Monotone Cubic Interpolation

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Given a set of data points $(x_1, y_1), \ldots, (x_n, y_n)$ in which both coordinates strictly increase moving down the list (i.e. $x_1 < \ldots < x_n$ and $y_1 < \ldots < y_n$), how can we define a function $f(x) = y$ which passes through all points and which has positive slope everywhere? Drawing line segments between adjacent points almost works, but the derivative is generally undefined at each point, since the slopes of the segments on either side may be different. The apparent solution would be to use cubic interpolation instead, which allows for the derivative at each point to be set, and cubic curves between each pair of adjacent points may be found.

(Hermite polynomials)

This raises the question of what the derivatives shoud be.

A common approach to cubic interpolation is, for every point, to connect the points on its left and right and use the slope of the resulting segment. If the point is on the far left or right, the slope of the segment between it and its adjacent point is used. However, this approach does not guarantee monotonicity for monotone inputs.

(Example)