



தேசிய வெளிக்கள் நிலையம் தொண்டமானாறு
ராண்டாம் தவணைப் பர்த்தசெ - 2024

National Field Work Centre, Thondaimanaru.
2nd Term Examination - 2024

Gr : 12 (2025)

இலைங்க கணிதம்

புள்ளித்திட்டம்

01] $P(x) = 2x^3 + ax^2 - 11x - 5$

$P(2) = 1 \quad (5)$

$2(2)^3 + a(2)^2 - 11(2) - 5 = 1$

$4a = 12$

$a = 3 \quad (5)$

$\therefore P(x) = 2x^3 + 3x^2 - 11x - 5$

$P(x)-1 = 2x^3 + 3x^2 - 11x - 6$
 $= (x-2)(2x^2 + Ax + 3) \quad (5)$

$x^2, 3 = A - 4 \quad (5)$
 $A = 7 \quad (5)$

$P(x)-1 = (x-2)(2x^2 + 7x + 3) \quad (5)$
 $= (x-2)(2x+1)(x+3) \quad (5)$

02] $k \in R, f(0) \quad (25)$

$\frac{1}{x(x-k)^2} = \frac{A}{x} + \frac{B}{x-k} + \frac{C}{(x-k)^2} \quad (5)$

$1 = A(x-k)^2 + Bx(x-k) + Cx$

$x^2, 0 = A+B \quad \left\{ \begin{array}{l} A = \frac{1}{k} \\ B = -\frac{1}{k} \end{array} \right. \quad (5)$

$x, 0 = -2Ak - Bk + C \quad \left\{ \begin{array}{l} B = -\frac{1}{k} \\ C = \frac{1}{k} \end{array} \right. \quad (5)$

$x^0, 1 = Ak^2 \quad (5)$

$\frac{1}{x(x-k)^2} = \frac{1}{k^2x} - \frac{1}{k^2(x-k)} + \frac{1}{k(x-k)^2} \quad (5)$

02] $x+2 > \frac{8}{x}, x \neq 0 \quad (25)$

$x+2 - \frac{8}{x} > 0 \quad (5)$

$\frac{x^2 + 2x - 8}{x} > 0$

$\frac{(x+4)(x-2)}{x} > 0 \quad (5)$

	$x < -4$	$-4 < x < 0$	$0 < x < 2$	$x > 2$
$x+4$	(-)	(+)	(+)	(+)
$x-2$	(-)	(-)	(-)	(+)
x	(-)	(-)	(+)	(+) (10)
$(x+4)(x-2)$	(-)	(+)	(-)	(+)
x				

$-4 \leq x < 0 \quad \frac{0x}{x} \quad x > 2 \quad (25)$

04] $\log_3 2 = a, \log_5 2 = b$

$\log_{2025} 2 = \frac{1}{\log_2 2025} \quad (5)$

$= \frac{1}{\log_2 \{5^2 \times 3^2\}} \quad (5)$

$= \frac{1}{2 \log_2 5 + 2 \log_2 3} \quad (5)$

$= \frac{1}{2 \{ \frac{1}{b} \} + 4 \{ \frac{1}{a} \}} \quad (5)$

$= \frac{ab}{2a+4b}$

$= \frac{ab}{2(a+2b)} \quad (5)$

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05]

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x - \cos 4x + \cos 2x \cos 4x}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(1 - \cos 4x)}{x^4} \quad (5)$$

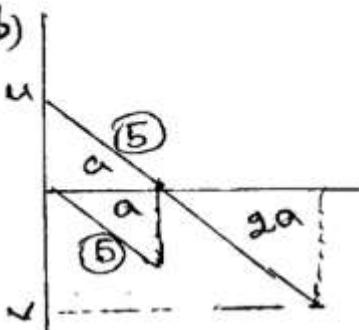
$$= \lim_{x \rightarrow 0} \frac{(2 \sin^2 x)(2 \sin^2 2x)}{x^4} \quad (5)$$

$$= 4 \times 4 \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} \right]^2 \left[\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \right]^2 \quad (25)$$

$$= 16 \times 1^2 \times 1^2 \quad (5)$$

$$= 16 \quad (5)$$

06)



$$\frac{1}{2} \frac{u^2}{g} = a \quad (5)$$

$$u^2 = 2ag$$

$$u = \sqrt{2ag} \quad (5)$$

$$\frac{1}{2} \frac{v^2}{g} = a \quad (5)$$

$$v^2 = 4ag$$

$$v = 2\sqrt{ag} \quad (25)$$

07)



$$u = v + \tan \theta$$

$$= 3v \quad (5)$$

$$13 = u t + \frac{1}{2} g t^2$$

$$0 = 3vt - \frac{1}{2} g t^2$$

$$t = \frac{6v}{g} \quad (5)$$

$$vt = 24R$$

$$\frac{6v^2}{g} = 24R$$

$$v = 2\sqrt{8R} \quad (5)$$

$$\uparrow v^2 = u^2 + 2as$$

$$0 = 9v^2 - 2gh \quad (5)$$

$$h = \frac{9v^2}{2g} = 144R \quad (5)$$

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$$\vec{AB} = 2\sqrt{3}\hat{i} - 2\hat{j} \quad (5)$$

$$\vec{OC} = x\hat{i} + y\hat{j}, x^2 + y^2 = 4$$

$$\vec{AB} \cdot \vec{OC} = 0 \quad (5)$$

$$(2\sqrt{3}\hat{i} - 2\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0 \quad (5)$$

$$2\sqrt{3}x - 2y = 0$$

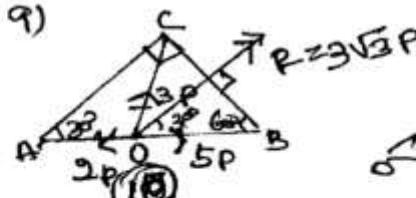
$$y = \sqrt{3}x$$

$$x^2 = 4 \quad \vec{OC} = \hat{i} + \sqrt{3}\hat{j}$$

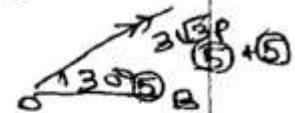
$$x = \pm 2 \quad (5) \quad \text{or} \quad (5)$$

$$\vec{OC} = -\hat{i} - \sqrt{3}\hat{j}$$

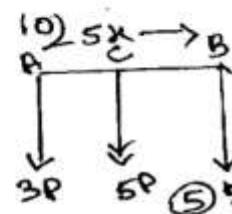
9)



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$$AC = \frac{3}{5} \times 5x = 3x \quad (5)$$

$$AD = \frac{3}{5} \times 5x = 3x \quad (5)$$

distance in 5th sec = $3x - 3x$
moving distance = x

25

11)

$$a) f(x) = 0$$

$$x^2 - (3k+1)x + (k+1)(k-2) = 0$$

Discriminant Δ

$$\Delta = \{-(3k+1)\}^2 - 4(1)(k+1)(k-2) \quad (1)$$

$$= (9k^2 + 6k + 1) - 4(k^2 - k - 2) \quad (2)$$

$$= 5k^2 + 10k + 9 \quad (3)$$

$$= 5\left\{k^2 + 2k + \frac{9}{5}\right\}$$

$$= 5\left\{(k+1)^2 + \frac{4}{5}\right\} \quad (4)$$

$$= 5(k+1)^2 + 4 \quad (5)$$

$$> 0 \quad (6)$$

\therefore the roots of $f(x) = 0$
are real and distinct

$$\begin{aligned} \alpha + \beta &= 3k+1 \\ \alpha\beta &= (k+1)(k-2) \end{aligned} \quad (7)$$

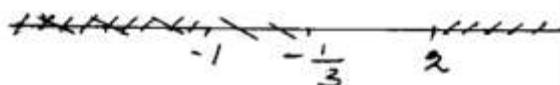
Both α, β are negative

$$\alpha < 0, \beta < 0$$

$$\Leftrightarrow \alpha + \beta < 0 \quad \& \quad \alpha\beta > 0 \quad (8)$$

$$\Leftrightarrow 3k+1 < 0 \quad \& \quad (k+1)(k-2) > 0$$

$$\Leftrightarrow k < -\frac{1}{3} \quad \& \quad k < -1 \text{ or } k > 2 \quad (9)$$



$$\text{So, } k < -1 \quad (10)$$

$$x^2 - (3k+1)x + (k+1)(k-2) = 0 \quad (11)$$

$$p \Delta t \quad y = x+1 \quad (12)$$

$$x = \alpha \Rightarrow y = \alpha + 1$$

$$x = \beta \Rightarrow y = \beta + 1$$

$$x \rightarrow y-1 \text{ substitute in (11)}$$

$$(11) \Rightarrow (y-1)^2 - (3k+1)(y-1) + (k+1)(k-2) = 0 \quad (13)$$

$$y^2 - 3(k+1)y + k(k+2) = 0 \quad (14)$$

The required eq is

$$x^2 - 3(k+1)x + k(k+2) = 0 \quad (15)$$

[20]

$$b) f(x) = (x-a)\phi_1(x) + A \quad (16)$$

$$\phi_1(x) = (x-b)\phi_2(x) + B \quad (17)$$

(1), (2) \Rightarrow

$$f(x) = (x-a)\{ (x-b)\phi_2(x) + B \} + A \quad (18)$$

$$= (x-a)(x-b)\phi_2(x) + B(x-a) + A \quad (19)$$

$$(1) \Rightarrow f(a) = A \quad (20)$$

$$(2) \Rightarrow \phi_1(b) = B \quad (21)$$

$$(1) \Rightarrow f(b) = (b-a)\phi_1(b) + A$$

$$f(b) = (b-a)B + f(a)$$

$$B = \frac{f(b) - f(a)}{b-a} \quad (22)$$

$$(3) \Rightarrow f(x) = (x-a)(x-b)\phi_2(x) + (x-a)\frac{f(b) - f(a)}{b-a}(x-a) + f(a) \quad (23)$$

$$\therefore \text{Remainder } \frac{f(b) - f(a)}{b-a}(x-a) + f(a) \quad (24)$$

$$f(x) = x^3 + px^2 + qx + r \quad (25)$$

$$\text{put } a = 1, b = 2$$

$$-10x+5 = \frac{f(2) - f(1)}{2-1}(x-1) + f(1) \quad (26)$$

$$x = 10 = f(2) - f(1) \quad (27)$$

$$-10 = (8 + 4p + 2q + 1) - (1 + p + q + 1) \quad (28)$$

$$3p + q = -17 \quad (29)$$

$$x = 5 = -\{f(2) - f(1)\} + f(1) \quad (30)$$

$$5 = 2f(1) - f(2) \quad (31)$$

$$5 = 2\{1 + p + q + 1\} - \{8 + 4p + 2q + 1\} \quad (32)$$

$$2p = -10 \quad (33)$$

$$p = -5 \quad (34)$$

$$(31) \Rightarrow q = -2 \quad (35)$$

[35]

12]

a) Let $f(x) = \sin x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h \{ \sin(x+h) + \sin x \}} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos(x+\frac{h}{2}) \sin \frac{h}{2}}{h \{ \sin(x+h) + \sin x \}} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \lim_{h \rightarrow 0} \frac{\cos(x+\frac{h}{2})}{\{ \sin(x+h) + \sin x \}} \quad (5)$$

$$= 1 \cdot \frac{\cos x}{\{ \sin x + \sin x \}} \quad (5)$$

$$= \frac{\cos x}{2 \sin x} \quad (6)$$

b) $y = \frac{x^2-1}{x^2+1} \quad [35]$

$$\frac{dy}{dx} = \frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2} \quad (5)$$

$$= \frac{4x}{(x^2+1)^2} \quad (5)$$

$$y = \frac{1+e^x}{1-e^x}$$

$$\frac{dy}{dx} = \frac{(1-e^x)e^x - (1+e^x)(-e^x)}{(1-e^x)^2} \quad (5)$$

$$= \frac{2e^x}{(1-e^x)^2} \quad (5)$$

$$y = x^{x^2+1} \quad x^{x^2+1}$$

$$\ln y = \ln x$$

$$\ln y = (x^2+1) \ln x \quad (5)$$

$$\frac{1}{y} \frac{dy}{dx} = (x^2+1) \cdot \frac{1}{x} + (\ln x) \{ 2x \} \quad (10)$$

$$\frac{dy}{dx} = x^{x^2+1} \left\{ \frac{2x^2+1}{x} + 2x \ln x \right\} \quad (5)$$

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c) $y = e^x \cos e^x \quad (1)$

$$\frac{dy}{dx} = e^x (-\sin e^x \cdot e^x + \cos e^x \cdot e^x) \quad (10)$$

$$\frac{dy}{dx} = -e^{2x} \sin e^x + y \quad (\text{by (1)})$$

$$\frac{dy}{dx} - y = -e^{2x} \sin e^x \quad (2)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -\left[e^{2x} \cos e^x e^x \right] \quad (10) + \sin e^x e^{-x}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -e^{2x} y + 2 \left\{ \frac{dy}{dx} - y \right\} \quad (5)$$

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + (e^{2x} + 2)y = 0 \quad (\text{by (1) (2)})$$

35

d) $x = \sin n\theta \quad - \quad y = \cos m\theta$

$$\frac{dx}{d\theta} = n \cos n\theta \quad (5) \quad \frac{dy}{d\theta} = -m \sin m\theta \quad (5)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{-m \sin m\theta}{n \cos n\theta} \quad (6)$$

$$n^2 \cos^2 n\theta \left(\frac{dy}{dx} \right)^2 = m^2 \sin^2 m\theta \quad (5)$$

$$n^2 (1-x^2) \left(\frac{dy}{dx} \right)^2 = m^2 (1-y^2) \quad (5)$$

$$n^2 \left\{ (1-x^2) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 (-2) \right\} = m^2 \left\{ -2y \frac{dy}{dx} \right\} \quad (10)$$

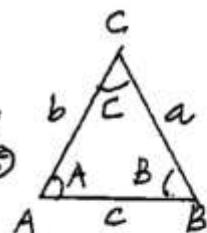
$$n^2 \left\{ (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} \right\} = -m^2 y$$

$$n^2 (1-x^2) \frac{d^2y}{dx^2} - n^2 x \frac{dy}{dx} + m^2 y = 0 \quad (5)$$

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[3] Sin rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (5)$$



proof - (15)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k \cos \gamma} \quad (6) \quad \boxed{20}$$

$$\frac{a+b-c}{a+b+c}$$

$$= \frac{k \sin A + k \sin B - k \sin C}{k \sin A + k \sin B + k \sin C} \quad (64x)$$

$$= \frac{2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right) - 2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right) + 2 \sin \frac{C}{2} \cos \frac{C}{2}} \quad (10)$$

$$= \frac{\cos \frac{C}{2} \cos \left(\frac{A-B}{2}\right) - \sin \frac{C}{2} \cos \frac{C}{2}}{\cos \frac{C}{2} \cos \left(\frac{A-B}{2}\right) + \sin \frac{C}{2} \cos \frac{C}{2}}$$

$$= \frac{\cos \left(\frac{C}{2} - \frac{I}{2}\right) - \sin \frac{C}{2}}{\cos \left(\frac{C}{2} + \frac{I}{2}\right) + \sin \frac{C}{2}} \quad (14x)$$

$$= \frac{\cos \left(\frac{C}{2} - \frac{I}{2}\right) - \sin \frac{C}{2}}{\cos \left(\frac{C}{2} + \frac{I}{2}\right) + \sin \frac{C}{2}} \quad (5)$$

$$= \frac{\cos \left(\frac{C}{2} - \frac{I}{2}\right) - \cos \left(\frac{C+I}{2}\right)}{\cos \left(\frac{C}{2} + \frac{I}{2}\right) + \cos \left(\frac{C-I}{2}\right)} \quad (5)$$

$$= \frac{2 \sin \frac{C}{2} \sin \frac{B}{2}}{2 \cos \frac{C}{2} \cos \frac{B}{2}} \quad (10)$$

$$= \tan \frac{C}{2} \tan \frac{B}{2} \quad (5)$$

$$b) \tan^{-1} \left(\frac{x}{2}\right) + \tan^{-1} \left(\frac{x}{3}\right) = \frac{\pi}{4} \quad (1)$$

$$\text{put } \alpha = \tan^{-1} \left(\frac{x}{2}\right) \Rightarrow \tan \alpha = \frac{x}{2}$$

$$\beta = \tan^{-1} \left(\frac{x}{3}\right) \Rightarrow \tan \beta = \frac{x}{3}$$

$$(1) \Rightarrow \alpha + \beta = \frac{\pi}{4}$$

$$\tan(\alpha + \beta) = 1 \quad (5)$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = 1 \quad (5)$$

$$\frac{\frac{x}{2} + \frac{x}{3}}{1 - \left(\frac{x}{2}\right)\left(\frac{x}{3}\right)} = 1 \quad (5)$$

$$\frac{5x}{6-x^2} = 1$$

$$x^2 + 5x - 6 = 0 \quad (5)$$

$$(x+6)(x-1) = 0 \quad (5)$$

$$x-1 = 0 \quad (\because x > 0)$$

$$x = 1 \quad (5)$$

$$(1) \Rightarrow \tan^{-1} \left(\frac{1}{2}\right) + \tan^{-1} \left(\frac{1}{3}\right) = \frac{\pi}{4} \quad (5)$$

$$\tan^{-1} \left(\frac{1}{2}\right) = \frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3}\right)$$

$$\tan^{-1} \left(\frac{1}{3}\right) = \frac{\pi}{4} - \tan^{-1} \left(\frac{1}{2}\right) \quad (5)$$

$$\frac{1}{\sqrt{3}} = \sin \left\{ \frac{\pi}{4} - \tan^{-1} \left(\frac{1}{2}\right) \right\} \quad (5) \quad \boxed{50}$$

$$c) 0 < x < 1, \quad x = \tan \alpha$$

$$\sin^{-1} \left\{ \frac{2x}{1+x^2} \right\} + \cos^{-1} \left\{ \frac{1-x^2}{1+x^2} \right\} + \tan^{-1} \left\{ \frac{2x}{1-x^2} \right\} =$$

$$\sin^{-1} \left\{ \frac{2+2\alpha}{1+2\alpha} \right\} + \cos^{-1} \left\{ \frac{1+2\alpha}{1+2\alpha} \right\} + \tan^{-1} \left\{ \frac{2+2\alpha}{1-2\alpha} \right\} =$$

$$\sin^{-1} \left\{ \sin 2\alpha \right\} + \cos^{-1} \left\{ \cos 2\alpha \right\} + \tan^{-1} \left\{ \tan 2\alpha \right\} = \pi$$

$$2\alpha + 2\alpha + 2\alpha = \pi \quad (5) \quad (0 < 2\alpha < \frac{\pi}{2}) \quad (5)$$

$$6\alpha = \pi$$

$$\alpha = \frac{\pi}{6} \quad (5)$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad (5)$$

50

30

14]

a)

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad (5) \quad (1)$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \quad (5) \quad (2)$$

(1) + (2) \Rightarrow

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B \quad (5) \quad (3)$$

$$\begin{aligned} A+B &= C \\ A-B &= D \end{aligned} \Rightarrow \begin{aligned} A &= \frac{C+D}{2} \\ B &= \frac{C-D}{2} \end{aligned} \quad (5)$$

$$(3) \Rightarrow \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right) \quad (5)$$

 $\sin C - \sin D$

$$= \sin C + \sin(-D) \quad (5)$$

$$= 2 \sin\left(\frac{C+(-D)}{2}\right) \cos\left(\frac{C-(-D)}{2}\right) \quad (5)$$

$$= 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right) \quad (5)$$

[40]

$$\sin 7x + \sin 5x - \cot x (\sin 7x - \sin 5x) = 0 \quad \therefore \theta = \frac{\pi}{8} \quad (5) \quad (4)$$

$$2 \sin 6x \cos x - \frac{\cos x}{\sin x} (2 \cos 6x \sin 2x) = 0 \quad c) \quad \frac{\tan 50 + \tan 30}{\tan 50 - \tan 30}$$

$$\sin 6x \cos x - \cos 6x \sin x = 0 \quad \frac{\sin 50}{\cos 50} + \frac{\sin 30}{\cos 30} \quad (5)$$

$$\cos x (\sin 6x - \cos 6x) = 0 \quad \frac{\sin 50}{\cos 50} - \frac{\sin 30}{\cos 30},$$

$$\cos x = 0 \text{ or } \sin 6x - \cos 6x = 0 \quad \frac{\sin 50 \cos 30 + \cos 50 \sin 30}{\sin 50 \cos 30 - \cos 50 \sin 30} \quad (5)$$

$$\cos x = \cos \frac{\pi}{2} \quad \tan 6x = 1 (-2k\pi + \frac{\pi}{4}) \quad = \frac{\sin 50 \cos 30 + \cos 50 \sin 30}{\sin 50 \cos 30 - \cos 50 \sin 30}$$

$$x = 2k\pi \pm \frac{\pi}{4} \quad (5)$$

 $n_1 \in \mathbb{Z}$

$$6x = n_2 \pi + \frac{\pi}{4}$$

$$x = \frac{n_2 \pi}{6} + \frac{\pi}{24} \quad (5)$$

 $n_2 \in \mathbb{Z}$

[40]

b) $\sin 7\theta + \sin \theta$

$$= 2 \sin 4\theta \cos 3\theta \quad (5)$$

$$= 2 \{ 2 \sin 2\theta \cos 2\theta \} \cos 3\theta \quad (5)$$

$$= 4 \{ 2 \sin \theta \cos \theta \} \cos 2\theta \cos 3\theta \quad (5)$$

$$= 8 \sin \theta \cos \theta \cos 2\theta \cos 3\theta \quad (5)$$

$$\sin 7\theta + \sin \theta = 8 \sin \theta \cos \theta \cos 2\theta \cos 3\theta$$

$$\sin 7\theta = \sin \theta \{ 8 \cos \theta \cos 2\theta \cos 3\theta - 1 \} \quad (5)$$

$$4 \cos \theta \cos 2\theta \cos 3\theta = 1$$

$$8 \cos \theta \cos 2\theta \cos 3\theta = 2$$

$$\frac{\sin 7\theta}{\sin \theta} + 1 = 2 \quad (5)$$

$$\sin 7\theta = \sin \theta \quad (5)$$

$$7\theta = n\pi + (-1)^n \theta, \quad n=0, \pm 1, \pm 2$$

$$n=0 \Rightarrow \theta=0 \quad (5)$$

$$n=1 \Rightarrow \theta=\frac{\pi}{7}$$

$$n=2 \Rightarrow \theta=\frac{2\pi}{7}$$

$$\therefore \theta=\frac{\pi}{8} \quad (5) \quad (4)$$

$$c) \quad \frac{\tan 50 + \tan 30}{\tan 50 - \tan 30}$$

$$\frac{\tan 50}{\sin 50} - \frac{\tan 30}{\sin 30}$$

$$\frac{\sin 50}{\cos 50} + \frac{\sin 30}{\cos 30} \quad (5)$$

$$\frac{\sin 50}{\cos 50} - \frac{\sin 30}{\cos 30},$$

$$\frac{\sin 50 \cos 30 + \cos 50 \sin 30}{\sin 50 \cos 30 - \cos 50 \sin 30} \quad (5)$$

$$\frac{\sin 80}{\sin 20} \quad (5)$$

$$\frac{2 \sin 4\theta \cos 4\theta}{\sin 20} \quad (5)$$

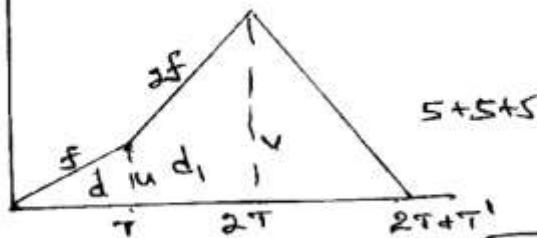
$$\frac{4 \sin 2\theta \cos 2\theta \cos 4\theta}{\sin 20}$$

$$= 4 \cos 2\theta \cos 4\theta \quad (5)$$

[25]

(15)

(1)



15

$$(ii) u = fT \quad (5)$$

$$\frac{1}{2} fT^2 = d \quad (5)$$

$$T = \sqrt{\frac{2d}{f}} \quad (5)$$

$$\frac{v-u}{T} = 2f \quad (5)$$

$$v = 3fT \quad (5)$$

$$d_1 = \frac{(u+v)T}{2} = \frac{2fT^2}{2} \quad (5) \\ = 4d \quad (5)$$

$$(iii) \frac{1}{2} v T' = 2d \quad (5)$$

$$T' = \frac{4d}{v} = \frac{4d}{3fT} = \frac{4d}{3} \cdot \frac{1}{T} \quad (5)$$

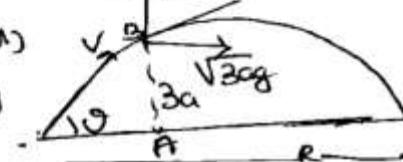
$$\text{Total time} = T + T + \frac{2}{3} T \\ = 8T \quad (5)$$

$$f' = \frac{v}{T'} = \frac{3v}{T} = 9f \quad (5)$$

(b)

$$\sqrt{3ag} \quad (75)$$

(i)



$$v \cos \theta = \sqrt{3ag} \rightarrow ① \quad (5)$$

$$\uparrow v^2 = u^2 + 2as$$

$$3ag = \frac{2}{3} u^2 \sin^2 \theta - 2gs \times 3a \quad (5)$$

$$v \sin \theta = 3\sqrt{ag} \rightarrow ② \quad (5)$$

$$\tan \theta = \sqrt{3}$$

$$\theta = \frac{\pi}{3} \quad (5)$$

$$v = 2\sqrt{3ag} \quad (5) \quad [25]$$

$$(i) s = ut + \frac{1}{2} a t^2$$

$$0 = v \sin \theta \cdot T - \frac{1}{2} g T^2 \quad (5)$$

$$T = \frac{2v \sin \theta}{g} \quad (5)$$

$$R = v \cos \theta \times T \quad (5)$$

$$= \frac{v \cos \theta \times 2v \sin \theta}{g}$$

$$= \frac{\sqrt{3ag} \times 2 \times 3\sqrt{ag}}{g}$$

$$= 6\sqrt{3}a. \quad (5) \quad [20]$$

(b) T_1 is the time p reaches B
 $\uparrow v = u + at$

$$\sqrt{3ag} = v \sin \theta - gT_1 \quad (5)$$

$$T_1 = \frac{v \sin \theta - \sqrt{3ag}}{g} \quad (5)$$

Time taken to reach C to Q

$$T_2 = T - T_1$$

$$= \frac{v \sin \theta + \sqrt{3ag}}{g} \quad (5)$$

$$= \frac{3\sqrt{ag} + \sqrt{3ag}}{g}$$

$$= (3+\sqrt{3})\sqrt{\frac{a}{g}} \quad (5)$$

$$u \cos \theta \cdot T_2 = R \quad (5)$$

$$u \frac{\sin \theta}{T_2} = \frac{g}{2} \quad (5)$$

$$\tan \theta = \frac{g T_2^2}{2}$$

$$d = \tan^{-1} \left(\frac{g T_2^2}{2} \right) \quad (5)$$

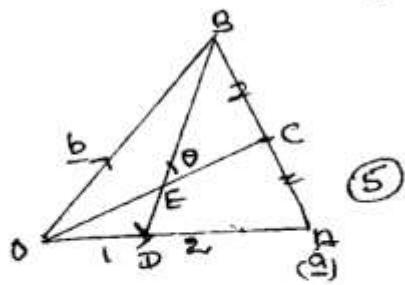
$$u = \sqrt{\frac{g^2 a^2}{4} + \frac{R^2}{T_2^2}} \quad (5)$$

$$\text{max height of p} = \frac{v^2 \sin^2 \theta}{2g} \quad (5) \quad [25]$$

$$\text{max height of Q} = \frac{u^2 \sin^2 \theta}{2g} \quad (5) \quad [15]$$

(16)

(a)



$$\begin{aligned} \text{(i) } \vec{OE} &= \vec{OA} + \vec{AE} \quad (5) \\ &= \vec{OA} + \frac{1}{2}\vec{AB} \\ &= \vec{a} + \frac{1}{2}(b - a) \\ &= \frac{a+b}{2} \quad (5) \end{aligned}$$

$$\vec{OD} = \frac{1}{3}\vec{OE} = \frac{1}{3}\vec{a} \quad (5) \quad [25]$$

$$\begin{aligned} \text{(ii) } \vec{OE} &= \vec{OB} + \vec{BE} \quad (5) \\ &= \vec{OB} + \lambda \vec{BC} \\ &= \vec{OB} + \lambda (\vec{OC} - \vec{OB}) \\ &= \vec{b} + \lambda(\vec{c} - \vec{b} + \frac{1}{3}\vec{a}) \\ &= (1-\lambda)\vec{b} + \frac{\lambda}{3}\vec{a} \quad (5) \end{aligned}$$

$$\text{(iii) } \vec{OE} = \mu \vec{OC}$$

$$(1-\lambda)\vec{b} + \frac{\lambda}{3}\vec{a} = \mu(\vec{a} + \vec{b}) \quad (5)$$

$$\text{(5) } \frac{1}{3} = \frac{\mu}{2}, \quad 1-\lambda = \frac{\mu}{2}$$

$$\begin{aligned} \frac{1}{3} &= 1-\lambda \\ 4\lambda &= 3 \\ \lambda &= \frac{3}{4} \quad (5) \end{aligned}$$

[26]

$$\text{(iv) } \vec{a} = 12 \cdot \frac{1}{2} \cdot \vec{b} = 2\vec{b} + 4\vec{a}$$

$$\text{(v) } \vec{OE} = 7\vec{b} + 4\vec{a} \quad (5)$$

$$\vec{OD} = 4\vec{a} \quad (5)$$

[10]

$$\text{(ii) } \vec{DE} = 7\vec{b} + 4\vec{a}$$

$$\begin{aligned} \vec{DB} &= \vec{DA} + \vec{AB} \quad (5) \\ &= -4\vec{a} + 2\vec{b} + \vec{b} \\ &= -2\vec{a} + 3\vec{b} \quad (5) \end{aligned}$$

$$\begin{aligned} \vec{DE} \cdot \vec{DB} &= (7\vec{b} + 4\vec{a}) \cdot (-2\vec{a} + 3\vec{b}) \\ &= -14 + 32 \quad (5) \\ &= 18 \quad (5) \end{aligned}$$

$$\begin{aligned} 18 &= |\vec{DE}| |\vec{DB}| \cos \theta \quad (5) \\ &= \sqrt{49+16} \sqrt{64+4} \cos \theta \quad (5) \\ &= \sqrt{65} \times \sqrt{68} \cos \theta \end{aligned}$$

$$|\vec{DE}| = \frac{18}{\sqrt{65 \times 68}} \quad (5)$$

$$\theta = \cos^{-1} \left(\frac{18}{\sqrt{65} \sqrt{68}} \right) \quad [35]$$

$$\text{(iii) } \vec{DP} = \beta \vec{b} \text{ எனக்க.} \quad (5)$$

$$\vec{AP} = \alpha \vec{a} \quad (5)$$

$$\vec{AO} + \vec{OP} = \alpha (\vec{a} + \vec{OE}) \quad (5)$$

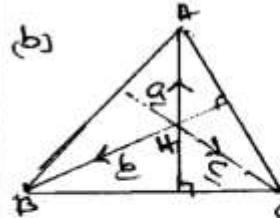
$$-\vec{a} + \beta \vec{b} = \alpha \left(-\frac{a}{2} + \frac{1}{2}\vec{b} + \frac{1}{3}\vec{a} \right) \quad (5)$$

$$-1 = -\frac{3}{4}\alpha, \quad \beta = \frac{8}{3}$$

$$\alpha = \frac{4}{3}, \quad \beta = \frac{8}{3} \quad (5)$$

$$\therefore \vec{OP} = \frac{1}{3}\vec{b} \quad (5)$$

[25]



$\vec{AP} \perp \vec{BC}$

$$\vec{AP} \cdot \vec{CB} = 0 \quad (5)$$

$$\vec{a} \cdot (\vec{b} - \vec{c}) = 0 \rightarrow (1)$$

$\vec{AP} \perp \vec{AC}$

$$\vec{AP} \cdot \vec{c} = 0 \quad (5)$$

$$\vec{b} \cdot (\vec{c} - \vec{a}) = 0 \rightarrow (2)$$

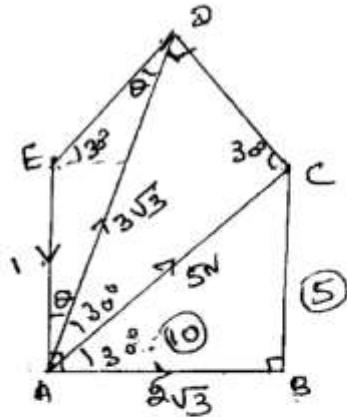
$$(1) + (2) \quad \vec{b} \cdot \vec{c} - \vec{a} \cdot \vec{c} = 0 \quad (5)$$

$$(\vec{b} - \vec{a}) \cdot \vec{c} = 0 \quad (5)$$

180

$\vec{AP} \perp \vec{AC}$ [25]

(17)(a)



$$\text{(i)} \quad 2AE \cos \theta = \sqrt{3}AE \quad (5) \\ \cos \theta = \frac{\sqrt{3}}{2} \quad (5) \\ \theta = 30^\circ \quad (5)$$

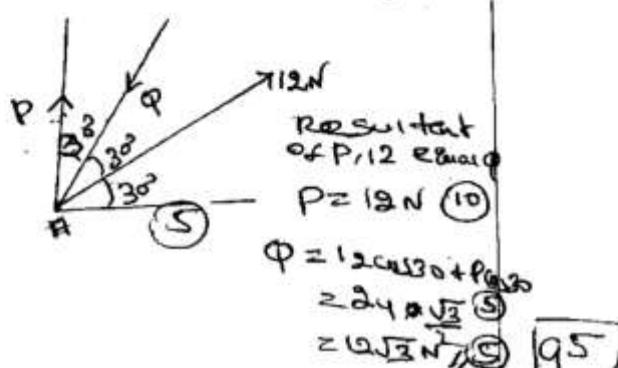
$$\text{(ii)} \rightarrow \\ x = 2\sqrt{3} + 5 \cos 30 + 2\sqrt{3} \cos 60 \\ = 2\sqrt{3} + \frac{5\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} \quad (10) \\ = 6\sqrt{3} \quad (5) \quad (10) \\ \uparrow y = 5 \sin 30 + 3\sqrt{3} \cos 30 \\ = \frac{5}{2} + \frac{9}{2} - 1 \\ = 6 \quad (5)$$

$R = \sqrt{6^2 + (6\sqrt{3})^2}$
 $= 12N \quad (5)$

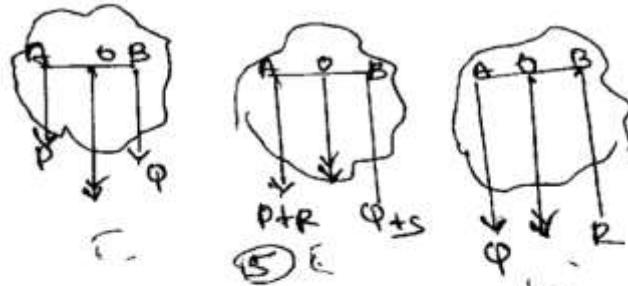
$\tan \theta = \frac{6}{6\sqrt{3}}$
 $= \frac{1}{\sqrt{3}}$
 $\theta = 30^\circ \quad (5)$

Resultant 12N along \vec{AC} .

(18)



(b)



$$\frac{AO}{OB} = \frac{Q}{P} \quad \frac{AO}{OB} = \frac{Q+S}{P+R} \quad \frac{AO}{OB} = \frac{R}{Q} \quad (5) + (5) \\ \therefore \frac{Q}{P} = \frac{Q+S}{P+R} = \frac{R}{Q}$$

$$\text{(i)} \quad \frac{Q}{P} = \frac{R}{Q} \\ \therefore Q^2 = RP \quad (5)$$

$$\text{(ii)} \quad \frac{Q}{P} = \frac{Q+S}{P+R} = \frac{R}{Q} = \frac{Q+S-Q-S}{P+R-P-R} \quad (5) \\ \therefore \frac{P}{Q} = \frac{Q}{R} = \frac{R}{S} \quad (5)$$

$$\text{(iii)} \quad Q^2 = RP, \quad R = \frac{PS}{Q} \quad (5) \\ \therefore Q^2 = \frac{PS}{Q} \times P \quad (5) \\ Q^3 = SP^2 //$$

$$\text{(iv)} \quad \frac{P}{Q} = \frac{P}{R} = \frac{R}{S} = \frac{P+R}{Q+S} = \frac{P+Q}{Q+R} \quad (5) \\ \therefore (R-S)(P-Q) = (P+R) \\ P-S = \frac{(P+R)^2}{P+Q} \quad (5)$$

[55]

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