Irrational Rhythms

Aresh Pourkavoos

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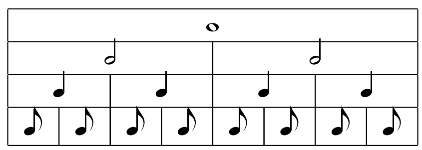
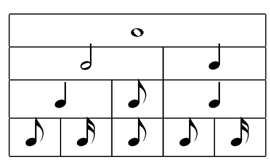
One of the most fundamental features of music (particularly in the West) is rhythm, the spacing between notes in time that come in multiples (or fractions) of a fixed interval. A whole note divides into two half notes, which divide into two quarter notes each, which divide into eighth notes, etc.



This binary system carries many interesting properties that we often take for granted. For example, it is possible to double or halve the speed of a melody just by changing the notation and keeping the tempo the same. This allows for so-called canons in augmentation or diminution. where two copies of the same music at different speeds are played at the same time. An instant in time may be given a "strength" based on how many binary subdivisions it takes to produce it: the start of a measure is stronger than the center of one, which is in turn stronger than the beats immediately to its left and right, which are stronger than the times in the center of each beat. In 4/4 time, this means that beat 1 is stronger than beat 3, which is stronger than beats 2 and 4, which are stronger than the "and"s of each beat.

Not all music follows this binary system exactly. Many pieces have time signatures other than powers of 2 like 3/4 or 5/4, and tuplets can in principle subdivide a time interval into any number of equal parts. In swing, each beat is subdivided unequally, and the dividing line is not exactly specified: musicians simply "feel" the rhythm and coordinate with each other.

However, all of these still fall within the realm of rational numbers: any piece of music using tuplets is based on some equally-spaced (isochronic) underlying beat of which all time intervals are a multiple.



0: length $1+\sqrt{2}$ (long), 1: length 1 (short), 2: length $\sqrt{2}$ (medium)

- $0 \to 001$, $1 \to 0$: $1, 0, 001, 0010010, 00100100010010001, \ldots$ (a)
  - $0 \hookrightarrow 21$: $1, 21, 21211, 212112121121, 21211212112121211212112121211, \ldots$ (b)
    No rule
  - $0 \to 12$: $1, 12, 12121, 121211212112, 12121121211212121121211212121, \ldots$ (c)
    $1 \to 12$, $2 \to 121$
- $0 \to 010$, $1 \to 0$: $1, 0, 010, 0100010, 01000100100100010, \ldots$ (d)
  - $0 \hookrightarrow 21$: $1, 21, 21121, 211212121121, 21121212112121121211212121121, \ldots$ (e)
    $1 \to 21$, $2 \to 211$
  - $0 \hookrightarrow 12$: $1, 12, 12112, 121121212112, 12112121211212112121121212112, \ldots$ (f)
    $1 \to 12$, $2 \to 112$

- $0 \to 100,\ 1 \to 0$: $1, 0, 100, 0100100, 10001001000100100, \ldots$ (g)
  - $0 \hookrightarrow 21$: $1, 21, 12121, 211212112121, 1212121121211212121121212112121, \ldots$ (h)
    $1 \to 21,\ 2 \to 121$
  - $0 \hookrightarrow 12$: $1, 12, 11212, 121121211212, 1121212112121121212121121211212, \ldots$ (i)
    No rule
- $1 \to 12, 2 \to 211$: $1, 12, 12211, 122112111212, 122112111212211121212221112211, \ldots$ (j)
- $1 \to 21, 2 \to 112$: $1, 21, 11221, 212111211221, 112211122121211122121112 11221, \ldots$ (k)

Reverses: (a)-(g), (b)-(i), (c)-(h), (d)-(d), (e)-(f), (j)-(k)
(g) and (k) oscillate between 2 limit words, all others converge
(j) and (k) do not have maximally even spacing (MOS-like), all others do
Personal favorite: (e) - derived from palindromic (d), long beat first (swing)