A Projection

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$$Q = \frac{1}{2\sqrt{2+\varphi}} \begin{pmatrix} \varphi & \varphi & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & \varphi & \varphi \\ 0 & 0 & \varphi & \varphi & 1 & -1 \\ -1 & -1 & \varphi & -\varphi & 0 & 0 \\ \varphi & -\varphi & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & \varphi & -\varphi \end{pmatrix}$$

The 6×6 matrix above is orthonormal, meaning that it represents a rotation of 6-dimensional space. φ is the golden ratio $\frac{1+\sqrt{5}}{2}$, the positive solution to the equation $\varphi^2 = \varphi + 1$. I found it while exploring the Penrose tiling(s) and the mathematical theory behind it, which I hope to summarize here.

Although Roger Penrose has multiple related aperiodic tilings in his name, When I mention "the" Penrose tiling, I refer to the one made of two golden rhombi, known as the P3 tiling. However, even this is insufficient to narrow it down, as depending on the method of construction, there might be uncountably many variations of this tiling. Rather than get bogged down in definitions too early on.