Convergence of Products from Sums

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March 3, 2022

$$\prod_{n=1}^{\infty} a_n:$$

- If $a_n = 0$ for some n, all partial products after n are 0: "diverges" to 0
- Otherwise, rewrite all a_n as $s_n \exp(b_n)$, where $s_n = \operatorname{sgn}(a_n) = \pm 1$ and $b_n = \ln |a_n|$, so the product is

$$\left(\prod_{n=1}^{\infty} s_n\right) \exp\left(\sum_{n=1}^{\infty} b_n\right)$$

- the latter sum is equivalent to

$$\sum_{n=1}^{\infty} (|a_n| - 1)$$