slide).

Interpreting Confidence Intervals

- In the previous lectures, we looked at confidence intervals, noting that these intervals are a pair of limits defining an interval.
- Often, we can use confidence intervals to make inferences on a population parameter.
- Consider the following example: Suppose that, when considering the leaving cert points of two groups of students A and B, the difference of the sample means was found to be $\bar{x}_B - \bar{x}_A = 30$ points.
- We would surmise than the average points level for group B is higher.
- Lets suppose that the 95% confidence interval was $(-15, 75)$ points. Consider what each of the two numbers mean,

Interpreting Confidence Intervals

- The upper bound (+75) inferences that those in group B could have, on average, 75 more points than those in group A.
- But the lower bound (−15) inferences that those in group A could have, on average, 15 more points than those in group B.
- Also, the confidence interval allows for the possibility of both groups having equal means (i.e. $\bar{x}_B - \bar{x}_A = 0$)
- Essentially we can not be 95% confident that group B has a higher mark than group A.

Confidence Intervals

- We have studied two types of confidence interval, a confidence interval for a sample mean and for a sample proportion (Later we will call these ***One Sample*** confidence Intervals.
- There are more types of confidence intervals that we will cover later in this course. (We shall refer to these confidence intervals as the ***Two Sample***confidence Intervals.

0.4 Difference in Two means

For this calculation, we will assume that the variances in each of the two populations are equal. This assumption is called the assumption of homogeneity of variance.

The first step is to compute the estimate of the standard error of the difference between means ().

Computing the Confidence Interval
Standard Error for difference of two means (large sample)

$$S.E.(\bar{x}-\bar{y}) = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$$

- s_x^2 and s_x^2 is the variance of samples X and Y respectively.
- n_x and n_y is the sample size of both samples.
- For small samples, the degrees of freedom is $df = n_x + n_y - 2$. If the sample size $n \leq 32$, we can find appropriate $t-$quantile, rather than assuming it is a $z-$quantile.

0.4.1 Computing the Confidence Interval (Small Samples)

Standard Error for difference of two means (small aggregate sample)

$$S.E.(\bar{x}-\bar{y}) = \sqrt{s_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$$

Pooled Variance s_p^2 is computed as:

$$s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{(n_x-1)+(n_y-1)}$$

0.4.2 CI for Difference in Two Means

A research company is comparing computers from two different companies, X-Cel and Yellow, on the basis of energy consumption per hour. Given the following data, compute a 95% confidence interval for the difference in energy consumption.

Type	sample size	mean	variance
X-cel	17	5.353	2.743
Yellow	17	3.882	2.985

Remark: It is reasonable to believe that the variances of both groups is the same. Be mindful of this.

0.4.3 CI for Difference in Two Means (Example)

From the previous example (comparing X-cel and Yellow) lets compute a 95% confidence interval when the sample sizes are $n_x = 10$ and $n_y = 12$ respectively. (Lets assume the other values remain as they are.)

Type	sample size	mean	variance
X-cel	10	5.353	2.743
Yellow	12	3.882	2.985

The point estimate $\bar{x} - \bar{y}$ remains as 1.469. Also we require that both samples have equal variance. As both X and Y have variances at a similar level, we will assume equal variance.

0.4.4 Computing the Confidence Interval

- Pooled variance s_p^2 is computed as:

$$s_p^2 = \frac{(10-1)2.743 + (12-1)2.985}{(10-1)+(12-1)} = \frac{57.52}{20} = 2.87$$

- Standard error for difference of two means is therefore

$$S.E.(\bar{x} - \bar{y}) = \sqrt{2.87\left(\frac{1}{10} + \frac{1}{12}\right)} = 0.726$$

- The aggregate sample size is small i.e. 22. The degrees of freedom is $n_x + n_y - 2 = 20$. From Murdoch Barnes tables 7, the quantile for a 95% confidence interval is 2.086.
- The confidence interval is therefore

$$1.469 \pm (2.086 \times 0.726) = 1.4699 \pm 1.514 = (-0.044, 2.984)$$

0.5 Two sample test

Suppose one has two independent samples, x1, ..., xm and y1, ..., yn, and wishes to test the hypothesis that the mean of the x population is equal to the mean of the y population:

$H0 : \mu_x = \mu_y$.

Alternatively this can be formulated as $H0 : \mu_x - \mu_y = 0$.

Let $\bar{X}$ and $\bar{Y}$ denote the sample means of the xs and ys and let S_x and S_y denote the respective standard deviations. The standard test of this hypothesis H_0 is based on the t statistic

$$T = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{1/n_1 + 1/n_2}} \tag{1}$$

where S_p is the pooled standard deviation.

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}} \tag{2}$$

Under the hypothesis H_0, the test statistic T has a t distribution with $m + n - 2$ degrees of freedom when

- both the xs and ys are independent random samples from normal distributions
- the standard deviations of the x and y populations, σ_x and σ_y, are equal

.

Suppose the level of significance of the test is set at α. Then one will reject H when $|T| < tn + m.2, \alpha/2$, where tdf, α is the $(1-\alpha)$ quantile of a t random variable with df degrees of freedom.

If the underlying assumptions of

Point Estimate

- Difference in samples means $\bar{x}_1$ - $\bar{x}_2$
- Suppose the sample mean for group 1 is 74, while the sample mean for group 2 is 69.
- The point estimate is $\bar{x}_1$ - $\bar{x}_2 = 29 - 25 = 4$.

Standard Error

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

	Mean	Sample Size	Std Deviation
Group 1	74	50	10
Group 2	69	48	12

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{2+3} = \sqrt{5}$$

95% **Confidence Intervals** The 95% confidence interval $\mu_X - \mu_Y$ is computed as

$$(\bar{X} - \bar{Y}) \pm 1.96\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$$

We will use this in an example shortly.

0.6 Part II: Two-Sample t-Test for Equal Means

0.6.1 Standard Error

$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{}(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}) \quad (3)$$

0.6.2 The Test Statistic

The $t-$statistic to test whether the means are different can be calculated as follows:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1X_2} \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where

$$S_{X_1X_2} = \sqrt{\frac{(n_1-1)S_{X_1}^2 + (n_2-1)S_{X_2}^2}{n_1+n_2-2}}$$

- A random variable, usually written X, is a variable whose possible values are numerical outcomes of a random event.

- There are two types of random variables, ***discrete*** and ***continuous***.

0.7 Hypothesis Tests for Two Means

If the population standard deviations σ_1 and σ_2 are known, the test statistic is of the form:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{4}$$

The critical value and p-value are looked up in the normal tables.

0.7.1 2 sided test

A two-sided test is used when we are concerned about a possible deviation in either direction from the hypothesized value of the mean. The formula used to establish the critical values of the sample mean is similar to the formula for determining confidence limits for estimating the population mean, except that the hypothesized value of the population mean m0 is the reference point rather than the sample mean.

$$(\bar{X} - \bar{Y}) \pm \left[\text{Quantile } \times S.E(\bar{X} - \bar{Y})\right]$$

- Assume that the mean (μ) and the variance (σ) of the distribution of people taking the drug are 50 and 25 respectively and that the mean (μ) and the variance (σ) of the distribution of people not taking the drug are 40 and 24 respectively.

Two Sample Inference Procedures

- Previously we looked at inference procedures (Confidence Intervals and Hypothesis Testing) for single samples.
- Yesterday we looked at ***paired*** samples, with two sets of paired measurements. With paired measurements, we are specifically interested in the ***case-wise*** differences.
- Although there are two sets of data, we consider the single data set of case-wise differences.
- Now we look at the case of two independent sample procedures.
- Independent samples are distinct from paired samples, in that data in one set are not paired with data in another set.

Two Sample Inference Procedures

- Firstly, we will look at the difference in the means of two independent populations.
- Let us assume that the both populations are normally distributed $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$

- The difference in means for two independent populations X and Y is denoted $\mu_X - \mu_Y$.
- Almost always, this value is unknown, and is instead estimated by the difference in sample means: $\bar{X} - \bar{Y}$.
- The sample sizes do not need to be equal necessarily. We denote the respective sample sizes n_X and n_Y.
- For the moment, we will assume that both n_X and n_Y are large samples (≥ 30).

Sampling

- The sampling distribution of the difference in means is normally distributed, when both samples sizes are greater than 30.
- The expected value of this distribution is $\mu_X - \mu_Y$.
- Importantly, the standard error of this distribution is

$$S.E(\bar{X} - \bar{Y}) = \sqrt{\frac{\sigma^2_X}{n_X} + \frac{\sigma^2_Y}{n_Y}}$$

- The standard deviations for populations X and Y are σ_X and σ_Y respectively.
- Usually these population standard deviations are estimated by the sample standard deviations s_X and s_Y respectively.

0.8 Two Small Samples Case

- Previously we have looked at large samples, now we will consider small samples.
- (For the sake of clarity, I will not use small samples that have a combined sample size of greater than 30.
- Additionally we require the assumption that both samples have equal variance. This assumption **must** be tested with another formal hypothesis test. We will revisit this later, and in the mean time, assume that the assumption of equal variance holds.
- The key differences between the large sample case and the small sample cases arise in the following steps.
  - The standard error is computed in a different way (see next slide).
  - The degrees of freedom used to compute the critical value is $(n_X - 1) + (n_Y - 1))$ or equivalently $(n_X + n_Y - 2)$.
  - Also - a formal test of equality of variances is required beforehand (End of Year Exam)

0.8.1 Two Small Samples Case: Standard Error

Computing the standard error requires a two step calculation. From the formulae, we have the two equations below. The first term s^2_p is called the ***pooled variance*** of the combined samples.

0.9 Working with Two Samples

Computing the Confidence Interval Standard Error for difference of two means (small aggregate sample)

$$S.E.(\bar{x} - \bar{y}) = \sqrt{s_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}$$

Pooled Variance s_p^2 is computed as:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_y^2}{(n_2 - 1) + (n_2 - 1)}$$

- Assume that the mean (μ) and the variance (σ) of the distribution of people taking the drug are 50 and 25 respectively and that the mean (μ) and the variance (σ) of the distribution of people not taking the drug are 40 and 24 respectively.

CI for Proportion: Example (1)

- $\hat{p} = 0.62$
- Sample Size $n = 250$
- Confidence level $1 - \alpha$ is 95%

CI for Proportion: Example (2)

- First, lets determine the quantile.
- The sample size is large, so we will use the Z distribution.
- (Alternatively we can uses the $t-$ distribution with ∞ degrees of freedom.

Although the sample mean is useful as an unbiased estimator of the population mean, there is no way of expressing the degree of accuracy of a point estimator. In fact, mathematically speaking, the probability that the sample mean is exactly correct as an estimator of the population mean is $P = 0$.

A confidence interval for the mean is an estimate interval constructed with respect to the sample mean by which the likelihood that the interval includes the value of the population mean can be specified.

The *level of confidence* associated with a confidence interval indicates the long-run percentage of such intervals which would include the parameter being estimated.

- Confidence intervals for the mean typically are constructed with the unbiased estimator $\bar{x}$ at the midpoint of the interval.
- The $\pm Z\sigma_x$ or $\pm Zs_x$ frequently is called the ***margin of error*** for the confidence interval.

We indicated that use of the normal distribution in estimating a population mean is warranted for any large sample ($n > 30$), **and** for a small sample ($n \leq 30$) only if the population is normally distributed and σ is known.

- Now we consider the situation in which the sample is small and the population is normally distributed, but σ is not known.
- The distribution is a family of distributions, with a somewhat different distribution associated with the degrees of freedom (df). For a confidence interval for the population mean based on a sample of size n, $df = n - 1$.

Computing the Standard Error

$$S.E.(\hat{p}) \;=\; \sqrt{\frac{\hat(p) \times (100 - \hat{p})}{n}}$$

$$\hat{p} = 144/200 \times 100\% = 0.72 \times 100\%. = 72$$

$100\% - \hat{p} = 100\% - 72\% = 28\%$
Computing the Standard Error

$$S.E.(\hat{p}) \;=\; \sqrt{\frac{72 \times 28}{200}}$$

- SE $= \sqrt{[p_1 \times (1 - p_1)/n_1] + [p_2 \times (1 - p_2)/n_2]}$
- SE $= \sqrt{[0.40 \times 0.60/400] + [0.30 \times 0.70/300]}$
- SE $= \sqrt{[(0.24/400) + (0.21/300)]} = \sqrt{(0.0006 + 0.0007)} = \sqrt{0.0013} = 0.036$

0.9.1 Example using R

Finding confidence intervals for the mean for the nitrate ion concentrations in Example 2.7.1.

```
#Typing data in
x=c(102,97,99,98,101,106)
mean(x)
sd(x)
n=length(x)
#setting the confidence level
CL=0.95
#computing confidence interval
pm=sd(x)*c(qt(0.025,n-1),qt(0.975,n-1))/sqrt(n)
CI=mean(x)+pm
```

0.10 Part II: Two-Sample t-Test for Equal Means

0.11 Hypothesis Tests for Two Means

If the population standard deviations σ_1 and σ_2 are known, the test statistic is of the form:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{5}$$

The critical value and p-value are looked up in the normal tables.

0.11.1 Standard Error

$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{(}\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}) \tag{6}$$

0.11.2 Two Sample t-test : Example

The mean height of adult males is 69 inches and the standard deviation is 2.5 inches. The mean height of adult females is 65 inches and the standard deviation is 2.5 inches. Let population 1 be the population of male heights, and population 2 the population of female heights. Suppose samples of 50 each are selected from both populations.

0.11.3 Example

A sample of 50 households in one community shows that 10 of them are watching a TV special on the national economy. In a second community, 15 of a random sample of 50 households are watching the TV special. We test the hypothesis that the overall proportion of viewers in the two communities does not differ, using the 1 percent level of significance, as follows:

0.12 Hypothesis Tests for Two Means

If the population standard deviations σ_1 and σ_2 are known, the test statistic is of the form:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{7}$$

The critical value and p-value are looked up in the normal tables.

0.13 Study Section : Example

A survey of study habits wishes to determine whether the mean study hours completed by women at a particular college is higher than for men at the same college. A sample of $n_1 = 10$ women and $n_2 = 12$ men were taken, with mean hours of study $\bar{x}_1 = 120$ and $\bar{x}_2 = 105$ respectively. The standard deviations were known to be $\sigma_1 = 28$ and $\sigma_2 = 35$.

The hypothesis being tested is:

$$H_0 : \mu_1 = \mu_2 \qquad (\mu_1 - \mu_2 = 0) \tag{8}$$
$$H_a : \mu_1 \neq \mu_2 \qquad (\mu_1 - \mu_2 \neq 0) \tag{9}$$

In R, the test statistic is calculated using:

```
xbar1 <- 120
xbar2 <- 105
sd1 <- 28
sd2 <- 35
n1 <- 10
n2 <-12
TS <- ( (xbar1 - xbar2) - (0) )/sqrt( (sd1^2/n1) + (sd2^2/n2) )
TS
[1] 1.116536
```

Now need to calculate the critical value or the p-value.

The critical value can be looked up using qnorm. Since this is a one-tailed test and there is a ¿ sign in H_1:

```
qnorm(0.95)
[1] 1.644854
```

Since the test statistic is less than the critical value (1.116536 ¡ 1:645)there is not enough evidence to reject H_0 and conclude that the population mean hours study for women is not higher than the population mean hours study for men.

The p-value is determined using pnorm.

Careful! Remember pnorm gives the probability of getting a value LESS than the value specified. We want the probability of getting a value greater than the test statistic.

```
1-pnorm(1.116536) # OR pnorm(1.116536, lower.tail=FALSE)
[1] 0.1320964
```

0.13.1 Difference of Two Means

- **Point Estimate**
- Difference in samples means $\bar{x}_1$ - $\bar{x}_2$
- Suppose the sample mean for group 1 is 74, while the sample mean for group 2 is 69.
- The point estimate is $\bar{x}_1$ - $\bar{x}_2 = 29 - 25 = 4$

Standard Error

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

	Mean	Sample Size	Std Deviation
Group 1	74	50	10
Group 2	69	48	12

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{10^2}{50} + \frac{12^2}{48}} = \sqrt{\frac{100}{50} + \frac{144}{48}}$$

$$S.E.(\bar{X}_1 - \bar{X}_2) = \sqrt{2+3} = \sqrt{5}$$

0.13.2 Difference of Two Means

- **Point Estimate**
- Difference in samples means $\bar{x}_1$ - $\bar{x}_2$
- Suppose the sample mean for group 1 is 74, while the sample mean for group 2 is 69.
- The point estimate is $\bar{x}_1$ - $\bar{x}_2$ $= 29 - 25 = 4$

0.14 F-test of equality of variances

The test statistic is

$$F = \frac{S_X^2}{S_Y^2} \tag{10}$$

has an F-distribution with $n-1$ and $m-1$ degrees of freedom if the null hypothesis of equality of variances is true.

95% **Confidence Intervals**

The 95% confidence interval $\mu_X - \mu_Y$ is computed as

$$(\bar{X} - \bar{Y}) \pm 1.96\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}$$

We will use this in an example shortly.

0.15 Hypothesis Testing

- Hypothesis testing works in much the same way as material we have covered already, in that we will use a four step process.
- The final two steps (critical value step and decision rule step) are precisely the same as previously.
- We will now discuss the first two steps.

Hypothesis Testing: Null and Alternative Hypothesis

We are often interested in whether or not two populations have equal mean values. Accordingly, we would construct the hypotheses accordingly.

H_0 $\mu_X = \mu_Y$

H_1 $\mu_X \neq \mu_Y$

Equivalently we may view in the context of the difference in the populations means, where a difference of zero indicates equality of means.

H_0 $\mu_X - \mu_Y = 0$

H_1 $\mu_X - \mu_Y \neq 0$

This second approach is more intuitive in the context of constructing the test statistic.

Hypothesis Testing: Test Statistic

- The standard error for difference in means has been introduced previously

$$S.E(\bar{X} - \bar{Y}) = \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$$

- **Null value**: The expected value of the difference under the null hypothesis $\mu_X - \mu_Y$ is always 0, when the equality of population means is in question.
- **Observed Value**: The observed difference between sample means is $\bar{X} - \bar{Y}$.

Interpreting p-values

The p-value is the probability of having observed our data (or more extreme data) when the null hypothesis is true

The smaller the p-value, the less likely it is that the sample results come from a situation where the null hypothesis H0 is true. If the p-value is sufficiently small, we reject the null hypothesis, and support the alternative hypothesis Ha.

One Sided Tests

p-value ¿ 0.05 : no evidence against H0 in favour of Ha

p-value ¡ 0.05 : evidence against H0 in favour of Ha

Two Sided Tests

p-value ¿ 0.025 : no evidence against H0 in favour of Ha

p-value ¡ 0.025 : evidence against H0 in favour of Ha