தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு
மூன்றாம் தவணைப் பரீட்சை – 2023
National Field Work Centre, Thondaimanaru.
3rd Term Examination - 2023

Grade 12(2023) **Combined Mathematics** **Marking Scheme**

1.

$$\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \quad (5)$$

$3x = A(x+2) + B(x-1)$

Comparing coefficients of powers of x;

$x^1 : \ 3 = A+B$, $x^0 : \ 0 = 2A-B$ $\Big\}$ $A=1$, $B=2$ (5)

$$\frac{3x}{(x-1)(x+2)} = \frac{1}{x-1} + \frac{2}{x+2} \quad (5)$$

Replacing x by $\frac{1}{x}$, we get

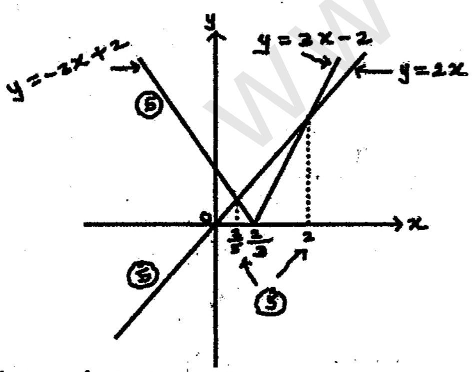
$$\frac{\frac{3}{x}}{(\frac{1}{x}-1)(\frac{1}{x}+2)} = \frac{1}{\frac{1}{x}-1} + \frac{2}{\frac{1}{x}+2} \quad (5)$$

$$\frac{3x}{(1-x)(1+2x)} = \frac{x}{1-x} + \frac{2x}{1+2x}$$

$$\frac{3}{(x-1)(2x+1)} = \frac{1}{x-1} - \frac{2}{2x+1} \quad (5)$$

$\boxed{25}$

2.



$3|x-2| \le 2x$

$\Leftrightarrow 3|3u-2| \le 2(3u)$, where $u = \frac{x}{3}$ (5)

$\Leftrightarrow |3u-2| \le 2u$

$\Leftrightarrow \frac{2}{5} \le u \le 2$ [∵ From the graphs]

$\Leftrightarrow \frac{2}{5} \le \frac{x}{3} \le 2$

$\Leftrightarrow \frac{6}{5} \le x \le 6$ (5)

$\boxed{25}$

3.

$$\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{3}-\sqrt{2+\sin x}}{(\frac{\pi}{2}-x)\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{3-(2+\sin x)}{(\frac{\pi}{2}-x)\cos x(\sqrt{3}+\sqrt{2+\sin x})} \quad (5)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{(\frac{\pi}{2}-x)\cos x(\sqrt{3}+\sqrt{2+\sin x})(1+\sin x)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos^{\cancel{2}} x}{(\frac{\pi}{2}-x)\cancel{\cos x}(\sqrt{3}+\sqrt{2+\sin x})(1+\sin x)}$$

$$= \lim_{(\frac{\pi}{2}-x)\to 0} \frac{\sin(\frac{\pi}{2}-x)}{\frac{\pi}{2}-x} \lim_{x\to\frac{\pi}{2}} \frac{1}{(\sqrt{3}+\sqrt{2+\sin x})(1+\sin x)} \quad (5)$$

$$= 1 \times \frac{1}{(\sqrt{3}+\sqrt{3})2} = \frac{1}{4\sqrt{3}} \quad (5)$$

(5) (5)

$\boxed{25}$

4. $x = 4\sin 2\theta$ $\qquad$ $y = 2+\cos 4\theta$

$\frac{dx}{d\theta} = 4\cos 2\theta \cdot 2$ $\qquad$ $\frac{dy}{d\theta} = -\sin 4\theta \cdot 4$

$= 8\cos 2\theta$ $\qquad$ $= -4\sin 4\theta$ (5)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-4\sin 4\theta}{8\cos 2\theta}$$

$$= -\frac{2\sin 2\theta\cos 2\theta}{2\cos 2\theta}$$

$$= -\sin 2\theta \quad (5)$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{8}} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \quad (5)$$

When $\theta = \frac{\pi}{8}$, $x = 2\sqrt{2}$ and $y = 2$ (5)

The equation of tangent at $(2\sqrt{2}, 2)$ is

$$y-2 = -\frac{1}{\sqrt{2}}(x-2\sqrt{2})$$

$$x+\sqrt{2}y-4\sqrt{2}=0 \quad (5)$$

$\boxed{25}$

5. $(2\sin\theta-\cos\theta)(1+\cos\theta) = \sin^2\theta$

$(2\sin\theta-\cos\theta)(1+\cos\theta) - (1-\cos^2\theta) = 0$ (5)

$(1+\cos\theta)(2\sin\theta-\cos\theta-1+\cos\theta) = 0$ (5)

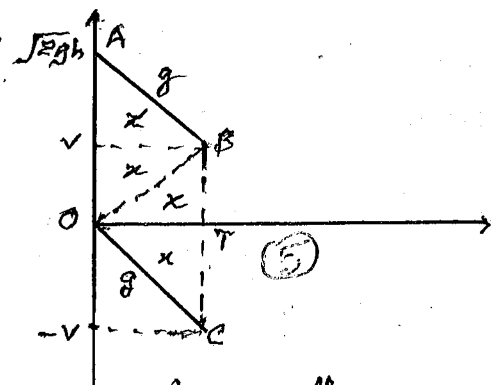
$(1+\cos\theta)(2\sin\theta-1) = 0$ (5)

$\cos\theta = -1$ or $\sin\theta = \frac{1}{2}$ (5)

$\theta = \pi$ or $\theta = \pi/6$ or $\theta = \frac{5\pi}{6}$ [∵ $0 \le \theta \le \pi$] (5)

$\boxed{25}$

6.



OABC is parallelogram

$2v = \sqrt{2gh}$ (5)

$v = \sqrt{\frac{gh}{2}}$

$T = \frac{v}{g}$

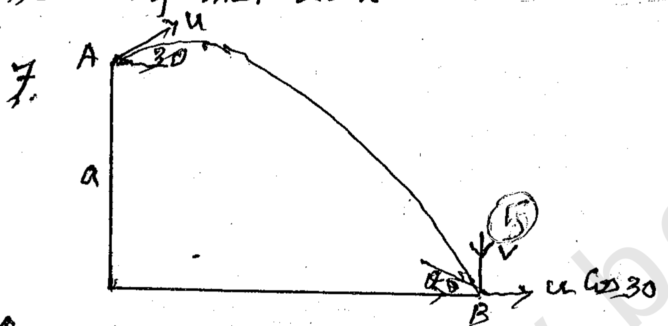
$= \sqrt{\frac{h}{2g}}$ (5)

(5) $4x = \sqrt{2gh}\cdot T$

$= h$

Hence they met each other (5) $\boxed{25}$

7.



↑ A→B $v^2 = u^2 + 2as$

$v^2 = (u\sin 30)^2 + 2ga$

$= \frac{u^2}{4} + 2ag$ —— ① (5)

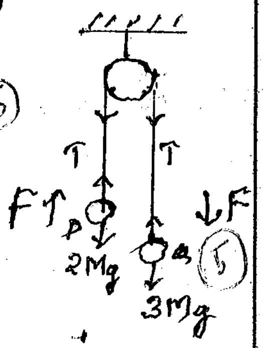
$3ag = v^2 + (u\cos 30)^2$

$= v^2 + \frac{3u^2}{4}$ —— ② (5)

①+② ⇒ $u = \sqrt{ag}$ (5) $\boxed{25}$

$\tan\theta = \frac{\sqrt{3}}{2}$ (5)

8.



$T = 24mg$

↓ $3Mg - 24mg = 3MF$ (5)

↑ $24mg - 2Mg = 2MF$ (5)

$F = \frac{g}{5}$ (5)

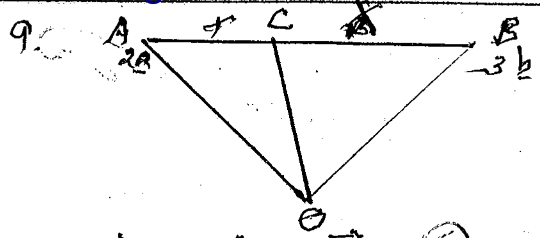
$3Mg - 24mg = 3M\frac{g}{5}$

$M = 10m$

Mass of P is 20m and Q is 30m

(5) $\boxed{25}$

9.



$\vec{OC} = \vec{OA} + \vec{AC}$ (5)

$= 2a + \lambda\vec{AB}$

$= 2a + \lambda(3b - 2a)$

$= (1-\lambda)2a + \lambda 3b$

Let $\lambda = \alpha$

$c = 3\alpha b + (1-\alpha)2a$ (5)

$\vec{OC} \perp \vec{AB}$

16. $\vec{OC}\cdot\vec{AB} = 0$ (5)

$[3\alpha b + (1-\alpha)2a]\cdot[3b - 2a] = 0$

$9\alpha|b|^2 - 4(1-\alpha)|a|^2 + [6(1-\alpha) - 6\alpha]a\cdot b = 0$

$9\alpha 2^2 - 4(1-\alpha).3^2 + 6(1-2\alpha).3.2\cos\alpha = 0$

$\alpha - 1 + \alpha + (1-2\alpha)\cos\alpha = 0$

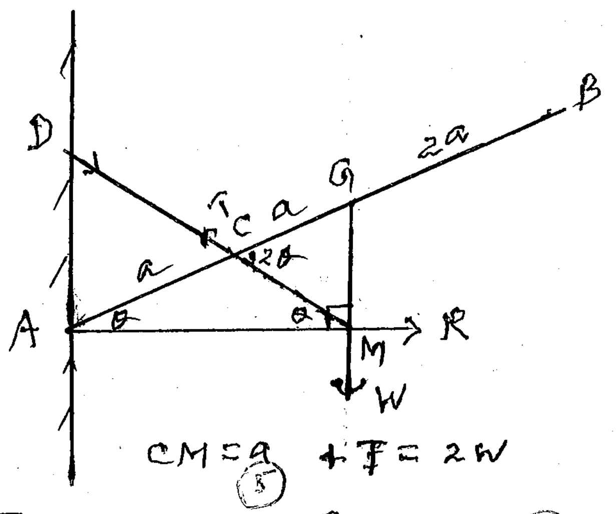
$2\alpha - 1 - (2\alpha - 1)\cos\alpha = 0$

$(2\alpha - 1)(1 - \cos\alpha) = 0$

$\cos\alpha \neq 1 \Rightarrow \alpha = \frac{1}{2}$ (5)

$c = \frac{3}{2}b + a$ (5) $\boxed{25}$

10.



$CM = a$ (5) $T = 2W$

$\frac{T}{\sin 90} = \frac{W}{\sin(180-\theta)} = \frac{R}{\sin(90+2\theta)}$ (5)

$2W = \frac{W}{\sin\theta} = \frac{R}{\cos\theta}$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ (5)

$R = \sqrt{3}W$ (5)

$A\hat{C}D = 60^\circ = D\hat{A}C$

$\therefore DC = a$ (5) $\boxed{25}$

11.(a) $f(x) = x^2+(2k+1)x+(2k-5)=0$

Discriminant Δ

$=(2k+1)^2-4(2k-5)$ (5)

$=4k^2+4k+1-8k+20$

$=4k^2-4k+1+20$

$=(2k-1)^2+20>0$ $[\because (2k-1)^2\geq 0]$ (5) (5)

$\therefore$ The equation $f(x)=0$ has two real distinct roots. (5)

$\boxed{20}$

$f(x)=x^2+(2k+1)x+(2k-5)=0$

$\alpha+\beta=-(2k+1)$ (5)

$\alpha\beta=2k-5$ (5)

$\alpha<0$ and $\beta<0$

$\Leftrightarrow \alpha+\beta<0$ and $\alpha\beta>0$ (5)

$\Leftrightarrow -(2k+1)<0$ and $2k-5>0$ (5)

$\Leftrightarrow k>-\frac{1}{2}$ and $k>\frac{5}{2}$ (5)

$\Leftrightarrow k>\frac{5}{2}$ (5)

$\boxed{30}$

$\left(\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}\right)^2=\frac{\alpha}{\beta}+2+\frac{\beta}{\alpha}$ (5)

$=\frac{\alpha^2+2\alpha\beta+\beta^2}{\alpha\beta}$

$=\frac{(\alpha+\beta)^2}{\alpha\beta}$ (5)

$=\frac{(2k+1)^2}{2k-5}$ (5)

$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}=\frac{2k+1}{\sqrt{2k-5}}$ $[\because k>3]$ (5)

$\sqrt{\frac{\alpha}{\beta}}\cdot\sqrt{\frac{\beta}{\alpha}}=1$ (5)

The equation whose roots are $\sqrt{\frac{\alpha}{\beta}}$ and $\sqrt{\frac{\beta}{\alpha}}$

is $\left(x-\sqrt{\frac{\alpha}{\beta}}\right)\left(x-\sqrt{\beta/\alpha}\right)=0$ (5)

ie, $x^2-\left(\sqrt{\frac{\alpha}{\beta}}+\sqrt{\beta/\alpha}\right)x+\sqrt{\frac{\alpha}{\beta}}\sqrt{\beta/\alpha}=0$

$x^2-\left(\frac{2k+1}{\sqrt{2k-5}}\right)x+1=0$ (5)

ie, $\sqrt{2k-5}\,x^2-(2k+1)x+\sqrt{2k-5}=0$ —— (*)

$\boxed{35}$

Since the roots of $f\left(-\frac{2}{x}\right)=0$ are γ and δ

$\alpha=-\frac{2}{\gamma}$ and $\beta=-\frac{2}{\delta}$ (say)

$\sqrt{\frac{2\gamma}{\delta}}=\sqrt{2}\sqrt{\beta/\alpha}$ and $\sqrt{\frac{2\delta}{\gamma}}=\sqrt{2}\sqrt{\alpha/\beta}$ (5)

Let $y=\sqrt{2}x$.

Putting $x=\frac{y}{\sqrt{2}}$ in the equation (*) (5)

$\sqrt{2k-5}\,\frac{y^2}{2}-(2k+1)\frac{y}{\sqrt{2}}+\sqrt{2k-5}=0$ (5)

$\sqrt{2k-5}\,y^2-\sqrt{2}(2k+1)y+2\sqrt{2k-5}=0$ $\boxed{15}$

(b) $f(x)=3x^3+ax^2+x+b$

$g(x)=x^3+cx^2+ax+1$

Since, the remainder when $f(x)$ is divided by $x+1$ is 1

$f(-1)=1 \Rightarrow -3+a-1+b=1$ (5)

$\Rightarrow a+b=5$ —— (1)

Since, the remainder when $g(x)$ is divided by x^2+x-2 is $2x+5$, we have

$g(x)=x^3+cx^2+ax+1=(x+2)(x-1)\phi(x)+2x+5$ (5)

$g(1)=1+c+a+1=7$ (5)

$a+c=5$ —— (2)

$g(-2)=-8+4c-2a+1=1$ (5)

$a-2c=-4$ —— (3)

(1), (2), (3) $\Rightarrow a=2,\ b=3,\ c=3$

(5) (5) (5) $\boxed{35}$

$3g(x)-f(x)$

$=3(x^3+3x^2+2x+1)-(3x^3+2x^2+x+3)$

$=7x^2+5x$ (5)

$=7\left(x^2+\frac{5}{7}x\right)$

$=7\left\{\left(x+\frac{5}{14}\right)^2-\frac{25}{196}\right\}$ (5)

$=7\left(x+\frac{5}{14}\right)^2-\frac{25}{28}\geq-\frac{25}{28}$ $\left[\because \left(x+\frac{5}{14}\right)^2\geq 0\right]$ (5)

$\boxed{15}$

12.(a) $f(x)=\frac{(x+2)(x-1)}{x^2}$

$f'(x)=\frac{x^2(2x+1)-(x^2+x-2)2x}{x^4}$ (10)

$=\frac{2x^3+x^2-2x^3-2x^2+4x}{x^4}$

$=\frac{-x^2+4x}{x^4}$ (5)

$=\frac{4-x}{x^3}$ (5)

$\boxed{20}$

$f'(x)=-\frac{x-4}{x^3}$

$f'(x)=0 \Leftrightarrow x=4$ (5)

	$-\infty<x<0$	$0<x<4$	$4<x<\infty$
Sign of $f'(x)$	$(-)$	$(+)$	$(-)$
$f(x)$ is	Decreasing	Increasing	Decreasing
	(5)	(5)	(5)

Turning point: $\left(4,\frac{9}{8}\right)$ is a local maximum. (5)

$f(x)$ is increasing on $(0,4]$ and decreasing on $(-\infty,0)$ and $[4,\infty)$. (5)

$\boxed{30}$

$f''(x)=\frac{2(x-6)}{x^4}$

$f''(x)=0 \Leftrightarrow x=6$ (5)

	$-\infty<x<0$	$0<x<6$	$6<x<\infty$
Sign of $f''(x)$	$(-)$	$(-)$	$(+)$
concavity	concave down	concave down	concave up
		(5)	(5)

$\left(6,\frac{10}{6}\right)$ is a inflection point. (5)

$\boxed{20}$

When $y=0 \Rightarrow x=1$ or $x=-2$ (5)

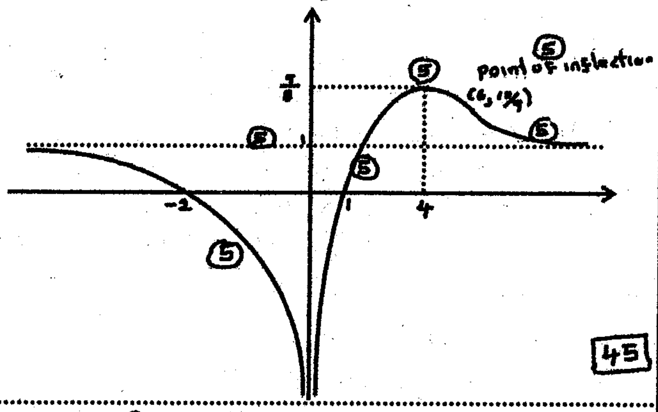
$(1,0), (-2,0)$ (5)

$\lim_{x\to 0} \frac{(x+2)(x-1)}{x^2} = -\infty$

$\therefore x=0$ is a vertical asymptote. (5)

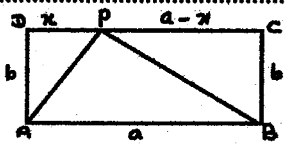
$\lim_{x\to\pm\infty} \frac{(x+2)(x-1)}{x^2} = \lim_{x\to\pm\infty} (1+\frac{2}{x})(1-\frac{1}{x}) = 1$

$y=1$ is a horizontal asymptote. (5)



$\boxed{45}$

(b)



Let $DP = x$ and $L(x) = PA^2 + PB^2$

$L(x) = PA^2 + PB^2$

$= b^2 + x^2 + (a-x)^2 + b^2$ (5)

$= 2b^2 + x^2 + (a-x)^2$ (5)

$L'(x) = 2x + 2(a-x)(-1) = 4(x - \frac{a}{2})$ (5)

$L'(x) = 0 \Leftrightarrow x = \frac{a}{2}$ (5)

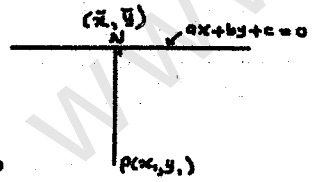
For $0 \le x < \frac{a}{2}$, $L'(x) < 0$ (5) and for $\frac{a}{2} < x \le a$, $L'(x) > 0$ (5)

$\therefore L(x)$ is minimum when $x = \frac{a}{2}$ (5)

$\therefore$ P must be mid point of CD.

$\boxed{35}$

13.



If $a \ne 0$ and $b \ne 0$

$\frac{\bar{y} - y_1}{\bar{x} - x_1} \times -\frac{a}{b} = -1$ (5)

$\frac{\bar{y} - y_1}{b} = \frac{\bar{x} - x_1}{a} = \lambda$ (say) (5)

$\bar{x} = x_1 + a\lambda$, $\bar{y} = y_1 + b\lambda$ (5)

This result is also true when $a=0$ and $b \ne 0$ or when $a \ne 0$ and $b=0$.

Since $N(\bar{x}, \bar{y})$ lies on $ax+by+c=0$, we have

$a(x_1 + a\lambda) + b(y_1 + b\lambda) + c = 0$ (5)

$\lambda = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$ (5)

Perpendicular distance $PN = \sqrt{(\bar{x} - x_1)^2 + (\bar{y} - y_1)^2}$ (5)

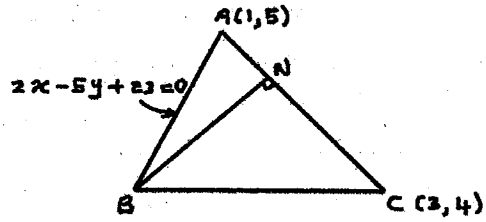
$= \sqrt{a^2\lambda^2 + b^2\lambda^2}$ (5)

$= |\lambda| \sqrt{a^2 + b^2}$ (5)

$PN = \frac{|ax_1 + by_1 + c|}{a^2 + b^2} \sqrt{a^2 + b^2}$ (5)

$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ (5)

$\boxed{50}$



$AC = \sqrt{2^2 + 1^2} = \sqrt{5}$ (5) (5)

$m_{AC} = \frac{5-4}{1-3} = -\frac{1}{2}$ (5)

Equation of AC is $y - 5 = -\frac{1}{2}(x-1)$ (5)

$x + 2y - 11 = 0$ (5)

$\boxed{25}$

$2x - 5y + 23 = 0$

$2(x-1) - 5(y-5) = 0$ (5)

$\frac{x-1}{5} = \frac{y-5}{2} = t$ (say) (5)

$x = 5t+1$, $y = 2t+5$

$(5t+1, 2t+5)$ (5)

$\boxed{15}$

$\frac{1}{2} \times AC \times BN = 4\frac{1}{2}$

$\frac{1}{2} \times \sqrt{5} \times BN = \frac{9}{2}$ (5)

$BN = \frac{9}{\sqrt{5}}$ (5)

Let $B \equiv (5t_0 + 1, 2t_0 + 5)$

$BN = \frac{|5t_0 + 1 + 2(2t_0 + 5) - 11|}{\sqrt{5}} = \frac{9}{\sqrt{5}}$ (10)

$|9t_0| = 9$

$|t_0| = 1$ (5)

$t_0 = \pm 1$ (5)

$t_0 = 1 \Rightarrow B \equiv (6, 7)$ (5)

$t_0 = -1 \Rightarrow B \equiv (-4, 3)$ (5)

Since B lies in the first quadrant

$\therefore B \equiv (6, 7)$ (5)

$\boxed{45}$

$m_{BC} = \frac{7-4}{6-3} = 1$ (5)

Equation of BC is $y - 7 = 1(x - 6)$ (5)

$x - y + 1 = 0$ (5)

$\boxed{15}$

14.

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ (5)

$\cos(A+B)$

$= \sin\left[\frac{\pi}{2}+(A+B)\right]$ (5)

$= \sin\left[\left(\frac{\pi}{2}+A\right)+B\right]$

$= \sin\left(\frac{\pi}{2}+A\right)\cos B + \cos\left(\frac{\pi}{2}+A\right)\sin B$

$= \cos A\cos B - \sin A\sin B$ (5) $\boxed{15}$

Putting $A=B=\theta$, we get

$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$ (5)

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$ (5)

$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$

$= 2\cos^2\theta - 1$ (5)

$\cos 3\theta = \cos(2\theta+\theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ (5)

$= (2\cos^2\theta-1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ (5)

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$ (5)

$= 4\cos^3\theta - 3\cos\theta$ (5) $\boxed{35}$

Putting $\theta = \frac{\pi}{12}$, we get (5)

$\cos\frac{3\pi}{12} = 4\cos^3\frac{\pi}{12} - 3\cos\frac{\pi}{12}$

$\frac{1}{\sqrt{2}} = 4\cos^3\frac{\pi}{12} - 3\cos\frac{\pi}{12}$ (5)

$4\sqrt{2}\cos^3\frac{\pi}{12} - 3\sqrt{2}\cos\frac{\pi}{12} - 1 = 0$

$\therefore \cos\frac{\pi}{12}$ is a root of $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$ (5)

$(\sqrt{2}x+1)(4x^2-2\sqrt{2}x-1)$

$= 4\sqrt{2}x^3 - 4x^2 - \sqrt{2}x + 4x^2 - 2\sqrt{2}x - 1$ (5)

$= 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$ (5)

$4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$

$\Rightarrow (\sqrt{2}x+1)(4x^2-2\sqrt{2}x-1) = 0$

$\Rightarrow \sqrt{2}x+1 = 0$ or $4x^2-2\sqrt{2}x-1 = 0$ (5)

$\Rightarrow x = -\frac{1}{\sqrt{2}}$ or $x = \frac{2\sqrt{2} \pm \sqrt{8-4(4)(-1)}}{2(4)}$

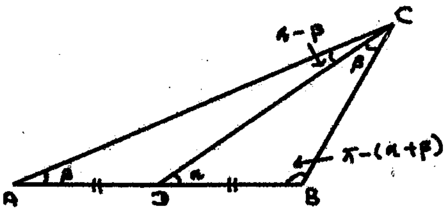
$\Rightarrow x = -\frac{1}{\sqrt{2}}$ or $x = \frac{\sqrt{2}\pm\sqrt{6}}{4}$ (5)

Since $\cos\frac{\pi}{12} > 0$

$\therefore \cos\frac{\pi}{12} = \frac{\sqrt{2}+\sqrt{6}}{4}$ (5)

$\boxed{40}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (5) $\boxed{05}$



using the Sine Rule

Triangle ACD : $\frac{CD}{\sin\beta} = \frac{AD}{\sin(\alpha-\beta)}$ (5) —— (1)

Triangle BCD : $\frac{CD}{\sin(\pi-(\alpha+\beta))} = \frac{BD}{\sin\beta}$ (5)

$\frac{CD}{\sin(\alpha+\beta)} = \frac{BD}{\sin\beta}$ —— (2) (5)

$\frac{(1)}{(2)} \Rightarrow \frac{\sin(\alpha+\beta)}{\sin\beta} = \frac{\sin\beta}{\sin(\alpha-\beta)}$ (5)

$\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\beta$

$(\sin\alpha\cos\beta + \cos\alpha\sin\beta)(\sin\alpha\cos\beta - \cos\alpha\sin\beta) = \sin^2\beta$ (5)

$\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \sin^2\beta$

$(1+\cos^2\alpha)\sin^2\beta = \sin^2\alpha\cos^2\beta$ (5)

$\tan^2\beta = \frac{\sin^2\alpha}{1+\cos^2\alpha}$ (5) $\boxed{35}$

(c) $\sin^{-1}\left(\frac{e^x}{5e^x-6}\right) + \cos^{-1}\left(\frac{1}{e^x}\right) = \frac{\pi}{2}$

Let $\alpha = \sin^{-1}\left(\frac{e^x}{5e^x-6}\right)$, $\beta = \cos^{-1}\left(\frac{1}{e^x}\right)$

$\alpha + \beta = \frac{\pi}{2}$

$\alpha = \frac{\pi}{2} - \beta$

$\sin\alpha = \sin\left(\frac{\pi}{2}-\beta\right) = \cos\beta$ (5)

$\frac{e^x}{5e^x-6} = \frac{1}{e^x}$ (5)

$(e^x)^2 - 5e^x + 6 = 0$ (5)

$(e^x-3)(e^x-2) = 0$ (5)

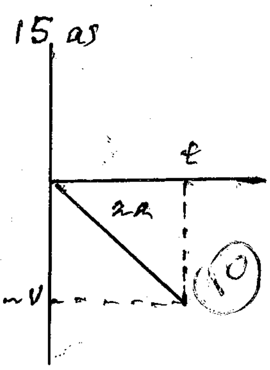
$e^x = 3$ or $e^x = 2$

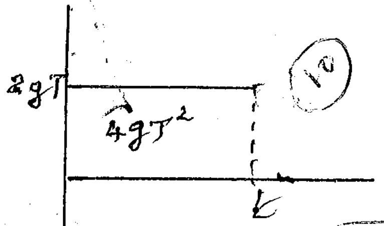
$x = \ln 3$ or $x = \ln 2$

(5) (5)

$\boxed{30}$

15. a)





(10)

[20]

$2gT \cdot t = 4gT^2$ (10)

$t = 2T$ (5)

[15]

$\frac{1}{2} \cdot t \cdot V = 2a$ (5)

$V = \frac{2a}{T}$

$\frac{V}{t} = g$ (5)

$a = gT^2$ (5)

[15]

velocity ω

$\omega^2 = V^2 + (2gT)^2$ (10)

(5) $= (2gT)^2 + (2gT)^2$

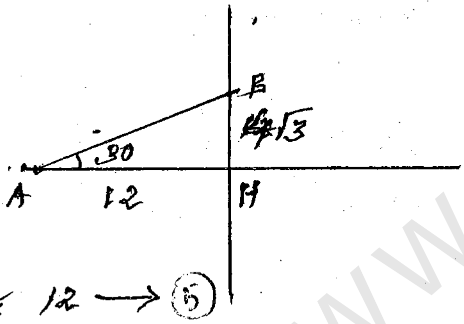
$\omega = 2\sqrt{2}\,gT$ (5)

$\tan^{-1}\sqrt{2}$ with horizontal (5)

[25]

70

b) In the frame of earth

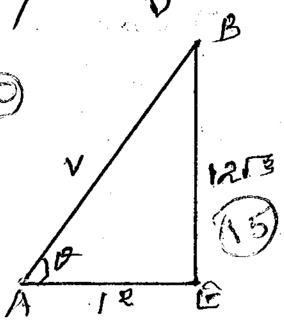


V_{AE} $12 \rightarrow$ (5)

V_{BE} $12\sqrt{3}$ $\uparrow$ (5)

$V_{AB} = V_{AE} + V_{EB}$ (10)

Relative Velocity Triangle



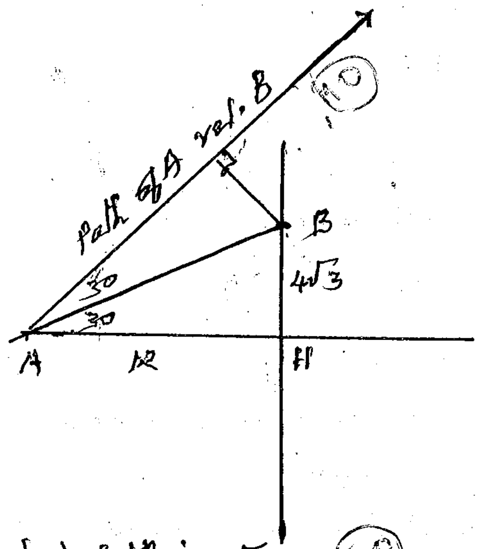
$V^2 = 12^2 + (12\sqrt{3})^2$ (10)

$V = 24$ (5)

(15)

[50]

In the frame of B



(10)

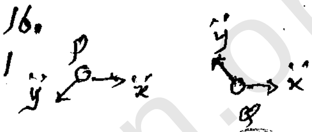
Shortest distⁿ is $4\sqrt{3}$ km (10)

time $\frac{12}{24}h = \frac{1}{2}h$ (5)

[25]

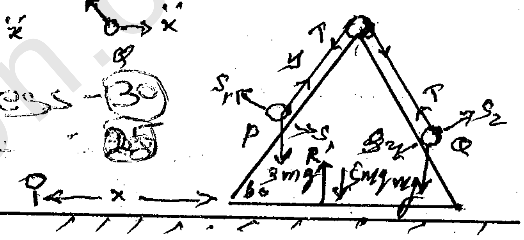
75

16.



Forces − 30

Acc − 25



for P ↙ $3mg\sin 60 - T = 3m(\ddot{y} - \ddot{x}\cos 60)$ (10)

Q ↖ $T - mg\sin 60 = m(\ddot{y} - \ddot{x}\cos 60)$ (10)

Wedge, P, Q → $0 = 5m\ddot{x} + 3m[\ddot{x} - \ddot{y}\cos 60] + m(\ddot{x} - \ddot{y}\cos 60)$ (5)

$\ddot{x} = \frac{2}{9}\ddot{y}$ (5)

$\ddot{x} = \frac{\sqrt{3}}{16}g$ (5)

$\ddot{y} = \frac{9\sqrt{3}}{32}g$ (5)

Let accⁿ of P $\frac{a}{?}$ a

$a^2 = [\ddot{x} - \ddot{y}\cos 60]^2 + (\ddot{y}\sin 60)^2$ (5)

$= \frac{268}{16}\ddot{x}^2$ (5)

$a = \frac{\sqrt{201}}{32}g$

↑ for the system

$R - 9mg = m\ddot{y}\sin 60 - 3m\ddot{y}\sin 60$ (10)

$R = \frac{261}{32}mg$ (5)

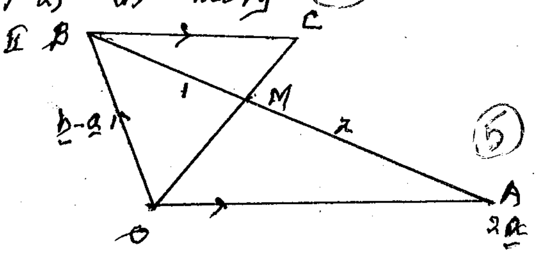
for P ↖ $S_1 - 3mg\cos 60 = 3m(-\ddot{x}\sin 60)$ (10)

for Q ↗ $S_2 - mg\cos 60 = m(\ddot{x}\sin 60)$ (10)

$\boxed{150}$

17 a) as Theory (10)

II



(5)

I $\vec{AB} = \vec{AO} + \vec{OB}$

$= -2a + b - a$

$= b - 3a$ (5)

II $\vec{MB} = \frac{1}{3}\vec{AB}$

$= \frac{1}{3}(b - 3a)$ (5)

III $\vec{OM} = \vec{OB} + \vec{BM}$ (5)

$= b - a + \frac{1}{3}(b - 3a)$

$= \frac{2}{3}b$ (5)

IV $\vec{BC} = \mu a + 0b$ (5)

V $\vec{OM} = \lambda\vec{OC}$

$= \lambda[\vec{OB} + \vec{BC}]$

$= \lambda[b - a + \mu a]$ (5)

VI $\frac{2}{3}b = \lambda[b - a + \mu a]$ (5)

$\Rightarrow \lambda(\mu - 1)a + (\lambda - \frac{2}{3})b = 0$

$\lambda(\mu - 1) = 0$ + $\lambda - \frac{2}{3} = 0$

$\mu = 1$ (5) $\lambda = \frac{2}{3}$ (5)

VII $\vec{OC} = \frac{1}{\lambda}\vec{OM}$

$= \frac{1}{\lambda}\cdot\frac{2}{3}b$

$= b$ (5)

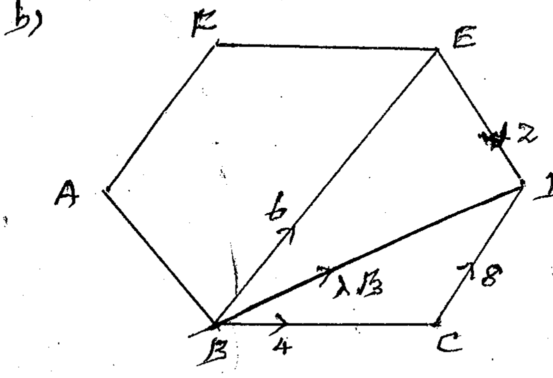
$\vec{BC} = a$ (5)

$\frac{OM}{OC} = \frac{2}{3}$

$OM : OC = 2 : 1$ (5)

$\boxed{75}$

b)



(5)

$\rightarrow$ AD

$X = 4 + 8\cos 60 + 2\cos 60 + 6\cos 60 - \lambda\sqrt{3}\cos 30$ (5)

$= 12 - \lambda\frac{3}{2}$ (5)

$\uparrow$ BF

$Y = 6\sin 60 + 8\sin 60 - 2\sin 60 - \lambda\sqrt{3}\sin 30$ (5)

$= 6\sqrt{3} - \lambda\frac{\sqrt{3}}{2}$ (5)

$R \perp AD \Rightarrow X = 0$ (5)

$\lambda = 8$ (5)

$Y = 6\sqrt{3} - 4\sqrt{3}$

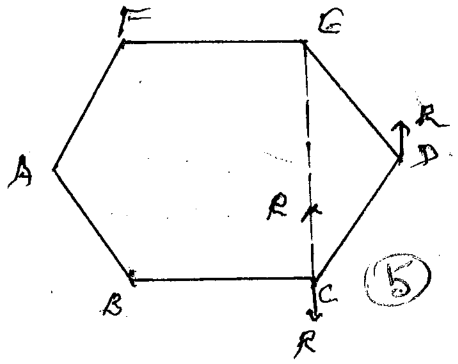
$= 2\sqrt{3}$ (5)

B↺ $2\sqrt{3}x = 8.2a\sin 60 - 4a\sin 60$ (10)

$= 8a\sin 60$

$x = 2a$ (5)

$\Rightarrow R$, lies on CE (5)



(5)

G↺ to be added

G↺ $= R.2a\cos 60$ (5)

$= 2\sqrt{3}a$ (5)

$\boxed{75}$