KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2015–04–08, 14.00–19.00

Literature:

- Course text book:
  - Diniz, da Silva & Netto: *Digital Signal Processing; System Analysis and Design*

    or

  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or

  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. Some mixed shorter questions to warm up. Remember to motivate your answers.

a) Which of the following operations on a signal $x[n]$ is not time invariant? (2p)

i. Delaying the signal by L samples, i.e., $y[n] = x[n-L]$ for $n \in \mathbb{Z}$

ii. Inverting the signal, i.e., $y[n] = -x[n]$ for $n \in \mathbb{Z}$

iii. Downsampling the signal by a factor of 2, i.e., $y[m] = x[2m]$ for $m \in \mathbb{Z}$

b) Name something that can be done in order to improve the sensitivity of a long FIR filter to coefficient quantization effects. (2p)

c) Assume that `x` is a MATLAB vector of length `N` that contains consecutive samples of a wide sense stationary process, and that we at the MATLAB prompt write

```
>> Px = (1/N)*abs(fft(X)).^2;
```

What type of spectrum estimate did we then just compute? (2p)

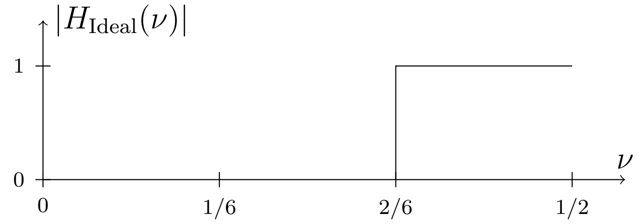
d) Assume that

$$x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 1, 2, 3, 4\} \quad \text{and} \quad h[n] = \{\underset{\uparrow}{1}, 1, 1, 0, 0, 0, 0, 0\},$$

and let $X[k] = \mathcal{F}_8\{x[n]\}$ and $H[k] = \mathcal{F}_8\{h[n]\}$ where $\mathcal{F}_8\{\cdot\}$ denotes the 8-point DFT. Let $Y[n] = H[k]X[k]$ and $y[n] = \mathcal{F}_8^{-1}\{Y[k]\}$ where $\mathcal{F}_8^{-1}\{\cdot\}$ denotes the inverse 8-point DFT. What is then $y[n]$?. Give the numerical values. (2p)

e) Assume that $x[n]$ is given for $n = 0, \ldots, 7$ as in part d, and that $x[n] = 0$ for $n < 0$ and $n \geq 8$. Assume that we wish to numerically compute the discrete time Fourier transform (DTFT) $X(\nu)$ for $\nu = \nu_k = k/16$, where $k = 0, \ldots, 15$. Explain how to do this efficiently using the FFT algorithm. You do not need to do the numerical computation; just explain how to do it. (2p)

2. In this problem you should describe the design and implementation of a high pass FIR filter that approximate the following magnitude response.



a) Using the window design method, approximate the requested magnitude response by designing a causal Type I FIR filter of length N with impulse response $h[n]$, i.e., N is an odd number, $h[n] = 0$ if $n < 0$ and $n \geq N$, and $h[n] = h[N-1-n]$ for $n = 0, \ldots, N-1$. Assume that you have a given symmetric and properly normalized window function $w[n]$, where $w[n] = w[N-1-n]$, and express $h[n]$ in terms of $w[n]$, N, and standard functions and constants. (6p)

b) Assume that you can afford a complexity of at most 12 complex valued multiplications per sample of the filtered signal when filtering a long input signal. If you use the overlap and add implementation of the filter using a radix-2 FFT, what is then the maximum filter length N that you can use? If you do not have a calculator, explain well how you would solve the problem if you had one and you will still get full points. (4p)

3. Consider the signal model of the form

$$x[n] = \sum_{k=1}^{3} a_k \sin(2\pi\nu_k n + \varphi_k) + v[n]$$

where a_k, ν_k and φ_k, for $k = 1, \ldots, 3$ are unknown amplitudes, frequencies and phases of three sinus-signals, and where $v[n]$ is additive white Gaussian noise of some unknown variance σ^2. You obtain $N = 512$ samples of this signal, and which to determine the unknown parameters. In order to do so, you form the periodogram estimate of the signal based on your $N = 512$ samples, and obtain the result seen on the next page. As it is hard to see more than two distinct peaks in the Periodogram, you also use the modified periodogram with a Chebyshev window, and use Welch's method with a block-length of $L = 128$, 50% overlap, and a Chebyshev window. You can still only see at most two clear peaks in any one spectrum estmate, but should now have enough information in order to obtain the all unknown parameters except the phases.

a) Explain why you can only see two clear peaks in the Periodogram. (1p)

b) Explain why you can only see two clear peaks in the Modified periodogram. (1p)

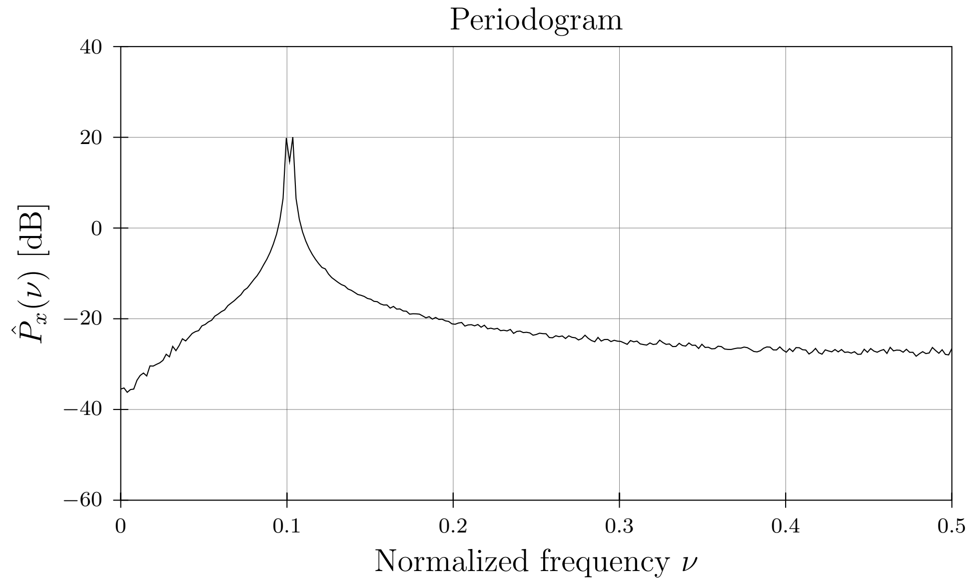
c) What are the frequencies ν_k, for $k = 1, \ldots, 3$. (2p)

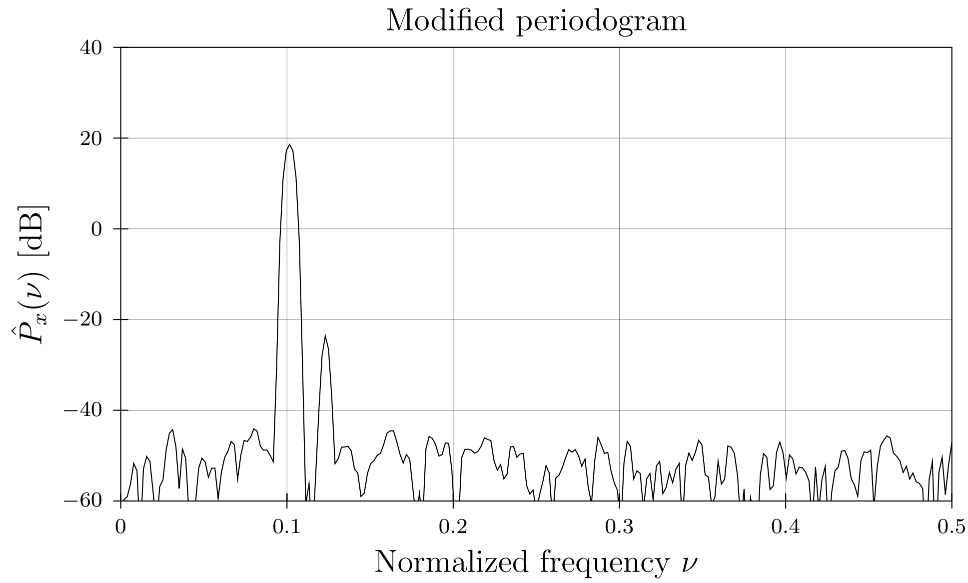
d) What is the noise power σ^2. (2p)

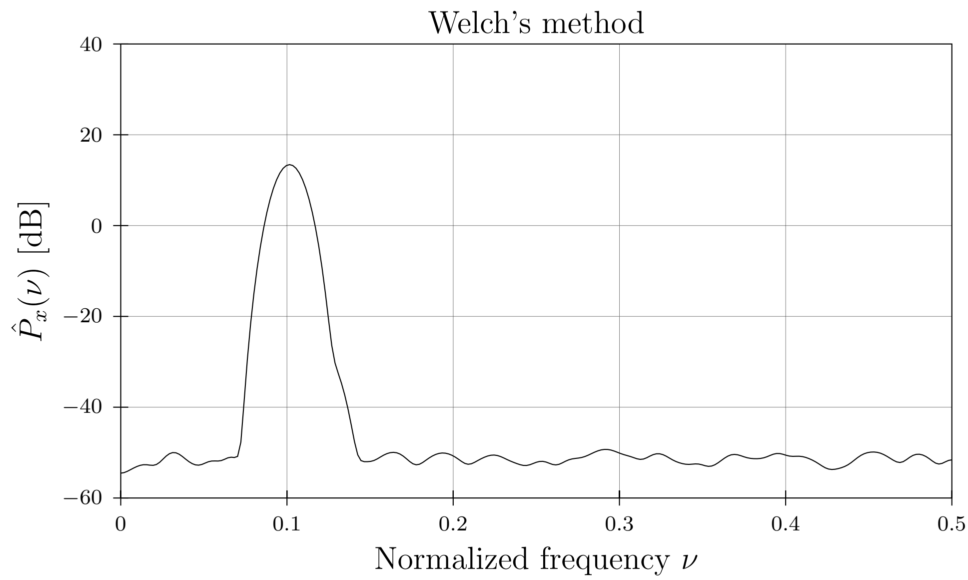
e) What are the amplitudes a_k, for $k = 1, \ldots, 3$. (4p)

Hint: As it is difficult to read the printed plots exactly, approximate answers are ok given that you clearly explain how you read the figures in order to obtain the required information. Use sketches if it helps to explain this.

Hint: The highest sidelobes of the Chebyshev window used are 100 dB below the main lobe, and the 3 dB bandwidth of $|W(\nu)|^2$, where $W(\nu) = \mathcal{F}\{w[n]\}$, is approximately $1.85/L$ for a window $w[n]$ of length L. The distance in frequency between the location of the main peak and the first sidelobe is approximately $4.6/L$.

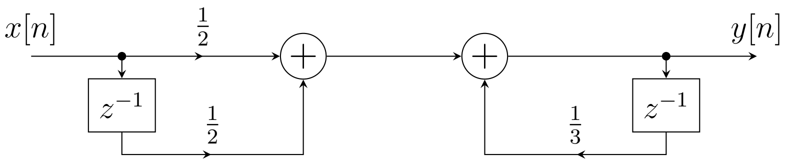

Periodogram
$\hat{P}_x(\nu)$ [dB]
40
20
0
−20
−40
−60
0
0.1
0.2
0.3
0.4
0.5
Normalized frequency ν


Modified periodogram
$\hat{P}_x(\nu)$ [dB]
40
20
0
−20
−40
−60
0
0.1
0.2
0.3
0.4
0.5
Normalized frequency ν

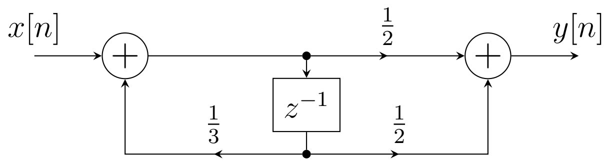

Welch's method
$\hat{P}_x(\nu)$ [dB]
40
20
0
−20
−40
−60
0
0.1
0.2
0.3
0.4
0.5
Normalized frequency ν

4. In this problem we will consider two equivalent circuits implemented in fixed point using a $B+1$ bit signed magnitude representation of the range $[-1, 1]$. You can assume throughout that no overflow occurs.

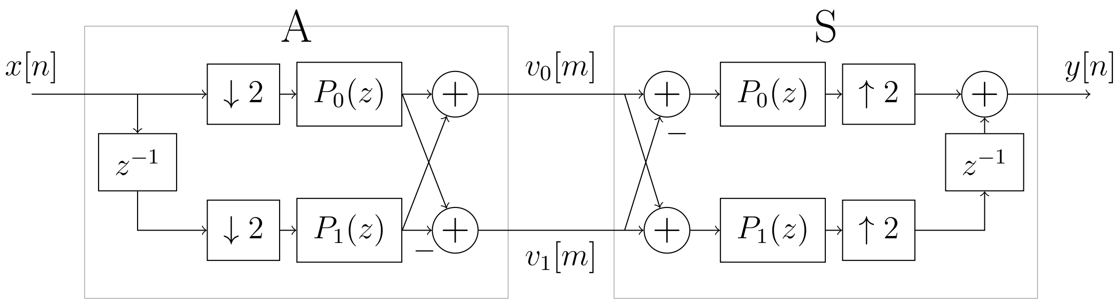
a) The circuit below corresponds to the concatenation of an MA system and an AR system. Calculate the power of the total quantization noise at the output of the circuit. (5p)



b) The so-called canonical form of the previously considered circuit is given below. Calculate the power of the total quantization noise at the output of this implementation, and compare it to the implementation in part a. Which one yields more overall fixed point quantization noise? (5p)



5. The following system depicts the polyphase implementation of a QMF filterbank, where the block labelled A is the analysis section and where the block labelled S is the syntesis section.



a) The arguably simplest choice of the filters $P_0(z)$ and $P_1(z)$ are $P_0(z) = P_1(z) = \frac{1}{\sqrt{2}}$, i.e., the filters are just constants amplifications (or more correctly attenuations). Prove that this choice of filters yield perfect reconstruction in the sense that $y[n] = x[n-l]$ for some delay l, and determine l. (6p)

b) Consider the new system shown below, which is formed by combining the building blocks A and B. Using the result of part a, prove that this system provides perfect reconstruction and obtain the total delay L of the system. If you were not able to solve part a, you can express L in terms of l without any numerical values. (4p)