The Continuation Monad

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Within System F, there is a monad Cont which maps type α to $\mathrm{Cont}\,\alpha := (\omega : \mathrm{Type}) \to (\alpha \to \omega) \to \omega$, i.e. a polymorphic function parameterized by type ω. In this monad, $\mathrm{return} : (\alpha : \mathrm{Type}) \to \alpha \to \mathrm{Cont}\,\alpha$ is defined by $\mathrm{return}\,\alpha\ a\ \omega\ f := f\ a$, where $a : \alpha$ and $f : \alpha \to \omega$. The operation bind has type

$$(\alpha : \mathrm{Type}) \to (\beta : \mathrm{Type}) \to (\alpha \to \mathrm{Cont}\,\beta) \to \mathrm{Cont}\,\alpha \to \mathrm{Cont}\,\beta.$$

In order to see how it should be defined, we may unpack its arguments:

$$\begin{aligned} \alpha &: \mathrm{Type} \\ \beta &: \mathrm{Type} \\ f &: \alpha \to \mathrm{Cont}\,\beta \\ g &: \mathrm{Cont}\,\alpha \end{aligned}$$

The return type, then, is $\mathrm{Cont}\,\beta = (\omega : \mathrm{Type}) \to (\alpha \to \omega) \to \omega$, so we may take two more arguments:

$$\begin{aligned} \omega &: \mathrm{Type} \\ h &: \beta \to \omega \end{aligned}$$

So the new return type is ω. Given what we have, there are a few potential ways to obtain a term of type ω:

- The type of f may be expanded to $\alpha \to l$

is defined by

$$\mathrm{bind}\,\alpha\ \beta\ f\ g\ \omega\ h := g\ \omega\ (\lambda a.(f\ a\ \omega\ h)).$$