SOUTH EASTERN UNIVERSITY OF SRI LANKA
Faculty of Applied Sciences
Department of Mathematical Sciences

MTM 22031 ELEMENTARY DIFFERENTIAL EQUATIONS

WORKSHEET 1

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1. Classify each of the following DEs as per type, order, degree and linearity. Also determine whether the equation is homogeneous or not.

   1. $\frac{dy}{dx}+\frac{d^3y}{dx^3}=5\left(\frac{dy}{dx}\right)^2$.
   2. $\frac{d^2y}{dx^2}+e^x=\tan y$.
   3. $\frac{d^3y}{dx^3}=\left(1+\left(\frac{d^2y}{dx^2}\right)^2\right)^{5/2}$.
   4. $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=0$.
   5. $\left(\frac{dr}{ds}\right)^3=\left(\frac{d^4r}{ds^4}+1\right)^2$.
   6. $\sqrt{\frac{d^2\rho}{d\theta^2}+\rho}=\sin\rho$.
   7. $\frac{dy}{dx}=\left(\frac{1+x}{1+y}\right)^{1/3}$.
   8. $\sin x\frac{d^2y}{dx^2}-(1-y^2)\frac{dy}{dx}+5y=0$.
   9. $\frac{d^2y}{dx^2}-2y\frac{dy}{dx}+xy^2=\frac{d^3}{dx^3}(e^{-2x})$.
   10. $\frac{d^2y}{dx^2}+\sqrt{\frac{dy}{dx}}+xy=0$

2. For what values of the constant m will $y=e^{mx}$ be a solution of the differential equation $2y'''+y''-5y'+2y=0$.
   What is the general solution of the equation?

3. Verify and reconcile that $y=\ln x+A$ and $\sinh y+\cosh y=Cx$ are primitives of
   $$\frac{dy}{dx}=\frac{1}{x}.$$

4. Determine whether or not each of the following function is a solution of the differential equation $(x+1)y''+xy'-y=(x+1)^2$:
   (a) $y_1(x)=e^{-x}+x^2+1$.
   (b) $y_2(x)=x^2+1$.

5. Verify that $x^2=2y^2\ln y$ is a solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$.

6. Verify that $x^2+xy=C$ is a solution of the differential equation
   $$x^2\frac{d^2y}{dx^2}-2(x+y)=0$$
   for any value of the constant C.

7. Verify that $y=4\,e^{3x}\sin x$ is a solution of the differential equation
   $$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+10\,y=0.$$
   What is this solution? Justify your answer.