Minimal Polynomials of Real Parts of Roots of Unity

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$$\begin{aligned}
r &= 2\cos\left(\frac{m}{n}\tau\right) \\
p(r) &= 0 \\
z &= \operatorname{cis}\left(\frac{m}{n}\tau\right) \\
z^{-1} &= \operatorname{cis}\left(-\frac{m}{n}\tau\right) \\
r &= z + z^{-1} \\
q(z) &= 0
\end{aligned}$$

$$\begin{aligned}
n &= 7 \\
0 &= z^3 + z^2 + z + 1 + z^{-1} + z^{-2} + z^{-3} \\
&= a + br + cr^2 + dr^3 \\
&= a + b(z + z^{-1}) + c(z + z^{-1})^2 + d(z + z^{-1})^3 \\
&= a + b(z + z^{-1}) + c(z^2 + 2 + z^{-2}) + d(z^3 + 3z + 3z^{-1} + z^{-3}) \\
&= dz^3 + cz^2 + (b + 3d)z + (a + 2c) + (b + 3d)z^{-1} + cz^{-2} + dz^{-3}
\end{aligned}$$

$$\begin{aligned}
\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\
\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
\begin{bmatrix} a \\ c \end{bmatrix} &= \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\
\begin{bmatrix} b \\ d \end{bmatrix} &= \begin{bmatrix} -2 \\ 1 \end{bmatrix}
\end{aligned}$$

$$r^3 + r^2 - 2r - 1 = 0$$

1	$r-2$
2	$r+2$
3	$r+1$
4	r
5	r^2+r-1
6	$r-1$
7	r^3+r^2-2r-1
8	r^2-2
9	r^2-3r+1
10	$r^2\pm r-1$