SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 91 **Digital Signalbehandling**, 2E1340

Final Examination 2003–08–29, 14.00–19.00

Literature: Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
On popular demand: Copies of the slides, distributed with the exam.
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Josefsson: formel- och tabellsamling i matematik
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. Consider upsampling of a signal by a factor two. In order to remove the high-frequency spectral image of the signal, a low-pass filter $H(z)$ is used.

a) Let

$$H(z) = \frac{0.125 + 0.25z^{-1} + 0.125z^{-2}}{1 - 0.5z^{-1}}.$$

Write down expressions for the two component polyphase decomposition of $H(z)$. (6p)

b) Compared with a direct implementation of $H(z)$, how many arithmetic operations (multiplications and additions, respectively) per output sample can be saved by using the polyphase decomposition? (4p)

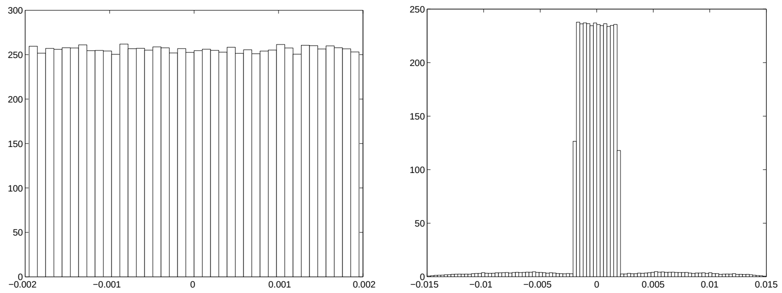

Figure 1: Estimated PDF of the quantization error from two groups.

2. a) In a lab, a signal generator is connected to a DSP card equipped with a 9 bit A/D converter. The signal generator is adjusted to generate a sine wave with approximately 1 V amplitude. Figure 1 shows the estimated probability density function of the quantization errors obtained by two different lab groups. Explain the difference between the results of the two groups. (3p)

b) Figure 2 shows two possible implementations for sampling rate conversion by a factor I/D. Explain which of the two implementations is better and why you should not use the other one. (4p)

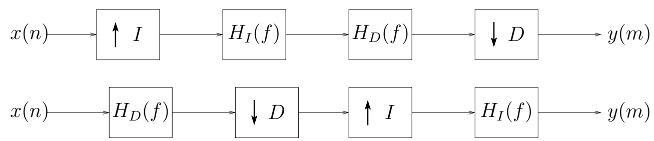

Figure 2: Two possible implementations of sampling rate conversion.

c) Figure 3 shows the true spectrum of a signal and two spectral estimates of the signal, obtained using Bartlett's method. One of the estimates has a large bias since the length of each segment was too small, the other has a large variance since the number of segments was too small. Which estimate is which? (3p)

3. A DSP program groups samples from a signal $x(n)$ into groups of 5 samples each. For every such group, the program produces a single output value $y(k)$ which is the average of the 5 samples. Determine a mathematical relationship between the discrete-time Fourier transforms (DTFT) of the output $y(k)$ and the input $x(n)$. (10p)

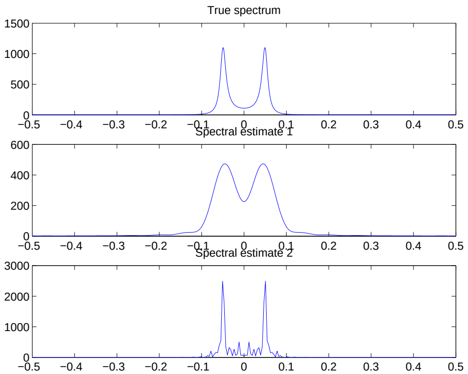

Figure 3: True spectrum (top) and two different spectral estimates (middle and bottom).

4. A fellow student of yours has found two different ways to implement the same filter in MATLAB. To his disappointment, both implementations turned out to be equally efficient (they both require the same number of multiplications, additions and memory).

Show that one of the implementations in Figure 4 has better performance in terms of round-off noise in the filter. *Assume that fix-point arithmetic is used.* Multiplications result in round-off errors. Additions do not cause overflow.

```
function y=my_filter1(x)

uold=0;
for n=1:length(x)
   u = x(n) + 0.6*uold;
   y(n) = 0.3*u + 0.1*uold;
   uold = u;
end
```

```
function y=my_filter2(x)

uold=0;
for n=1:length(x)
   y(n) = 0.3*x(n) + uold;
   uold = 0.1*x(n) + 0.6*y(n);
end
```

Figure 4: Filter realizations for problem 4

(10p)

5. A sensor is collecting samples $y(n)$ of a signal which is a time delayed version of a known waveform $x(t)$. If the time delay is τ_0 [s], we can therefore write

$$y(n) = x(nT_s - \tau_0) + \nu(n) ,$$

where T_s is the sampling period of the signal $y(n)$ and $\nu(n)$ denotes measurement noise. We want to estimate the delay τ_0. Below, you can assume that the noise is zero, $\nu(n) = 0$.

a) A natural method to find a copy of a waveform in a signal is to correlate the signal with the waveform, i.e. to calculate

$$z(\tau) = \sum_n y(n)x(nT_s - \tau) .$$

The correlation should be highest for the τ value that corresponds to the true delay. Show that this estimation method is equivalent to the Least Squares principle, i.e. to find the τ value that minimizes

$$\varepsilon(\tau) = \sum_n |y(n) - x(nT_s - \tau)|^2 .$$

(2p)

b) One problem of the methods described above is that τ is a continuous variable. Assume that we only have access to a sampled version $x_d(n) = x(nT_s)$ of the the waveform $x(t)$. Then a direct implementation of the methods in 5a) can only determine τ_0 within integer steps of T_s. However, this can be improved by converting the problem to the frequency domain.
Let $z_d(n)$ denote the discrete time correlation between $y(n)$ and $x_d(n)$,

$$z_d(n) = \sum_k y(k)x_d(k - n)$$

Assume that $x(t)$ is band-limited within half the sampling frequency. Show that the discrete-time Fourier transform (DTFT) of $z_d(n)$ is given by

$$Z_d(f) = \left|\frac{1}{T_s}X(f/T_s)\right|^2 e^{-j2\pi f\frac{\tau_0}{T_s}} , \qquad |f| < \frac{1}{2} ,$$

where $X(F)$ denotes the (time continuous) Fourier transform of $x(t)$.
Hint: remember that if $v(t) = x(t - \tau_0)$, then $V(F) = X(F)e^{-j2\pi F\tau_0}$. (3p)

c) Assume that $x(t)$ (and consequently $y(n)$) have a finite duration and that the correlation $z_d(n)$ contains at most N non-zero values. Show that the discrete Fourier transform (DFT), $Z_d(k)$ of length N of $z_d(n)$ is given by

$$Z_d(k) = \left|\frac{1}{T_s}X\left(\frac{k}{NT_s}\right)\right|^2 e^{-j2\pi k\frac{\tau_0}{NT_s}} , \qquad k = 0, \ldots, N-1 .$$

(To be precise, the transform should be calculated for $z_d((n)_{\text{mod } N})$ but you can pretend that $z_d(n)$ is zero for negative n, the result will be the same.) (2p)

d) If $|X(k/NT_s)|^2$ is reasonably constant for at least K consecutive k values. Describe briefly at least two methods that you have learned during the course, that can be used to estimate the delay τ_0 from the frequency domain data $Z_d(k)$. (3p)

Good luck!