UNIT-6

Graph → A graph written as

$G = G(V, E)$ consists of two components:

(i) The finite set of vertices V, also called points or nodes and

(ii) The finite set of (directed) edges E, also called lines or arcs connecting pair of vertices.

I.e in a graph there is a mapping from the set of edges E to the set of vertices V such that each $e \in E$ is associated with ordered or unordered pair of elements of V.

Basic Properties of a Graph]

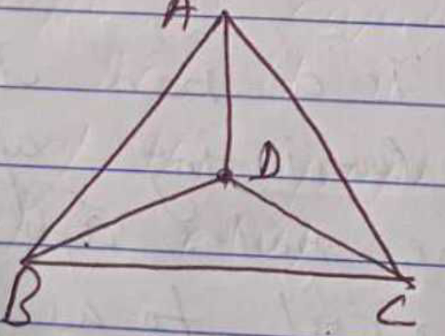
A Graph is a non linear data structure consisting of nodes and edges. The nodes are sometimes also refrred to as vertices and the edges are lines or arcs that connect any two nodes in the graph.

properties of Graph are basically used for characterization of graph depending on their Structures.

The properties of graph these are as follows

1. **Distance between two vertices ?**

It is basically the Number of edges that are available in the shortest path between vertices A and vertice B. If there is more than one edge which is used two connect two vertices then we basically considered the shortest path as the distance between these two vertices.

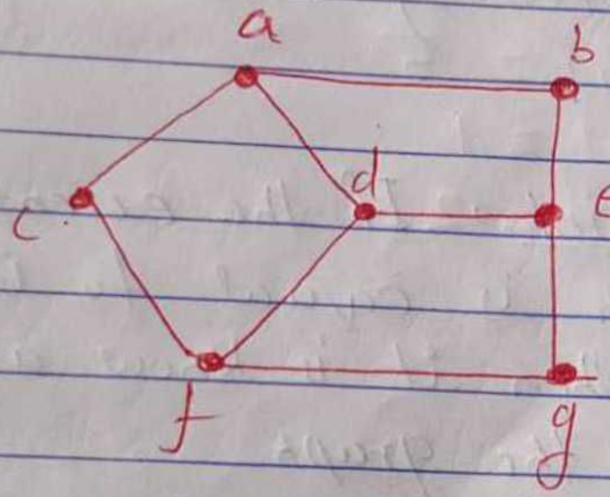


There is many paths from vertex B to vertex D.

B → C → A → D length = 3
B → D length = 1
B → A → D length = 2
B → C → D length = 2
B → C → A → D length = 3
Hence the minimum distance between vertex B and vertex D is 1.

2- Eccentricity of a vertex :-

Eccentricity of a vertex is the Maximum distance between a vertex to all other vertex. It is denoted by e(V)



- The distance from vertex a to b is 1
- The distance from vertex a to c is 1
- The distance from vertex a to f is 2
- The distance from vertex a to b is 1
- The distance from vertex a to e is 2.
- The distance from vertex a to g is 3.

Hence the maximum eccentricity of vertex is 3.

3. Radius of Connected Graph]

The Radius of a Connected graph is the minimum eccentricity from all the vertices.

4. Diameter of a Graph]

Diameter of a graph is the maximum eccentricity from all the vertices. It is denoted by $d(G)$

5. Central point → If the eccentricity of the graph is equal to its radius, then it is know as central point of the graph

$$r(v) = e(v)$$

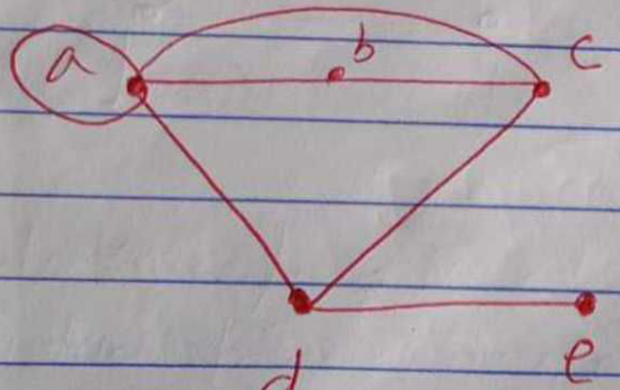
(6) Centre The set of all centre point of the graph is know as centre of the graph.

7. Circumference - The total Number of edges in the logest cycle of graph G is know as the circumference of G.

8. Girth - The total Number of edge in the shortest cycle of graph G is know as girth. it is denoted by g(G)

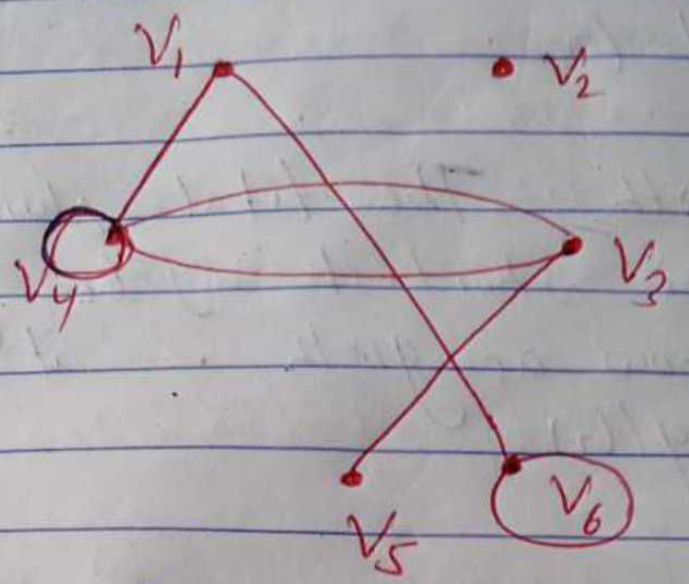
Degree of a Vertex :

The degree of a vertex denoted by d(v) or deg(v) is the Number of edges (or arcs) connected with it



Vertex a has degree = 5
degree of vertex b = 2
degree of vertex d and c = 3

Q. find the degree of each in the following figure



$deg(V_1) = 2$

$degree(V_2) = 0$

$degree(V_3) = 3$

$deg(V_4) = 5$

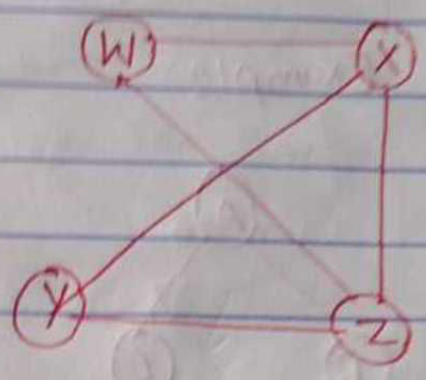
$deg(V_5) = 1$

$deg(V_6) = 2$

PATH → A path is a sequence of edges that begins at a vertex, and travels from vertex to vertex along edges of the graph. The number of edges on the path is called the length of the path.

Example

Consider the graph on the right.

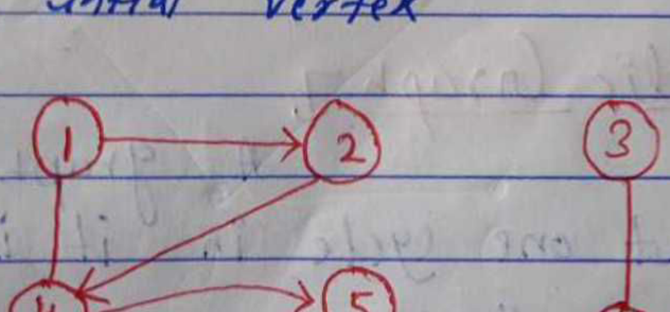


W → X → Y → Z → X

Corresponds to a path of length 4.

* SIMPLE PATH → A path is simple if each vertex is distinct.

* CIRCUIT → A circuit is a path in which the terminal vertex coincides with the intial vertex



Simple path : [1, 2, 4, 5]

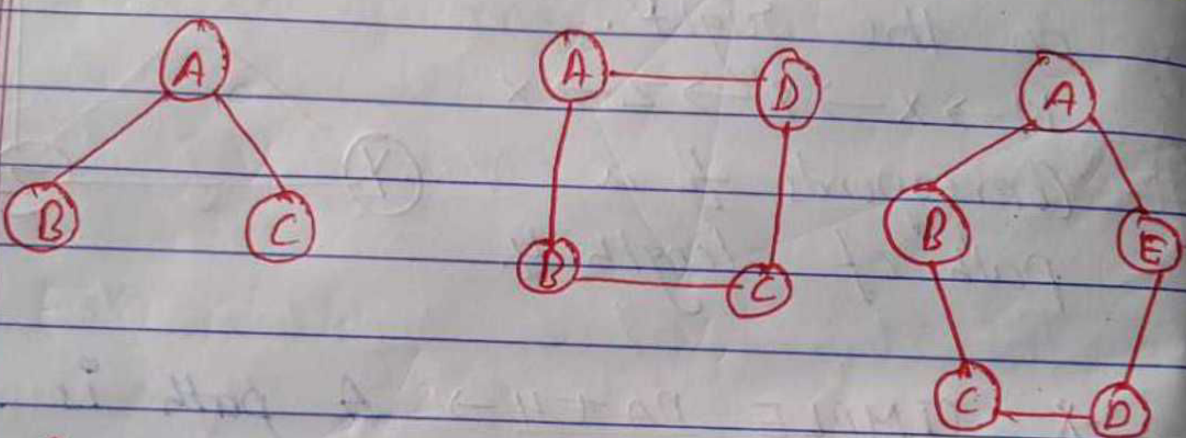
path : [1, 2, 4, 5, 4]

circuit : [1, 2, 4, 5, 4, 1]

Cycle Graph

A simple graph of 'n' vertices ($n \geq 3$) and n edges forming a cycle of length 'n' is called as a cycle graph.
In a cycle graph all the vertices are of degree 2.

Example

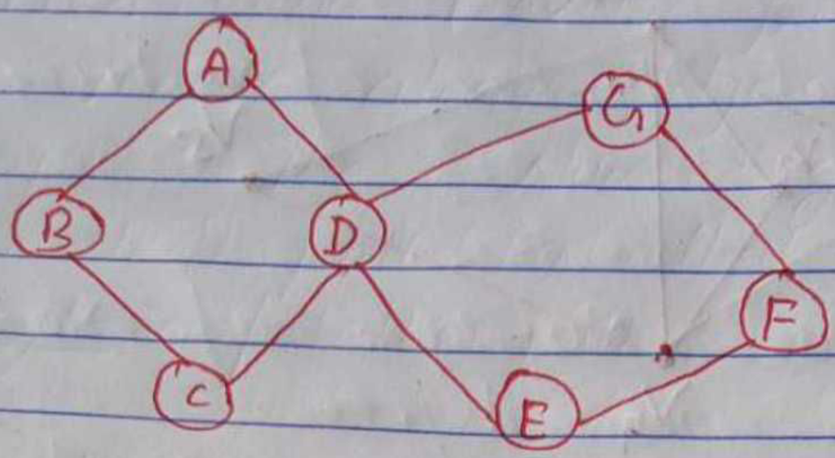


In these graph
* Each vertex is having degree 2.
* Therefore, they are cycle graph.

Cyclic Graph

A graph containing at least one cycle in it is called as a cyclic graph.

Example



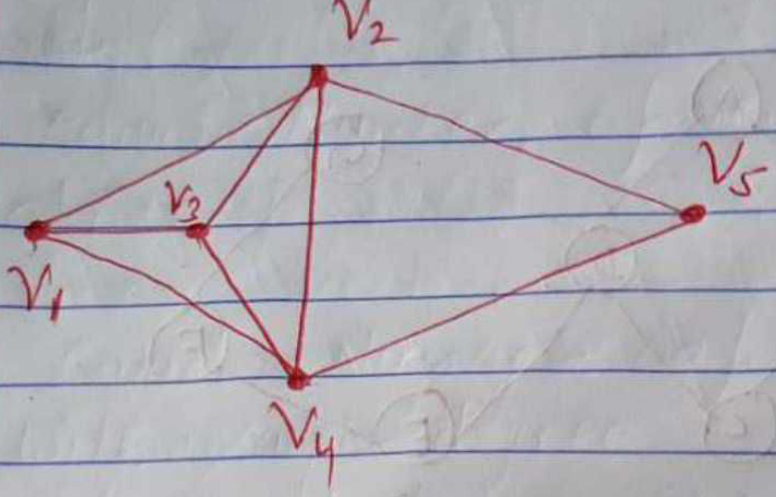
* This graph contains two cycle in it Therefore it is a cyclic graph.

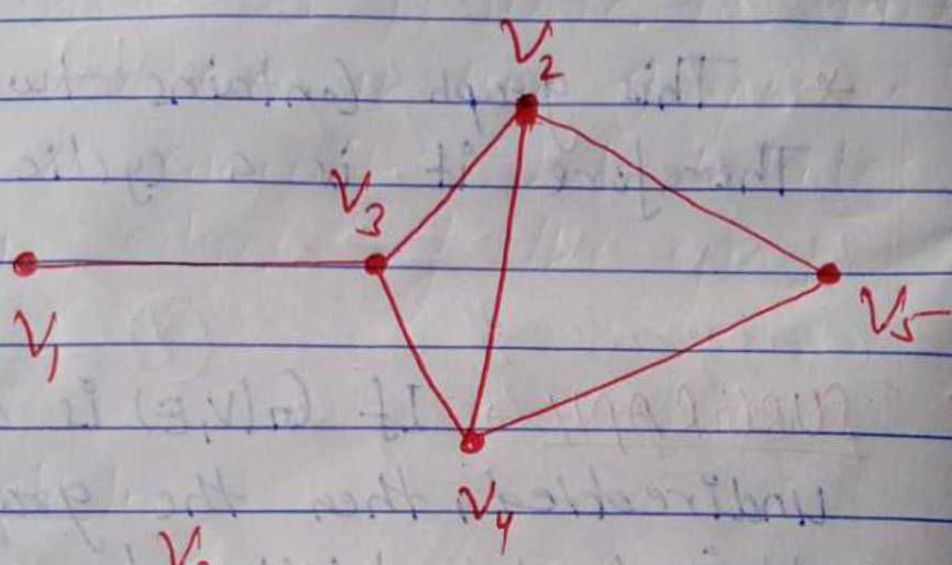
SUBGRAPH → If G(V,E) is a directed or undirected, then the graph H(V,' E') obtained by deleting few vertices and edges from G is called a subgraph of G. provided the set of vertices V' in H contains (at least) all the end points. of edges in E'

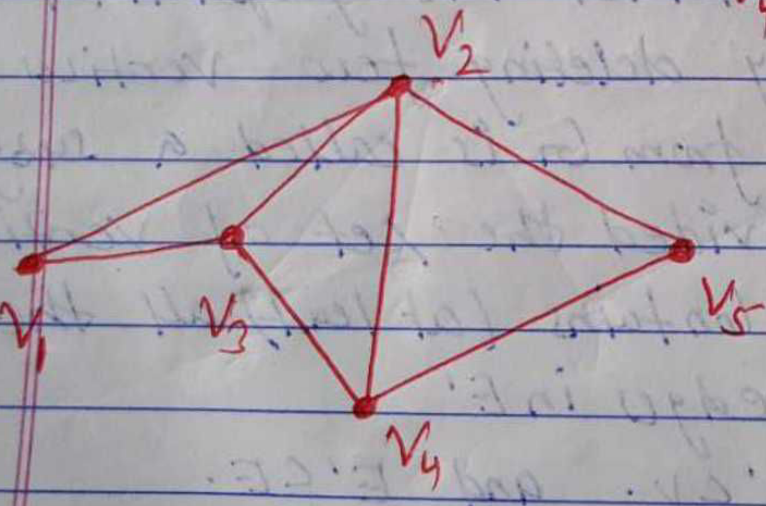
i.e $\phi = V' \subseteq V$. and $E' \subseteq E$.

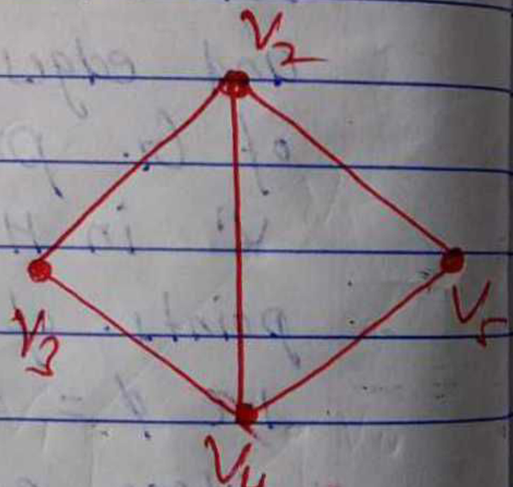
where each edge in E' is incident with vertices in V'.

Example









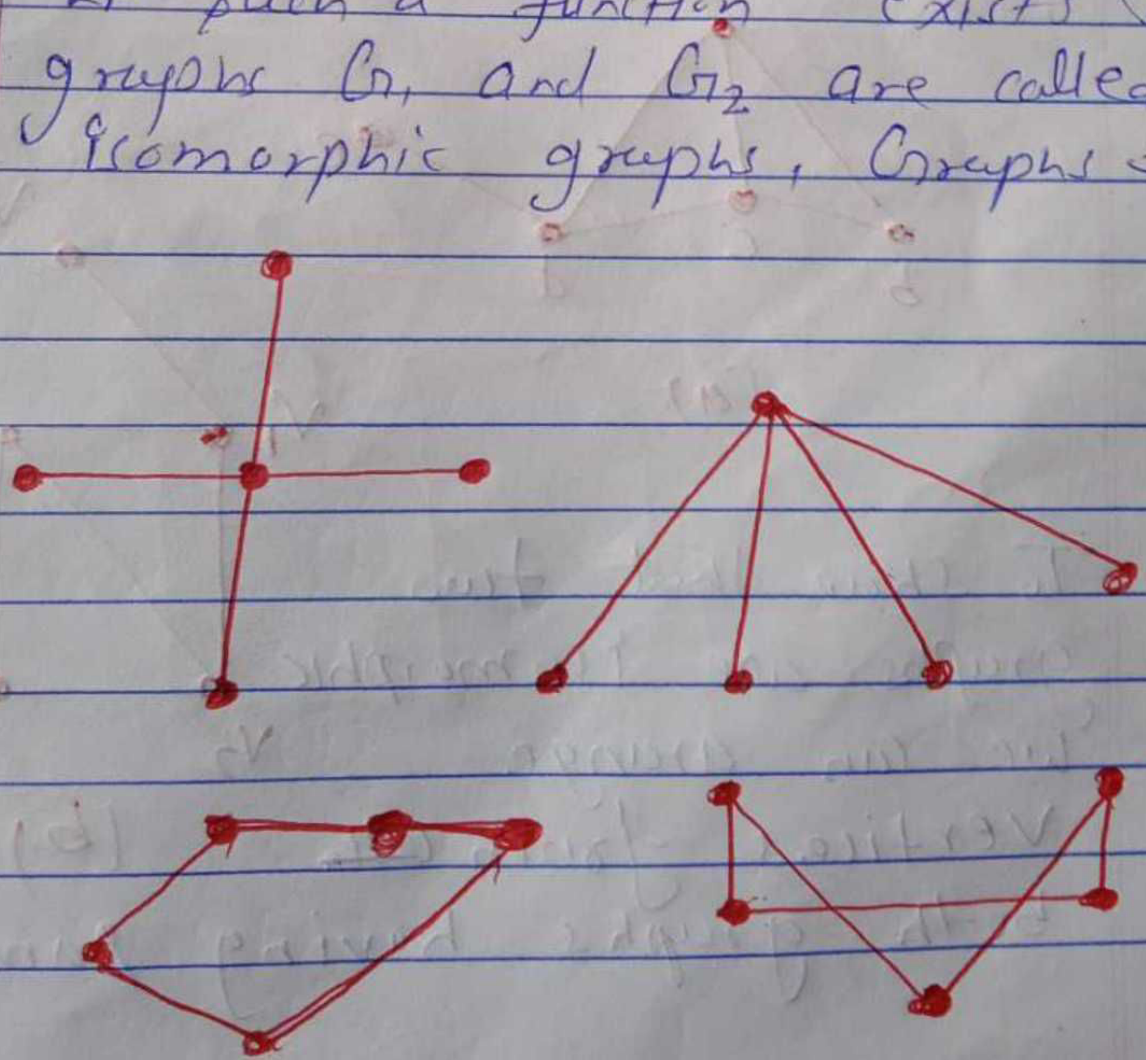
ISOMORPHIC-GRAPHS

Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two undirected graphs. A function $f: V_1 \to V_2$ is called a graph isomorphism if

(1) f is one-one and onto i.e there exist a one to one correspondence between their vertices as well as edges (Both the graphs have equal number of vertices and ~~eg~~ edges however, vertices may have different levels and

(2) for all $u, v \in V_1$, $\{u, v\} \in E_1$ if and only if $\{f(u), f(v)\} \in E_2$

If such a function exists then graphs G_1 and G_2 are called isomorphic graphs, Graphs shown



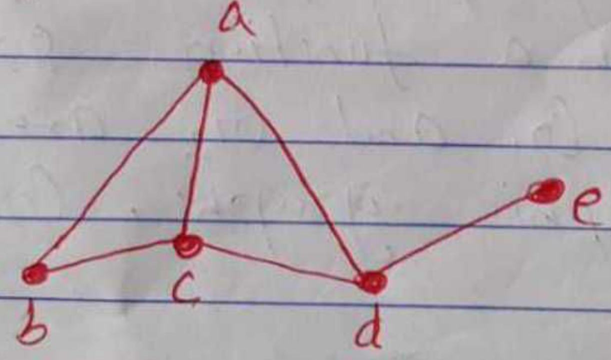
* Remark two isomorphic graph must have

① → equal Number of vertices and edges.

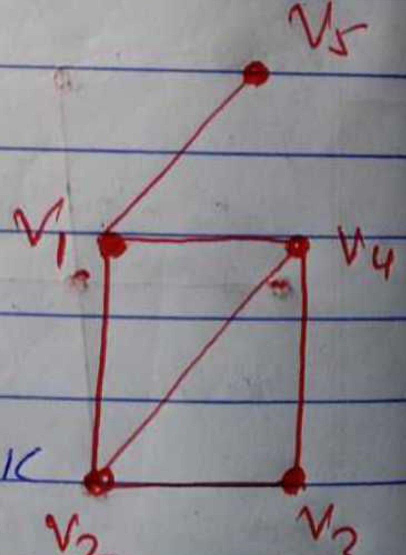
② → equal Number of vertices with same degree

③ → It should be possible to start at any vertex in two graphs and find a circuit that includes every edges of the graph.

Example 1.



(a)



(b)

To show that two graphs are isomorphic we can arange vertices from ~~oth~~ both graphs having same degrees in decreasing order of degree. If both graphs contain vertices having same degree then they are isomorphic otherwise Not.

degree of vertices both graph

$deg(c) = deg(v_2) = 3$

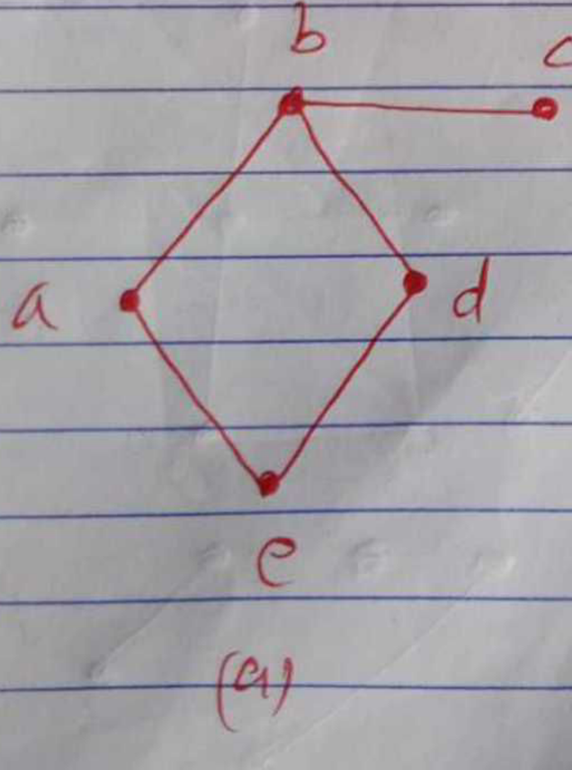
$deg(a) = deg(v_1) = 3$

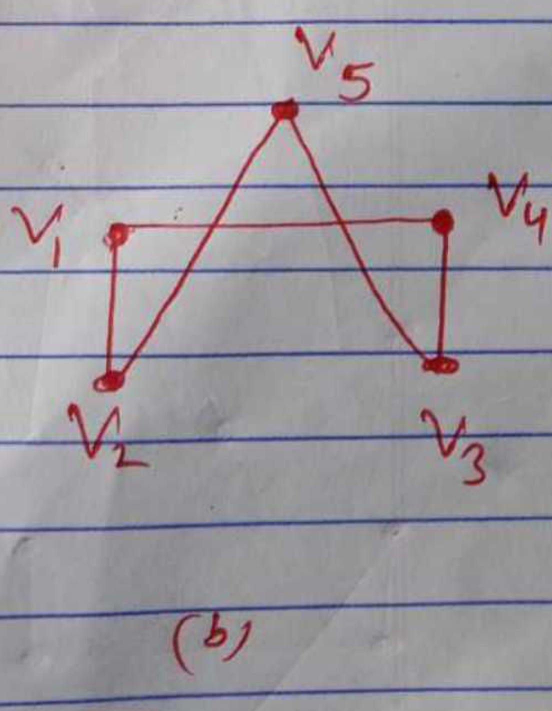
$deg(d) = deg(v_4) = 3$

$deg(b) = deg(v_3) = 2$

$deg(e) = deg(v_1) = 1$

Example 2.





Sol^n In graph (a)

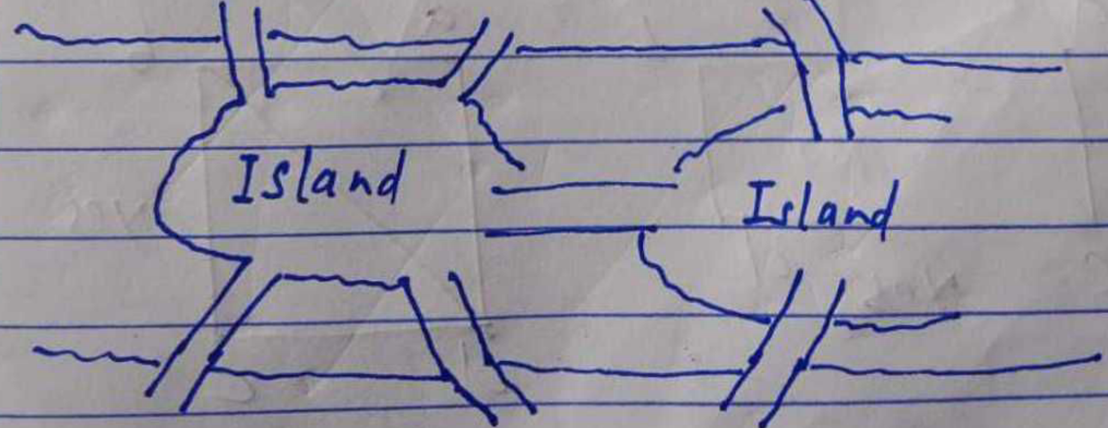
deg (vertex (b)) = 3

In graph (b) there is no such vertex.

Hence graphs are not Isomorphic.

Eulerian Graph ↓

A graph that contains an Euler tour (path or circuit) is called Eulerian graph.
One of the oldest problem involving graph is the problem of seven bridges of Konigsberg. A Map of the city of Königsberg which is divided into four section by the Pregel River. Seven briges connect of these regions. The probleme was to take a walk about the city so as to cross each briges exactly one and returning to the start point. This problem of crossing each of bridges only once is equivalent to find a closed walk through a Multi-graph



* Connected graph]

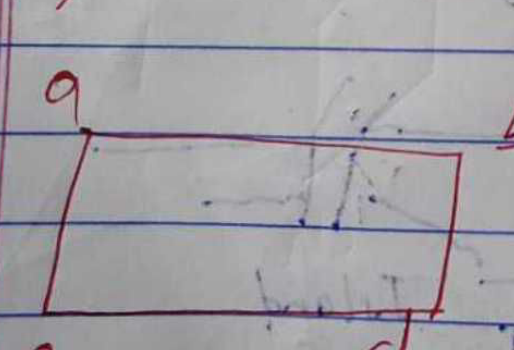
A graph is said to be connected when there is a path between every pair of vertex.

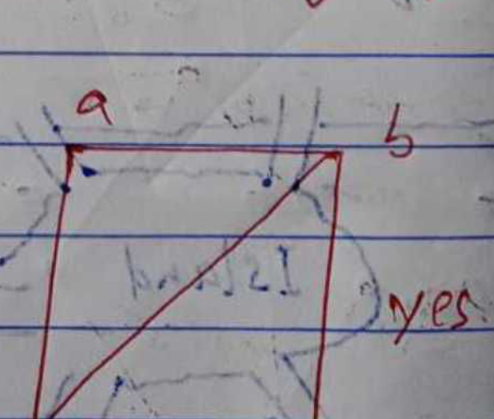
* Euler circuite / cycle]

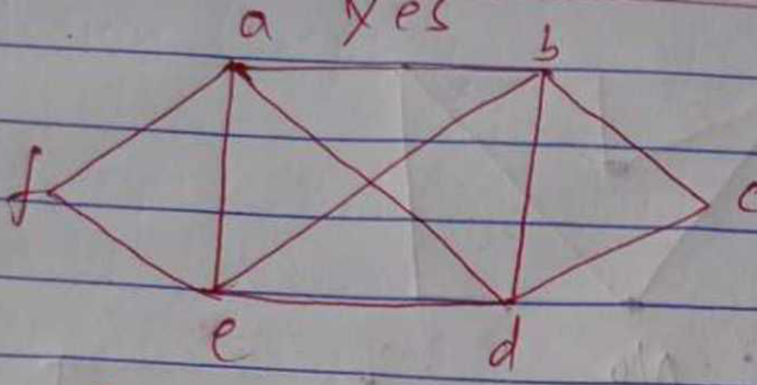
A closed walk which visit every Edges of the graph exactly once.

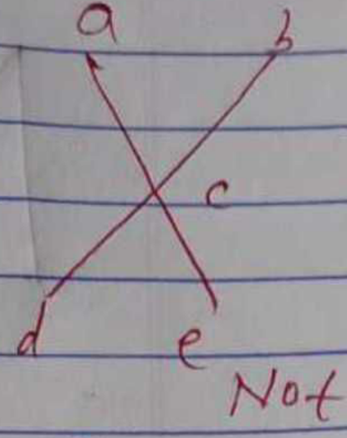
* Euler graph

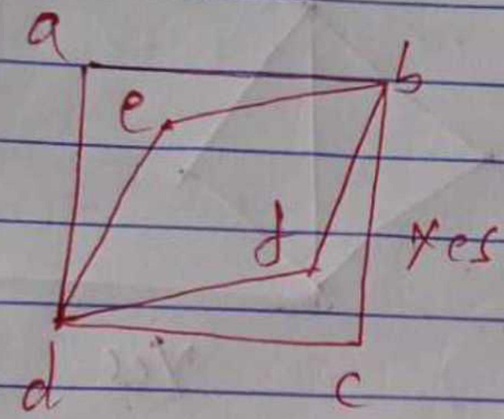
A graph which contains a Euler cycle is called Euler graph

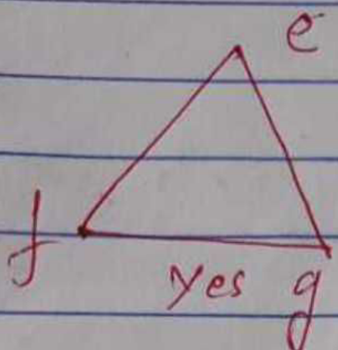






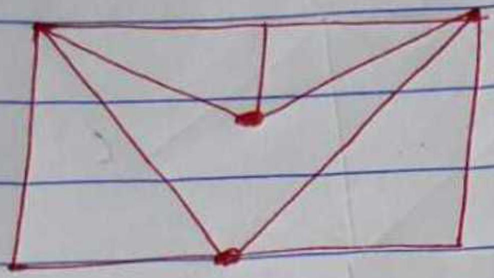






One Euler walk → A open walk visit every edge of the graph exacty once.

Semi Euler graph if a graph Contain a open Euler walk.



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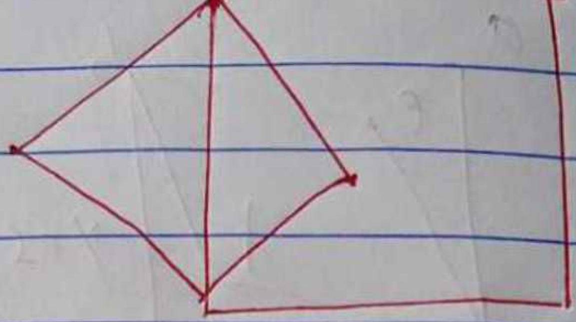
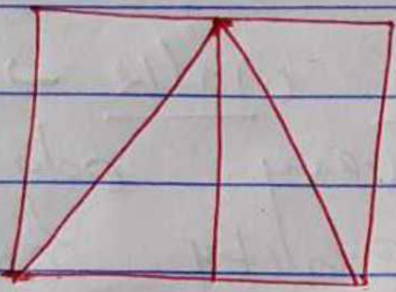


Yes



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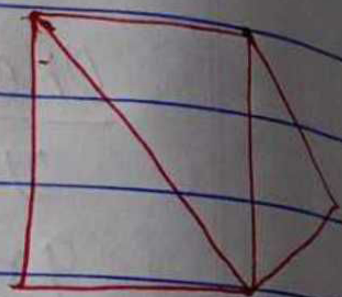

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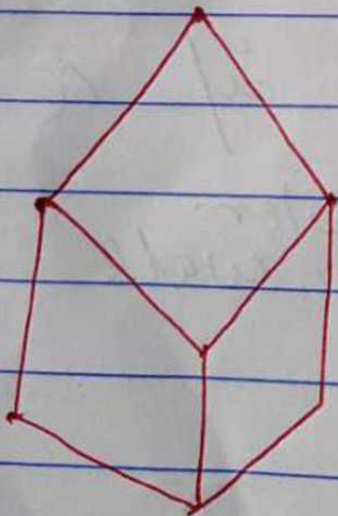

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