Uncertainty of Probability

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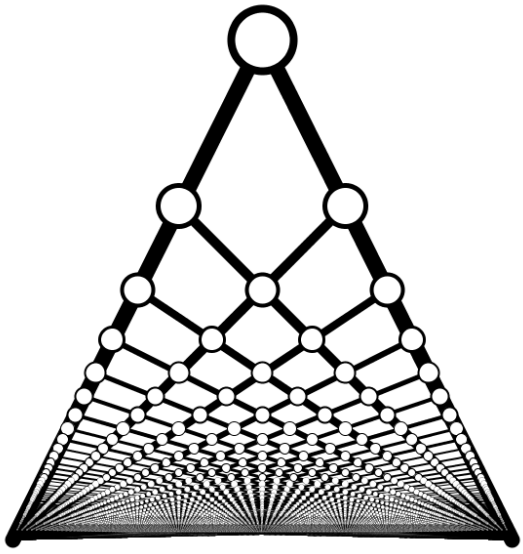
Experiment with unknown probability of success p
Prior distribution is uniform (1) over interval $[0, 1]$
Conduct experiments, obtain j successes and k failures
New distribution is $p^j(1-p)^k$ (normalized to have integral 1)
Mean is $\frac{j+1}{j+k+2}$, as if adding a "virtual" success and failure
May visualize all possible "paths" as experiments are performed as below



Initial state is the top vertex, each experiment moves down a "layer"
Failure moves left, success moves right
Horizontal position is expected probability, e.g. fail-fail-succeed is $\frac{1}{2} \to \frac{1}{3} \to \frac{1}{4} \to \frac{2}{5}$
State approaches position on bottom edge corresponding to probability

Diagram may also be obtained by perspective transformation of a square grid
Straight lines are preserved: adding successes/failures moves straight
Start with point $(j, k, 2)$ (success, failure, unknown)
Normalize (make sum 1) by dividing coordinates by $j + k + 2$
Use coefficients to combine $(1, 0)$ (success), $(0, 0)$ (failure), $\left(\frac{1}{2}, 1\right)$ (unknown)
Vertical position ranges from 0 to 1 like probability: measures "uncertainty"
Given by $\frac{2}{j+k+2}$: depends only on total number of trials
Horizontal position may be obtained from distribution alone: expected value generalizes to all distributions
How to generalize uncertainty?