தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு

இரண்டாம் தவணைப் பரீட்சை - 2024

National Field Work Centre, Thondaimanaru.

2nd Term Examination - 2024

Gr : 12 (2025) இணைந்த கணிதம் புள்ளித்திட்டம்

01]
$P(x)=2x^3+ax^2-11x-5$

$P(2)=1$ (5)

$2(2)^3+a(2)^2-11(2)-5=1$

$4a=12$

$a=3$ (5)

$\therefore P(x)=2x^3+3x^2-11x-5$

$P(x)-1=2x^3+3x^2-11x-6$

$=(x-2)(2x^2+Ax+3)$ (5)

$x^2;\ 3=A-4$

$A=7$ (5)

$P(x)-1=(x-2)(2x^2+7x+3)$ (5)

$=(x-2)(2x+1)(x+3)$

$\boxed{25}$

02] $k\in\mathbb{R}\setminus\{0\}$

$\dfrac{1}{x(x-k)^2}=\dfrac{A}{x}+\dfrac{B}{x-k}+\dfrac{C}{(x-k)^2}$ (5)

$1=A(x-k)^2+Bx(x-k)+Cx$

$x^2;\ 0=A+B$

$x;\ 0=-2Ak-Bk+C$

$x^0;\ 1=Ak^2$

$A=\dfrac{1}{k^2}$ (5)

$B=-\dfrac{1}{k^2}$ (5)

$C=\dfrac{1}{k}$ (5)

$\dfrac{1}{x(x-k)^2}=\dfrac{1}{k^2x}-\dfrac{1}{k^2(x-k)}+\dfrac{1}{k(x-k)^2}$ (5)

$\boxed{25}$

03] $x+2\geq\dfrac{8}{x}$, $x\neq0$

$x+2-\dfrac{8}{x}\geq0$ (5)

$\dfrac{x^2+2x-8}{x}\geq0$

$\dfrac{(x+4)(x-2)}{x}\geq0$ (5)

	$x<-4$	$-4<x<0$	$0<x<2$	$2<x$
$x+4$	(−)	(+)	(+)	(+)
$x-2$	(−)	(−)	(−)	(+)
x	(−)	(−)	(+)	(+) (10)
$\dfrac{(x+4)(x-2)}{x}$	(−)	(+)	(−)	(+)

$-4\leq x<0$ or $x\geq2$ (5)

$\boxed{25}$

04] $\log_3 2=a$, $\log_5 2=b$

$\log_{2025}2=\dfrac{1}{\log_2 2025}$ (5)

$=\dfrac{1}{\log_2\{5^2\times3^4\}}$ (5)

$=\dfrac{1}{2\log_2 5+4\log_2 3}$ (5)

$=\dfrac{1}{2\{\frac{1}{b}\}+4\{\frac{1}{a}\}}$ (5)

$=\dfrac{ab}{2a+4b}$

$=\dfrac{ab}{2(a+2b)}$ (5)

$\boxed{25}$

05]

$$\lim_{x\to 0} \frac{1-\cos 2x-\cos 4x+\cos 2x\cos 4x}{x^4}$$

$$=\lim_{x\to 0} \frac{(1-\cos 2x)(1-\cos 4x)}{x^4} \quad 5$$

$$=\lim_{x\to 0} \frac{(2\sin^2 x)(2\sin^2 2x)}{x^4} \quad 5$$

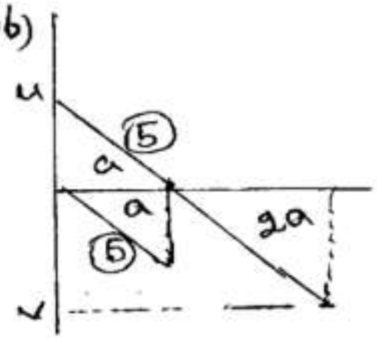
$$=4\times 4\left\{\lim_{x\to 0}\frac{\sin x}{x}\right\}^2\left\{\lim_{2x\to 0}\frac{\sin 2x}{2x}\right\}^2 \quad 5$$

$$=16\times 1^2\times 1^2 \quad 5$$

$$=16 \quad 5$$

$\boxed{25}$

06)



$$\frac{1}{2}\frac{u^2}{g}=a \quad 5$$

$$u^2=2ag$$

$$u=\sqrt{2ag} \quad 5$$

$$\frac{1}{2}\frac{v^2}{g}=2a \quad 5$$

$$v^2=4ag$$

$$v=2\sqrt{ag}$$

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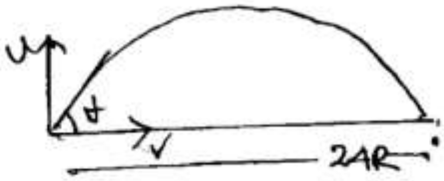
07)



$u=v\tan\alpha$

$=3v \quad 5$

$\uparrow s=ut+\frac{1}{2}at^2$

$0=3vt-\frac{1}{2}gt^2$

$t=\frac{6v}{g} \quad 5$

$vt=24R$

$$\frac{6v^2}{g}=24R$$

$$v=2\sqrt{gR} \quad 5$$

$\uparrow v^2=u^2+2as$

$0=9v^2-2gh \quad 5$

$$h=\frac{9v^2}{2g}=18R \quad 5$$

$\boxed{25}$

08)

$\overrightarrow{AB}=2\sqrt{3}\underline{i}-2\underline{j} \quad 5$

$\overrightarrow{OC}=x\underline{i}+y\underline{j}$, $x^2+y^2=4 \quad 5$

$\overrightarrow{AB}\cdot\overrightarrow{OC}=0$

$(2\sqrt{3}\underline{i}-2\underline{j})\cdot(x\underline{i}+y\underline{j})=0 \quad 5$

$2\sqrt{3}x-2y=0$

$y=\sqrt{3}x$

$4x^2=4$

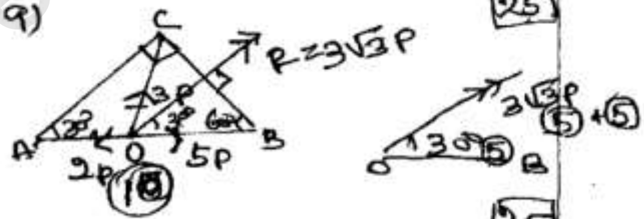
$x=\pm 1 \quad 5$

$\overrightarrow{OC}=\underline{i}+\sqrt{3}\underline{j}$ or $\quad 5$

$\overrightarrow{OC}=-\underline{i}-\sqrt{3}\underline{j}$

$\boxed{25}$

9)



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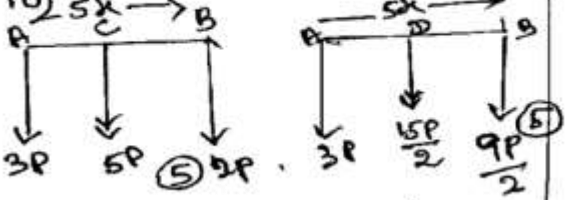
10)



$AC=\frac{2}{5}\times 5x$

$=2x \quad 5$

$AD=\frac{3}{5}\times 5x$

$=3x \quad 5$

விசையின் நகர்வு தூரம் $=3x-2x$

(moving distance $=x$ //

5

$\boxed{25}$

11)

a) $f(x)=0$

$$x^2-(3k+1)x+(k+1)(k-2)=0$$

Discriminant Δ

$$\Delta=\{-(3k+1)\}^2-4(1)(k+1)(k-2) \quad (10)$$

$$=(9k^2+6k+1)-4(k^2-k-2) \quad (5)$$

$$=5k^2+10k+9 \quad (5)$$

$$=5\left\{k^2+2k+\frac{9}{5}\right\}$$

$$=5\left\{(k+1)^2+\frac{4}{5}\right\} \quad (5)$$

$$=5(k+1)^2+4$$

$$>0 \quad (5)$$

$\therefore$ the roots of $f(x)=0$ are real and distinct $\boxed{30}$

$$\alpha+\beta=3k+1$$
$$\alpha\beta=(k+1)(k-2) \quad (5)$$

Both α, β are negative

$$\alpha<0\ ,\ \beta<0$$

$$\Leftrightarrow \alpha+\beta<0 \ \& \ \alpha\beta>0 \quad (5)$$

$$\Leftrightarrow 3k+1<0 \ \& \ (k+1)(k-2)>0$$

$$\Leftrightarrow k<-\frac{1}{3} \ \& \ k<-1 \text{ or } k>2 \quad (5)$$

$-1 \qquad -\frac{1}{3} \qquad 2$

sol^n $k<-1$ (5) $\boxed{20}$

$$x^2-(3k+1)x+(k+1)(k-2)=0 \quad \text{—}(*)$$

put $y=x+1$ (5)

$x=\alpha \Rightarrow y=\alpha+1$

$x=\beta \Rightarrow y=\beta+1$

$x \rightarrow y-1$ substitute in $(*)$

$$(*)\rightarrow (y-1)^2-(3k+1)(y-1)+(k+1)(k-2)=0 \quad (5)$$

$$y^2-3(k+1)y+k(k+2)=0 \quad (5)$$

The required eq^n is (5)

$$x^2-3(k+1)x+k(k+2)=0$$ $\boxed{20}$

b) $f(x)=(x-a)\phi_1(x)+A$ —(1) (5)

$\phi_1(x)=(x-b)\phi_2(x)+B$ —(2) (5)

(1), (2) $\Rightarrow$

$$f(x)=(x-a)\{(x-b)\phi_2(x)+B\}+A \quad (5)$$

$$=(x-a)(x-b)\phi_2(x)+B(x-a)+A \quad \text{—}(3) \quad (5)$$

(1) $\Rightarrow f(a)=A$ (5)

(2) $\Rightarrow \phi_1(b)=B$ (5)

(1) $\Rightarrow f(b)=(b-a)\phi_1(b)+A$

$f(b)=(b-a)B+f(a)$

$$B=\frac{f(b)-f(a)}{b-a} \quad (5)$$

(3) $\Rightarrow$

$$f(x)=(x-a)(x-b)\phi_2(x)+(x-a)\left\{\frac{f(b)-f(a)}{b-a}\right\}+f(a) \quad (5)$$

$\therefore$ Reminder

$$\left\{\frac{f(b)-f(a)}{b-a}\right\}(x-a)+f(a) \quad (5)$$ $\boxed{45}$

$f(x)=x^3+px^2+qx+1$

put $a=1$, $b=2$

then

$$-10x+5=\left\{\frac{f(2)-f(1)}{2-1}\right\}(x-1)+f(1) \quad (5)$$

$x;\ -10=f(2)-f(1)$ (5)

$$-10=(8+4p+2q+1)-(1+p+q+1)$$

$$3p+q=-17 \quad (5) \quad \text{—}(*)$$

$x^0;\ 5=-\{f(2)-f(1)\}+f(1)$

$$5=2f(1)-f(2) \quad (5)$$

$$5=2\{1+p+q+1\}-\{8+4p+2q+1\} \quad (5)$$

$$2p=-10$$

$$p=-5 \quad (5)$$

$(*)\rightarrow q=-2$ (5) $\boxed{35}$

1 x]

a) Let $f(x)=\sqrt{\sin x}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad (5)$$

$$=\lim_{h\to 0}\frac{\sqrt{\sin(x+h)}-\sqrt{\sin x}}{h} \quad (5)$$

$$=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h\{\sqrt{\sin(x+h)}+\sqrt{\sin x}\}} \quad (5)$$

$$=\lim_{h\to 0}\frac{2\cos(x+\frac{h}{2})\sin\frac{h}{2}}{h\{\sqrt{\sin(x+h)}+\sqrt{\sin x}\}} \quad (5)$$

$$=\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}} \quad (5) \cdot \lim_{h\to 0}\frac{\cos(x+\frac{h}{2})}{2\{\sqrt{\sin(x+h)}+\sqrt{\sin x}\}}$$

$$=1 \cdot \frac{\cos x}{\sqrt{\sin x}+\sqrt{\sin x}} \quad (5)$$

$$=\frac{\cos x}{2\sqrt{\sin x}} \quad (5)$$

$\boxed{35}$

b) $y=\dfrac{x^2-1}{x^2+1}$

$$\frac{dy}{dx}=\frac{(x^2+1)2x-(x^2-1)2x}{(x^2+1)^2} \quad (5)$$

$$=\frac{4x}{(x^2+1)^2} \quad (5)$$

$$y=\frac{1+e^x}{1-e^x}$$

$$\frac{dy}{dx}=\frac{(1-e^x)e^x-(1+e^x)(-e^x)}{(1-e^x)^2} \quad (5)$$

$$=\frac{2e^x}{(1-e^x)^2} \quad (5)$$

$$y=x^{x^2+1}$$

$$\ln y=\ln x^{x^2+1}$$

$$\ln y=(x^2+1)\ln x \quad (5)$$

$$\frac{1}{y}\frac{dy}{dx}=(x^2+1)\cdot\frac{1}{x}+(\ln x)\{2x\} \quad (10)$$

$$\frac{dy}{dx}=x^{x^2+1}\left\{\frac{x^2+1}{x}+2x\ln x\right\} \quad (5)$$

$\boxed{40}$

c) $y=e^x\cos e^x$ ——(1)

$$\frac{dy}{dx}=e^x(-)\sin e^x\cdot e^x+\cos e^x\cdot e^x \quad (10)$$

$$\frac{dy}{dx}=-e^{2x}\sin e^x+y \quad (\text{by }(1))$$

$$\frac{dy}{dx}-y=-e^{2x}\sin e^x \quad (2) \quad (5)$$

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}=-\{e^{2x}\cos e^x e^x+\sin e^x e^{2x}\cdot 2\} \quad (10)$$

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}=-e^{2x}y+2\left\{\frac{dy}{dx}-y\right\} \quad (5)$$

(by (1),(2))

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+(e^{2x}+2)y=0 \quad (5)$$

$\boxed{35}$

d) $x=\sin n\theta \qquad y=\cos m\theta$

$$\frac{dx}{d\theta}=n\cos n\theta \quad (5) \qquad \frac{dy}{d\theta}=-m\sin m\theta \quad (5)$$

$$\frac{dy}{dx}=\frac{dy}{d\theta}\cdot\frac{d\theta}{dx}=\frac{-m\sin m\theta}{n\cos n\theta} \quad (5)$$

$$n^2\cos^2 n\theta\left(\frac{dy}{dx}\right)^2=m^2\sin^2 m\theta \quad (5)$$

$$n^2(1-x^2)\left(\frac{dy}{dx}\right)^2=m^2(1-y^2) \quad (5)$$

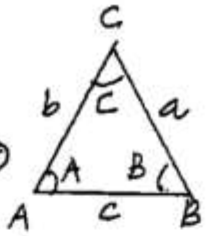
$$n^2\left\{(1-x^2)2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2(-2x)\right\}=m^2\left\{-2y\frac{dy}{dx}\right\} \quad (10)$$

$$n^2\left\{(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}\right\}=-m^2y$$

$$n^2(1-x^2)\frac{d^2y}{dx^2}-n^2x\frac{dy}{dx}+m^2y=0 \quad (5)$$

$\boxed{40}$

13] a) Sin rule



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 5

proof - 15

20

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k}$ —(x) (say) 5

$\frac{a+b-c}{a+b+c}$

$= \frac{k\sin A + k\sin B - k\sin C}{k\sin A + k\sin B + k\sin C}$ (by x) 5

$= \frac{2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2}) - 2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2}) + 2\sin\frac{C}{2}\cos\frac{C}{2}}$ 10

$= \frac{\cos\frac{C}{2}\cos(\frac{A-B}{2}) - \sin\frac{C}{2}\cos\frac{C}{2}}{\cos\frac{C}{2}\cos(\frac{A-B}{2}) + \sin\frac{C}{2}\cos\frac{C}{2}}$

$\left(\frac{C}{2} = \frac{\pi}{2} - (\frac{A+B}{2})\right)$

5 $\left(\cos\frac{C}{2} = \sin(\frac{A+B}{2})\right)$

$= \frac{\cos(\frac{A-B}{2}) - \sin\frac{C}{2}}{\cos(\frac{A-B}{2}) + \sin\frac{C}{2}}$ 5

$= \frac{\cos(\frac{A-B}{2}) - \cos(\frac{A+B}{2})}{\cos(\frac{A-B}{2}) + \cos(\frac{A+B}{2})}$ 5

$= \frac{2\sin\frac{A}{2}\sin\frac{B}{2}}{2\cos\frac{A}{2}\cos\frac{B}{2}}$ 10

$= \tan\frac{A}{2}\tan\frac{B}{2}$ 5

50

b) $\tan^{-1}(\frac{x}{2}) + \tan^{-1}(\frac{x}{3}) = \frac{\pi}{4}$ —(1)

put $\alpha = \tan^{-1}(\frac{x}{2}) \Rightarrow \tan\alpha = \frac{x}{2}$

$\beta = \tan^{-1}(\frac{x}{3}) \Rightarrow \tan\beta = \frac{x}{3}$ 5

$(1) \Rightarrow \alpha + \beta = \frac{\pi}{4}$

$\tan(\alpha+\beta) = 1$ 5

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$ 5

$\frac{\frac{x}{2} + \frac{x}{3}}{1 - (\frac{x}{2})(\frac{x}{3})} = 1$ 5

$\frac{5x}{6 - x^2} = 1$

$x^2 + 5x - 6 = 0$ 5

$(x+6)(x-1) = 0$ 5

$x - 1 = 0$ ($\because x > 0$)

$x = 1$ 5

$(1) \Rightarrow \tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$ 5

$\tan^{-1}(\frac{1}{2}) = \frac{\pi}{4} - \tan^{-1}(\frac{1}{3})$

$\sin^{-1}(\frac{1}{\sqrt{5}}) = \frac{\pi}{4} - \tan^{-1}(\frac{1}{3})$ 5

$\frac{1}{\sqrt{5}} = \sin\{\frac{\pi}{4} - \tan^{-1}(\frac{1}{3})\}$ 5

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c) $0 < x < 1$, $x = \tan\alpha$

$\sin^{-1}\{\frac{2x}{1+x^2}\} + \cos^{-1}\{\frac{1-x^2}{1+x^2}\} + \tan^{-1}\{\frac{2x}{1-x^2}\} = \pi$

$\sin^{-1}\{\frac{2\tan\alpha}{1+\tan^2\alpha}\} + \cos^{-1}\{\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\} + \tan^{-1}\{\frac{2\tan\alpha}{1-\tan^2\alpha}\} = \pi$ 5

$\sin^{-1}\{\sin 2\alpha\} + \cos^{-1}\{\cos 2\alpha\} + \tan^{-1}\{\tan 2\alpha\} = \pi$ 5

$(0 < 2\alpha < \frac{\pi}{2})$ 5

$2\alpha + 2\alpha + 2\alpha = \pi$ 5

$6\alpha = \pi$

$\alpha = \frac{\pi}{6}$ 5

$x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 5

30

14]

a)

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ (5) —(1)

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ (5) —(2)

$(1)+(2) \Rightarrow$

$\sin(A+B)+\sin(A-B) = 2\sin A\cos B$ (5) —(3)

$\left.\begin{array}{l} A+B=C \\ A-B=D \end{array}\right\} \Rightarrow A = \frac{C+D}{2}$, $B = \frac{C-D}{2}$ (5)

$(3) \Rightarrow \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ (5)

$\sin C - \sin D$

$= \sin C + \sin(-D)$ (5)

$= 2\sin\left\{\frac{C+(-D)}{2}\right\}\cos\left\{\frac{C-(-D)}{2}\right\}$ (5)

$= 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$ (5)

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$\sin 7x + \sin 5x - \cot x(\sin 7x - \sin 5x) = 0$

$2\sin 6x\cos x - \frac{\cos x}{\sin x}(2\cos 6x\sin x) = 0$ (10)

$\sin 6x\cos x - \cos 6x\cos x = 0$ (5)

$\cos x(\sin 6x - \cos 6x) = 0$ (5)

$\cos x = 0$ or $\sin 6x - \cos 6x = 0$

$\cos x = \cos\frac{\pi}{2}$ (5) $\qquad \tan 6x = 1 \; (\because \cos 6x \neq 0)$

$\tan 6x = \tan\frac{\pi}{4}$ (5)

$x = 2n_1\pi \pm \frac{\pi}{2}$ (5), $n_1 \in \mathbb{Z}$

$6x = n_2\pi + \frac{\pi}{4}$

$x = \frac{n_2\pi}{6} + \frac{\pi}{24}$ (5)

$n_2 \in \mathbb{Z}$

40

b) $\sin 7\theta + \sin\theta$

$= 2\sin 4\theta\cos 3\theta$ (5)

$= 2\{2\sin 2\theta\cos 2\theta\}\cos 3\theta$ (5)

$= 4\{2\sin\theta\cos\theta\}\cos 2\theta\cos 3\theta$ (5)

$= 8\sin\theta\cos\theta\cos 2\theta\cos 3\theta$ (5)

$\sin 7\theta + \sin\theta = 8\sin\theta\cos\theta\cos 2\theta\cos 3\theta$

$\sin 7\theta = \sin\theta\{8\cos\theta\cos 2\theta\cos 3\theta - 1\}$ (5)

$4\cos\theta\cos 2\theta\cos 3\theta = 1$

$8\cos\theta\cos 2\theta\cos 3\theta = 2$

$\frac{\sin 7\theta}{\sin\theta} + 1 = 2$ (5)

$\sin 7\theta = \sin\theta$ (5)

$7\theta = n\pi + (-1)^n\theta$ (5), $n = 0, \pm 1, \pm 2$

$n=0 \Rightarrow \theta = 0$

$n=1 \Rightarrow \theta = \frac{\pi}{8}$

$n=2 \Rightarrow \theta = \frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{8}$ (5) $\left(\because 0 < \theta < \frac{\pi}{4}\right)$

45

c) $\frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta}$

$= \dfrac{\frac{\sin 5\theta}{\cos 5\theta} + \frac{\sin 3\theta}{\cos 3\theta}}{\frac{\sin 5\theta}{\cos 5\theta} - \frac{\sin 3\theta}{\cos 3\theta}}$ (5)

$= \frac{\sin 5\theta\cos 3\theta + \cos 5\theta\sin 3\theta}{\sin 5\theta\cos 3\theta - \cos 5\theta\sin 3\theta}$ (5)

$= \frac{\sin 8\theta}{\sin 2\theta}$ (5)

$= \frac{2\sin 4\theta\cos 4\theta}{\sin 2\theta}$ (5)

$= \frac{4\sin 2\theta\cos 2\theta\cos 4\theta}{\sin 2\theta}$

$= 4\cos 2\theta\cos 4\theta$ (5)

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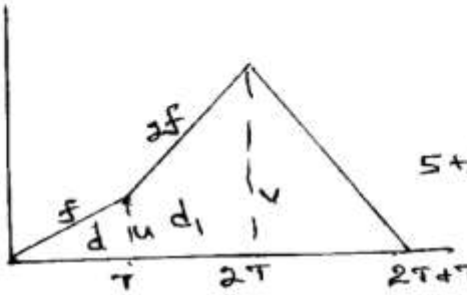
(15)

(i)



5+5+5

$\boxed{15}$

(ii) $u = fT$ (5)

$\frac{1}{2} f T^2 = d$ (5)

$T = \sqrt{\frac{2d}{f}}$ (5)

$\frac{v-u}{T} = 2f$ (5)

$v = 3fT$ (5)

$d_1 = \frac{(u+v)}{2} \times T$ (5) $= 2fT^2$ (5)

$= 4d$ (5)

$\boxed{40}$

(iii) $\frac{1}{2} v T' = 2d$ (5)

$T' = \frac{4d}{v} = \frac{4d}{3fT} = \frac{2}{3}T$ (5)

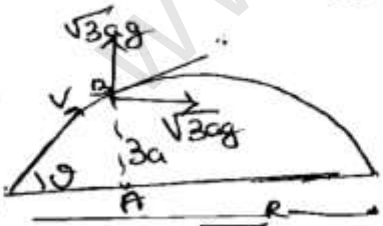
Total time $= T + T + \frac{2}{3}T$

$= \frac{8T}{3}$ (5)

$f' = \frac{v}{T'} = \frac{3v}{2T} = \frac{9f}{2}$ (5) $\boxed{20}$

(75)

(b)



(i)

$\rightarrow v\cos\theta = \sqrt{3ag} \rightarrow (1)$ (5)

$\uparrow v^2 = u^2 + 2as$

$3ag = v^2\sin^2\theta - 2g \times 3a$ (5)

$v\sin\theta = 3\sqrt{ag} \rightarrow (2)$ (5)

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$ (5)

$v = 2\sqrt{3ag}$ (5) $\boxed{25}$

(ii) $\uparrow S = ut + \frac{1}{2}at^2$.

$0 = v\sin\theta\, T - \frac{1}{2} g T^2$ (5)

$T = \frac{2v\sin\theta}{g}$ (5)

$R = v\cos\theta \times T$. (5)

$= v\cos\theta \times \frac{2v\sin\theta}{g}$

$= \frac{\sqrt{3ag} \times 2 \times 3\sqrt{ag}}{g}$

$= 6\sqrt{3}a$. (5) $\boxed{20}$

(iii) T_1 is the time P reach B

$\uparrow v = u + at$

$\sqrt{3ag} = v\sin\theta - gT_1$ (5)

$T_1 = \frac{v\sin\theta - \sqrt{3ag}}{g}$ (5)

Time taken to reach C to Q

$T_2 = T - T_1$

$= \frac{v\sin\theta + \sqrt{3ag}}{g}$ (5)

$= \frac{3\sqrt{ag} + \sqrt{3ag}}{g}$

$= (3+\sqrt{3})\sqrt{\frac{a}{g}}$ (5).

$u\cos\alpha\, T_2 = R$

$\frac{u\sin\alpha}{T_2} = \frac{g}{2}$ (5)

$\tan\alpha = \frac{gT_2^2}{2}$

$\alpha = \tan^{-1}\left(\frac{gT_2^2}{2}\right)$ (5)

$u = \sqrt{\frac{T_2^2 g^2}{4} + \frac{R^2}{T_2^2}}$ (5)

max hight of P $= \frac{v^2\sin^2\theta}{2g}$

max hight of Q $= \frac{u^2\sin^2\alpha}{2g}$ (5)

$\boxed{35}$

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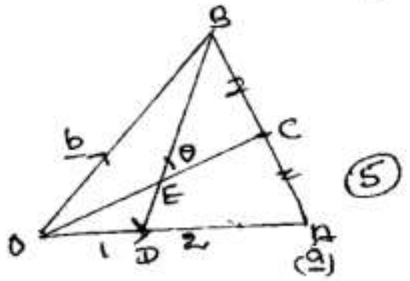
16)
(a)



(5)

(a) (i) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ (5)

$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= \frac{\underline{a}+\underline{b}}{2}$ (5)

$\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OA} = \frac{1}{3}\underline{a}$ (5) (5) $\boxed{25}$

(ii) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$ (5)

$= \overrightarrow{OB} + \lambda\overrightarrow{BD}$

$= \overrightarrow{OB} + \lambda(\overrightarrow{BO} + \overrightarrow{OD})$

$= \underline{b} + \lambda(-\underline{b} + \frac{1}{3}\underline{a})$ (5)

$= (1-\lambda)\underline{b} + \frac{\lambda}{3}\underline{a}$ (5) $\boxed{15}$

(iii) $\overrightarrow{OE} = \mu\overrightarrow{OC}$

$(1-\lambda)\underline{b} + \frac{\lambda}{3}\underline{a} = \frac{\mu}{2}(\underline{a}+\underline{b})$ (5)

(5) $\frac{\lambda}{3} = \frac{\mu}{2}$, $1-\lambda = \frac{\mu}{2}$

$\frac{\lambda}{3} = 1-\lambda$

$4\lambda = 3$

$\lambda = \frac{3}{4}$ (5), $\mu = \frac{1}{2}$ (5) $\boxed{20}$

(b) $\underline{a} = 12\underline{i}$ · $\underline{b} = 2\underline{i} + 8\underline{j}$

(i) $\overrightarrow{OC} = 7\underline{i} + 4\underline{j}$ (5)

$\overrightarrow{OD} = 4\underline{i}$ (5) $\boxed{10}$

(ii) $\overrightarrow{OC} = 7\underline{i} + 4\underline{j}$

$\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB}$

$= -4\underline{i} + 2\underline{i} + 8\underline{j}$

$= -2\underline{i} + 8\underline{j}$ (5)

$\overrightarrow{OC} \cdot \overrightarrow{DB} = (7\underline{i} + 4\underline{j}) \cdot (-2\underline{i} + 8\underline{j})$ (5)

$= -14 + 32$

$= 18$ (5)

$18 = |\overrightarrow{OC}|\,|\overrightarrow{DB}|\cos\theta$ (5)

$= \sqrt{49+16}\,\sqrt{64+4}\cos\theta$ (5) + (5)

$= \sqrt{65} \times \sqrt{68}\cos\theta$

$\cos\theta = \frac{18}{\sqrt{65}\sqrt{68}}$ (5) $\boxed{35}$

$\theta = \cos^{-1}\left(\frac{18}{\sqrt{65}\sqrt{68}}\right)$

(iii) $\overrightarrow{OP} = \beta\underline{b}$ என்க. (5)

$\overrightarrow{AP} = \gamma\overrightarrow{AE}$ (5)

$\overrightarrow{AO} + \overrightarrow{OP} = \gamma(\overrightarrow{AO} + \overrightarrow{OE})$ (5)

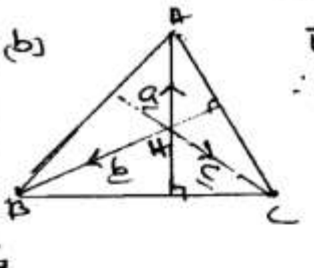
$-\underline{a} + \beta\underline{b} = \gamma(-\underline{a} + \frac{1}{4}\underline{b} + \frac{1}{4}\underline{a})$

$-1 = -\frac{3}{4}\gamma$, $\beta = \frac{\gamma}{4}$

$\gamma = \frac{4}{3}$ $\quad = \frac{1}{3}$ (5)

$\therefore \overrightarrow{OP} = \frac{1}{3}\underline{b}$ (5) $\boxed{25}$

(b)



$\overrightarrow{AH} \perp \overrightarrow{CB}$

$\therefore \overrightarrow{AH} \cdot \overrightarrow{CB} = 0$ (5)

$\underline{a} \cdot (\underline{b} - \underline{c}) = 0 \rightarrow (1)$

$\overrightarrow{BH} \perp \overrightarrow{AC}$

$\overrightarrow{BH} \cdot \overrightarrow{AC} = 0$ (5)

$\underline{b} \cdot (\underline{c} - \underline{a}) = 0 \rightarrow (2)$

(1) + (2) $\underline{b}\cdot\underline{c} - \underline{a}\cdot\underline{c} = 0$ (5)

$(\underline{b} - \underline{a}) \cdot \underline{c} = 0$ (5)

$\overrightarrow{AB} \perp \overrightarrow{CH}$ $\boxed{20}$

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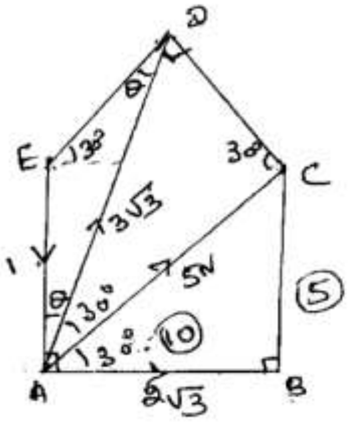
(17) (a)



(i) $2\,AE\cos\theta = \sqrt{3}\,AE$ (5)

$\cos\theta = \frac{\sqrt{3}}{2}$ (5)

$\theta = 30^\circ$ (5)

(ii) $\rightarrow$

$x = 2\sqrt{3} + 5\cos 30 + 3\sqrt{3}\cos 60$ (10)

$= 2\sqrt{3} + \frac{5\sqrt{3}}{2} + \frac{3\sqrt{3}}{2}$

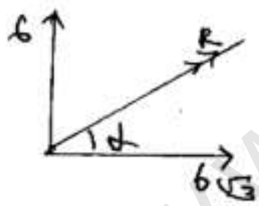
$= 6\sqrt{3}$ (5)

(10)

$\uparrow y = 5\sin 30 + 3\sqrt{3}\cos 30 - 1$

$= \frac{5}{2} + \frac{9}{2} - 1$

$= 6$ (5)



$R = \sqrt{6^2 + (6\sqrt{3})^2}$

$= 12\,N$ (5)

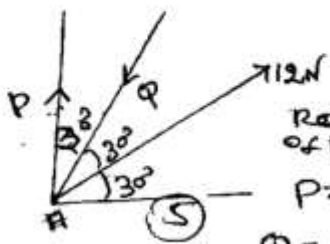
$\tan\alpha = \frac{6}{6\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$\alpha = 30^\circ$ (5)

Resultant 12N along $\overrightarrow{AC}$.

(iii)



(5)

Resultant of P, 12 equal

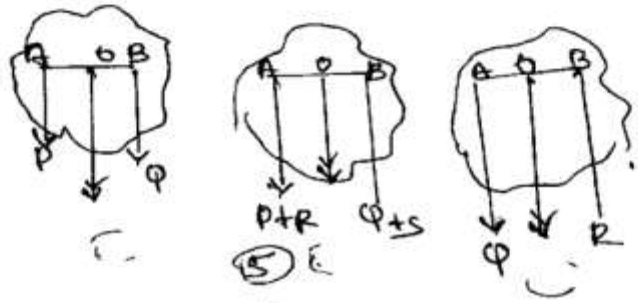
$P = 12\,N$ (10)

$Q = 12\cos 30 + P\cos 30$

$= 24 \times \frac{\sqrt{3}}{2}$ (5)

$= 12\sqrt{3}\,N$ (5) $\boxed{95}$

(b)



(5)

$\frac{AO}{OB} = \frac{Q}{P}$ $\qquad \frac{AO}{OB} = \frac{Q+S}{P+R}$ $\qquad \frac{AO}{OB} = \frac{R}{Q}$

(5)+(5)

$\therefore \frac{Q}{P} = \frac{Q+S}{P+R} = \frac{R}{Q}$

(i) $\frac{Q}{P} = \frac{R}{Q}$

$\therefore Q^2 = RP$ (5)

(ii) $\frac{Q}{P} = \frac{Q+S}{P+R} = \frac{R}{Q} = \frac{Q+S-Q}{P+R-P} = \frac{S}{R}$ (5)

$\therefore \frac{P}{Q} = \frac{Q}{R} = \frac{R}{S}$ (5)

(iii) $Q^2 = RP$, $R = \frac{PS}{Q}$ (5)

$\therefore Q^2 = \frac{PS}{Q} \times P$ (5)

$Q^3 = SP^2$ //

(iv) $\frac{P}{Q} = \frac{Q}{R} = \frac{R}{S} = \frac{Q-R}{R-S} = \frac{P-Q}{Q-R}$

(5) (5)

$\therefore (R-S)(P-Q) = (Q-R)^2$

$R - S = \frac{(Q-R)^2}{P-Q}$ (5)

$\boxed{55}$

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