A Generating Function Problem

Aresh Pourkavoos

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$$f(x,y)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{(j+k+1)!}x^jy^k$$

$$\frac{d^{j+k}f}{dx^jdy^k}(0,0)=\frac{j!k!}{(j+k+1)!}=B(j+1,k+1)$$

$$B(j+1,k+1)=B(j+1,k+2)+B(j+2,k+1)$$

$$\frac{d^{j+k}f}{dx^jdy^k}(0,0)=\frac{d^{j+k+1}f}{dx^{j+1}dy^k}(0,0)+\frac{d^{j+k+1}f}{dx^jdy^{k+1}}(0,0)=\frac{d}{dx}\left(\frac{d^{j+k}f}{dx^jdy^k}\right)(0,0)+\frac{d}{dy}\left(\frac{d^{j+k}f}{dx^jdy^k}\right)(0,0)$$

$$f=\frac{df}{dx}+\frac{df}{dy}$$

$$B(j+1,1)=\frac{1}{j+1}$$

$$\frac{d^jf}{dx^j}(0,0)=\frac{1}{j+1}$$

$$f(x,0)=\sum_{j=0}^{\infty}\frac{1}{j+1}\frac{x^j}{j!}=\sum_{j=0}^{\infty}\frac{x^j}{(j+1)!}=\frac{1}{x}\sum_{j=0}^{\infty}\frac{x^{j+1}}{(j+1)!}=\frac{1}{x}\left(\sum_{j=0}^{\infty}\left(\frac{x^j}{j!}\right)-\frac{x^0}{0!}\right)=\frac{e^x-1}{x}$$

$$f(0,y)=\frac{e^y-1}{y}$$