Deriving a Trig Identity

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There are some lesser-known trigonometric identities relating the sum of two functions with different frequencies to a product of two functions.

$$\cos(ax) + \cos(bx) = 2\cos\left(\frac{a+b}{2}x\right)\cos\left(\frac{a-b}{2}x\right)$$
$$\sin(ax) + \sin(bx) = 2\sin\left(\frac{a+b}{2}x\right)\cos\left(\frac{a-b}{2}x\right)$$

$$\cos(2ax) + \cos(2bx) = 2\cos((a+b)x)\cos((a-b)x)$$

$$\sin(2ax) + \sin(2bx) = 2\sin((a+b)x)\cos((a-b)x)$$

$$(\cos(2a) + \cos(2b)) + i(\sin(2a) + \sin(2b)) = 2\cos(a+b)\cos(a-b) + 2i\sin(a+b)\cos(a-b)$$
$$\cos(2a) + \cos(2b) = 2\cos(a+b)\cos(a-b)$$

$$\operatorname{cis}(2a) + \operatorname{cis}(2b) = \operatorname{cis}(a+b)(\operatorname{cis}(a-b) + \operatorname{cis}(b-a))$$