E 86 **Digital Signalbehandling**, 2E1340

Final Examination 2001–12–21, 14.00–18.00

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: At most one problem should be treated per page.
Motivate each step in the solution.
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

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Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. In contrast to problems 2–4, for the questions in this problem, it is sufficient to give the answer without motivation.

a) The signal $x(n)$ has the double-sided Z-transform $X(z)$. Which one of the following alternatives is correct? If we know that $x(n) = 0$ for all $n < 0$, the signal $x(n)$ is uniquely determined by

A. $X(z)$ alone
B. $X(z)$ together with the region of convergence (ROC)
C. The region of convergence alone
D. None of the above (2p)

b) A signal $x(n)$ (possibly complex-valued), $n = 0, 1, \ldots, 63$ has the $N = 64$ point DFT $X(k)$. What is the DFT of $y(n) = |x(n)|^2$?

A. $Y(k) = \frac{1}{N} X(k) \textcircled{N} X(k)$
B. $Y(k) = \frac{1}{N} |X(k)|^2$
C. $Y(k) = \frac{1}{N} X(k) \textcircled{N} X^*(N-k)$
D. None of the above (3p)

c) A time-discrete signal contains the sum of four different (real-valued) sinusoidal signals with normalized frequencies 0.1, 0.11, 0.3 and 0.32. Assume that you have 64 samples available. Using the periodogram with zero-padding it is impossible to resolve the peaks at 0.1 and 0.11 whereas the two peaks at 0.3 and 0.32 are clearly distinguishable. What happens if you use the Bartlett method instead, dividing the data into two segments?

A. None of the pairs of frequencies can be resolved (i.e. only two wide peaks are seen in the frequency range $[0, 0.5]$.
B. The peaks at 0.1 and 0.11 can be resolved but not those at 0.3 and 0.32.
C. The peaks at 0.3 and 0.32 can be resolved but not those at 0.1 and 0.11.
D. All four frequencies can be resolved.

(2p)

d) A signal is given in vector form by $\mathbf{x} = \mathbf{A}\mathbf{b} + \mathbf{w}$, where $\mathbf{A}$ is a known matrix, $\mathbf{b}$ is a vector of unknown parameters and $\mathbf{w}$ is a vector of N independent noise samples, each with variance σ_w^2. Assuming that $\mathbf{A}$ is full rank, it is well-known that the least squares estimate of $\mathbf{b}$ is given by $\hat{\mathbf{b}} = (\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{x}$. Which of the following expressions is the best estimate of σ_w^2?

A. $\hat{\sigma}_w^2 = \frac{1}{N}\mathbf{x}^*\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{x}$
B. $\hat{\sigma}_w^2 = \frac{1}{N}\mathbf{x}^*(\mathbf{I} - \mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*)\mathbf{x}$
C. $\hat{\sigma}_w^2 = \frac{1}{N}\mathbf{x}^*\mathbf{x}$
D. $\hat{\sigma}_w^2 = \frac{1}{N}\hat{\mathbf{b}}^*\hat{\mathbf{b}}$

(3p)

2.

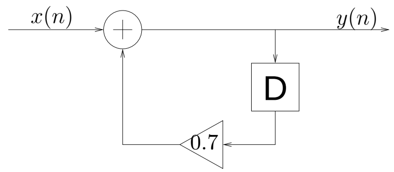

Figure 1: Filter with quantization

The circuit in Figure 1 is implemented using fixed point arithmetic, where b bits plus one sign-bit are used to represent the range $[-1, 1]$. The input signal is $x(n) = A\cos(2\pi f_0 n)$. The amplitude A can be adjusted to use the dynamic range of the circuit but must be kept small enough that all signals in the algorithm are guaranteed to fall within the range $[-1, 1]$. Determine the maximum signal to quantization noise ratio on the output $y(n)$. The signal to noise ratio is defined as the average signal power divided by the average quantization noise power.

Note: To be precise, the range is limited to $[-1, 1-2^{-b}]$, but you can use the simplified assumption stated above.

(10p)

3. It is well-known that a filter bank, as shown in Figure 2, provides perfect recon-

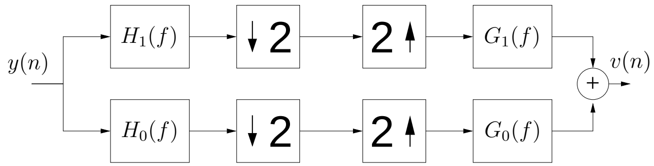

Figure 2: Standard filterbank

struction, i.e. $v(n) = y(n - L)$, if the following conditions are fulfilled:

$$\begin{aligned} H_1(f) &= G_0(f - \frac{1}{2}) \\ G_1(f) &= -H_0(f - \frac{1}{2}) \\ G_0(f)H_0(f) - G_0(f - \frac{1}{2})H_0(f - \frac{1}{2}) &= 2e^{-j2\pi fL} \end{aligned} \tag{1}$$

In the second project during this years course, you examined how the output is affected if quantization is applied to the decimated signals. Here, you should analyze what happens when a linear time-invariant operation is applied instead.

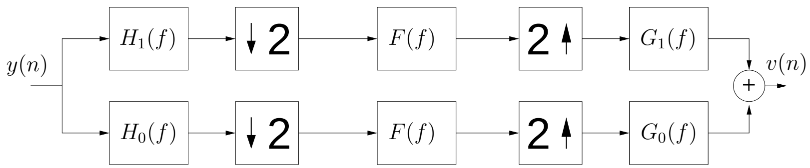

Figure 3: Filterbank in Problem 3

a) Assuming that the conditions (1) hold, determine a frequency domain relationship between $V(f)$ and $Y(f)$ for the system in Figure 3, where a linear time-invariant system with transfer function $F(f)$ is applied to the two decimated signals. (4p)

b) Determine the corresponding time-domain relationship between $v(n)$ and $y(n)$. (3p)

c) In particular, determine the time-domain relationship between $v(n)$ and $y(n)$ if $F(f) = e^{-j2\pi f\Delta}$. (3p)

4. An engineer has used the Levinson-Durbin algorithm, in an attempt to find an AR model for a measured signal. The resulting estimated reflection coefficients are shown in Table 1. The estimates where obtained from a set of $N = 25$ samples and the estimated power of the signal was $\hat{\sigma}_y^2 = 1.0$.

K_1	K_2	K_3	K_4	K_5
0.9	−0.8	−0.6	0.2	−0.3

Table 1: Reflection coefficients

a) Determine the optimal choice of model order, using the Akaike Information Criterion (AIC). (5p)

b) Determine the filter coefficients of the corresponding AR model (if you didn't manage to solve a), I'm afraid you have to determine the answer for all possible model orders). (5p)

Note: The AIC method simply means to determine the model order p that minimizes the cost function

$$\mathrm{AIC}(p) = \ln(\hat{\sigma}_p^2) + \frac{2p}{N},$$

where $\hat{\sigma}_p^2$ denotes the estimated prediction error of an one-step-ahead predictor of order p and N is the number of samples.

5. A time-continuous signal $x(t)$, consisting of two sinusoids at frequencies, $F_1 = 208$ Hz and $F_2 = 707$ Hz, is sampled, using a sampling frequency $F_s = 1000$ Hz, generating $x(n)$, $n = 0 \ldots 99$. An FFT is performed on the sampled signal, zero-padded to $N = 1024$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}} \qquad k = 0 \ldots N-1 \tag{2}$$

a) At what values of k do the maxima of $X(k)$ occur? (4p)

b) The signal $x(n)$, is decimated by a factor $D = 2$ and then reconstructed into a time-continuous signal. In the decimation circuit an ideal filter adapted to the decimator is included. What frequencies are present in this decimated reconstructed signal? Assume that the reconstruction is done corresponding to a sampling frequency of $\frac{F_s}{2}$. (2p)

The Pisarenko method is used to estimate the frequency of a sampled sinusoidal signal. The eigenvector, corresponding to the smallest eigenvalue of the covariance matrix, turned out to be $\mathbf{a} = \begin{bmatrix} 1.000 & -1.854 & 9.000 \end{bmatrix}^T$.

c) Determine the frequency of the signal, assuming the same sampling frequency as in a) above? (4p)