தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு
முதலாம் தவணைப் பரீட்சை - 2024
National Field Work Centre, Thondaimanaru
1st Term Examination - 2024

FWC

இணைந்த கணிதம்	Gr. 12	புள்ளித்திட்டம்

01]

$(x-1)(x-2)-(x-2)+4(x-1)$

$= x^2-3x+2-x+2+4x-4$

$= x^2$ (5)

$\frac{x^2}{(x-1)(x-2)} = \frac{(x-1)(x-2)-(x-2)+4(x-1)}{(x-1)(x-2)}$ (5)

$= 1 - \frac{1}{x-1} + \frac{4}{x-2}$ (5)

$\frac{x^2}{(x-1)(x-2)} = 1 - \frac{1}{x-1} + \frac{4}{x-2}$

$\frac{x^2}{(x-1)(x-2)} - 1 = \frac{4}{x-2}$ (5) $- \frac{1}{x-1}$

$\frac{3x-2}{(x-1)(x-2)} = \frac{4}{x-2}$ (5) $- \frac{1}{x-1}$

$\boxed{25}$

02]

$\sqrt{x+1} = \sqrt{x+6} - 1$

$x+1 = x+6-2\sqrt{x+6}+1$ (10)

$2\sqrt{x+6} = 6$

$\sqrt{x+6} = 3$ (5)

$x+6 = 9$

$x = 3$ (5)

when $x=3$

L.H.S $= \sqrt{x+1} = \sqrt{3+1} = 2$

R.H.S $= \sqrt{x+6} - 1$

$= \sqrt{3+6} - 1$ (5)

$= 2$

Sol^n: $x = 2$

$\boxed{25}$

03]

$\frac{(x-1)(x-9)}{x} < 0$

	$x<0$	$0<x<1$	$1<x<9$	$9<x$
$(x-1)$	$(-)$	$(-)$	$(+)$	$(+)$
$(x-9)$	$(-)$	$(-)$	$(-)$	$(+)$
x	$(-)$	$(+)$	$(+)$	$(+)$
$\frac{(x-1)(x-9)}{x}$	$(-)$	$(+)$	$(-)$	$(+)$ (10)

$x<0$ or $1<x<9$ (5)

$\frac{4x^2+9}{x} < 20$

$\frac{4x^2-20x+9}{x} < 0$

$\frac{(2x-1)(2x-9)}{2x} < 0$ (5)

by above part

$2x<0$ or $1<2x<9$

$x<0$ or $\frac{1}{2}<x<\frac{9}{2}$ (5)

$\boxed{25}$

04]

$x, y, z \in \mathbb{Z}^+$

$y = x+1 \qquad z = y+1$ (5)

$\Rightarrow z = (x+2)$ (5)

$\log\{1+xz\} = \log\{1+x(x+2)\}$ (5)

$= \log\{(x+1)^2\}$ (5)

$= \log y^2$

$= 2\log y$ (5)

$\boxed{25}$

05] $(x+1)(2x-7)(2x+1)(x-3)=45$

$(2x^2-5x-7)(2x^2-5x-3)=45$

$t=2x^2-5x-7$ (say) (5) — (1)

(1) $\Rightarrow t(t+7-3)=45$

$t^2+4t-45=0$ (5)

$(t+9)(t-5)=0$

$t=-9$ or $t=5$ (5)

when $t=-9$

$2x^2-5x-7=-9$

$2x^2-5x+2=0$

$(2x-1)(x-2)=0$

$x=\frac{1}{2}, 2$ (5)

when $t=5$

$2x^2-5x-7=5$

$2x^2-5x-12=0$

$(2x+3)(x-4)=0$ (9)

$x=-\frac{3}{2}, 4$ (5)

$\boxed{25}$

06] $\sin\theta+\sin^2\theta=1$

$\sin\theta=\cos^2\theta$ (5)

$\cos^{16}\theta+4\cos^{14}\theta+6\cos^{12}\theta+4\cos^{10}\theta+\cos^8\theta$

$=\sin^8\theta+4\sin^7\theta+6\sin^6\theta+4\sin^5\theta+\sin^4\theta$ (5)

$=(\sin^2\theta)^4+4(\sin^2\theta)^3(\sin\theta)+6(\sin^2\theta)^2(\sin\theta)^2+4(\sin^2\theta)(\sin\theta)^3+(\sin\theta)^4$ (5)

$=(\sin^2\theta+\sin\theta)^4$ (5)

$=1^4$

$=1$ (5)

$\boxed{25}$

7) $\underline{a}\cdot\underline{b}=|\underline{a}||\underline{b}|\cos\theta$, $0\le\theta\le\pi$ (5)

$\underline{a}\cdot\underline{b}=|\underline{a}||\underline{b}|\cos\theta$ — (1) (5)

$\underline{b}\cdot\underline{a}=|\underline{b}||\underline{a}|\cos\theta$ — (2) (5)

from (1)+(2) $\Rightarrow \underline{a}\cdot\underline{b}=\underline{b}\cdot\underline{a}$ (5)

$\Rightarrow \underline{a}\cdot\underline{b}$ and $\underline{b}\cdot\underline{a}$ are commutative (5)

$\boxed{25}$

(8) Forces are in equilibrium (5)

$\Rightarrow$ Resultant is zero

$2\underline{i}+3\underline{j}+4\underline{i}+\alpha\underline{j}-7\underline{i}+8\underline{j}+\beta\underline{i}-5\underline{j}=\underline{0}$ (5)

$(2+4-7+\beta)\underline{i}+(3+\alpha+8-5)\underline{j}=\underline{0}$

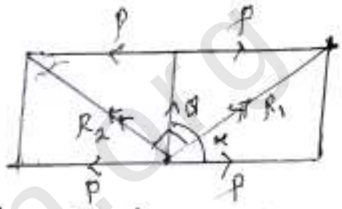
$\Rightarrow 2+4-7+\beta=0$ (5)

$3+\alpha+8-5=0$ (5)

$\Rightarrow \beta=1, \alpha=-6$ (5)

$\boxed{25}$

(9)



(5)

$R_1^2=P^2+Q^2+2PQ\cos\alpha$ (5)

$R_2^2=P^2+Q^2+2PQ\cos(\pi-\alpha)$

$=P^2+Q^2-2PQ\cos\alpha$ (5)

$R_1^2+R_2^2=2(P^2+Q^2)$ — (α) (5)

But in the diagram

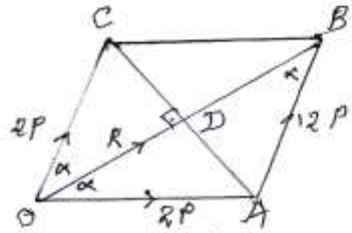
$R_1^2+R_2^2=(2P)^2$ — (β) (5)

(α)+(β) $\Rightarrow 2(P^2+Q^2)=4P^2$

$Q=P$

$\boxed{25}$

(10)



(5)

OABC is a rhombus. (5)

$\Rightarrow OD=2P\cos\alpha$ (5)

$DB=2P\cos\alpha$ (5)

$OD+DB=R$ (5)

$4P\cos\alpha=R$

$\boxed{25}$

11] a) $f(x) = x^2 - bx + 1 = 0$

(i) Discriminant Δ

$\Delta = (-b)^2 - 4(1)(1)$ (5)

$= b^2 - 4$

for the roots of $f(x)=0$ are real

$\Delta \geq 0$ (5)

$b^2 - 4 \geq 0$ (5)

$(b-2)(b+2) \geq 0$ (5)

$b \leq -2$ or $b \geq 2$ (5)

$\boxed{25}$

(ii) $x^2 - bx + 1 = 0$ $\quad \alpha, \beta$

$\left.\begin{array}{l}\alpha+\beta = b \\ \alpha\beta = 1\end{array}\right\}$ (5)

$\alpha(\alpha+1) + \beta(\beta+1)$

$= \alpha^2 + \beta^2 + \alpha + \beta$ (5)

$= (\alpha+\beta)^2 - 2\alpha\beta + (\alpha+\beta)$ (5)

$= b^2 - 2\times 1 + b$ (5)

$= b^2 + b - 2$

$\alpha(\alpha+1)\beta(\beta+1)$

$= \alpha\beta(\alpha+1)(\beta+1)$

$= \alpha\beta\{\alpha\beta + \alpha + \beta + 1\}$ (5)

$= 1\{1 + b + 1\}$

$= b + 2$ (5)

The required quadratic equation is given by

$\{x - \alpha(\alpha+1)\}\{x - \beta(\beta+1)\} = 0$ (5)

$x^2 - \{\alpha(\alpha+1) + \beta(\beta+1)\}x + \alpha(\alpha+1)\beta(\beta+1) = 0$ (5)

$x^2 - (b^2+b-2)x + (b+2) = 0$ (5) ——— (*)

$\boxed{45}$

$\alpha(\alpha+\beta+1) = \alpha(\alpha+1) + \alpha\beta$

$= \alpha(\alpha+1) + 1$

$\beta(\alpha+\beta+1) = \beta(\beta+1) + \alpha\beta$

$= \beta(\beta+1) + 1$ (5)

put $y = x + 1$ (5)

$x = \alpha(\alpha+1) \Rightarrow y = \alpha(\alpha+\beta+1)$

$x = \beta(\beta+1) \Rightarrow y = \beta(\alpha+\beta+1)$

$x \to (y-1)$ substitute in (*)

$(y-1)^2 - (b^2+b-2)(y-1) + (b+2) = 0$ (5)

$y^2 - 2y + 1 - (b^2+b-2)(y-1) + b + 2 = 0$ (5)

$y^2 - b(b+1)y + (b+1)^2 = 0$

$\therefore$ The required eq^n is

$x^2 - b(b+1)x + (b+1)^2 = 0$ (5)

$\boxed{25}$

b) $x^2 + ax + b = 0$ $\quad \alpha, \beta$

$x^2 + bx + a = 0$ $\quad \alpha, \gamma$

then (5)

$\alpha + \beta = -a$ — (1), $\quad \alpha + \gamma = -b$ — (3)

$\alpha\beta = b$ — (2) (5) $\quad \alpha\gamma = a$ — (4)

since α is common root

$\alpha^2 + a\alpha + b = 0$ — (5) (5)

$\alpha^2 + b\alpha + a = 0$ — (6) (5)

$(5) - (6) \Rightarrow (a-b)\alpha + b - a = 0$

$\alpha = 1$ (5) $(b \neq a)$

$(5) \Rightarrow 1^2 + a + b = 0$

$1 + a + b = 0$ (5)

$(1) + (3) \Rightarrow 2\alpha + \beta + \gamma = -(b+a)$ (5)

$2 + \beta + \gamma = -(a+b)$

$\beta + \gamma = -(a+b+2)$

$= -1$ (5)

$(2) \times (4) \Rightarrow \alpha^2\beta\gamma = ab$

$\beta\gamma = ab$ (5) $(\because \alpha = 1)$

The required eq^n is

$x^2 - (\beta+\gamma)x + \beta\gamma = 0$ (5)

$x^2 - (-1)x + ab = 0$

$x^2 + x + ab = 0$ (5)

$\boxed{55}$

12]

a] $f(x) = 2x^3 + ax^2 + bx - 6$

Since $(x-1)$ is a factor of $f(x)$

$f(1) = 0$ (5)

$2 + a + b - 6 = 0$

$a + b = 4$ (5) (1)

$|||^{ly}$ $f(2) = 0$ (5)

$2(2)^3 + a(2)^2 + b(2) - 6 = 0$

$2a + b = -5$ (5) (2)

(1), (2) $\Rightarrow$ $a = -9$ (5), $b = 13$ (5)

$\boxed{30}$

$f(x) = (x-1)(x-2)(2x+B)$ (5)

x^0; $-6 = 2B$

$B = -3$

$f(x) = (x-1)(x-2)(2x-3)$ (5)

$\boxed{10}$

$f(x) = (x^2 - 3x + 2)(2x-3)$

$= \{x^2 - 3x + \frac{9}{4} - \frac{1}{4}\}(2x-3)$ (5)

$= \{(x-\frac{3}{2})^2 - \frac{1}{4}\}(2x-3)$ (5)

$= (x-\frac{3}{2})^2(2x-3) - \frac{1}{4}(2x-3)$ (5)

Quotient $= (2x-3)$ (5)

Reminder $= -\frac{1}{4}(2x-3)$ (5)

$\boxed{25}$

b] $y = \frac{x^2+5}{x-2}$

$x^2 - yx + (5+2y) = 0$ (5)

This is quadratic eq^n in x

for real value of x

$\Delta \geq 0$ (5)

$(-y)^2 - 4(1)(5+2y) \geq 0$ (5)

$y^2 - 8y - 20 \geq 0$ (5)

$(y-10)(y+2) \geq 0$ (5)

$y \leq -2$ or $y \geq 10$ (5)

$\therefore$ y doesnot take any values between -2 and 10

$\boxed{30}$

c] $f(x) = ax^2 + bx + c$ (say)

$= a\{x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 + \frac{c}{a} - (\frac{b}{2a})^2\}$ (5)

$= a\{(x+\frac{b}{2a})^2 + (\frac{4ac-b^2}{4a^2})\}$

$= a(x+\frac{b}{2a})^2 + (\frac{4ac-b^2}{4a})$ (5)

If $b^2 - 4ac < 0$ & $a > 0$

then $\frac{4ac-b^2}{4a} > 0$ (5)

$a > 0 \Rightarrow a(x+\frac{b}{2a})^2 \geq 0$ (5)

$\therefore$ $a(x+\frac{b}{2a})^2 + \frac{4ac-b^2}{4a} > 0$

$ax^2 + bx + c > 0$ (5)

$\boxed{25}$

$2x^2 + 4x - 22 + m(x+5) > 0$

$2x^2 + (4+m)x - (22-5m) > 0$ (5)

Since the coefficients of x^2 is $2 > 0$, Δ must be negative

ie) $\Delta < 0$ (5)

$(4+m)^2 - 4(2)(5m-22) < 0$ (5)

$m^2 - 32m + 192 < 0$ (5)

$(m-24)(m-8) < 0$ (5)

$8 < m < 24$ (5)

$\boxed{30}$

13] a)

(i) $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$

$=\dfrac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$ (5)

$=\dfrac{2}{1-\cos^2\theta}$

$=\dfrac{2}{\sin^2\theta}$ (5)

$=2\,\mathrm{cosec}^2\theta$ (5) $\boxed{15}$

(ii) $\tan\theta+\cot\theta$

$=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$ (5)

$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ (5)

$=\dfrac{1}{\cos\theta\sin\theta}$ (5)

$=\sec\theta\,\mathrm{cosec}\theta$ (5) $\boxed{20}$

b) $\cos\theta+\sin\theta=a$ —(1)

$\cos2\theta+\sin2\theta=b$ —(2)

$(1)^2\Rightarrow 1+2\sin\theta\cos\theta=a^2$ (5)

$\sin2\theta=a^2-1$ (5)

$(2)\Rightarrow \cos2\theta+a^2-1=b$

$\cos2\theta=b+1-a^2$ (5)

$\cos^2\theta-\sin^2\theta=b+1-a^2$

$(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)^2=(b+1-a^2)^2$ (5)

$(1-\sin2\theta)\,a^2=(b+1-a^2)^2$ (5)

$\{1-(a^2-1)\}a^2=(b+1-a^2)^2$ (5)

$(2-a^2)a^2=(b+1-a^2)^2$ (5)

$\boxed{35}$

c) $\sin(A-B)=\sin A\cos B-\cos A\sin B$ (5)

$\dfrac{\sin5\theta}{\sin\theta}-\dfrac{\cos5\theta}{\cos\theta}$

$=\dfrac{\sin5\theta\cos\theta-\cos5\theta\sin\theta}{\sin\theta\cos\theta}$ (5)

$=\dfrac{\sin4\theta}{\sin\theta\cos\theta}$ (5)

$=\dfrac{2\sin2\theta\cos2\theta}{\frac{1}{2}\times\sin2\theta}$ (5)

$=4\cos2\theta$ (5) $\boxed{25}$

put $\theta=18^\circ$

$\dfrac{\sin90^\circ}{\sin18^\circ}-\dfrac{\cos90^\circ}{\cos18^\circ}=4\cos36^\circ$ (5)

$\dfrac{1}{\sin18^\circ}=4\cos36^\circ$ (5)

$4\sin18^\circ\cos36^\circ=1$ (5)

$4\sin18^\circ\{1-2\sin^2 18^\circ\}=1$ (5)

$8\sin^3 18-4\sin18+1=0$ (5)

$8x^3-4x+1=0$ (5) where $x=\sin18^\circ$

$\therefore \sin18^\circ$ is a root of eqⁿ $8x^3-4x+1=0$ (5)

$\boxed{35}$

$8x^3-4x+1=0$

$(2x-1)(4x^2+2x-1)=0$

$2x-1=0$ or $4x^2+2x-1=0$

$x=\frac{1}{2}$

i.e.) $\sin18^\circ=\frac{1}{2}$ (5)

$\neq$ ($\because \sin18^\circ<\frac{1}{2}$)

$\therefore \sin18^\circ$ must be root of $4x^2+2x-1=0$

$x=\dfrac{-2\pm\sqrt{2^2-4(4)(-1)}}{2\times4}$ (5)

$x=\dfrac{-1\pm\sqrt5}{4}$ (5)

$\sin18^\circ=x=\dfrac{-1+\sqrt5}{4}$ (5) ($\because \sin18^\circ>0$)

$\boxed{20}$

14)

a)

$\sin(A+B) = \sin A\cos B + \cos A \sin B \quad (1)$ 5

$\cos(A+B) = \sin\left\{\frac{\pi}{2} + (A+B)\right\}$ 5

$= \sin\left\{\left(\frac{\pi}{2}+A\right)+B\right\}$

$= \sin\left(\frac{\pi}{2}+A\right)\cos B + \cos\left(\frac{\pi}{2}+A\right)\sin B$ 5

$= \cos A\cos B - \sin A\sin B \quad (2)$ 5

$A = B = \theta$ ஆகுக

$(1) \Rightarrow \sin 2\theta = \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2\sin\theta\cos\theta$ 5

$(2) \Rightarrow \cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - (1-\cos^2\theta)$ 5

$= 2\cos^2\theta - 1$ 5

$\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)\cos\theta - 2\cos\theta(1-\cos^2\theta)$ 5

$= 4\cos^3\theta - 3\cos\theta$ 5 $\boxed{45}$

$\cos 3\theta - 4\cos 2\theta + 2\cos\theta - 2 = 0$

$4\cos^3\theta - 3\cos\theta - 4\{2\cos^2\theta - 1\} + 2\cos\theta - 2 = 0$

$4\cos^3\theta - 8\cos^2\theta - \cos\theta + 2 = 0$ 5

$4\cos^2\theta(\cos\theta - 2) - (\cos\theta - 2) = 0$ 5

$(\cos\theta - 2)(4\cos^2\theta - 1) = 0$ 5

$\cos\theta - 2 = 0$ or $4\cos^2\theta - 1 = 0$

$\cos\theta = 2$ 5 $\quad \cos\theta = \pm\frac{1}{2}$ 5

$\#^n (-1 \le \cos\theta \le 1)$

$\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\theta = 2n_1\pi \pm \frac{\pi}{3} \quad n_1 \in \mathbb{Z}$ 5

$\cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\theta = 2n_2\pi \pm \frac{2\pi}{3}$ 5

$n_2 \in \mathbb{Z}$

$\boxed{35}$

b) $2\sqrt{2}\cos\theta(\cos\theta + \sin\theta) = \sqrt{2} + 1$

$2\cos\theta(\cos\theta + \sin\theta) = 1 + \frac{1}{\sqrt{2}}$

$2\cos^2\theta - 1 + 2\sin\theta\cos\theta = \frac{1}{\sqrt{2}}$ 5

$\cos 2\theta + \sin 2\theta = \frac{1}{\sqrt{2}}$ 5

$\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta = \frac{1}{2}$ 5

$\cos\frac{\pi}{4}\cos 2\theta + \sin\frac{\pi}{4}\sin 2\theta = \frac{1}{2}$ 5

$\cos\left(2\theta - \frac{\pi}{4}\right) = k \qquad k = \frac{1}{2}$ 5

$\cos\left(2\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}$ 5

$2\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3} \quad n \in \mathbb{Z}$

$\theta = \frac{1}{2}\left\{2n\pi + \frac{\pi}{4} \pm \frac{\pi}{3}\right\}$ 5

$\boxed{35}$

c)

$\frac{1 - \cos 2A + \cos 2B - \cos(2A+2B)}{1 + \cos 2A - \cos 2B - \cos(2A+2B)}$

$= \frac{1 - \cos(2A+2B) - \{\cos 2A - \cos 2B\}}{1 - \cos(2A+2B) + \{\cos 2A - \cos 2B\}}$ 5

$= \frac{2\sin^2(A+B) - \{2\sin(A+B)\sin(B-A)\}}{2\sin^2(A+B) + \{2\sin(A+B)\sin(B-A)\}}$ 10

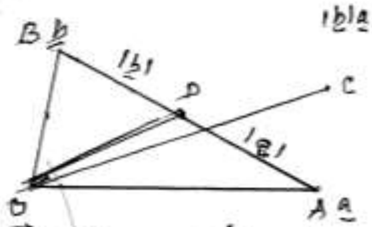
$= \frac{\sin(A+B) - \sin(B-A)}{\sin(A+B) + \sin(B-A)}$ 5 $(\sin(A+B) \neq 0)$

$= \frac{2\cos B\sin A}{2\sin B\cos A}$ 5 5

$= \cot B\tan A$ 5

$\boxed{35}$

(15) (a)



$\vec{OA} = \underline{a}$ $\quad \vec{OB} = \underline{b}$

$\vec{OC} = |\underline{b}|\underline{a} + |\underline{a}|\underline{b}$

$\vec{OD} = \vec{OA} + \vec{AD}$ (10)

$= \underline{a} + \frac{|\underline{a}|}{|\underline{a}|+|\underline{b}|}\vec{AB}$ (10)

$= \underline{a} + \frac{|\underline{a}|}{|\underline{a}|+|\underline{b}|}(\vec{AO}+\vec{OB})$ (10)

$= \underline{a} + \frac{|\underline{a}|}{|\underline{a}|+|\underline{b}|}(-\underline{a}+\underline{b})$

$= \frac{|\underline{b}|\underline{a}+|\underline{a}|\underline{b}}{|\underline{a}|+|\underline{b}|}$ (10)

$[|\underline{a}|+|\underline{b}|]\vec{OD} = |\underline{b}|\underline{a}+|\underline{a}|\underline{b}$ (10)

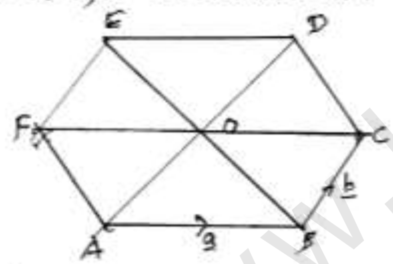
$= \vec{OC}$ (10)

$\lambda \vec{OD} = \vec{OC}$

$\Rightarrow$ O, D, C are Collinear. (10)

$\boxed{70}$

b)



I $\quad AO = OD = BC$ (10)

$AOD \parallel BC$

$\Rightarrow \vec{AD} = 2\vec{BC} = 2\underline{b}$ (10)

II $\quad \vec{AC} = \vec{AB} + \vec{BC}$

$= \underline{a} + \underline{b}$ (10)

$\vec{FO} = \vec{AB} = \underline{a}$

$\vec{AF} = \vec{AO} + \vec{OF}$

$= \underline{b} - \underline{a}$ (10)

$BO = OE = AF$ + $BOE \parallel \vec{AF}$

$\vec{BE} = \vec{BO} + \vec{OE}$

$= (\underline{b}-\underline{a}) + (\underline{b}-\underline{a})$

$= 2(\underline{b}-\underline{a})$ (10)

iii $\quad \underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}| \cos 60$

$= |\underline{a}||\underline{b}| \frac{1}{2}$ (5)

$\vec{AB}\cdot\vec{AF} = AB.AF \cos 120$

$\underline{a}\cdot(\underline{b}-\underline{a}) = |\underline{a}||\underline{b}-\underline{a}|(-\frac{1}{2})$ (5)

$|\underline{a}|^2 - \underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}-\underline{a}|\frac{1}{2}$

$2\left[|\underline{a}|^2 - \frac{|\underline{a}||\underline{b}|}{2}\right] = |\underline{a}||\underline{b}-\underline{a}|$

$2|\underline{a}| - |\underline{b}| = |\underline{b}-\underline{a}|$

$[2|\underline{a}| - |\underline{b}|]^2 = |\underline{b}-\underline{a}|^2$

$= (\underline{b}-\underline{a})\cdot(\underline{b}-\underline{a})$ (10)

$4|\underline{a}|^2 + |\underline{b}|^2 - 4|\underline{a}||\underline{b}| = |\underline{b}|^2 + |\underline{a}|^2 - 2\underline{a}\cdot\underline{b}$

$4|\underline{a}|^2 - 4|\underline{a}||\underline{b}| = |\underline{a}|^2 - |\underline{a}||\underline{b}|$

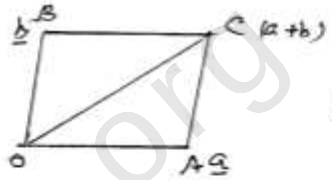
$3[|\underline{a}|^2 - |\underline{a}||\underline{b}|] = 0$

$3|\underline{a}|[|\underline{a}| - |\underline{b}|] = 0$ (10)

$|\underline{a}| \neq 0 \Rightarrow |\underline{a}| - |\underline{b}| = 0$

$|\underline{a}| = |\underline{b}|$ $\boxed{80}$

(16) a)



(10)

i $\quad \vec{AC} = \vec{AO} + \vec{OC}$

$= -\underline{a} + \underline{a} + \underline{b}$

$= \underline{b}$ (10)

$\vec{BC} = \vec{BO} + \vec{OC}$

$= -\underline{b} + \underline{a} + \underline{b}$

$= \underline{a}$ (10)

$\vec{AB} = \vec{AO} + \vec{OB}$

$= -\underline{a} + \underline{b}$ (10)

ii $\quad AB = OC$

$|\underline{b}-\underline{a}| = |\underline{a}+\underline{b}|$ (10)

$|\underline{b}-\underline{a}|^2 = |\underline{a}+\underline{b}|^2$

$(\underline{b}-\underline{a})\cdot(\underline{b}-\underline{a}) = (\underline{a}+\underline{b})\cdot(\underline{a}+\underline{b})$ (10)

$|\underline{b}|^2 + |\underline{a}|^2 - 2\underline{a}\cdot\underline{b} = |\underline{a}|^2 + |\underline{b}|^2 + 2\underline{a}\cdot\underline{b}$

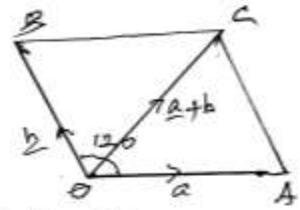
$\Rightarrow 4\underline{a}\cdot\underline{b} = 0$ (10)

$\underline{a}\cdot\underline{b} = 0$

$\underline{a} \perp \underline{b}$ (10)

$OA \perp AC$. $\boxed{80}$

b)



(10)

$|\underline{a}+\underline{b}| = |\underline{a}|$

$\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}| \cos 120$

$= -|\underline{a}||\underline{b}| \frac{1}{2}$ (10)

$|\underline{a}+\underline{b}|^2 = |\underline{a}|^2$ (10)

$(\underline{a}+\underline{b}).(\underline{a}+\underline{b}) = \underline{a}.\underline{a}$ (10)

$|\underline{a}|^2+|\underline{b}|^2+2\underline{a}.\underline{b} = |\underline{a}|^2$ (10)

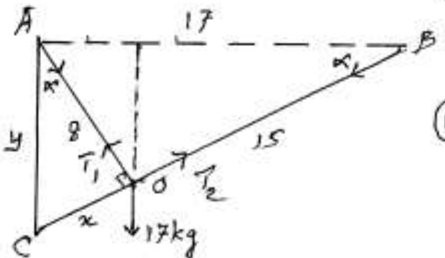
$|\underline{b}|^2 - |\underline{a}||\underline{b}| = 0$

$|\underline{b}|\,[|\underline{b}| - |\underline{a}|] = 0$ (10)

$|\underline{b}| \neq 0 \Rightarrow |\underline{b}| - |\underline{a}| = 0$ (10)

$|\underline{b}| = |\underline{a}|$ $\boxed{70}$

(17) a)



(10)

$8^2+15^2 = 17^2$ (10)

$\Rightarrow A\hat{O}B = \frac{\pi}{2}$ (10)

$\tan\alpha = \frac{8}{17}$

$CO = x = 8\tan\alpha = 8.\frac{8}{15} = \frac{64}{15}$ (10)

$AC = y = 17\tan\alpha = 17.\frac{8}{15} = 8.\frac{17}{15}$ (10)

$T_1 \parallel OA,\ 17 \parallel AC,\ T_2 \parallel CO$ (10)

$\Rightarrow$ ACO is a triangle of force

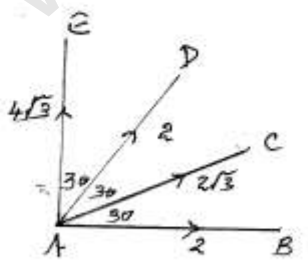
$\frac{T_1}{AO} = \frac{17}{AC} = \frac{T_2}{CO}$ (10)

$\frac{T_1}{8} = \frac{17}{8.\frac{17}{15}} = \frac{T_2}{64/15}$ (10)

$T_1 = 15,\quad T_2 = 8$ (10)

$\boxed{90}$

b)



(10)

$\overset{AB}{\longrightarrow}$ $X = 2 + 2\sqrt{3}\cos 30 + 2\cos 60$ (10)

$= 2+3+1$

$= 6$ (5)

$AE \uparrow$ $Y = 4\sqrt{3} + 2\cos 30 + 2\sqrt{3}\cos 60$ (10)

$= 4\sqrt{3} + \sqrt{3} + \sqrt{3}$

$= 6\sqrt{3}$ (5)

$R^2 = X^2 + Y^2$

$= 6^2 + (6\sqrt{3})^2$

$R = 12$ (10)

$\tan\theta = \frac{6\sqrt{3}}{6}$

$= \sqrt{3}$

$\theta = 60^\circ$ (10)

Resultant acts along $\overrightarrow{AD}$. $\boxed{60}$