SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 88 **Digital Signalbehandling**, 2E1340

Final Examination 2002–08–23, 14.00–18.00

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solution (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. During the course, we have gone through a number of different methods for spectral estimation and frequency estimation.

Give a list of different methods. For each method, describe in a few words the main properties, advantages and disadvantages of each method. These methods can be classified into different general categories. Describe these categories and specify which methods belongs to which category.

5 different methods is sufficient to get full credits for this question. Use your own words; full sentences copied from the book or included mathematical formulas will **not** give any credits. (10p)

2. The circuit in Figure 1 uses fix-point arithmetics with b bits plus sign bit. The results of the multiplications are rounded, giving quantization noise. No round-off errors occur due to overflow.

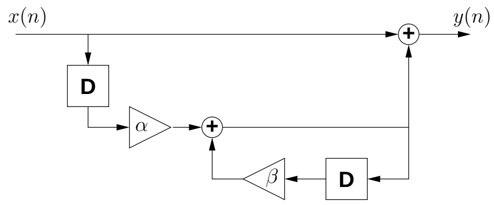

Figure 1:

a) Determine the power of the quantization noise at the output $y(n)$ as a function of b. (7p)

b) Plot the result with a practically useful range of b values on the horizontal axis and the corresponding noise power in dB on the vertical axis. Assume that $\alpha = 5/7$ and $\beta = 2/3$. (3p)

3. Polyphase implementations are often used for interpolation (see Figure 2) or decimation. Assume that (for some strange reason) we want a polyphase implementation of the time-discrete filter $H(f) = |f|$, $|f| \leq 1/2$. Determine analytical expressions for $P_0(f)$ and $P_1(f)$.

Hint: It is not necessary (and will probably take too long time) to determine the inverse transform of $H(f)$. Find a frequency domain relationship instead. (10p)

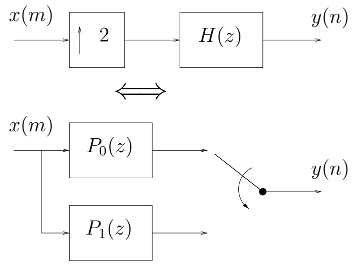

Figure 2: Example of polyphase implementation of interpolation.

4. A rotating machine, running with 20 rps (rounds per second), is vibrating. The vibration may be described by its fundamental frequency and up to 9 harmonics. A sensor is attached to the machine and the measured signal $x(t)$ is sampled, using an appropriate sampling frequency, and then analyzed with a 512-point FFT. Denoting the resulting transform $X(k)$, it is found the the only non-zero values are: $X(5)$, $X(10)$, $X(15)$, $\ldots$,$X(50)$ and $X(462)$, $X(467)$, $X(472)$, $\ldots$,$X(507)$.

Note that the vibration signal may be modeled with the Fourier series,

$$x(t) = \sum_{m=-10}^{10} c_m e^{j2\pi \frac{mt}{T}} \,,$$

since it is periodic and has only 9 harmonics.

a) What sampling frequency was used in the analysis? (5p)

b) Determine the coefficients c_m in the complex Fourier series of $x(t)$, as a function of $X(k)$. (5p)

5. Consider a signal model:

$$x(n) = \begin{cases} \theta_1 & n = 0 \\ \theta_2 & n = 1 \\ 0 & n = 2 \end{cases}$$

where we observe

$$\mathbf{x} = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \end{bmatrix}$$

a) Formulate the data model as $\mathbf{x} = \mathbf{H}\boldsymbol{\theta}$ and use the least squares (LS) method to estimate $\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix}$. (3p)

If we have a priori knowledge that the parameters are linearly related, we get a so-called constrained LS problem. In general, if we have a LS problem with a constraint $\mathbf{A}\boldsymbol{\theta} = \mathbf{b}$, we can solve this problem by minimizing the following Lagrangian

$$J_c = (\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) + \boldsymbol{\lambda}^T(\mathbf{A}\boldsymbol{\theta} - \mathbf{b}) \,.$$

Note that the first derivative of the Lagrangian is

$$\frac{\partial J_c}{\partial \boldsymbol{\theta}} = -2\mathbf{H}^T\mathbf{x} + 2\mathbf{H}^T\mathbf{H}\boldsymbol{\theta} + \mathbf{A}^T\boldsymbol{\lambda} \,.$$

b) Assume that we know a priori that $\theta_1 = \theta_2$. Write this constraint in the form $\mathbf{A}\boldsymbol{\theta} = \mathbf{b}$. (2p)

c) Find the estimates of this constrained LS problem. (3p)

d) An alternative approach to solve this problem is to reformulate the data model in order to incorporate the constraint into the new data model. Consider

$$\mathbf{H}_T = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

write down the new data model and solve the problem. Do you get the same estimates? (2p)