DD2448 Foundations of Cryptography
Lecture 2

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Last Lecture: Summary of Simple Ciphers

- Caesar cipher and affine cipher: $m_i \mapsto am_i + b$.
- Substitution cipher (generalize Ceasar/affine):

$$m_i \mapsto \sigma(m_i)$$

- Vigénère cipher (more uniform frequency table):

$$m_i \mapsto m_i + k_{i \bmod l}$$

- Hill cipher (invertible linear map):

$$(m_1, \ldots, m_l) \mapsto (m_1, \ldots, m_l)A$$

- Transposition cipher (permutation):

$$\begin{aligned} (m_1, \ldots, m_l) &\mapsto (m_{\pi(1)}, \ldots, m_{\pi(l)}) \\ (m_1, \ldots, m_l) &\mapsto (m_1, \ldots, m_l)M_\pi \qquad \text{(equivalently)} \end{aligned}$$

Simple Ciphers are Bad, What is a Good Block Cipher?

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- What should we look for instead?

Something Smaller

Idea. Compose smaller weak ciphers into a large one. Mix the components "thoroughly".

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Shannon (1948) calls this:

- **Diffusion.** "In the method of diffusion the statistical structure of M which leads to its redundancy is dissipated into long range statistics..."
- **Confusion.** "The method of confusion is to make the relation between the simple statistics of E and the simple description of K a very complex and involved one."

Substitution-Permutation Networks

Substitution-Permutation Networks (1/2)

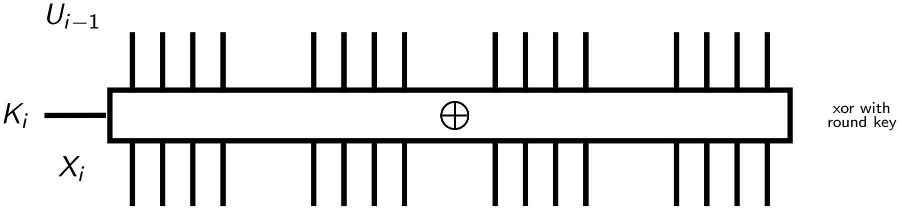
- **Block-size.** We use a block-size of $n = \ell \times m$ bits.
- **Key Schedule.** Round r uses its own round key K_r derived from the key K using a key schedule.
- **Each Round.** In each round we invoke:
  1. **Round Key.** xor with the round key.
  2. **Substitution.** ℓ substitution boxes each acting on one m-bit word (m-bit S-Boxes).
  3. **Permutation.** A permutation π_i acting on $\{1, \ldots, n\}$ to reorder the n bits.

Substitution-Permutation Networks (2/2)

U_{i-1}

K_i

Substitution-Permutation Networks (2/2)



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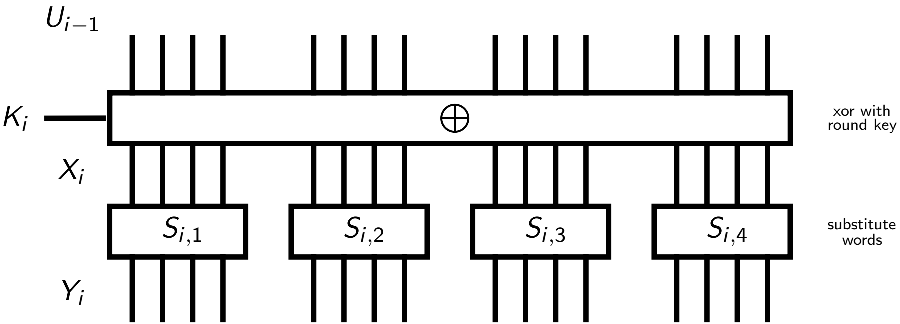

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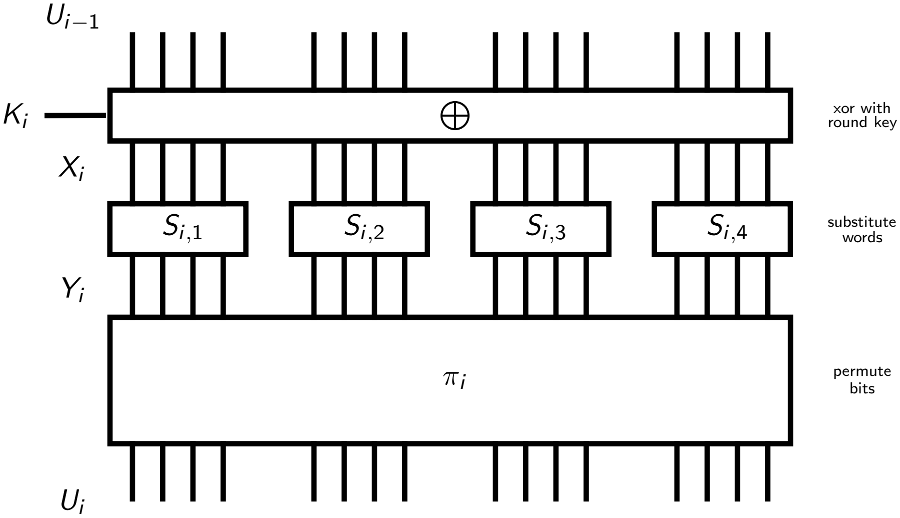
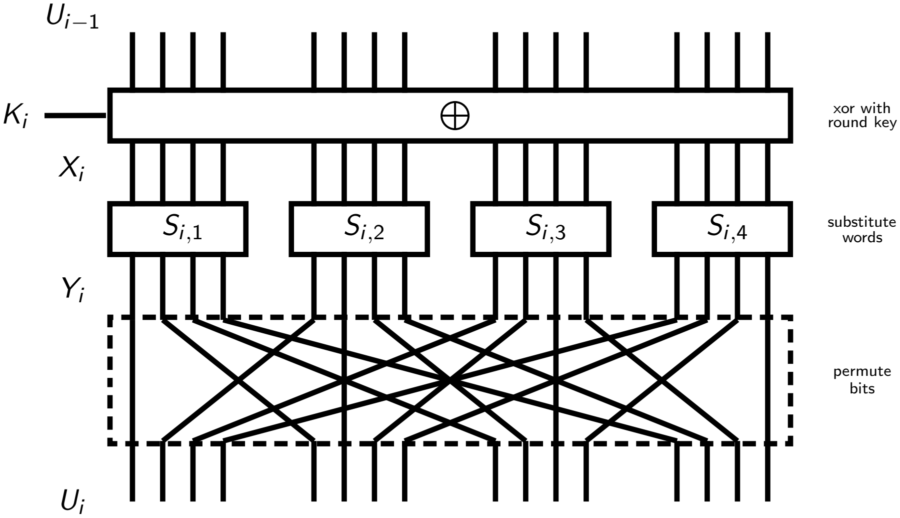
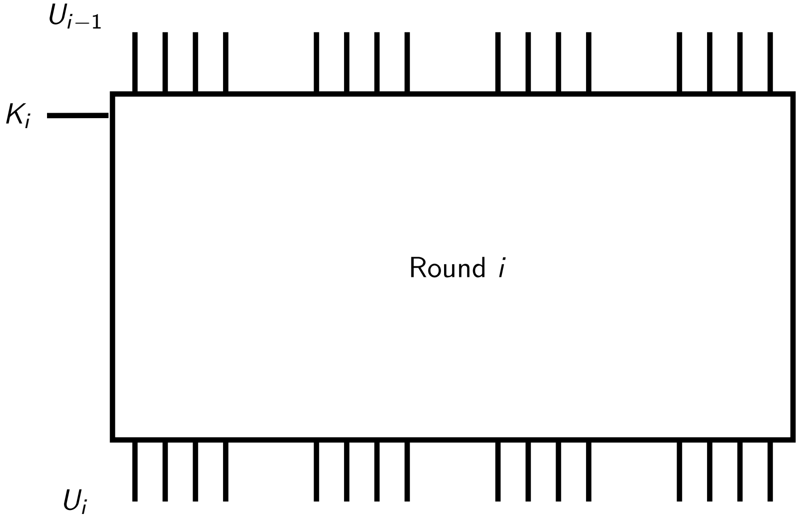
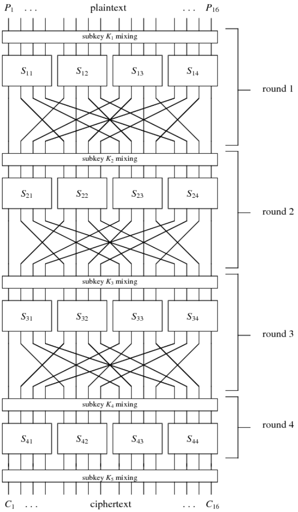



U_{i-1}
K_i
$\oplus$
xor with round key
X_i
$S_{i,1}$
$S_{i,2}$
$S_{i,3}$
$S_{i,4}$
substitute words
Y_i
permute bits
U_i

Substitution-Permutation Networks (2/2)



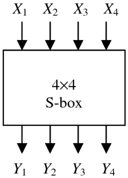
A Simple Block Cipher (1/2)



- $|P| = |C| = 16$
- 4 rounds
- $|K| = 32$
- rth round key K_r consists of the $4r$th to the $(4r + 16)$th bits of key K.
- 4-bit S-Boxes

A Simple Block Cipher (2/2)

S-Boxes the same ($S \neq S^{-1}$)



- $Y = S(X)$
- Can be described using 4 boolean functions

Input	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Output	E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7

A Simple Block Cipher (2/2)

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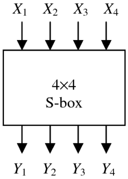

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16-bit permutation ($\pi = \pi^{-1}$)

Input	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Output	1	5	9	13	2	6	10	14	3	7	11	15	4	8	12	16

AES

Advanced Encryption Standard (AES)

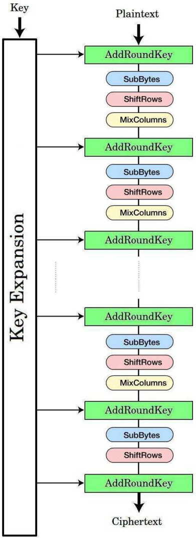
- Chosen in worldwide **public competition** 1997-2000. Probably no backdoors. Increased confidence!
- Winning proposal named "Rijndael", by Rijmen and Daemen
- Family of 128-bit block ciphers:

Key bits	128	192	256
Rounds	10	12	14

- The first key-recovery attacks on full AES due to Bogdanov, Khovratovich, and Rechberger, published **2011**, is faster than brute force by a factor of about **4**.
- ... algebraics of AES make some people uneasy, but they have been uneasy for years now.

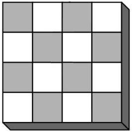
AES

- **AddRoundKey**: xor with round key.
- **SubBytes**: substitution of bytes.
- **ShiftRows**: permutation of bytes.
- **MixColumns**: linear map.



Similar to SPN

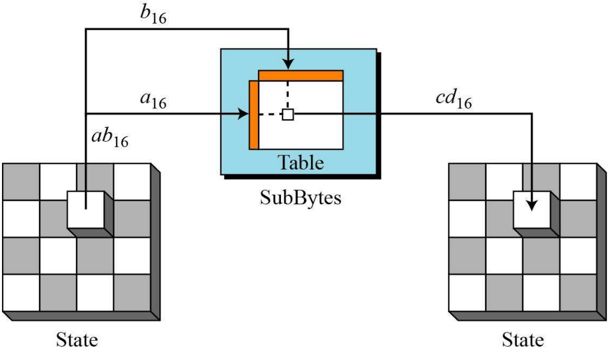
The 128 bit state is interpreted as a 4×4 matrix of bytes.



Something like a mix between substitution, permutation, affine version of Hill cipher. In each round!

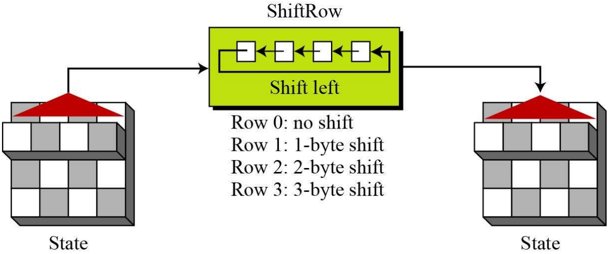
SubBytes

`SubBytes` is field inversion in $\mathbb{F}_{2^8}$ plus affine map in $\mathbb{F}_2^8$.



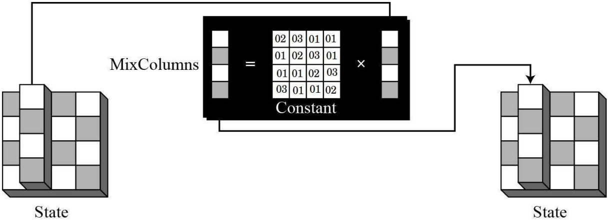
ShiftRows

`ShiftRows` is a cyclic shift of bytes with offsets: 0, 1, 2, and 3.



MixColumns

`MixColumns` is an invertible linear map over $\mathbb{F}_{2^8}$ (with irreducibile polynomial $x^8 + x^4 + x^3 + x + 1$) with good diffusion.



Decryption

Uses the following transforms:

- **AddRoundKey**
- **InvSubBytes**
- **InvShiftRows**
- **InvMixColumns**

Feistel Networks

Feistel Networks

- Identical rounds are iterated, but with different round keys.
- The input to the ith round is divided in a left and right part, denoted L^{i-1} and R^{i-1}.
- f is a function for which it is somewhat hard to find pre-images, but f is **not invertible**!
- One round is defined by:

$$\begin{aligned} L^i &= R^{i-1} \\ R^i &= L^{i-1} \oplus f(R^{i-1}, K^i) \end{aligned}$$

where K^i is the ith round key.

Feistel Round

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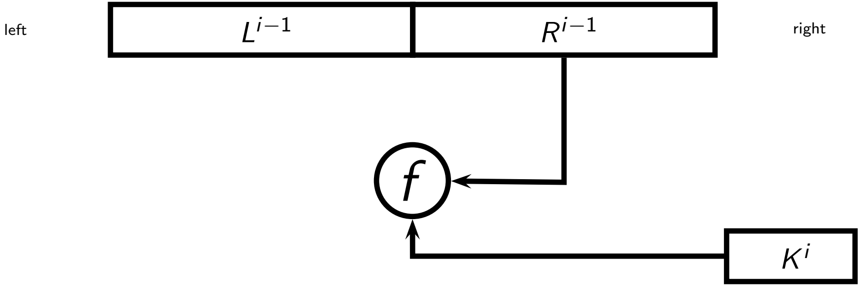

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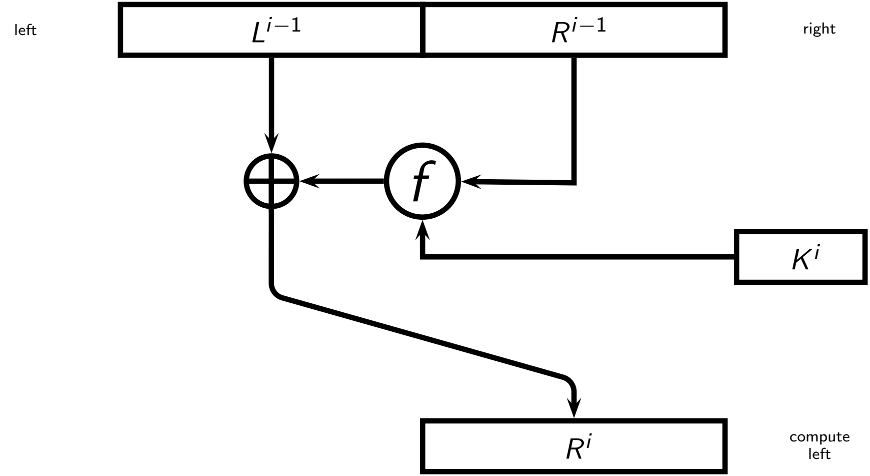

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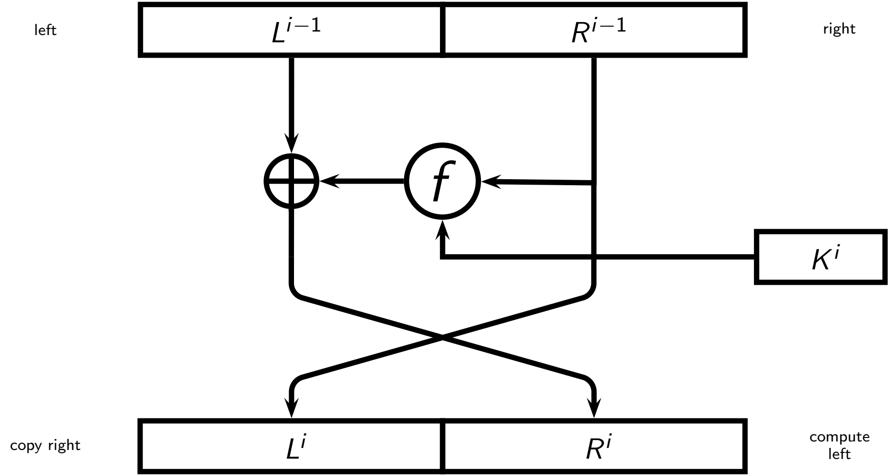
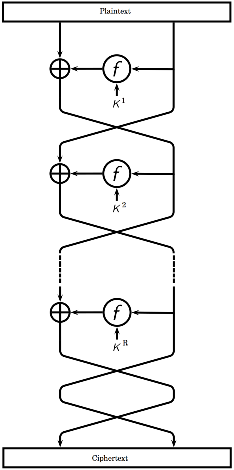


Feistel Cipher



Inverse Feistel Round

Feistel Round.

$$L^i = R^{i-1}$$
$$R^i = L^{i-1} \oplus f(R^{i-1}, K^i)$$

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$$

Inverse Feistel Round.

$$
\begin{aligned}
L^{i-1} &= R^i \oplus f(L^i, K^i) \\
R^{i-1} &= L^i
\end{aligned}
$$

Reverse direction and swap left and right!

DES

Quote

The news here is not that DES is insecure, that hardware algorithm-crackers can be built, or that a 56-bit key length is too short. ... The news is how long the government has been denying that these machines were possible. As recently as 8 June 98, Robert Litt, principal associate deputy attorney general at the Department of Justice, denied that it was possible for the FBI to crack DES. ... My comment was that the FBI is either incompetent or lying, or both.

– Bruce Schneier, 1998

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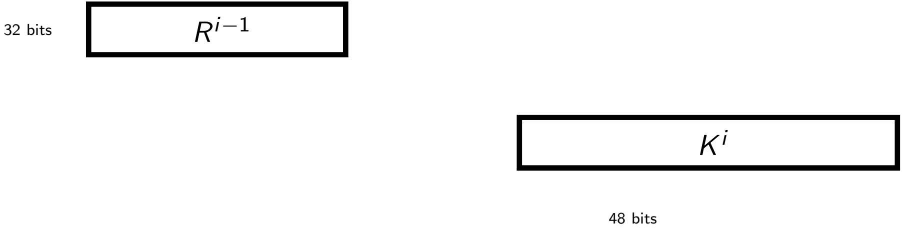
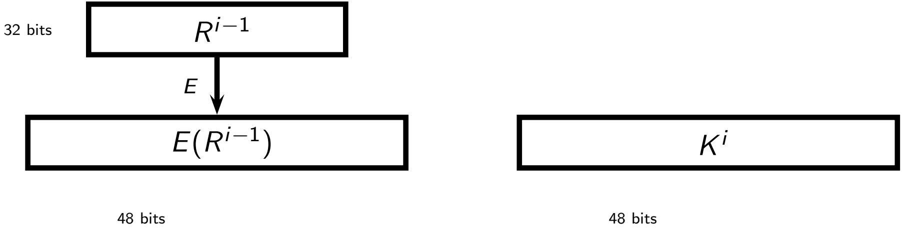
- Key schedule derives permuted bits for each round key from a 56-bit key. Supposedly not 64-bit due to parity bits.
- Let us look a little at the Feistel-function f.


32 bits
R^{i-1}
K^{i}
48 bits

DES's f-Function



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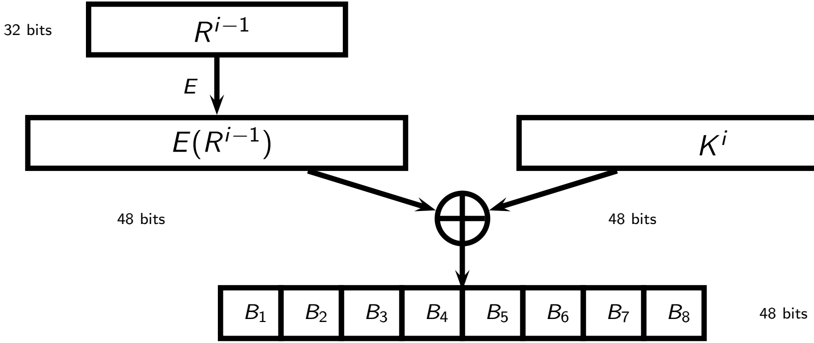

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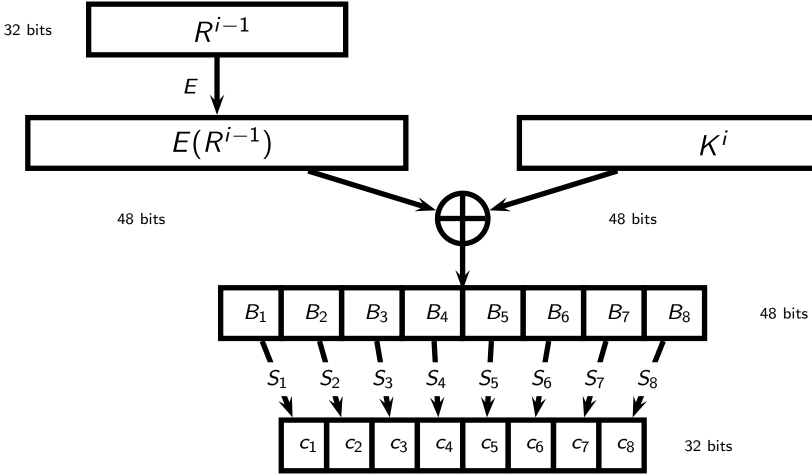

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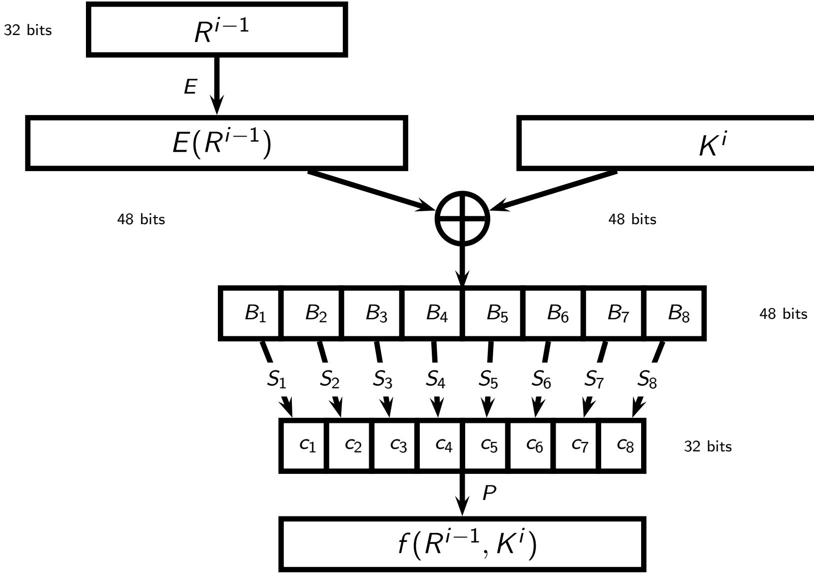

Security of DES

- **Brute Force.** Try all 2^{56} keys. Done in practice with special chip by Electronic Frontier Foundation, 1998. Likely much earlier by NSA and others.
- **Differential Cryptanalysis.** 2^{47} chosen plaintexts, Biham and Shamir, 1991. (approach: late 80'ies). Known earlier by IBM and NSA. DES is surprisingly resistant!
- **Linear Cryptanalysis.** 2^{43} known plaintexts, Matsui, 1993. Probably **not** known by IBM and NSA!

Double DES

We have seen that the key space of DES is too small. One way to increase it is to use DES twice, so called "double DES".

$$\mathrm{2DES}_{k_1,k_2}(x) = \mathrm{DES}_{k_2}(\mathrm{DES}_{k_1}(x))$$

Is this more secure than DES?

This question is valid for any cipher.