Notes on Type Theory

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Base types, sum/product/function types, dependent types, inductive types, universes
Base types: $\bot$ AKA false, $\top$ AKA true, bool
#:$\top$, 0:bool, 1:bool ($\bot$ has no terms), $\bot$:Type, $\top$:Type, bool:Type

Dependent types: $\sum$ AKA $\exists$, $\prod$ AKA $\forall$
$\exists$

$$A + B \qquad A \vee B \tag{1}$$
$$A \times B \qquad A \wedge B \tag{2}$$
$$B^A \qquad A \to B \tag{3}$$

(1): The union type of types A and B has terms which are either of type A or of type B. For example, if A had 2 terms (and thus stored one bit of information) and B had 3 terms (one trit), there would be $2 + 3 = 5$ possible values of the union.

(2): The pair type of A and B has terms which contain both a term of type A and a term of type B. Using the previous examples of A and B, there would be $2 \times 3 = 6$ possible pairs.

(3): The function type that goes from A to B has terms which associate exactly one (non-unique) element of B with each element of A. Since for each of the 2 terms of A (input), there are 3 choices for the corresponding element of B (output), there would be $3^2 = 9$ possible functions.

$$A \times (B + C) = (A \times B) + (A \times C) \qquad A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C) \tag{4}$$
$$C^{A+B} = C^A \times C^B \qquad (A \vee B) \to C \equiv (A \to C) \wedge (B \to C) \tag{5}$$
$$(B \times C)^A = B^A \times C^A \qquad A \to (B \wedge C) \equiv (A \to B) \wedge (A \to C) \tag{6}$$
$$C^{A \times B} = (C^B)^A \qquad (A \wedge B) \to C \equiv A \to (B \to C) \tag{7}$$
$$A + 0 = A \qquad A \vee F \equiv A \tag{8}$$
$$A \times 1 = A \qquad A \wedge T \equiv A \tag{9}$$

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