தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு

இரண்டாம் தவணைப் பரீட்சை - 2023

National Field Work Centre, Thondaimanaru.

2nd Term Examination - 2023

NFWC

Gr : 12 (2024) இணைந்த கணிதம் புள்ளித்திட்டம்

1. $f(x) = x^4 + ax^3 + bx^2 - x + 2$

$f(x) = (x+1)^2\phi(x) + 5$ (5)

$f'(x) = (x+1)^2\phi'(x) + \phi(x)\,2(x+1)$ (5)

$f'(x) = 4x^3 + 3ax^2 + 2bx - 1$ (5)

$f(-1) = 5 \Rightarrow 1 - a + b + 1 + 2 = 5$

$\Rightarrow -a + b = 1$ —— (1)

$f'(-1) = 0 \Rightarrow -4 + 3a - 2b - 1 = 0$ (5)

$\Rightarrow 3a - 2b = 5$ —— (2)

(1),(2) $\Rightarrow a = 7;\ b = 8$ (5)

$\boxed{25}$

2. $x^2 \geq \dfrac{4x^2}{x+3}$

$x^2 - \dfrac{4x^2}{x+3} \geq 0$ (5)

$\dfrac{x^2(x-1)}{x+3} \geq 0$ (5)

	$x<-3$	$-3<x<0$	$0<x<1$	$x>1$
x^2	(+)	(+)	(+)	(+)
$x-1$	(−)	(−)	(−)	(+)
$x+3$	(−)	(+)	(+)	(+)
$\dfrac{x^2(x-1)}{x+3}$	(+)	(−)	(−)	(+)

(10)

$x < -3$ or $x \geq 1$ or $x = 0$ (5)

$\boxed{25}$

3. $\dfrac{2x}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$ (5)

$2x = A(x+2) + B(x+1)$

$x^1:\ 2 = A + B$

$x^0:\ 0 = 2A + B$

$A = -2,\ B = 4$ (5)

$\dfrac{2x}{(x+1)(x+2)} = \dfrac{-2}{x+1} + \dfrac{4}{x+2}$ (5)

$\dfrac{x}{(x+1)(2x+1)}$

$= \dfrac{y/2}{(\frac{y}{2}+1)(y+1)}$; where $2x = y$ (5)

$= \dfrac{y}{(y+1)(y+2)}$

$= \dfrac{1}{2}\left\{\dfrac{-2}{y+1} + \dfrac{4}{y+2}\right\}$

$= \dfrac{-1}{2x+1} + \dfrac{2}{2x+2}$ $[\because y = 2x]$ (5)

$= \dfrac{-1}{2x+1} + \dfrac{1}{x+1}$

$\boxed{25}$

4. $\log_x 5 - (\log_5\sqrt{3})(\log_3 x) = \log_3\sqrt{3}$

$\dfrac{1}{\log_5 x} - \dfrac{1}{2}\log_5 3 \times \dfrac{\log_5 x}{\log_5 3} = \dfrac{1}{2}$ (5)(5)

$\dfrac{1}{t} - \dfrac{1}{2}t = \dfrac{1}{2}$; where $t = \log_5 x$ (5)

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$ (5)

$t = -2$ or $t = 1$

$\log_5 x = -2$ or $\log_5 x = 1$

$x = 5^{-2} = \dfrac{1}{25}$ or $x = 5^1 = 5$ (5)

$\boxed{25}$

5. $\displaystyle\lim_{x\to\frac{\pi}{6}} \dfrac{2\sin x - 1}{\sqrt{\pi} - \sqrt{6x}}$

$= \displaystyle\lim_{x\to\pi/6} \dfrac{2(\sin x - \frac{1}{2})(\sqrt{\pi} + \sqrt{6x})}{\pi - 6x}$ (5)

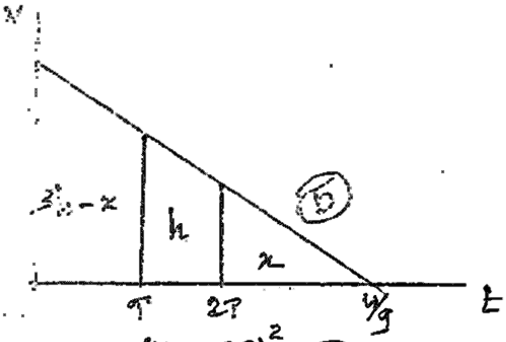
$= \displaystyle\lim_{x\to\pi/6} \dfrac{2\cos(\frac{x+\pi/6}{2})\sin(\frac{x-\pi/6}{2})(\sqrt{\pi}+\sqrt{6x})}{-6(x-\pi/6)}$ (5)

$= \displaystyle\lim_{\frac{x-\pi/6}{2}\to 0} \dfrac{\sin(\frac{x-\pi/6}{2})}{\frac{x-\pi/6}{2}} \times \lim_{x\to\pi/6} \dfrac{\cos(\frac{x+\pi/6}{2})(\sqrt{\pi}+\sqrt{6x})}{-6}$ (5)

$= 1 \times \dfrac{\sqrt{3}}{2} \cdot \dfrac{2\sqrt{\pi}}{-6} = -\dfrac{\sqrt{3\pi}}{6}$
(5) (5)

$\boxed{25}$

(7)



(5)

$$\frac{x}{h+h} = \frac{\left(\frac{u}{g} - 2T\right)^2}{\left(\frac{u}{g}\right)^2} \quad (5)$$

$$\frac{x}{h+x} = \frac{\left(\frac{u}{g} - 2T\right)^2}{\left(\frac{u}{g} - T\right)^2} \quad (5)$$

$$\frac{x}{h} = \frac{\left(\frac{u}{g} - 2T\right)^2}{T\left(\frac{2u}{g} - 3T\right)}$$

$$\frac{1}{4} = \frac{T\left(\frac{2u}{g} - 3T\right)}{\left(\frac{u}{g}\right)^2}$$

$$12g^2T^2 - 8guT + u^2 = 0 \quad (5)$$

$$(6gT - u)(2gT - u) = 0$$

$$T = \frac{u}{6g}, \ \frac{u}{2g}$$

$$T < \frac{u}{2g} \Rightarrow T = \frac{u}{6g} \quad (5)$$

(8)

$$|\underline{a}+\underline{b}|^2 = |\underline{a}-\underline{b}|^2$$

$$(\underline{a}+\underline{b})\cdot(\underline{a}+\underline{b}) = (\underline{a}-\underline{b})\cdot(\underline{a}-\underline{b}) \quad (10)$$

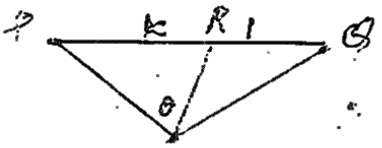
$$a^2 + b^2 + 2\underline{a}\cdot\underline{b} = a^2 + b^2 - 2\underline{a}\cdot\underline{b} \quad (5)$$

$$4\underline{a}\cdot\underline{b} = 0$$

$$\underline{a}\cdot\underline{b} = 0 \quad (5)$$

$$\Rightarrow \underline{a} \perp \underline{b} \quad (5)$$

(8)



$$\overrightarrow{PQ} = -6\underline{i} + 4\underline{j} - (2\underline{i} + 3\underline{j}) \quad (5)$$

$$= -8\underline{i} + \underline{j}$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \overrightarrow{OP} + \frac{k}{k+1}(\overrightarrow{PQ})$$

$$= \left(2 - \frac{8k}{k+1}\right)\underline{i} + \left(3 + \frac{k}{k+1}\right)\underline{j} \quad (5)$$

$$\overrightarrow{OP}\cdot\overrightarrow{OR} = (2\underline{i} + 3\underline{j})\cdot\left[\left(2 - \frac{8k}{k+1}\right)\underline{i} + \left(3 + \frac{k}{k+1}\right)\underline{j}\right]$$

$$\sqrt{13}\,|\overrightarrow{OR}|\cos\theta = 4 - \frac{16k}{k+1} + 9 + \frac{3k}{k+1} \quad (5)$$

$$\overrightarrow{OQ}\cdot\overrightarrow{OR} = -6\left(2 - \frac{8k}{k+1}\right) + 4\left(3 + \frac{k}{k+1}\right)$$

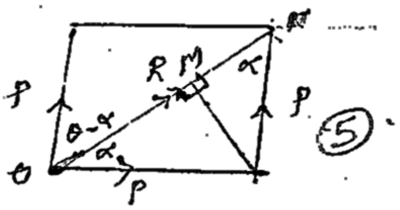
$$\sqrt{52}\,|\overrightarrow{OR}|\cos\theta = -12 + \frac{48k}{k+1} + 12 + \frac{4k}{k+1} \quad (5)$$

$$\Rightarrow 13 - \frac{13k}{k+1} = \frac{26k}{k+1}$$

$$\frac{39k}{k+1} = 13$$

$$k = \frac{1}{2} \quad (5)$$

(9)



(5)

$$\theta - \alpha = \alpha$$

$$\alpha = \frac{\theta}{2} \quad (5)$$

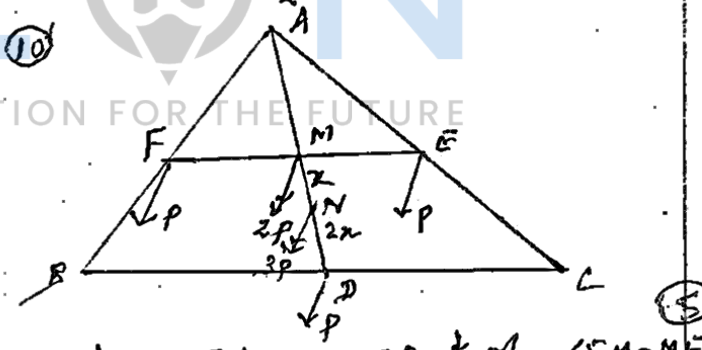
$$R = OM + MN \quad (5)$$

$$= P\cos\alpha + P\cos\alpha \quad (5)$$

$$= 2P\cos\alpha$$

$$= 2P\cos\frac{\theta}{2} \quad (5)$$

(10)



(5)

P at F + P at E $\equiv$ $2P$ at M $\quad (FM = ME)$

$2P$ at M + P at D $\equiv$ $3P$ at N $\quad \left(\frac{MN}{ND} = \frac{1}{2}\right)$ (5)

$AM = MD = 3x$ (5)

$\therefore AN = 4x, \ ND = 2x$

$\therefore AN : ND = 2:1$ (5)

$\therefore N$ is centre of Gravity of ABC (5)

11. (a)

$f(x) = x^2 - 2px + q^2 = 0$ (5)

$\Delta_1 = 4p^2 - 4(1)q^2$

$= 4(p^2 - q^2) > 0$ (5) $[\because p > q > 0]$

$\therefore$ the roots of $f(x) = 0$ are real and distinct.

$g(x) = x^2 - 2qx + p^2 = 0$

$\Delta_2 = 4q^2 - 4(1)p^2$ (5)

$= 4(q^2 - p^2) < 0$ (5) $[\because p > q > 0]$

$\therefore$ The roots of $g(x) = 0$ are imaginary.

$\boxed{20}$

$\alpha + \beta = 2p$ $\quad \gamma + \delta = 2q$ (5)

$\alpha\beta = q^2$ (5) $\quad \gamma\delta = p^2$

$(\alpha\gamma + \beta\delta) + (\alpha\delta + \beta\gamma)$

$= \alpha(\gamma + \delta) + \beta(\gamma + \delta)$ (5)

$= (\gamma + \delta)(\alpha + \beta)$

$= (2q)(2p)$ (5)

$= 4pq$

$(\alpha\gamma + \beta\delta)(\alpha\delta + \beta\gamma)$

$= \alpha^2\gamma\delta + \alpha\beta\gamma^2 + \alpha\beta\delta^2 + \beta^2\gamma\delta$ (5)

$= (\alpha^2 + \beta^2)\gamma\delta + \alpha\beta(\gamma^2 + \delta^2)$

$= \gamma\delta\{(\alpha+\beta)^2 - 2\alpha\beta\} + \alpha\beta\{(\gamma+\delta)^2 - 2\gamma\delta\}$ (5)

$= p^2(4p^2 - 2q^2) + q^2(4q^2 - 2p^2)$

$= 4(p^4 - p^2q^2 + q^4)$ (5)

The required equation is

$(x - (\alpha\gamma + \beta\delta))(x - (\alpha\delta + \beta\gamma)) = 0$ (5)

$x^2 - 4pqx + 4(p^4 - p^2q^2 + q^4) = 0$ (5)

$\boxed{45}$

$\Delta = 16p^2q^2 - 4(4)(p^4 - p^2q^2 + q^4)$ (5)

$= -16(p^4 - 2p^2q^2 + q^4)$

$= -16(p^2 - q^2)^2 < 0$ (5) (5) $[\because p > q > 0]$

$\therefore$ the roots of this equation are imaginary.

$\boxed{15}$

$f(x) + g(x)$

$= 2x^2 - 2(p+q)x + p^2 + q^2$ (5)

$= 2\{(x - \frac{1}{2}(p+q))^2 - \frac{1}{4}(p+q)^2\} + p^2 + q^2$ (5)

$= 2(x - \frac{1}{2}(p+q))^2 + \frac{1}{2}(p^2 - 2pq + q^2)$

$= 2(x - \frac{1}{2}(p+q))^2 + \frac{1}{2}(p-q)^2$ (5)

$\geq \frac{1}{2}(p-q)^2$ (5) $[\because (x - \frac{1}{2}(p+q))^2 \geq 0]$

$\therefore f(x) + g(x) \geq \frac{1}{2}(p-q)^2$

$\boxed{20}$

(b) $p(x) = x^4 + ax^3 + bx^2 - 4x + c$

Since x and $x-1$ are factors of $p(x)$

$p(0) = 0$ and $p(1) = 0$ (5)

$c = 0$ — ① (5)

$1 + a + b - 4 + c = 0$ (5)

$a + b = 3$ — ②

Since, when $p(x)$ is divided by $x+1$, the remainder is 10

$p(-1) = 10$

$1 - a + b + 4 = 10$ (5)

$-a + b = 5$ — ③

②,③ $\Rightarrow$ $a = -1, b = 4$ (5)

$\boxed{25}$

$p(x) = x^4 - x^3 + 4x^2 - 4x$ (5)

$= x^3(x-1) + 4x(x-1)$

$= x(x-1)(x^2+4)$ (5)

$\boxed{10}$

$p(x) = x(x-1)(x^2+4)$

$= x(x-1)(x^2-4+8)$

$= x(x-1)(x-2)(x+2) + 8x(x-1)$ (5)

Quotient $= x+2$. (5)

Remainder $= 8x(x-1)$. (5)

$\boxed{15}$

12 (a)

(i) $\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$

$= \lim_{x\to 2} \frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$ (5)

$= \lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$

$= \lim_{x\to 2} \frac{1}{(\sqrt{2x+5}+\sqrt{x+7})}$ (5)

$= \frac{1}{3+3}$

$= \frac{1}{6}$. (5)

$\boxed{15}$

(ii) $\lim_{x\to\infty} \frac{x^3-4x^2-3}{(x-1)(2x-1)}$

$= \lim_{x\to\infty} x\left(\frac{x^2-4x-\frac{3}{x}}{(x-1)(2x-1)}\right)$ (5)

$= \lim_{x\to\infty} x\left(\frac{1-\frac{4}{x}-\frac{3}{x^3}}{(1-\frac{1}{x})(2-\frac{1}{x})}\right)$ (5)

$= \infty$ (5) $\left[\because \lim_{x\to\infty} x = \infty \text{ and } \lim_{x\to\infty}\left(\frac{1-\frac{4}{x}-\frac{3}{x^3}}{(1-\frac{1}{x})(2-\frac{1}{x})}\right) = \frac{1}{2}.\right]$

$\boxed{15}$

(b) $f(x) = \sin x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$ (5)

$= \lim_{h\to 0} \frac{2\cos(x+\frac{h}{2})\sin\frac{h}{2}}{h}$ (5)

$= \lim_{\frac{h}{2}\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \cdot \lim_{h\to 0} \cos(x+\frac{h}{2})$ (5)

$= 1\times\cos x$ (5)

$= \cos x$

$\boxed{20}$

Let $y = \sin^{-1}x$, $-1<x<1$

$\sin y = x$, $-\frac{\pi}{2}<y<\frac{\pi}{2}$ (5)

Differentiate w.r.t. x

$\cos y \frac{dy}{dx} = 1$ (5)

$\frac{dy}{dx} = \frac{1}{\cos y}$

$= \frac{1}{\sqrt{1-\sin^2 y}}$ (5) $\left[\because -\frac{\pi}{2}<y<\frac{\pi}{2} \Rightarrow \cos y>0\right]$

$= \frac{1}{\sqrt{1-x^2}}$ (5)

$\boxed{20}$

$y = \sin(m\sin^{-1}x)$

$\frac{dy}{dx} = \cos(m\sin^{-1}x)\, m \frac{1}{\sqrt{1-x^2}}$ (10)

$\sqrt{1-x^2}\frac{dy}{dx} = m\cos(m\sin^{-1}x)$

$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}(-2x) = -m\sin(m\sin^{-1}x)\cdot m\frac{1}{\sqrt{1-x^2}}$ (10)

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m^2y$ (5)

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$ (5)

$\boxed{30}$

(c). $x = 2\sin\theta - \sin 2\theta$

$\frac{dx}{d\theta} = 2\cos\theta - \cos 2\theta \cdot 2$ (5)

$y = 2\cos\theta - \cos 2\theta$

$\frac{dy}{d\theta} = -2\sin\theta + \sin 2\theta \cdot 2$ (5)

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$
$$= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$
$$= \frac{2(\sin 2\theta - \sin\theta)}{2(\cos\theta - \cos 2\theta)} \quad (5)$$
$$= \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}} \quad (5)\ (5)$$
$$= \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\frac{3\theta}{2}\cdot\frac{3}{2}\frac{d\theta}{dx} \quad (5)$$
$$= -\operatorname{cosec}^2\frac{3\theta}{2}\ \frac{3}{2}\ \frac{1}{2(\cos\theta-\cos 2\theta)}$$
$$= \frac{-3\operatorname{cosec}^2\frac{3\theta}{2}}{(4)\,2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}} \quad (5)$$
$$= -\frac{3}{8}\operatorname{cosec}^3\frac{3\theta}{2}\operatorname{cosec}\frac{\theta}{2} \quad (5)$$

$\boxed{40}$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{3}} = -\frac{3}{8}(1)^3(2) = -\frac{3}{4} \quad (5)\ (5)$$

$\boxed{10}$

13 (a)

(i) $$\frac{\cos\theta}{1-\sin\theta} + \frac{1-\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta + (1-\sin\theta)^2}{(1-\sin\theta)\cos\theta} \quad (5)$$
$$= \frac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{(1-\sin\theta)\cos\theta} \quad (5)$$
$$= \frac{2(1-\sin\theta)}{(1-\sin\theta)\cos\theta} \quad (5)$$
$$= 2\sec\theta \quad (5)$$

$\boxed{20}$

(ii) $$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$
$$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)} \quad (5)$$
$$= \frac{\sin\theta\cos 2\theta}{\cos\theta\cos 2\theta} \quad (5)$$
$$= \tan\theta \quad (5)$$

$\boxed{15}$

(b) $$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} \quad (5)$$
$$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2}} \quad (5)$$
$$= \frac{\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}}}{\frac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}} + \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$
$$= \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \quad (5)$$
$$= \frac{2t}{1+t^2}\ ;\ \text{where } t = \tan\frac{x}{2}$$

$\boxed{15}$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \quad (5)$$
$$= \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2}} \quad (5)$$
$$= \frac{\frac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}} - \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}{\frac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}} + \frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}}$$
$$= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \quad (5)$$
$$= \frac{1-t^2}{1+t^2}\ ;\ \text{where } t = \tan\frac{x}{2}$$

$\boxed{15}$

$$(a^2-b^2)\sin\alpha + 2ab\cos\alpha = a^2+b^2$$
$$(a^2-b^2)\frac{2t}{1-t^2} + 2ab\left(\frac{1-t^2}{1+t^2}\right) = a^2+b^2 \quad (5)\ (5)$$
$$(a^2-b^2)2t + 2ab(1-t^2) = (a^2+b^2)(1+t^2)$$
$$(a^2+2ab+b^2)t^2 - 2(a^2-b^2)t + a^2-2ab+b^2 = 0 \quad (5)$$
$$(a+b)^2t^2 - 2(a+b)(a-b)t + (a-b)^2 = 0 \quad (5)$$
$$[(a+b)t-(a-b)]^2 = 0 \quad (5)$$

$t = \frac{a-b}{a+b}$ (5)

$\sin\alpha = \frac{2t}{1+t^2}$

$= \frac{2\left(\frac{a-b}{a+b}\right)}{1+\left(\frac{a-b}{a+b}\right)^2}$ (5)

$= \frac{a^2-b^2}{a^2+b^2}$ (5)

$\cos\alpha = \frac{1-t^2}{1+t^2}$

$= \frac{1-\left(\frac{a-b}{a+b}\right)^2}{1+\left(\frac{a-b}{a+b}\right)^2}$ (5)

$= \frac{2ab}{a^2+b^2}$ (5)

$\boxed{50}$

(c) $2\sin x - 2\sqrt{3}\cos x - \sqrt{3}\tan x + 3 = 0$

$2\cos x(\tan x - \sqrt{3}) - \sqrt{3}(\tan x - \sqrt{3}) = 0$ (5)

$(\tan x - \sqrt{3})(2\cos x - \sqrt{3}) = 0$ (5)

$\tan x = \sqrt{3}$ or $\cos x = \frac{\sqrt{3}}{2}$ (5)

$\tan x = \tan\frac{\pi}{3}$ (5) or $\cos x = \cos\frac{\pi}{6}$ (5)

$x = n\pi + \frac{\pi}{3}, n\in\mathbb{Z}$ (5) or $x = 2n\pi \pm \frac{\pi}{6}, n\in\mathbb{Z}$ (5)

$\boxed{35}$

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14. (a)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ (5)

$\boxed{5}$

$\cos(A-B)$

$= \cos(A+(-B))$ (5)

$= \cos A\cos(-B) - \sin A\sin(-B)$ (5)

$= \cos A\cos B + \sin A\sin B$ (5)

$\boxed{15}$

$\cos\frac{\pi}{12}$

$= \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ (5)

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$ (5)

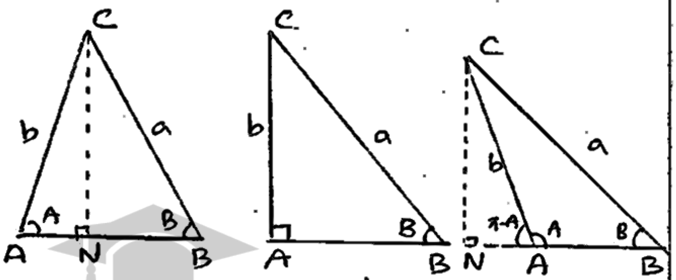
$= \frac{1}{2}\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}$ (5)

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\boxed{15}$

(b) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (5)

$\boxed{05}$



$CN = b\sin A$
$CN = a\sin B$ (5)

$CN = b$
$= b\sin\frac{\pi}{2}$
$CN = b\sin A$
$CN = a\sin B$ (5)

$CN = b\sin(\pi - A)$
$= b\sin A$
$CN = a\sin B$ (5)

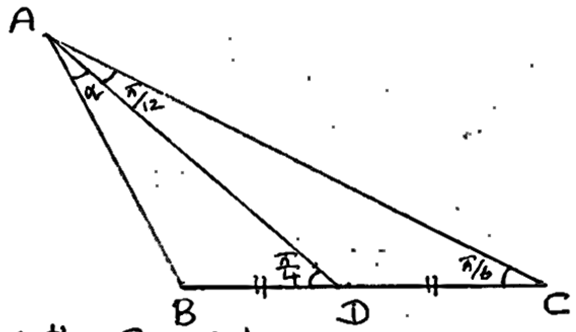
$b\sin A = a\sin B$

$\frac{\sin A}{a} = \frac{\sin B}{b}$ (5)

Similarly $\frac{\sin A}{a} = \frac{\sin C}{c}$ (5)

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\boxed{25}$



Using the Sine Rule:

for ABD; $\frac{\sin\alpha}{BD} = \frac{\sin(\pi-(\alpha+\frac{\pi}{4}))}{AD}$ (10)

$\frac{\sin\alpha}{BD} = \frac{\sin(\alpha+\frac{\pi}{4})}{AD}$ —(1) (5)

for ADC; $\frac{\sin\frac{\pi}{12}}{DC} = \frac{\sin\frac{\pi}{6}}{AD}$ —(2) (10)

$$\frac{\sin\alpha}{\sin\frac{\pi}{12}} = \frac{\sin(\alpha+\frac{\pi}{4})}{\sin\pi/6} \quad (5)$$

$$\frac{\sin\alpha}{\sin\frac{\pi}{12}} = \frac{\sin(\alpha+\frac{\pi}{4})}{2\sin\frac{\pi}{12}\cos\frac{\pi}{12}} \quad (5)$$

$$2\sin\alpha\cos\frac{\pi}{12} = \sin(\alpha+\frac{\pi}{4}) \quad (5) \qquad \boxed{40}$$

$$2\sin\alpha\cos\frac{\pi}{12} = \sin\alpha\cos\frac{\pi}{4}+\cos\alpha\sin\frac{\pi}{4} \quad (5)$$

$$2\sin\alpha\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin\alpha\frac{1}{\sqrt{2}} + \cos\alpha\frac{1}{\sqrt{2}} \quad (5)$$

$$\sqrt{3}\sin\alpha = \cos\alpha \quad (5)$$

$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6} \quad (5) \qquad \boxed{20}$$

(c) $x>0$

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$$

Let $\alpha = \tan^{-1}x$ (5)

$\tan\alpha = x$, $0<\alpha<\pi/2$

$$\tan^{-1}\left(\frac{1-\tan\alpha}{1+\tan\alpha}\right) = \frac{1}{2}\alpha \quad (5)$$

$$\tan^{-1}\left(\tan(\frac{\pi}{4}-\alpha)\right) = \frac{1}{2}\alpha \quad (5)$$

$$\frac{\pi}{4}-\alpha = \frac{1}{2}\alpha \quad (5) \qquad \left[\because 0<\alpha<\pi/2 \Rightarrow -\frac{\pi}{4}<\frac{\pi}{4}-\alpha<\frac{\pi}{4}\right]$$

$$\frac{3}{2}\alpha = \frac{\pi}{4}$$

$$\alpha = \frac{\pi}{6} \quad (5) \qquad \boxed{25}$$

Alter Let $\beta = \tan^{-1}\left(\frac{1-x}{1+x}\right)$, $\alpha = \tan^{-1}x$ (5)

$$\beta = \frac{1}{2}\alpha$$

$$2\beta = \alpha$$

$$\tan 2\beta = \tan\alpha$$

$$\frac{2\tan\beta}{1-\tan^2\beta} = \tan\alpha \quad (5)$$

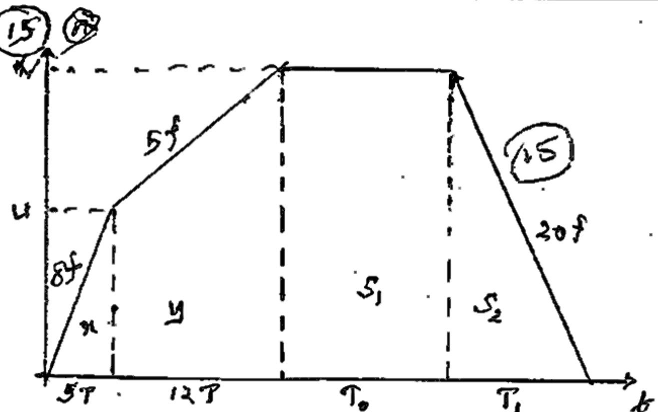
$$\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = x \quad (5)$$

$$\frac{2(1-x)(1+x)}{4x} = x \qquad \boxed{25}$$

$$x^2 = \frac{1}{3}$$

(5) $x = \frac{1}{\sqrt{3}}$ $[\because x>0]$ $\Rightarrow \alpha = \frac{\pi}{6}$ (5)

15 (a)



(ii) $\frac{u}{5T} = 8f$ (5)

$u = 40fT$ (5)

$\frac{v-u}{12T} = 5f$ (5)

$v = 100fT$ (5)

(iii) $x+y = \frac{1}{2}\cdot 5T.u + \frac{1}{2}(u+v)\cdot 12T$ 5+5

$= 940fT^2$ 5

(iv) $\frac{v}{T_1} = 20f$ 5

$T_1 = 5T$ 5

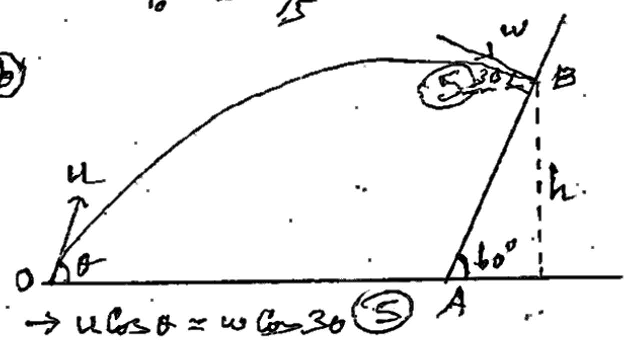
$S_1+S_2 = 5(x+y)$ (5)

$vT_0 + \frac{1}{2}vT_1 = 5\times 940fT^2$ (5)

$100fT T_0 + \frac{1}{2}100f.5T^2 = 5\times 940fT^2$

$T_0 = 89/2\, T$ (5) $\qquad \boxed{75}$

(b)



$\rightarrow$ $u\cos\theta = w\cos 30$ (5)

$= \sqrt{3ag}\cdot\frac{\sqrt{3}}{2} = \frac{3}{2}\sqrt{ag}$ (5)

$\uparrow$ $-w\cos 60 = u\sin\theta - g\cdot\sqrt{\frac{12a}{g}}$ (5)

$-\sqrt{3ag}\cdot\frac{1}{2} = u\sin\theta - 2\sqrt{3ag}$

$u\sin\theta = \frac{3}{2}\sqrt{3ag}$ (5)

$\tan\theta = \sqrt{3}$ (5)

$\theta = \frac{\pi}{3}$ (5)

$u\sin 60 = \frac{3}{2}\sqrt{3ag}$ (5)

$u = 3\sqrt{ag}$ (5)

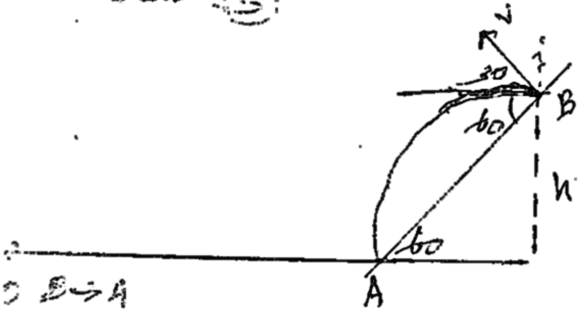
$\uparrow h = u\sin\alpha\, t - \frac{1}{2} g t^2$

$h = 3\sqrt{2ag}\cdot \sin 60\, t - \frac{1}{2} g \cdot \left(\frac{12a}{g}\right)$ (5)

$= 3\sqrt{2ag}\cdot \frac{\sqrt{3}}{2}\cdot\sqrt{\frac{12a}{g}} - \frac{1}{2} g \frac{12a}{g}$

$= 9a - 6a$

$= 3a$ (5)



$B \rightarrow A$

$\uparrow -3a = v\sin 30\, t - \frac{1}{2} g t^2$ (5)

$\rightarrow h\tan 30 = v\cos 30\, t$ (5)

$t = \frac{2a}{v}$ (5)

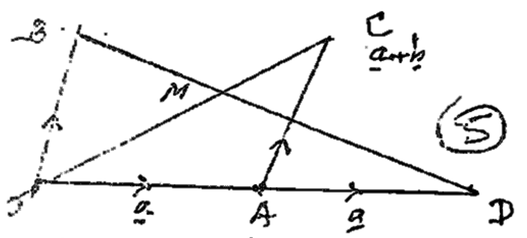
$-3a = \frac{v}{2}\left(\frac{2a}{v}\right) - \frac{1}{2} g \left(\frac{2a}{v}\right)^2$

$v^2 = \frac{ag}{2}$

$v = \sqrt{\frac{ag}{2}}$ (5)

$\boxed{75}$

(15) a)



(5)

i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ (5)

$= -\underline{a} + \underline{a} + \underline{b}$

$= \underline{b} = \overrightarrow{OB}$ (5)

$\overrightarrow{OB} = \underline{b}$

ii) $\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB}$ (5)

$= -2\underline{a} + \underline{b}$ (5)

iii) $\overrightarrow{OM} = \lambda(\underline{a} + \underline{b})$ (5)

$\overrightarrow{MB} = \mu\overrightarrow{DB}$

$= \mu(\underline{b} - 2\underline{a})$ (5)

$\overrightarrow{OM} + \overrightarrow{MB} = \overrightarrow{OB}$ (5)

$\lambda(\underline{a}+\underline{b}) + \mu(\underline{b} - 2\underline{a}) = \underline{b}$ (5)

$(\lambda - 2\mu)\underline{a} + (\lambda + \mu - 1)\underline{b} = \underline{0}$ (5)

$\lambda - 2\mu = 0 \qquad \lambda + \mu - 1 = 0$ (5)

$\mu = \frac{1}{3}$ (5)

$\lambda = \frac{2}{3}$ (5)

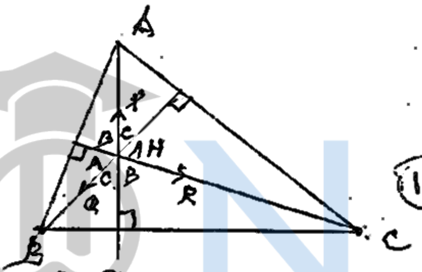
$\frac{MB}{DB} = \frac{1}{3}$

$\frac{MB}{MD} = \frac{1}{2} \Rightarrow DM : MB = 2 : 1$ (5)

$\frac{OM}{OC} = \frac{2}{3} \Rightarrow OM : MC = 2 : 1$ (5)

$\boxed{75}$

b)



(10)

By Lami's Theorem

$\frac{P}{\sin(B+C)} = \frac{Q}{\sin(A+C)} = \frac{R}{\sin(A+B)}$ (30)

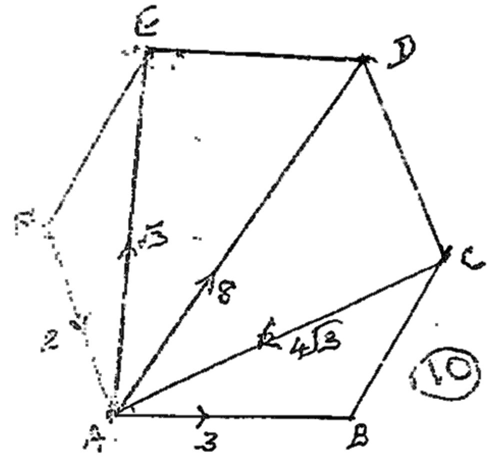
$\frac{P}{\sin(\pi - A)} = \frac{Q}{\sin(\pi - B)} = \frac{R}{\sin(\pi - C)}$ (15)

$\frac{P}{\sin A} = \frac{Q}{\sin B} = \frac{R}{\sin C}$ (5)

Sin Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (10)

$\Rightarrow \frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$ (5)

$\boxed{75}$



10

$\rightarrow X = 3 - 4\sqrt{3}\cos 30 + 8\cos 60 + 2\cos 60$ 20

$= 2$ 5

$\uparrow Y = \sqrt{3} + 8\cos 30 - 4\sqrt{3}\cos 60 - 2\cos 30$ 20

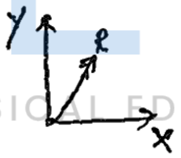
$= 2\sqrt{3}$ 5

$R^2 = 2^2 + (2\sqrt{3})^2$ 5

$= 4$ 5

$\tan\theta = \sqrt{3}$ 5

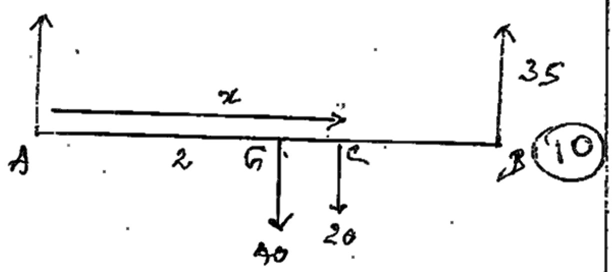
$\theta = \frac{\pi}{3}$ 5



Resultant acts along AD 10

4 N Force should be added along $\vec{DA}$ to bring the System Equilibrium. 10

b)



10

$A\curvearrowright \quad 2.40 + x.20 - 35 \cdot 4 = 0$ 25

$80 + 20x - 140 = 0$ 10

$x = 3$ 5