Rendering zonotope cross-sections

Aresh Pourkavoos

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This project is the culmination of a train of thought that started around a year ago with the question of whether I could build an icosahedron in Minecraft. I had learned that the coordinates of its vertices were based on the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$, the positive solution to the equation $\varphi^2 = \varphi + 1$. More specifically, an icosahedron can be constructed with three perpendicular golden rectangles.

[Image]

If the icosahedron has a side length of 2, then the rectangles are $2 \times 2\varphi$, and the coordinates of their vertices are

$$(0, \pm 1, \pm \varphi), (\pm 1, \pm \varphi, 0), (\pm \varphi, 0, \pm 1),$$

or more succinctly, all permutations and sign changes of

$$(0, 1, \varphi).$$

Since φ is irrational, there is no way to scale these coordinates such that they fall exactly on integers. (For the readers who are considering rotations, the proof that they don't work either is left as an exercise.)

Approximation of phi
What is a zonotope?
Construction method
C program
The omnitruncated 120-cell
The golden ring
The omnitruncated 24-cell
The $\sqrt{2}$ ring