Extending Pascal's Triangle

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Pascal's triangle contains binomial coefficients:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}, n \in \mathbb{N}, k \in \mathbb{N}, k \le n$$

Satisfies Pascal's identity for all arguments in range:

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

How to extend to all integers, i.e. fill the grid with numbers?
Entries where $n \ge 0$ are unambiguous: all numbers outside of triangle are 0
$n = -1$: can choose a single number freely, after which the rest of the row is forced

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        0           0        0        0      0    1    0     0         0          0          0
   -1           1        -1       1      -1   1    1   -1      1         -1          1                -1
        14          -11       8       -5      2    1    2    -5       8        -11         14
  158         -102       58      -26      6    2    2    6      -26       58        -102             158
       -954        444      -154     24      6    4    6     24      -154      444        -954
-9432        3708      -1044    120     24   12   12   24     120     -1044      3708            -9432
       33984       -8028     720     120     48   36   48    120     720      -8028       33984
341136       -69264     5040     720     240  144  144  240    720     5040      -69264          341136
       -663696     40320    5040    1440    720  576  720   1440    5040     40320      -663696
6999840       362880    40320   10080    4320 144  144 4320   10080   40320     362880          -6999840
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Row labels (left margin, top to bottom): $\frac{\circ}{1!}$, $\frac{\circ}{2!}$, $\frac{\circ}{3!}$, $\frac{\circ}{4!}$, $\frac{\circ}{5!}$, $\frac{\circ}{6!}$, $\frac{\circ}{7!}$, $\frac{\circ}{8!}$, $\frac{\circ}{9!}$, $\frac{\circ}{10!}$