New Idea

Aresh Pourkavoos

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Let F be a set of sets of vertices, $V = \bigcup F$. F is a polytope if there exists a function $d : F \to \mathbb{Z}$ such that:

1. Null face: $\varnothing \in F$, and $d(\varnothing) = -1$.
2. Ordering: for all $a, b \in F$, if there exists no $c \in F$ such that $a \subset c$ and $c \subset b$, then $d(b) = d(a) + 1$.
3. Recursion: define a *facet* of P to be a triplet (f, F', d') where $f \in F$, $d(f) = d(V) - 1$, $F' = F \cap \mathscr{P}(V)$, and $d' = d\big|_{F'}$. Then all facets of P are also polytopes.