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தேசிய வெளிக்கள் நிலையம் தொகைப்பானாறு
முதலாம் தவணைப் பரிசீலனை - 2023
National Field Work Centre, Thondaimanaru.
1st Term Examination - 2023

Grade - 12 (2024)

Combined Mathematics

Marking Scheme

$$1. \frac{3x^2}{(x-1)(x+2)} = A + \frac{B}{x-1} + \frac{C}{x+2} \quad (5)$$

$$3x^2 = A(x-1)(x+2) + B(x+2) + C(x-1) \quad (5)$$

Comparing coefficients of powers of x :

$$x^2: 3 = A$$

$$x: 0 = A+B+C \quad (5)$$

$$x^0: 0 = -2A + 2B - C \quad (5)$$

$$A = 3$$

$$B = 1$$

$$C = -4$$

$$\frac{3x^2}{(x-1)(x+2)} = 3 + \frac{1}{x-1} - \frac{4}{x+2} \quad (5)$$

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$$2. \frac{x+1}{x} < \frac{2}{x}$$

$$\frac{x+1}{x} - \frac{2}{x} < 0$$

$$\frac{x-1}{x} < 0 \quad (5)$$

	$x < 0$	$0 < x < 1$	$x > 1$
$x-1$	(-)	(-)	(+)
x	(-)	(+)	(+)
$\frac{x-1}{x}$	(+)	(-)	(+)

(5)

$$0 < x < 1 \quad (5)$$

$$\frac{x+1}{x} \geq \frac{2}{x}$$

$$\Leftrightarrow x < 0 \text{ or } x \geq 1$$

(5)

(5)

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$$3. f(x) = ax^4 + bx^3 + x^2 + 2$$

$$f(x) = (x^2-1)\phi(x) + x+5 \quad (5)$$

$$f(x) = (x-1)(x+1)\phi(x) + x+5$$

$$f(1) = a+b+1+2 = 0+1+5 \quad (5)$$

$$\Rightarrow a+b = 3 \quad (1)$$

$$f(-1) = a-b+1+2 = 0-1+5 \quad (5)$$

$$\Rightarrow a-b = 1 \quad (2)$$

$$(1), (2) \Rightarrow a=2, b=1 \quad (5) \quad (5)$$

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$$4. \frac{\log_a bc}{1+\log_a bc} + \frac{\log_b ca}{1+\log_b ca} + \frac{\log_c ab}{1+\log_c ab}$$

$$= \frac{\log_a bc}{\log_a + \log_b bc} + \frac{\log_b ca}{\log_b + \log_b ca} + \frac{\log_c ab}{\log_c + \log_c ab} \quad (5)$$

$$= \frac{\log_a bc}{\log abc} + \frac{\log_b ca}{\log abc} + \frac{\log_c ab}{\log abc} \quad (5)$$

$$= \log_{abc} (abc)^2 \quad (5)$$

$$= 2 \quad (5)$$

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$$5. 5^{2x+1} - 26 \times 5^x + 5 = 0$$

$$5(5^x)^2 - 26 \cdot 5^x + 5 = 0 \quad (5)$$

$$5t^2 - 26t + 5 = 0, \text{ where } t = 5^x \quad (5)$$

$$(5t-1)(t-5) = 0 \quad (5)$$

$$t = \frac{1}{5} \text{ or } t = 5 \quad (5)$$

$$5^x = 5^{-1} \text{ or } 5^x = 5^1$$

$$x = -1 \text{ or } x = 1 \quad (5)$$

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$$6. \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$= \frac{1 - (1 - 2\sin^2 A)}{1 + (2\cos^2 A - 1)} \quad (5)$$

$$= \frac{2\sin^2 A}{2\cos^2 A} \quad (5)$$

$$= \tan^2 A \quad (5)$$

$$\tan^2 \frac{\pi}{12} = \frac{1 - \cos \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}} \quad (5)$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} \quad (5)$$

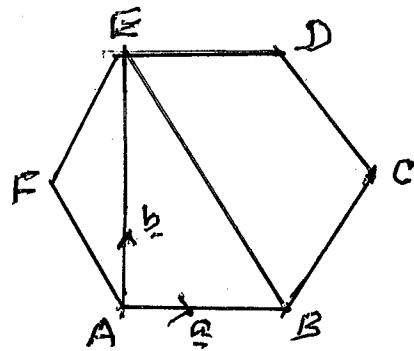
$$= \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \quad (5)$$

$$= \frac{(2 - \sqrt{3})^2}{4 - 3} \quad (5)$$

$$\tan^2 \frac{\pi}{12} = (2 - \sqrt{3})^2 \quad (5)$$

$$\tan \frac{\pi}{12} = 2 - \sqrt{3} \quad [\because \tan \frac{\pi}{12} > 0] \quad (5) \quad 25$$

7.



$$\vec{BAE} = \vec{BA} + \vec{AE}$$

$$= -\vec{a} + \vec{b} \quad (5)$$

$$\vec{AD} = \vec{a} + \vec{b} \quad (5)$$

$$\vec{BC} = \frac{1}{2} \vec{AD} = \frac{1}{2} (\vec{a} + \vec{b}) \quad (5)$$

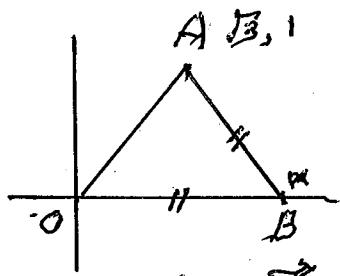
$$\vec{AC} = \vec{AB} + \vec{BC} \quad (5)$$

$$= \vec{a} + \frac{1}{2} (\vec{a} + \vec{b})$$

$$= \frac{3}{2} \vec{a} + \frac{1}{2} \vec{b}$$

$$= \frac{1}{2} (3\vec{a} + \vec{b}) \quad (5)$$

8.



$$\vec{OA} = \sqrt{3} \hat{i} + \hat{j}, \quad \vec{OB} = \alpha \hat{i}$$

$$\vec{AB} = \vec{AO} + \vec{OB} \quad (5)$$

$$= -\sqrt{3} \hat{i} - \hat{j} + \alpha \hat{i}$$

$$= (\alpha - \sqrt{3}) \hat{i} - \hat{j} \quad (5)$$

$$AB = OB$$

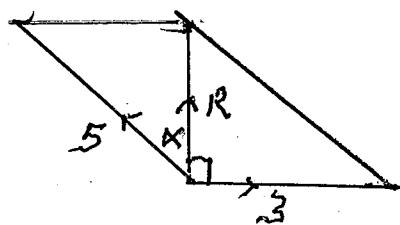
$$|\vec{AB}|^2 = |\vec{OB}|^2 \quad (5)$$

$$(\alpha - \sqrt{3})^2 + 1^2 = \alpha^2 \quad (5)$$

$$-2\sqrt{3}\alpha + 3 = 0$$

$$\alpha = \frac{2}{\sqrt{3}} \quad (5)$$

9.



$$\theta = \frac{\pi}{2} + \sin^{-1} \frac{3}{5}$$

$$= \frac{\pi}{2} + \alpha$$

$$\sin \alpha = \frac{3}{5} \quad (5)$$

$$R^2 = 5^2 + 3^2 + 2 \cdot 5 \cdot 3 \cos\left(\frac{\pi}{2} + \alpha\right) \quad (5)$$

$$= 25 + 9 - 30 \sin \alpha$$

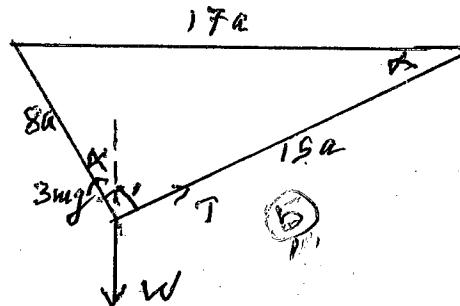
$$= 34 - 30 \cdot \frac{3}{5} \quad (5)$$

$$= 16$$

$$R = 4 \quad (5)$$

$$R^3 = \frac{\pi}{2} \quad (5)$$

10.



Using Sine Rule:

$$\frac{T}{\sin \frac{\pi}{2} - \alpha} = \frac{W}{\sin \frac{3}{2} \pi} = \frac{3mg}{\sin(90 + \alpha)} \quad (10)$$

$$\frac{T}{\sin \alpha} = W = \frac{3mg}{\cos \alpha}$$

$$T = 3mg \tan \alpha = \frac{8}{5} mg \quad (5)$$

$$W = 3mg \frac{12}{15} = \frac{17}{5} mg \quad (5)$$

11.(a) $0 < k < 3$, $f(x) = (3-k)x^2 - kx + 1$

$$\begin{aligned} \text{(i) Discriminant } \Delta &= k^2 - 4(3-k)(1) \quad (10) \\ &= k^2 + 4k - 12 \quad (5) \\ &= (k+6)(k-2) \quad (5) \end{aligned}$$

$$\Delta \geq 0 \quad (5)$$

$$\Leftrightarrow (k+6)(k-2) \geq 0$$

$$\Leftrightarrow k \leq -6 \text{ or } k \geq 2 \quad (5)$$

Since $0 < k < 3$

$$\therefore 2 \leq k < 3 \quad (5)$$

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(ii) $\alpha + \beta = \frac{k}{3-k} \quad (5)$

$$\alpha\beta = \frac{1}{3-k} \quad (5)$$

Since α and β are real

$$2 \leq k < 3$$

$$\alpha + \beta = \frac{k}{3-k} > 0 \quad [\because 2 \leq k < 3] \quad (5)$$

$$\alpha\beta = \frac{1}{3-k} > 0 \quad [\because 2 \leq k < 3] \quad (5)$$

$\therefore \alpha$ and β are both positive $\quad (5)$

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$$\begin{aligned} (\alpha+2) + (\beta+2) &\quad (\alpha+2)(\beta+2) \\ = (\alpha+\beta) + 4 \quad (5) &\quad = \alpha\beta + 2(\alpha+\beta) + 4 \quad (5) \\ = \frac{k}{3-k} + 4 \quad (5) &\quad = \frac{1}{3-k} + \frac{2k}{3-k} + 4 \quad (5) \\ = \frac{3(4-k)}{3-k} &\quad = \frac{13-2k}{3-k} \end{aligned}$$

The equation whose roots are

$$\alpha+2 \text{ and } \beta+2 \text{ is } (x-(\alpha+2))(x-(\beta+2)) = 0 \quad (5)$$

$$x^2 - [(\alpha+2) + (\beta+2)]x + (\alpha+2)(\beta+2) = 0 \quad (5)$$

$$x^2 - \frac{3(4-k)}{3-k}x + \frac{13-2k}{3-k} = 0$$

$$(3-4k)x^2 - 3(4-k)x + 13-2k = 0 \quad (5)$$

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(b) $x + \frac{1}{x} = t$

$$(x + \frac{1}{x})^2 = t^2 \quad (5)$$

$$x^2 + 2 + \frac{1}{x^2} = t^2$$

$$x^2 + \frac{1}{x^2} = t^2 - 2 \quad \text{--- (i)} \quad (5)$$

$$6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0$$

$$6x^2 + 5x - 38 + \frac{5}{x} + \frac{6}{x^2} = 0 \quad (5)$$

$$6(x^2 + \frac{1}{x^2}) + 5(x + \frac{1}{x}) - 38 = 0 \quad (5)$$

$$6(t^2 - 2) + 5t - 38 = 0 \quad [\because \text{by (i)}] \quad (5)$$

$$6t^2 + 5t - 50 = 0 \quad (5)$$

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$$6t^2 + 5t - 50 = 0$$

$$(3t+10)(2t-5) = 0$$

$$t = -\frac{10}{3} \text{ or } t = \frac{5}{2} \quad (5)$$

$$t = -\frac{10}{3} \Rightarrow x + \frac{1}{x} = -\frac{10}{3}$$

$$\Rightarrow 3x^2 + 10x + 3 = 0 \quad (5)$$

$$\Rightarrow (3x+1)(x+3) = 0$$

$$\Rightarrow x = -\frac{1}{3} \text{ or } x = -3. \quad (5)$$

$$t = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2}$$

$$\Rightarrow 2x^2 - 5x + 2 = 0 \quad (5)$$

$$\Rightarrow (2x-1)(x-2) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2. \quad (5)$$

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12.(a) $f(x) = 2x^2 + 8x + 13$

$$= 2(x^2 + 4x + \frac{13}{2}) \quad (5)$$

$$= 2\{(x+2)^2 - 4 + \frac{13}{2}\}$$

$$= 2(x+2)^2 + 5 \quad (5)$$

$$= a(x+b)^2 + c,$$

where $a = 2, b = 2, c = 5 \quad (10)$

$$\text{since } (x+2)^2 \geq 0 \quad (5)$$

$$\therefore f(x) \geq 5$$

$$(f(x))_{\min} = 5 \quad (5)$$

$$\left(\frac{2}{1+f(x)}\right)_{\max} = \frac{2}{1+(f(x))_{\min}} = \frac{2}{1+5} = \frac{1}{3}. \quad (5)$$

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(b) $g(x) = 3x^3 + px^2 + qx + r$

Since, when $g(x)$ is divided by $x+1$, the remainder is -9 , we have $g(-1) = -9$

$$-3+p-9+r = -9 \quad (5)$$

$$p-2 = -13 \quad \text{--- (i)} \quad (5)$$

Since, when $g(x)$ is divided by $x^2 - 3x + 2$,

the remainder is $2x + r$, we have

$$g(x) = (x^2 - 3x + 2) \phi(x) + 2x + r \quad (10)$$

$$3x^3 + px^2 + qx + r = (x-1)(x-2) \phi(x) + 2x + r \quad (5)$$

Substituting $x=1$, we have

$$3+p+q+r = 2+r \quad (5)$$

$$p+q = r-8 \quad (2) \quad (5)$$

Substituting $x=2$, we have

$$24+4p+2q+r = 4+r \quad (5)$$

$$4p+2q = r-27 \quad (3)$$

$$(3)-(2) \Rightarrow 3p+q = -19 \quad (4) \quad (5)$$

$$(1), (4) \Rightarrow p = -8, q = 5 \quad (5) \quad (5)$$

$$(2) \Rightarrow r = 5 \quad (5)$$

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$$g(x) = 3x^3 - 8x^2 + 5x + 7$$

$$3x^3 - 8x^2 + 5x + 7 \equiv (x^2 + 2)(Ax + B) + Cx + D \quad (5)$$

Comparing coefficients of powers of x :

$$\left. \begin{array}{l} x^3: 3 = A \\ x^2: -8 = B \\ x: 5 = 2A + C \\ x^0: 7 = 2B + D \end{array} \right\} \begin{array}{l} A = 3 \\ B = -8 \\ C = -1 \\ D = 23 \end{array} \quad (10)$$

$$\text{Quotient} = 3x - 8, \text{ Remainder} = -x + 23 \quad (5) \quad 25$$

Aliter

$$\begin{array}{r} 3x - 8 \\ \hline x^2 + 0x + 2 \quad | \quad 3x^3 - 8x^2 + 5x + 7 \\ \underline{-} 3x^3 + 0x^2 + 6x \\ \hline -8x^2 - x + 7 \\ \underline{-} -8x^2 + 0x - 16 \\ \hline -x + 23 \end{array} \quad (15)$$

$$\text{Quotient} = 3x - 8, \text{ Remainder} = -x + 23 \quad (5) \quad 25$$

$$g(x) = (x^2 + 2)(3x - 8) - x + 23$$

$$g(-2x) = (4x^2 + 2)(-6x - 8) - (-2x) + 23, \quad (5)$$

$$= (2x^2 + 1)(-12x - 16) + 2x + 23 \quad (5)$$

$$\text{Quotient} = -12x - 16, \text{ Remainder} = 2x + 23 \quad (5) \quad 20$$

13.

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad (5)$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad (5)$$

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$$(i) \sin(A-B)$$

$$= \sin(A + (-B)) \quad (5)$$

$$= \sin A \cos(-B) + \cos A \sin(-B) \quad (5)$$

$$= \sin A \cos B - \cos A \sin B \quad (5)$$

$$\cos(A-B)$$

$$= \cos(A + (-B)) \quad (5)$$

$$= \cos A \cos(-B) - \sin A \sin(-B) \quad (5)$$

$$= \cos A \cos B + \sin A \sin B \quad (5)$$

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$$\sin 15^\circ = \sin(45^\circ - 30^\circ) \quad (5)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \quad (5)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \quad (5)$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4} \quad (5)$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) \quad (5)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \quad (5)$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \quad (5)$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4} \quad (5) \quad 40$$

$$(ii) \sin 2A = \sin(A+A) \quad (5)$$

$$= \sin A \cos A + \cos A \sin A \quad (5)$$

$$= 2 \sin A \cos A$$

$$\cos 2A = \cos(A+A) \quad (5)$$

$$= \cos A \cos A - \sin A \sin A \quad (5)$$

$$= \cos^2 A - \sin^2 A$$

$$= 1 - \sin^2 A - \sin^2 A \quad (5)$$

$$= 1 - 2 \sin^2 A$$

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$$\begin{aligned}
 & \frac{1 - \cos 2A}{\sin 2A} \\
 &= \frac{1 - (1 - 2\sin^2 A)}{2\sin A \cos A} \quad (5) \\
 &= \frac{2\sin^2 A}{2\sin A \cos A} \quad (5) \\
 &= \tan A
 \end{aligned}$$

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$$\begin{aligned}
 \tan 7\frac{1}{2}^\circ &= \frac{1 - \cos 15^\circ}{\sin 15^\circ} \quad (5) \\
 &= \frac{1 - \frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{6} - \sqrt{2}}{4}} \quad (5) \\
 &= \frac{4 - (\sqrt{6} + \sqrt{2})}{\sqrt{6} - \sqrt{2}} \\
 &= \frac{[4 - (\sqrt{6} + \sqrt{2})][\sqrt{6} + \sqrt{2}]}{[\sqrt{6} - \sqrt{2}][\sqrt{6} + \sqrt{2}]} \quad (5) \\
 &= \frac{4\sqrt{6} + 4\sqrt{2} - (6 + 2\sqrt{6}\sqrt{2} + 2)}{6 - 2} \quad (5) \\
 &= \frac{4\sqrt{6} + 4\sqrt{2} - 8 - 4\sqrt{3}}{4} \quad (5) \\
 &= \sqrt{6} - \sqrt{3} + \sqrt{2} - 2. \quad (5)
 \end{aligned}$$

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14.(a) (i) $\cot^4 \theta + \cot^2 \theta$

$$\begin{aligned}
 &= \cot^2 \theta (\cot^2 \theta + 1) \quad (5) \\
 &= (\cosec^2 \theta - 1) \cosec^2 \theta \quad (5) \\
 &= \cosec^4 \theta - \cosec^2 \theta \quad (5)
 \end{aligned}$$

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$$\begin{aligned}
 & \text{(ii)} \frac{\sec \theta}{\sec \theta - 1} + \frac{\sec \theta}{\sec \theta + 1} \\
 &= \frac{\sec \theta (\sec \theta + 1) + \sec \theta (\sec \theta - 1)}{(\sec \theta - 1)(\sec \theta + 1)} \quad (5) \\
 &= \frac{2\sec^2 \theta}{\sec^2 \theta - 1} \quad (5) \\
 &= \frac{2\sec^2 \theta}{\tan^2 \theta} \quad (5) \\
 &= \frac{\frac{2}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}} \quad (5) \\
 &= \frac{2}{\sin^2 \theta} \quad (5) \\
 &= 2 \cosec^2 \theta \quad (5)
 \end{aligned}$$

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(b) (i) $\sin x + \sin tx = \sin 4x$

$$\begin{aligned}
 2\sin 4x \cos 3x &= \sin 4x \quad (5) \\
 \sin 4x(2\cos 3x - 1) &= 0 \quad (5)
 \end{aligned}$$

$$\sin 4x = 0 \text{ or } \cos 3x = \frac{1}{2} \quad (5)$$

$$\sin 4x = \sin 0 \text{ or } \cos 3x = \cos \frac{\pi}{3} \quad (5)$$

$$4x = n\pi; n \in \mathbb{Z} \text{ or } 3x = 2m\pi \pm \frac{\pi}{3}, m \in \mathbb{Z}$$

$$x = \frac{n\pi}{4}; n \in \mathbb{Z} \text{ or } x = \frac{2m\pi}{3} \pm \frac{\pi}{9}; m \in \mathbb{Z} \quad (5)$$

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$$(ii) 3\sin 8x - 4\sin^3 8x = \sin 3x$$

$$\Rightarrow \sin 24x = \sin 3x \quad (i) \quad (5)$$

$$\cos 3x, \cos 6x, \cos 12x,$$

$$= \frac{1}{2\sin 3x} 2\sin 3x \cos 3x \cos 6x \cos 12x \quad (5)$$

$$= \frac{1}{2\sin 3x} \sin 6x \cos 6x \cos 12x \quad (5)$$

$$= \frac{1}{4\sin 3x} \sin 12x \cos 12x \quad (5)$$

$$= \frac{1}{8\sin 3x} \sin 24x \quad (5)$$

$$= \frac{1}{8} [\because \text{by (i)}] \quad (5)$$

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$$(iii) \sin x = k \sin(A-x)$$

$$\Rightarrow k = \frac{\sin x}{\sin(A-x)}$$

$$\frac{k-1}{k+1} = \frac{\frac{\sin x}{\sin(A-x)} - 1}{\frac{\sin x}{\sin(A-x)} + 1} \quad (5)$$

$$= \frac{\sin x - \sin(A-x)}{\sin x + \sin(A-x)} \quad (5)$$

$$= \frac{2\cos \frac{A}{2} \sin(\frac{x-A}{2})}{2\sin \frac{A}{2} \cos(\frac{x-A}{2})} \quad (5)$$

$$= \frac{\tan(x - \frac{A}{2})}{\tan \frac{A}{2}} \quad (5)$$

$$\tan(x - \frac{A}{2}) = \frac{k-1}{k+1} \tan \frac{A}{2} \quad (5)$$

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$$\sin x = 2\sin(\frac{2\pi}{3} - x)$$

$$\Rightarrow \tan(x - \frac{\pi}{3}) = \frac{1}{3} \tan \frac{\pi}{3} \quad [k = z \text{ and } A = \frac{2\pi}{3}]$$

$$\Rightarrow \tan(x - \frac{\pi}{3}) = \frac{1}{3} \sqrt{3} \quad (5)$$

$$\Rightarrow \tan(x - \frac{\pi}{3}) = \frac{1}{\sqrt{3}}$$

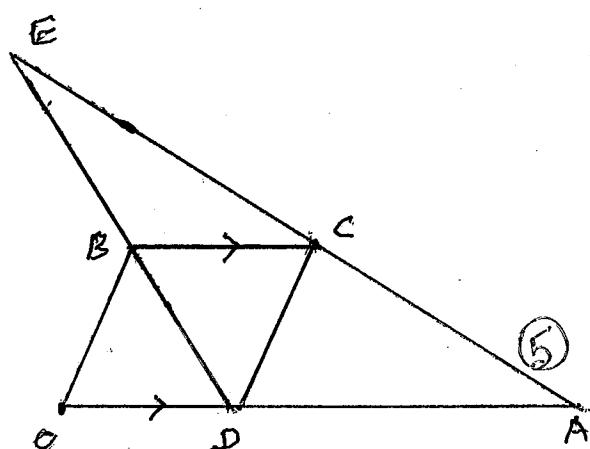
$$\Rightarrow x - \frac{\pi}{3} = \frac{\pi}{6} \quad [\because 0 < x < \pi] \quad (5)$$

$$\Rightarrow x = \frac{\pi}{2} \quad (5)$$

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15 a) Theory

10



$$\text{I} \quad \vec{OA} = b\hat{i}, \quad \vec{OB} = \hat{i} + \sqrt{3}\hat{j}$$

$$\vec{OD} = 2\hat{i}$$

$$\vec{BE} = 2\hat{i}$$

$$\vec{OC} = \vec{OB} + \vec{BC}$$

$$= \hat{i} + \sqrt{3}\hat{j} + \hat{i}$$

$$= 2\hat{i} + \sqrt{3}\hat{j}$$

$$\text{II} \quad \vec{DB} = \vec{DO} + \vec{OB}$$

$$= -2\hat{i} + \hat{i} + \sqrt{3}\hat{j}$$

$$= -\hat{i} + \sqrt{3}\hat{j}$$

$$\vec{AC} = \vec{AO} + \vec{OC}$$

$$= -b\hat{i} + \hat{i} + \sqrt{3}\hat{j}$$

$$= -3\hat{i} + \sqrt{3}\hat{j}$$

$$\text{III} \quad \vec{DE} = \pi(-\hat{i} + \sqrt{3}\hat{j})$$

$$\vec{AE} = \pi(-3\hat{i} + \sqrt{3}\hat{j})$$

$$\text{IV} \quad \vec{DE} = \vec{DA} + \vec{AE}$$

$$\mu(-\hat{i} + \sqrt{3}\hat{j}) = 4\hat{i} + \pi(-3\hat{i} + \sqrt{3}\hat{j})$$

$$(3\pi - \mu - 4)\hat{i} + (\sqrt{3}\mu - \pi\sqrt{3})\hat{j} = 0$$

$$3\pi - \mu - 4 = 0 \quad \text{and} \quad \sqrt{3}\mu - \pi\sqrt{3} = 0$$

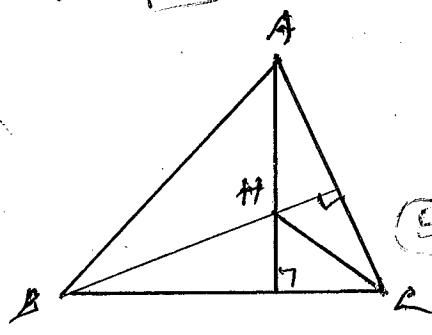
$$\pi = 2\hat{i}$$

$$\mu = \pi$$

$$DE = 2DB, \quad BC = 2AC$$

$$AC:CE = 1:1, \quad DB:BE = 1:1$$

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$$\vec{HA} = \underline{a}, \quad \vec{HB} = \underline{b}, \quad \vec{HC} = \underline{c}$$

$$HA \perp BC \quad + \quad HB \perp AC$$

$$\vec{HA} \cdot \vec{BC} \approx 0$$

$$\underline{a} \cdot (\underline{c} - \underline{b}) = 0$$

$$\underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = 0$$

$$\vec{HB} \cdot \vec{AC} \approx 0$$

$$\underline{b} \cdot (\underline{c} - \underline{a}) \approx 0$$

$$\underline{b} \cdot \underline{c} - \underline{b} \cdot \underline{a} \approx 0$$

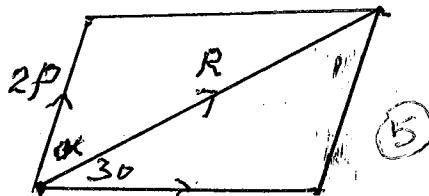
$$\text{I} + \text{II} \Rightarrow \underline{a} \cdot \underline{c} - \underline{b} \cdot \underline{c} = 0$$

$$(\underline{a} - \underline{b}) \cdot \underline{c} = 0$$

$$\vec{BA} \cdot \vec{HC} \approx 0$$

$$BA \perp HC$$

16 (a) Theory 10



$$\frac{kp}{\sin \alpha} = \frac{2P}{\sin 30} = \frac{1}{\sqrt{3}} \frac{R}{\sin(\alpha + 30)}$$

$$\text{I} \quad k = 4$$

$$\frac{4P}{\sin \alpha} = \frac{4P}{\sin \theta} = \frac{R}{\sin(\alpha + 30)}$$

$$\sin \alpha = 1$$

$$\alpha = \frac{\pi}{2}$$

$$R = 2\sqrt{3}P$$

$$\text{II} \quad k = 2\sqrt{3}$$

$$\frac{2\sqrt{3}P}{\sin \alpha} = 4P = \frac{R}{\sin(\alpha + 30)}$$

$$\sin \alpha = \frac{\sqrt{3}}{2}$$

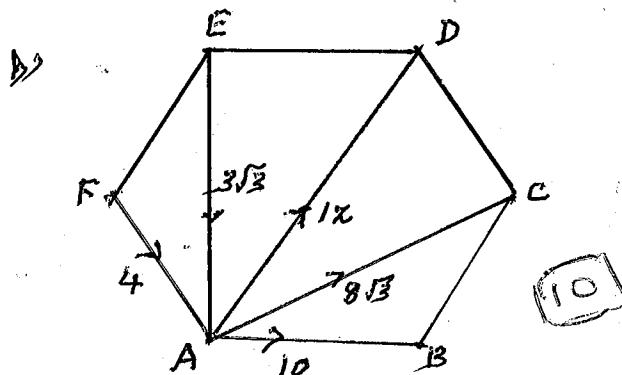
$$\alpha = \frac{\pi}{3}$$

$$R = 4P$$

$$\underline{65} \quad 3a = 6$$

$$\frac{6P}{\Delta m_{eff}} = 4P \Rightarrow \frac{R}{\sin(\theta + 30^\circ)} \quad (5)$$

$$\sin \alpha = \frac{3}{2} \text{ it's not possible.}$$



$$AB \rightarrow x = -10 + 8\sqrt{3} \cos 30^\circ + 12 \cos 60^\circ + 4 \cos 0^\circ$$

$$= -10 + 12 + 6 + 2$$

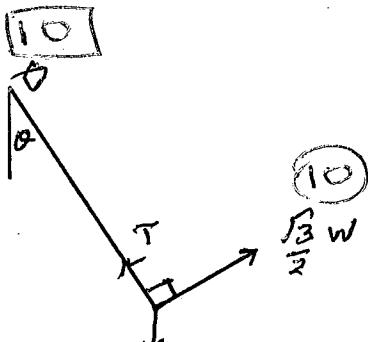
$$= 10$$

$$\begin{aligned} \text{TAKE } y &= -3\sqrt{3} + 12 \cos 30 + 8\sqrt{3} \cos 60 \\ &\quad - 4 \cos 30 \\ &\approx -3\sqrt{3} + 6\sqrt{3} + 4\sqrt{3} - 2\sqrt{3} \\ &= 5\sqrt{3} \quad (5) \end{aligned}$$

$$R^2 = 10^2 + (5\sqrt{3})^2 \quad (5)$$

$$R = 5\sqrt{2} \quad (5) \qquad \tan \alpha = \frac{\sqrt{3}}{2} \quad (5)$$

17 a) Theorem



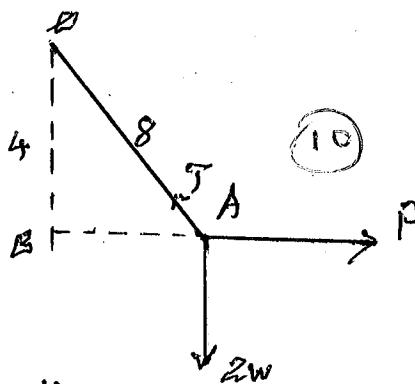
$$\frac{W}{\sin(\theta)} = \frac{T}{\sin(\theta + \phi)} = \frac{\frac{\sqrt{3}}{2} W}{\sin(\pi - \phi)}$$

$$W = \frac{T}{GSD} = \frac{\sqrt{3} W}{2 \sigma_{MP}} \quad (5)$$

$$\sin \theta = \frac{\sqrt{3}}{2} \quad (5)$$

$$J = \frac{W}{2} \quad (5) \quad | \overline{45}$$

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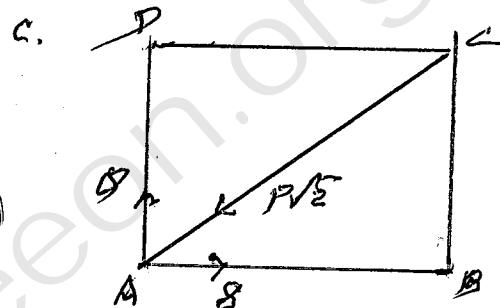


\Rightarrow OAB Tr. of Force (15)

$$\frac{F}{BA} = \frac{ZV}{VB} = \frac{T}{AB}$$

$$\frac{F}{4\sqrt{3}} = \frac{2W}{4} \approx \frac{T}{8}$$

$$T = 4w \quad F = 2\sqrt{3}w$$



$$\rightarrow 8 - \rho\sqrt{2} \cos 45^\circ = 0 \quad (15)$$

$$P = 8 \quad \textcircled{5}$$

$$\uparrow \quad B - P\sqrt{2} \sin 45^\circ = 0, \quad (15)$$

$$S = P$$