Irrational Rhythms

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One of the most fundamental features of Western music is rhythm, the spacing between notes in time that come in multiples (or fractions) of a fixed interval. A whole note divides into two half notes, which divide into two quarter notes each, which divide into eighth notes, etc. This binary system carries many interesting properties that we often take for granted. For example, it is possible to speed up or slow down a melody by a factor of 2 just by changing the notation and keeping the tempo the same, so two copies of the same music at different speeds may be placed on the same score. This allows for so-called canons in augmentation or diminution. An instant in time may be given a "strength" based on how many binary subdivisions it takes to produce it: the start of a measure is stronger than the center of one, which is in turn stronger than the beats immediately to its left and right. (In 4/4 time, theis means that beat 1 is stronger than 3, which is stronger than 2 and 4).

0: length $1+\sqrt{2}$ (long), 1: length 1 (short), 2: length $\sqrt{2}$ (medium)

- $0 \to 001$, $1 \to 0$: $1, 0, 001, 0010010, 00100100010010001, \ldots$ (a)
  - $0 \hookrightarrow 21$: $1, 21, 21211, 212112121121, 21211212112121211212112121211, \ldots$ (b)
    No rule
  - $0 \to 12$: $1, 12, 12121, 121211212112, 12121121211212121121211212121, \ldots$ (c)
    $1 \to 12$, $2 \to 121$
- $0 \to 010$, $1 \to 0$: $1, 0, 010, 0100010, 01000100100100010, \ldots$ (d)
  - $0 \hookrightarrow 21$: $1, 21, 21121, 211212121121, 21121212112121121211212121121, \ldots$ (e)
    $1 \to 21$, $2 \to 211$
  - $0 \hookrightarrow 12$: $1, 12, 12112, 121121212112, 12112121211212112121121212112, \ldots$ (f)
    $1 \to 12$, $2 \to 112$
- $0 \to 100$, $1 \to 0$: $1, 0, 100, 0100100, 10001001000100100, \ldots$ (g)
  - $0 \hookrightarrow 21$: $1, 21, 12121, 211212112121, 12121211212112121211212112121, \ldots$ (h)
    $1 \to 21$, $2 \to 121$
  - $0 \hookrightarrow 12$: $1, 12, 11212, 121121211212, 11212121121211212121121211212, \ldots$ (i)
    No rule
- $1 \to 12, 2 \to 211$: $1, 12, 12211, 122112111212, 12211211121221112121221112211, \ldots$ (j)
- $1 \to 21, 2 \to 112$: $1, 21, 11221, 212111211221, 11221112212121112212111211221, \ldots$ (k)

Reverses: (a)-(g), (b)-(i), (c)-(h), (d)-(d), (e)-(f), (j)-(k)
(g) and (k) oscillate between 2 limit words, all others converge
(j) and (k) do not have maximally even spacing (MOS-like), all others do
Personal favorite: (e) - derived from palindromic (d), long beat first (swing)