SIGNAL PROCESSING

DEPARTMENT OF ELECTRICAL ENGINEERING

E 105 **Digital Signalbehandling** EQ2300/ 2E1340

Final Examination 2009–06–04, 14.00–19.00

Literature:

- Hayes: *Statistical Digital Signal Processing and Modeling*
  or
  Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson: *Complementary Reading in Digital Signal Processing*
- *Copies of the slides and lecture hand outs*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in Swedish or English.
- At most one problem should be treated per page.
- Motivate each step in the solutions (also for the multi-choice questions) unless otherwise mentioned.
- Answers without motivation / justification carry no rewards unless otherwise mentioned
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

Contact: Bhavani Shankar/ Mats Bengtsson, Signal Processing, 08-790 84 63

Results: Will be reported within three working weeks at “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. Consider the implementation given in figure 1 where the output of each multiplier is rounded off to $B+1$ bits (including a sign bit), used to represent the range $[-1, 1[$. Assuming no overflow during additions, determine B to achieve a SQNR of 30dB at the output. The output signal power is 0.1. (10p)

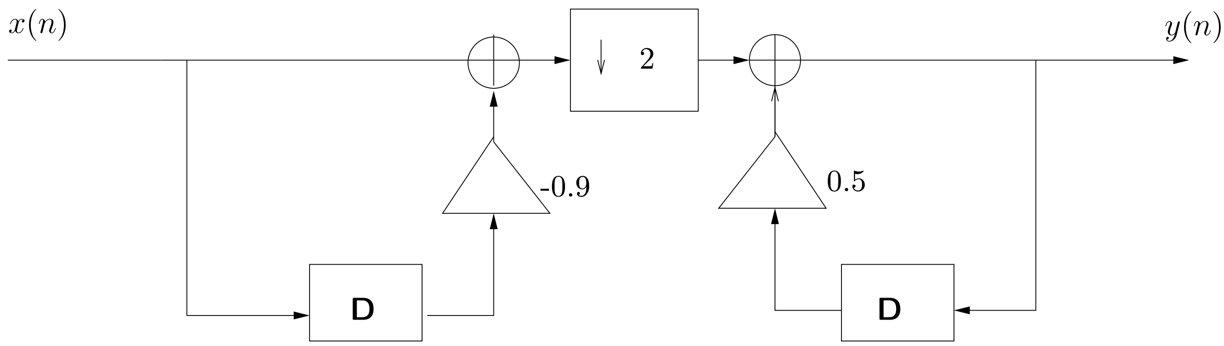

Figure 1: Calculating the Quantization noise

2. Consider filtering an infinite length sequence, $x(n)$, using the filter $H(z) = 1 - 2z^{-1}$ to obtain the output signal $v(n)$ as shown in Figure 2. The signal $v(n)$ is corrupted with noise $w(n)$. The noise samples, $w(n)$, are assumed to be independent identically distributed random variables with zero mean and unit variance. The objective is to devise a simple procedure to obtain a good estimate of $x(n)$ from $y(n)$.

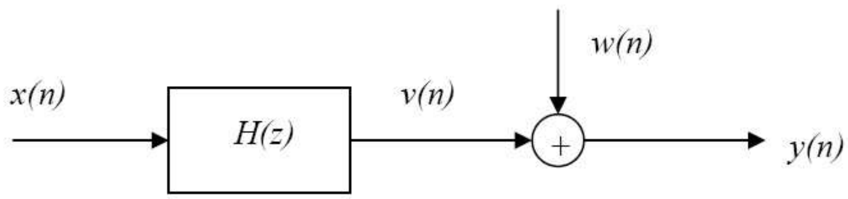

Figure 2: System Model

(a) The simplest procedure is to ignore noise and filter $y(n)$ using $\frac{1}{H(z)}$. However this is not a good idea. Show that $\frac{1}{H(z)}$ is causally unstable and that the noise variance after filtering $w(n)$ by $\frac{1}{H(z)}$ would be infinite. (2p)

(b) Show that $\frac{1}{H(z^{-1})}$ is causally stable and obtain the noise variance when $w(n)$ is filtered by $\frac{1}{H(z^{-1})}$. Compare this with step (a) above. (2p)

(c) Motivated by part (b) above and since $Y(z^{-1}) = X(z^{-1})H(z^{-1})$ in the absence of noise, one can consider obtaining $x(n)$ using $\frac{Y(z^{-1})}{H(z^{-1})}$. Is the scheme of obtaining $x(n)$ using $X(z^{-1}) = \frac{Y(z^{-1})}{H(z^{-1})}$ practical, even if noise is assumed to be zero? (3p)

Hint: First try to find a time-domain interpretation of the filtering $\frac{Y(z^{-1})}{H(z^{-1})}$.

(d) Assume that $x(n)$ is now a length N sequence. Exploiting this information, show that $\frac{Y(z^{-1})}{H(z^{-1})}$ can be practically implemented while incurring a delay. Also, determine this delay. (3p)

3. (a) Let $y(t)$ be a signal of the form

$$y(t) = \sin(2\pi f_0 t + \phi_0) + \sin(2\pi(f_0 + \triangle f)t + \phi_1) + w(t), \quad t \in [0, T]$$

where $w(t)$ is white Gaussian noise process of unit variance and $\phi_i, i \in [0, 1]$ are unknown phase components. You are provided with T seconds of this signal and the task is to estimate its spectrum using the Bartlett method. The parameter that you can control is the sampling frequency, f_s, which should be determined to attain the following objectives:

i. The different frequencies are sufficiently resolved

ii. Variance of the spectral estimate is less than 1% of the square of the true spectral density.

Derive an expression for the lowest possible f_s that attains these objectives. (6p)

Hint: The resolution of Barlett's method with M samples per block is $0.89\frac{2\pi}{M}$ radians, while the variance with K blocks is $\frac{P_x^2(e^{j\omega})}{K}$ with $P_x(e^{j\omega})$ being the true spectral density.

(b) Now, assume that the noise variance σ_w^2 is unknown and that the sampling frequency is given. Assume also that you have the option to choose between using the Periodogram method and using the Bartlett method (dividing the signal into 50 segments). Which of these two alternatives would you recommend if you are primarily interested in estimating the noise level σ_w^2? (2p)

(c) Same question as in (b), but assume instead that you are primarily interested in getting good estimates of f_0 and Δf and that Δf may be very small. (2p)

Do not forget to motivate your answers!

4. Assume that we have a real valued stationary stochastic process $y(n)$, where $\mathrm{E}[y(n)y(n + 2k + 1)] = 0$, for all integers k.

a) Determine the coefficients of a fourth order, one-step ahead MMSE predictor for $y(n)$ on the form

$$\hat{y}_1(n) = a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3) + a_4 y(n-4)$$

Exploit the structure in the signal to find simple closed-form expressions for the coefficients a_k. (3p)

b) Determine the coefficients of a fourth order, two-step ahead MMSE predictor for $y(n)$ on the form

$$\hat{y}_2(n+1) = b_1 y(n-1) + b_2 y(n-2) + b_3 y(n-3) + b_4 y(n-4)$$

(3p)

c) Let $u(n) = y(n) + y(n+1)$ and determine the coefficients of the optimal estimator for $u(n)$ of the special form

$$\hat{u}(n) = c_1\big(y(n) + y(n-1)\big) + c_2\big(y(n-2) + y(n-3)\big)$$

(4p)

5. Let $P_0(z)$ and $P_1(z)$ be the second order polyphase components of $H(z)$ so that,

$$H(z) = P_0(z^2) + z^{-1}P_1(z^2)$$

Let $Q_0(z)$ and $Q_1(z)$ denote the second order polyphase components of another filter $F(z)$. It is trivial to show that polyphase components of $\alpha H(z) + \beta F(z)$ are $\alpha P_0(z) + \beta Q_0(z)$ and $\alpha P_1(z) + \beta Q_1(z)$ where α, β are constants. In this problem we shall explore additional properties of polyphase implementation.

(a) Express $Q_0(z)$ and $Q_1(z)$ in terms of $P_0(z), P_1(z)$ if (3p)

i. $F(z) = \alpha z^{-2k}H(z)$, k being a positive integer and α being an arbitrary constant.

ii. $F(z) = \alpha z^{-(2k+1)}H(z)$, k, α chosen as above.

(b) Determine $Q_0(z)$ and $Q_1(z)$ if $F(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - cz^{-1}}, |z| > c, 0 < c < 1.$ (4p)

Hint: Choose $H(z) = \frac{1}{1-cz^{-1}}, |z| > c, 0 < c < 1$ and use the earlier results.

(c) Consider the decimator of figure 3 with $F(z)$ chosen as in step (b) above with $a_1 = 0$. Determine the number of multiplications required per sample of $y(n)$ when the system is implemented (i) directly and (ii) using polyphase decomposition. What do you observe and why? (3p)

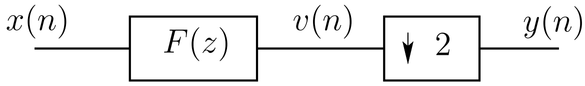

Figure 3: Decimation Filter