



# தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு

இரண்டாம் தவணைப் பரீட்சை - 2024

National Field Work Centre, Thondaimanaru.

2<sup>nd</sup> Term Examination - 2024

Gr : 12 (2025)

இணைந்த கணிதம்

புள்ளித்திட்டம்

01]  $P(x) = 2x^3 + ax^2 - 11x - 5$   
 $P(2) = 1$  (5)  
 $2(2)^3 + a(2)^2 - 11(2) - 5 = 1$   
 $4a = 12$   
 $a = 3$  (5)  
 $\therefore P(x) = 2x^3 + 3x^2 - 11x - 5$   
 $P(x) - 1 = 2x^3 + 3x^2 - 11x - 6$   
 $= (x-2)(2x^2 + Ax + 3)$  (5)  
 $x^2, 3 = A - 4$   
 $A = 7$  (5)  
 $P(x) - 1 = (x-2)(2x^2 + 7x + 3)$  (5)  
 $= (x-2)(2x+1)(x+3)$  (5)

	$x < -4$	$-4 < x < 0$	$0 < x < 2$	$x < 2$
$x+4$	(-)	(+)	(+)	(+)
$x-2$	(-)	(-)	(-)	(+)
$x$	(-)	(-)	(+)	(+)
$\frac{(x+4)(x-2)}{x}$	(-)	(+)	(-)	(+)

$-4 \leq x < 0$  or  $x > 2$  (5)

04]  $\log_3 2 = a, \log_5 2 = b$

$\log_{2025} 2 = \frac{1}{\log_2 2025}$  (5)

$= \frac{1}{\log_2 \{5^4 \times 3^4\}}$  (5)

$= \frac{1}{2\log_2 5 + 4\log_2 3}$  (5)

$= \frac{1}{2\{\frac{1}{b}\} + 4\{\frac{1}{a}\}}$  (5)

$= \frac{ab}{2a + 4b}$  (5)  
 $= \frac{ab}{2(a + 2b)}$  (5)

02]  $k \in \mathbb{R} \setminus \{0\}$

$\frac{1}{x(x-k)^2} = \frac{A}{x} + \frac{B}{x-k} + \frac{C}{(x-k)^2}$  (5)

$1 = A(x-k)^2 + Bx(x-k) + Cx$

$x^2, 0 = A + B$  (5)

$x, 0 = -2Ak - Bk + C$  (5)

$x^0, 1 = Ak^2$  (5)

$\frac{1}{x(x-k)^2} = \frac{1}{k^2x} - \frac{1}{k^2(x-k)} + \frac{1}{k(x-k)^2}$  (5)

02]  $x+2 \geq \frac{8}{x}, x \neq 0$

$x+2 - \frac{8}{x} \geq 0$  (5)

$\frac{x^2 + 2x - 8}{x} \geq 0$

$\frac{(x+4)(x-2)}{x} \geq 0$  (5)

05]

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x - \cos 4x + \cos 2x \cos 4x}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(1 - \cos 4x)}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{(2 \sin^2 x)(2 \sin^2 2x)}{x^4}$$

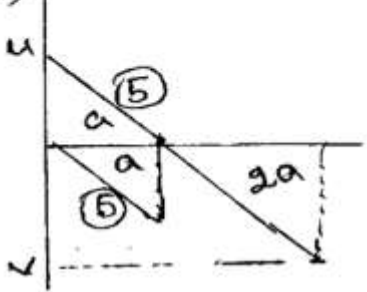
$$= 4 \times 4 \left\{ \lim_{x \rightarrow 0} \frac{\sin x}{x} \right\}^2 \left\{ \lim_{2x \rightarrow 0} \frac{\sin 2x}{2x} \right\}^2$$

$$= 16 \times 1^2 \times 1^2$$

$$= 16$$

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06)



$$\frac{1}{2} \frac{u^2}{g} = a$$

$$u^2 = 2ag$$

$$u = \sqrt{2ag}$$

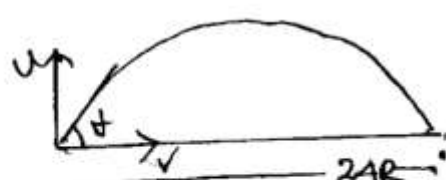
$$\frac{1}{2} \frac{v^2}{g} = 2a$$

$$v^2 = 4ag$$

$$v = 2\sqrt{ag}$$

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07)



$$u = v \cos \theta$$

$$0 = 3vt - \frac{1}{2}gt^2$$

$$t = \frac{6v}{g}$$

$$vt = 24R$$

$$\frac{6v^2}{g} = 24R$$

$$v = 2\sqrt{gR}$$

$$\uparrow v^2 = u^2 + 2as$$

$$0 = 9v^2 - 2gh$$

$$h = \frac{9v^2}{2g} = 18R$$

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$$\vec{AB} = 2\sqrt{3}\hat{i} - 2\hat{j}$$

$$\vec{OC} = x\hat{i} + y\hat{j}, x^2 + y^2 = 4$$

$$\vec{AB} \cdot \vec{OC} = 0$$

$$(2\sqrt{3}\hat{i} - 2\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$$

$$2\sqrt{3}x - 2y = 0$$

$$y = \sqrt{3}x$$

$$4x^2 = 4$$

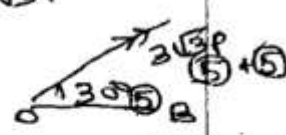
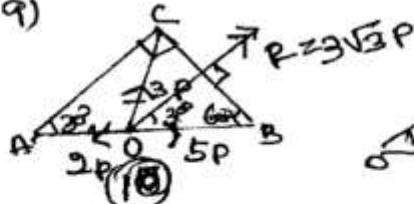
$$x = \pm 1$$

$$\vec{OC} = \hat{i} + \sqrt{3}\hat{j}$$

$$\vec{OC} = -\hat{i} - \sqrt{3}\hat{j}$$

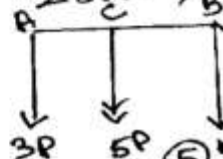
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9)



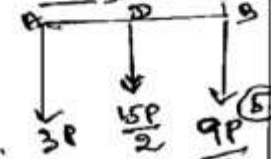
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10) 5x → B



$$AC = \frac{2}{5} \times 5x = 2x$$

5x → B



$$AD = \frac{3}{5} \times 5x = 3x$$

moving distance = 2x

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11)

a)  $f(x) = 0$

$$x^2 - (3k+1)x + (k+1)(k-2) = 0$$

Discriminant  $\Delta$

$$\Delta = \{-(3k+1)\}^2 - 4(1)(k+1)(k-2)$$

$$= (9k^2 + 6k + 1) - 4(k^2 - k - 2)$$

$$= 5k^2 + 10k + 9$$

$$= 5 \left\{ k^2 + 2k + \frac{9}{5} \right\}$$

$$= 5 \left\{ (k+1)^2 + \frac{4}{5} \right\}$$

$$= 5(k+1)^2 + 4$$

$$> 0$$

$\therefore$  the roots of  $f(x) = 0$  are real and distinct

$$\left. \begin{aligned} \alpha + \beta &= 3k+1 \\ \alpha\beta &= (k+1)(k-2) \end{aligned} \right\}$$

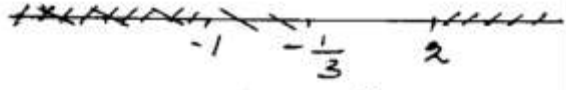
Both  $\alpha, \beta$  are negative

$$\alpha < 0, \beta < 0$$

$$\Rightarrow \alpha + \beta < 0 \text{ \& \& } \alpha\beta > 0$$

$$\Rightarrow 3k+1 < 0 \text{ \& \& } (k+1)(k-2) > 0$$

$$\Rightarrow k < -\frac{1}{3} \text{ \& \& } k < -1 \text{ or } k > 2$$



sol<sup>n</sup>  $k < -1$

$$x^2 - (3k+1)x + (k+1)(k-2) = 0$$

put  $y = x+1$

$$x = \alpha \Rightarrow y = \alpha+1$$

$$x = \beta \Rightarrow y = \beta+1$$

$x \rightarrow y-1$  substitute in (\*)

$$(y-1)^2 - (3k+1)(y-1) + (k+1)(k-2) = 0$$

$$y^2 - 3(k+1)y + k(k+2) = 0$$

The required eq<sup>n</sup> is

$$x^2 - 3(k+1)x + k(k+2) = 0$$

b)  $f(x) = (x-a)\phi_1(x) + A$

$\phi_1(x) = (x-b)\phi_2(x) + B$

(1), (2)  $\Rightarrow$

$$f(x) = (x-a)\{(x-b)\phi_2(x) + B\} + A$$

$$= (x-a)(x-b)\phi_2(x) + B(x-a) + A$$

$$(1) \Rightarrow f(a) = A$$

$$(2) \Rightarrow \phi_1(b) = B$$

$$(1) \Rightarrow f(b) = (b-a)\phi_1(b) + A$$

$$f(b) = (b-a)B + f(a)$$

$$B = \frac{f(b) - f(a)}{b-a}$$

(3)  $\Rightarrow$

$$f(x) = (x-a)(x-b)\phi_2(x) + (x-a)\left\{\frac{f(b)-f(a)}{b-a}\right\} + f(a)$$

$\therefore$  Remainder

$$\left\{\frac{f(b)-f(a)}{b-a}\right\}(x-a) + f(a)$$

$$f(x) = x^3 + px^2 + qx + 1$$

put  $a=1, b=2$

then

$$-10x + 5 = \left\{\frac{f(2)-f(1)}{2-1}\right\}(x-1) + f(1)$$

$$x; -10 = f(2) - f(1)$$

$$-10 = (8 + 4p + 2q + 1) - (1 + p + q + 1)$$

$$3p + q = -17$$

$$x^0; 5 = -\left\{\frac{f(2)-f(1)}{2-1}\right\} + f(1)$$

$$5 = 2f(1) - f(2)$$

$$5 = 2(1 + p + q + 1) - (8 + 4p + 2q + 1)$$

$$2p = -10$$

$$p = -5$$

$$(*) \Rightarrow q = -2$$

12]

a) Let  $f(x) = \sqrt{\sin x}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\sin(x+h)} - \sqrt{\sin x}}{h} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h \{ \sqrt{\sin(x+h)} + \sqrt{\sin x} \}} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos(x+\frac{h}{2}) \sin \frac{h}{2}}{h \{ \sqrt{\sin(x+h)} + \sqrt{\sin x} \}} \quad (5)$$

$$= \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \lim_{h \rightarrow 0} \frac{\cos(x+\frac{h}{2})}{\{ \sqrt{\sin(x+h)} + \sqrt{\sin x} \}} \quad (5)$$

$$= 1 \cdot \frac{\cos x}{\sqrt{\sin x} + \sqrt{\sin x}} \quad (5)$$

$$= \frac{\cos x}{2\sqrt{\sin x}} \quad (5)$$

[35]

b)  $y = \frac{x^2-1}{x^2+1}$

$$\frac{dy}{dx} = \frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2} \quad (5)$$

$$= \frac{4x}{(x^2+1)^2} \quad (5)$$

$$y = \frac{1+e^x}{1-e^x}$$

$$\frac{dy}{dx} = \frac{(1-e^x)ex - (1+e^x)(-e^x)}{(1-e^x)^2} \quad (5)$$

$$= \frac{2e^x}{(1-e^x)^2} \quad (5)$$

$$y = x^{\frac{1}{x^2+1}}$$

$$\ln y = \ln x$$

$$\ln y = (x^2+1) \ln x \quad (5)$$

$$\frac{1}{y} \frac{dy}{dx} = (x^2+1) \cdot \frac{1}{x} + (\ln x) \{2x\} \quad (10)$$

$$\frac{dy}{dx} = x^{\frac{1}{x^2+1}} \left\{ \frac{x^2+1}{x} + 2x \ln x \right\} \quad (5)$$

[40]

c)  $y = e^x \operatorname{Cose}^x \quad (1)$

$$\frac{dy}{dx} = e^x(-) \operatorname{Sine}^x \cdot e^x + \operatorname{Cose}^x \cdot e^x \quad (10)$$

$$\frac{dy}{dx} = -e^{2x} \operatorname{Sine}^x + y \quad (\text{by (1)})$$

$$\frac{dy}{dx} - y = -e^{2x} \operatorname{Sine}^x \quad (2) \quad (5)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -\{e^{2x} \operatorname{Cose}^x \cdot e^x + \operatorname{Sine}^x \cdot e^{2x}\} \quad (10)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -e^{2x} y + 2 \left\{ \frac{dy}{dx} - y \right\} \quad (5)$$

(by (1), (2))

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + (e^{2x} + 2)y = 0$$

[35]

d)  $x = \sin n\theta \quad y = \cos m\theta$

$$\frac{dx}{d\theta} = n \cos n\theta \quad (5) \quad \frac{dy}{d\theta} = -m \sin m\theta \quad (5)$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{-m \sin m\theta}{n \cos n\theta} \quad (5)$$

$$n^2 \cos^2 n\theta \left( \frac{dy}{dx} \right)^2 = m^2 \sin^2 m\theta \quad (5)$$

$$n^2 (1-x^2) \left( \frac{dy}{dx} \right)^2 = m^2 (1-y^2) \quad (5)$$

$$n^2 \{ (1-x^2) 2 \left( \frac{dy}{dx} \right) \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 (-2x) \} = m^2 \{ -2y \frac{dy}{dx} \} \quad (10)$$

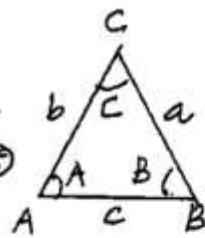
$$n^2 \{ (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} \} = -m^2 y$$

$$n^2 (1-x^2) \frac{d^2y}{dx^2} - n^2 x \frac{dy}{dx} + m^2 y = 0 \quad (5)$$

[40]

13] Sin rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad (5)$$



proof - (15)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{k \cos \frac{C}{2}} \quad (11)$$

$$\frac{a+b-c}{a+b+c}$$

$$= \frac{k \sin A + k \sin B - k \sin C}{k \sin A + k \sin B + k \sin C} \quad (64 \times)$$

$$= \frac{2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) - 2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) + 2 \sin \frac{C}{2} \cos \frac{C}{2}} \quad (10)$$

$$= \frac{\cos \frac{C}{2} \cos \left( \frac{A-B}{2} \right) - \sin \frac{C}{2} \cos \frac{C}{2}}{\cos \frac{C}{2} \cos \left( \frac{A-B}{2} \right) + \sin \frac{C}{2} \cos \frac{C}{2}}$$

$$\left( \frac{C}{2} = \frac{\pi}{2} - \left( \frac{A+B}{2} \right) \right)$$

$$\cos \frac{C}{2} = \sin \left( \frac{A+B}{2} \right)$$

$$= \frac{\cos \left( \frac{A-B}{2} \right) - \sin \frac{C}{2}}{\cos \left( \frac{A-B}{2} \right) + \sin \frac{C}{2}} \quad (5)$$

$$= \frac{\cos \left( \frac{A-B}{2} \right) - \sin \frac{C}{2}}{\cos \left( \frac{A-B}{2} \right) + \sin \frac{C}{2}} \quad (5)$$

$$= \frac{\cos \left( \frac{A-B}{2} \right) - \cos \left( \frac{A+B}{2} \right)}{\cos \left( \frac{A-B}{2} \right) + \cos \left( \frac{A+B}{2} \right)}$$

$$= \frac{2 \sin \frac{A}{2} \sin \frac{B}{2}}{2 \cos \frac{A}{2} \cos \frac{B}{2}} \quad (10)$$

$$= \tan \frac{A}{2} \tan \frac{B}{2} \quad (5)$$

$$b) \tan^{-1} \left( \frac{x}{2} \right) + \tan^{-1} \left( \frac{x}{3} \right) = \frac{\pi}{4} \quad (1)$$

$$\text{put } \alpha = \tan^{-1} \left( \frac{x}{2} \right) \Rightarrow \tan \alpha = \frac{x}{2} \quad (5)$$

$$\beta = \tan^{-1} \left( \frac{x}{3} \right) \Rightarrow \tan \beta = \frac{x}{3}$$

$$(1) \Rightarrow \alpha + \beta = \frac{\pi}{4}$$

$$\tan(\alpha + \beta) = 1 \quad (5)$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = 1 \quad (5)$$

$$\frac{\frac{x}{2} + \frac{x}{3}}{1 - \left( \frac{x}{2} \right) \left( \frac{x}{3} \right)} = 1 \quad (5)$$

$$\frac{5x}{6 - x^2} = 1$$

$$x^2 + 5x - 6 = 0 \quad (5)$$

$$(x+6)(x-1) = 0 \quad (5)$$

$$x-1=0 \quad (\because x > 0)$$

$$x=1 \quad (5)$$

$$(1) \Rightarrow \tan^{-1} \left( \frac{1}{2} \right) + \tan^{-1} \left( \frac{1}{3} \right) = \frac{\pi}{4} \quad (5)$$

$$\tan^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{4} - \tan^{-1} \left( \frac{1}{3} \right)$$

$$\sin^{-1} \left( \frac{1}{\sqrt{5}} \right) = \frac{\pi}{4} - \tan^{-1} \left( \frac{1}{3} \right) \quad (5)$$

$$\frac{1}{\sqrt{5}} = \sin \left[ \frac{\pi}{4} - \tan^{-1} \left( \frac{1}{3} \right) \right] \quad (5)$$

$$c) 0 < x < 1, \quad x = \tan \alpha$$

$$\sin^{-1} \left\{ \frac{2x}{1+x^2} \right\} + \cos^{-1} \left\{ \frac{1-x^2}{1+x^2} \right\} + \tan^{-1} \left\{ \frac{2x}{1-x^2} \right\} = \pi$$

$$\sin^{-1} \left\{ \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \right\} + \cos^{-1} \left\{ \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} \right\} + \tan^{-1} \left\{ \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \right\} = \pi$$

$$\sin^{-1} \{ \sin 2\alpha \} + \cos^{-1} \{ \cos 2\alpha \} + \tan^{-1} \{ \tan 2\alpha \} = \pi$$

$$2\alpha + 2\alpha + 2\alpha = \pi \quad (0 < 2\alpha < \frac{\pi}{2}) \quad (5)$$

$$6\alpha = \pi$$

$$\alpha = \frac{\pi}{6} \quad (5)$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad (5)$$

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14]

a)

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \quad \text{--- (1)}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B \quad \text{--- (2)}$$

(1) + (2)  $\Rightarrow$ 

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B \quad \text{--- (3)}$$

$$\left. \begin{array}{l} A+B=C \\ A-B=D \end{array} \right\} \Rightarrow \begin{array}{l} A = \frac{C+D}{2} \\ B = \frac{C-D}{2} \end{array}$$

$$(3) \Rightarrow \sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right) \quad \text{--- (4)}$$

$$\sin C - \sin D$$

$$= \sin C + \sin(-D) \quad \text{--- (5)}$$

$$= 2 \sin\left(\frac{C+(-D)}{2}\right) \cos\left(\frac{C-(-D)}{2}\right) \quad \text{--- (6)}$$

$$= 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right) \quad \text{--- (7)}$$

$$\sin 7x + \sin 5x - \cos x (\sin 7x - \sin 5x) = 0 \quad \text{--- (8)}$$

$$2 \sin 6x \cos x - \frac{\cos x}{\sin x} (2 \cos 6x \sin x) = 0 \quad \text{--- (9)}$$

$$\sin 6x \cos x - \cos 6x \cos x = 0 \quad \text{--- (10)}$$

$$\cos x (\sin 6x - \cos 6x) = 0 \quad \text{--- (11)}$$

$$\cos x = 0 \text{ or } \sin 6x - \cos 6x = 0$$

$$\cos x = \cos \frac{\pi}{2} \quad \text{--- (12)}$$

$$x = 2n\pi \pm \frac{\pi}{2} \quad \text{--- (13)}$$

$$n_1 \in \mathbb{Z}$$

$$6x = n_2\pi + \frac{\pi}{4}$$

$$x = \frac{n_2\pi}{6} + \frac{\pi}{24} \quad \text{--- (14)}$$

$$n_2 \in \mathbb{Z}$$

$$b) \sin 7\theta + \sin \theta$$

$$= 2 \sin 4\theta \cos 3\theta \quad \text{--- (15)}$$

$$= 2 \{ 2 \sin 2\theta \cos 2\theta \} \cos 3\theta \quad \text{--- (16)}$$

$$= 4 \{ 2 \sin \theta \cos \theta \} \cos 2\theta \cos 3\theta \quad \text{--- (17)}$$

$$= 8 \sin \theta \cos \theta \cos 2\theta \cos 3\theta \quad \text{--- (18)}$$

$$\sin 7\theta + \sin \theta = 8 \sin \theta \cos \theta \cos 2\theta \cos 3\theta$$

$$\sin 7\theta = \sin \theta \{ 8 \cos \theta \cos 2\theta \cos 3\theta - 1 \} \quad \text{--- (19)}$$

$$4 \cos \theta \cos 2\theta \cos 3\theta = 1$$

$$8 \cos \theta \cos 2\theta \cos 3\theta = 2$$

$$\frac{\sin 7\theta}{\sin \theta} + 1 = 2 \quad \text{--- (20)}$$

$$\sin 7\theta = \sin \theta \quad \text{--- (21)}$$

$$7\theta = n\pi + (-1)^n \theta, \quad n = 0, \pm 1, \pm 2 \quad \text{--- (22)}$$

$$n=0 \Rightarrow \theta = 0$$

$$n=1 \Rightarrow \theta = \frac{\pi}{2}$$

$$n=2 \Rightarrow \theta = \frac{\pi}{2}$$

$$\therefore \theta = \frac{\pi}{2} \quad \text{--- (23)}$$

$$c) \frac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta}$$

$$= \frac{\frac{\sin 5\theta}{\cos 5\theta} + \frac{\sin 3\theta}{\cos 3\theta}}{\frac{\sin 5\theta}{\cos 5\theta} - \frac{\sin 3\theta}{\cos 3\theta}} \quad \text{--- (24)}$$

$$= \frac{\sin 5\theta \cos 3\theta + \cos 5\theta \sin 3\theta}{\sin 5\theta \cos 3\theta - \cos 5\theta \sin 3\theta} \quad \text{--- (25)}$$

$$= \frac{\sin 8\theta}{\sin 2\theta} \quad \text{--- (26)}$$

$$= \frac{2 \sin 4\theta \cos 4\theta}{\sin 2\theta} \quad \text{--- (27)}$$

$$= \frac{4 \sin 2\theta \cos 2\theta \cos 4\theta}{\sin 2\theta} \quad \text{--- (28)}$$

$$= 4 \cos 2\theta \cos 4\theta \quad \text{--- (29)}$$

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$$T = \sqrt{\frac{2d}{g}}$$

$$V = 3 \text{ ft} \text{ (5)}$$

$$d_1 = \frac{(u+v) \times T}{2} = 2 \text{ ft} \quad \textcircled{5}$$

$$T' = \frac{4d}{v} = \frac{4d}{3f_T} = \frac{2}{3} T \quad (5)$$

$$\text{Total time} = T + T + \frac{2}{3}T$$

$$= \frac{8T}{3} \quad (5)$$

$$f' = \frac{v}{T-1} = \frac{2v}{T} = 9f \quad \boxed{90}$$



$$\bullet v \cos \theta = \sqrt{2ag} \rightarrow (1) (5)$$

$$\uparrow v^2 = u^2 + 2as$$

$$3a_g = \sqrt{5} \sin \theta - 2g \times 3a \quad (5)$$

$$v \sin \theta = 3\sqrt{ag} \rightarrow (2)(5)$$

$$\tan \theta = 2\sqrt{3}$$

⑤  $\approx \frac{1}{5}$  ⑤

$$v = \sqrt[3]{2\sqrt{3ag}} \quad (5)$$

$$(i) \uparrow S = ut + \frac{1}{2}at^2$$

$$0 = v \sin \theta T - \frac{1}{2} g T^2 \quad (5)$$

$$T = \frac{2v \sin \theta}{g} \quad (5)$$

$$R = v \cos \theta \times T \quad (5)$$

$$\frac{2V \cos \theta \times 2V \sin \theta}{g}$$

$$= \sqrt{2ag} \times 2 \times 3\sqrt{ag}$$

$$26\sqrt{3}a^8 \text{ (5)}$$

Qb)  $T_1$  is the time  $P$  reach  $B$

$$\uparrow v = u + at$$

$$\sqrt{3}cg = v \sin \theta - gT_1 \quad (5)$$

$$T_1 = \frac{v \sin \alpha - \sqrt{2}cg}{g} \quad (4)$$

Time taken to reach C to Q

$$T_2 = T - T_1$$

$$= \frac{v \sin \theta + \sqrt{20g}}{g} \quad (5)$$

$$= 3\sqrt{ag} + \sqrt{3ag}$$

$$= (2 + \sqrt{3}) \sqrt{\frac{9}{20}} \quad (5)$$

$$\left. \begin{aligned} u \cos \theta &= T_2 \\ \frac{u \sin \theta}{T_2} &= \frac{g}{2} \end{aligned} \right\} (5)$$

$$\tan \theta = \frac{a}{b}$$

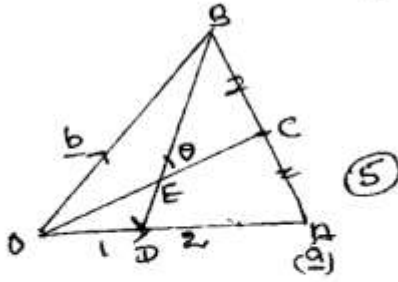
$$d = \tan^{-1} \left( \frac{g \sin \theta}{g \cos \theta} \right) \quad (5)$$

$$u = \sqrt{\frac{9^2 + 9^2}{4} + \frac{R^2}{12}} \quad (5)$$

$$\left. \begin{aligned} \text{max height of p} &= \frac{v^2 \sin^2 \theta}{2g} \\ \text{max height of q} &= \frac{u^2 \sin^2 \theta}{2g} \end{aligned} \right\} \textcircled{5}$$



16)  
(9)



(i)  $\vec{OE} = \vec{OA} + \vec{AE}$  (5)  
 $= \vec{OA} + \frac{1}{3}\vec{AB}$   
 $= \vec{a} + \frac{1}{3}(\vec{b} - \vec{a})$   
 $= \frac{\vec{a} + \vec{b}}{3}$  (5)

$OE = \frac{1}{3}|\vec{OA}| = \frac{1}{3}|\vec{a}|$  (5)

(ii)  $\vec{OE} = \vec{OB} + \vec{BE}$  (5)  
 $= \vec{OB} + \lambda \vec{BC}$   
 $= \vec{OB} + \lambda(\vec{OC} - \vec{OB})$   
 $= \vec{b} + \lambda(-\vec{b} + \frac{1}{3}\vec{a})$   
 $= (1-\lambda)\vec{b} + \frac{\lambda}{3}\vec{a}$  (5)

(iii)  $\vec{OE} = \mu \vec{OC}$

$(1-\lambda)\vec{b} + \frac{\lambda}{3}\vec{a} = \mu(\vec{a} + \vec{b})$  (5)

(5)  $\frac{\lambda}{3} = \mu, 1-\lambda = \mu$

$\frac{\lambda}{3} = 1-\lambda$

$4\lambda = 3$

$\lambda = \frac{3}{4}, \mu = \frac{1}{4}$  (5)

2)  $\vec{a} = 12\hat{i}, \vec{b} = 2\hat{i} + 8\hat{j}$

(i)  $\vec{OC} = 7\hat{i} + 4\hat{j}$  (5)

$\vec{OB} = 4\hat{j}$  (5)

(ii)  $\vec{OC} = 7\hat{i} + 4\hat{j}$

$\vec{OB} = \vec{OC} + \vec{CB}$  (5)  
 $= -4\hat{i} + 2\hat{j} + 8\hat{j}$   
 $= -4\hat{i} + 10\hat{j}$  (5)

$\vec{OC} \cdot \vec{OB} = (7\hat{i} + 4\hat{j}) \cdot (-4\hat{i} + 10\hat{j})$  (5)  
 $= -28 + 40$   
 $= 12$  (5)

$12 = |\vec{OC}| |\vec{OB}| \cos \theta$  (5)  
 $= \sqrt{49+16} \sqrt{16+100} \cos \theta$  (5)  
 $= \sqrt{65} \times \sqrt{116} \cos \theta$

$\cos \theta = \frac{12}{\sqrt{65} \sqrt{116}}$  (5)

$\theta = \cos^{-1}(\frac{12}{\sqrt{65} \sqrt{116}})$  (5)

(iii)  $\vec{OP} = \beta \vec{b}$  (5)

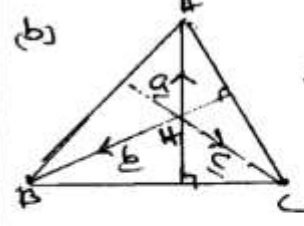
$\vec{AP} = \alpha \vec{AE}$  (5)

$\vec{AO} + \vec{OP} = \alpha(\vec{AO} + \vec{OE})$  (5)

$-\vec{a} + \beta \vec{b} = \alpha(-\vec{a} + \frac{1}{3}\vec{b} + \frac{1}{3}\vec{a})$

$-1 = -\frac{3}{4}\alpha, \beta = \frac{\alpha}{4}$  (5)  
 $\alpha = \frac{4}{3}, \beta = \frac{1}{3}$

$\therefore \vec{OP} = \frac{1}{3}\vec{b}$  (5)



$\vec{AB} \perp \vec{AC}$

$\therefore \vec{AB} \cdot \vec{AC} = 0$  (5)

$\vec{a} \cdot (\vec{b} - \vec{a}) = 0$  (5)

$\vec{AB} \perp \vec{AC}$

$\vec{AB} \cdot \vec{AC} = 0$  (5)

$\vec{b} \cdot (\vec{c} - \vec{b}) = 0$  (5)

(1)  $\vec{b} \cdot \vec{c} - \vec{b} \cdot \vec{b} = 0$  (5)

$(\vec{b} - \vec{a}) \cdot \vec{c} = 0$  (5)



$\vec{AB} \perp \vec{AC}$  (5)

(20)



