

# Recurrence Relations and Differential Equations

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What do these two have in common?

$$\begin{aligned}f''(x) &= f'(x) + f(x) \\ F_{n+2} &= F_{n+1} + F_n\end{aligned}$$

The first is a differential equation, which relates the value of a function to its derivatives, and the other is a recurrence relation, a rule for building a number sequence by looking at previous entries. Beyond their visual similarities, their solutions both involve the golden ratio and its conjugate,

$$\begin{aligned}\varphi &= \frac{1 + \sqrt{5}}{2} \approx 1.618 \\ \bar{\varphi} &= \frac{1 - \sqrt{5}}{2} \approx -0.618,\end{aligned}$$

the two solutions of the polynomial  $x^2 = x + 1$ . Specifically, the general solution to the first equation is

$$f(x) = a \exp(\varphi x) + b \exp(\bar{\varphi} x),$$

and the second is

$$F_n = a\varphi^n + b\bar{\varphi}^n.$$