KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2014–03–11, 14.00–19.00

Literature:

- Course text book:
  - Diniz, da Silva & Netto *Digital Signal Processing; System Analysis and Design*

    or
  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or
  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. Assume that you have a function (or circuit) capable of very efficiently computing the 16-point DFT (FFT) $X[k]$ where $k = 0, \ldots, 15$ of a finite length sequence $x[n]$ where $n = 0, \ldots, 15$.

a) Explain how you would use this function to compute the 16-point DFT $A[k]$ of the sequence $a[n] = x[15 - n]$ where $n = 0, \ldots, 15$. (2p)

b) Explain how you could use this function to compute the 8-point DFT $B[k]$ of a sequence $b[n]$ where $n = 0, \ldots, 7$, using no additional multiplications over what is already built into the function. (2p)

c) Explain how you could use 4 calls to this function and no more than 64 additional multiplications to compute the 64-point DFT $C[k]$ of a sequence $c[n]$ where $n = 0, \ldots, 63$. (6p)

2. Consider the filterbank structure below where the analysis filters are

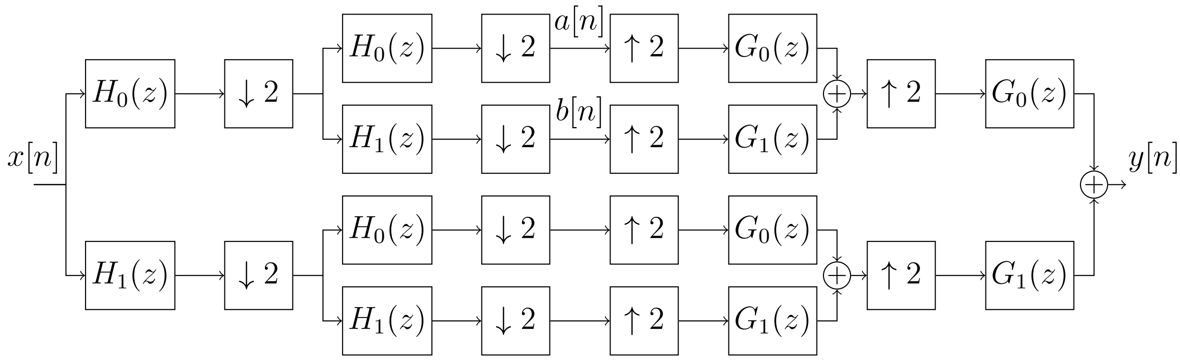
$$H_0(z) = \frac{1}{\sqrt{2}}(1 + z^{-1}), \quad H_1(z) = \frac{1}{\sqrt{2}}(1 - z^{-1})$$

and the synthesis filters are

$$G_0(z) = \frac{1}{\sqrt{2}}(1 + z^{-1}), \quad G_1(z) = \frac{1}{\sqrt{2}}(-1 + z^{-1}).$$

a) Prove that the filterbank provides perfect reconstruction, i.e., that $y[n] = x[n - L]$ for some integer L, and obtain L. (5p)

b) Obtain the power of $a[n]$ and $b[n]$ when $x[n]$ is a stochastic process given by $x[n] = A\sin(2\pi\nu n + \phi)$, where $\nu = 1/6$ and ϕ is uniformly distributed over $[0, 2\pi)$. Which signal, $a[n]$ or $b[n]$, contains most power? (5p)



3. The signal $x[n]$ has the form

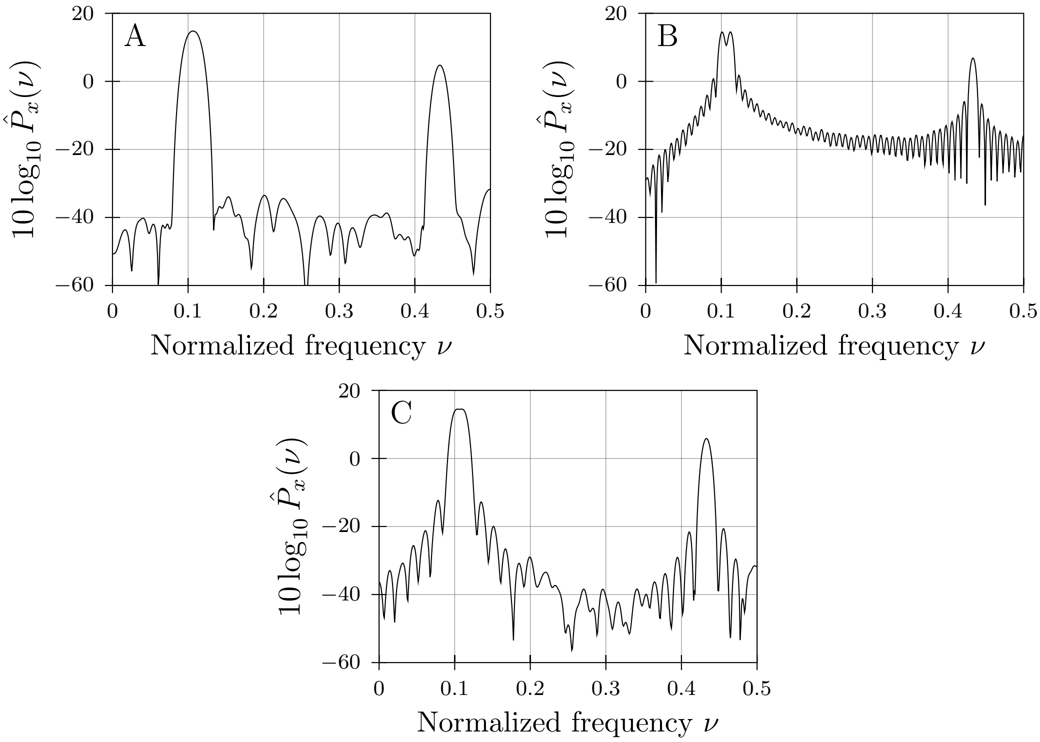
$$x[n] = \sum_{i=1}^{3} A_i \sin(2\pi\nu_i n + \phi_i) + e[n]$$

where $\nu_1 = 0.101$, $\nu_2 = 0.112$ and $\nu_3 = 0.433$; where $A_1 = 1$, $A_2 = 1$ and $A_3 = 0.4$; where ϕ_i for $i = 1, \ldots, 3$ are unknown; and where $e[n]$ is a white noise process with zero mean and variance $\sigma^2 = 10^{-4}$.

a) Assume that the power spectral density of $x[n]$ is estimated using modified periodograms with different windows, and that these yield the different figures below. Which window was used for each of the figures, and what property of the figure indicates this? Hint: Only windows from the table below were used, and you can get full points even if the answer is wrong if your motivation is solid. (4p)

b) If you did not know the noise variance σ^2, which spectrum estimate (A, B or C) would be most appropriate to use to estimate this variance, and why? (2p)

c) If you wanted to use Welch's method with a Blackman window to estimate the spectrum, have at least 20 blocks of data with 50% overlap to average over, and be able to resolve (see) all peaks due to to the sinus components in $x[n]$, how many data samples would you then need? (4p)



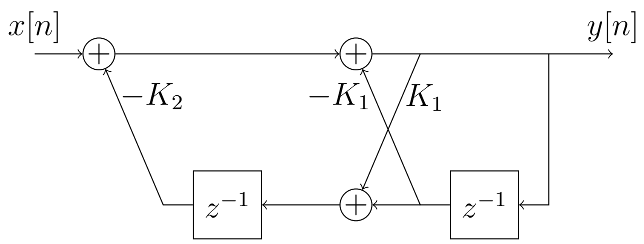
The table below (source: 'Hayes: *Statistical Digital Signal Processing and Modeling*) shows som key properties of different window functions when used in the length N modified periodogram.

Window	Sidelobe level (dB)	3 dB BW ($\Delta\nu$)
Rectangular	-13	$0.89/N$
Bartlett	-27	$1.28/N$
Hanning	-32	$1.44/N$
Hamming	-43	$1.30/N$
Blackman	-58	$1.68/N$

4. The lattice feedback IIR filter circuit in the picture below is implemented in $B+1$ bits fixed point precision (with one sign bit). All signals in the circuit are in the range $[-1,1]$, and $x[n]$ can be assumed to be a stochastic signal with broad spectral content. The multiplicative constants are

$$K_1 = \frac{5}{7} \quad \text{and} \quad K_2 = \frac{1}{6},$$

and multiplications are assumed to give rise to an additive roundoff error due to the fixed point implementation. Calculate the total power of the noise contained in $y[n]$ that is caused by the fixed point roundoff within the circuit. (10p)



5. The autocorrelation function (acf) of a stochastic process $x[n]$ has been estimated for lags $k = 0, 1, 2$ to be

$$\hat{r}_x(0) = 1, \quad \hat{r}_x(1) = \frac{5}{7}, \quad \hat{r}_x(2) = \frac{3}{7}.$$

a) Obtain the AR-2 based parametric power spectrum estimate of $x[n]$, and simplify the expression as much as possible (5p)

b) Obtain the Blackman-Tukey nonparametric spectrum estimate of $x[n]$ using a window $w[k]$ where $w[k] = 1 - |k|/3$ for $|k| \le 2$ and $w[k] = 0$ for $|k| > 2$. (3p)

c) The two figures below shows the AR-2 based spectrum estimate and the Blackman-Tukey spectrum estimate for this process, but which is which? (2p)