KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2014–01–15, 08.00–13.00

Literature:

- Course text book:
  - Diniz, da Silva & Netto *Digital Signal Processing; System Analysis and Design*

    or
  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or
  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

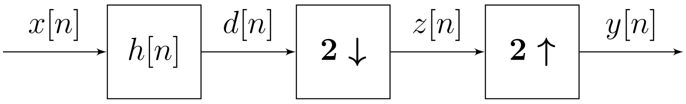
Contact: Joakim Jaldén, Signal Processing Lab, 08-790 77 88

Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. Let $x[n] = \cos(2\pi\nu_1 n) + \cos(2\pi\nu_2 n)$, with $\nu_1 = 1/16$, $\nu_2 = 1/8$ and $-\infty < n < \infty$, and consider that $x[n]$ passes through the system



where $h[n] = \{\underset{\uparrow}{1}, -\sqrt{2}, 1\}$.

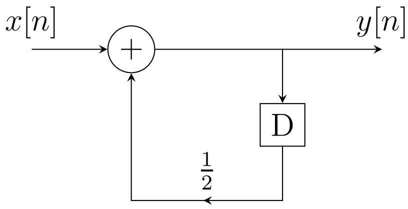
(*i*) Prove that the signal $d[n]$ (at the output of the filter $h[n]$) is going to be

$$d[n] = \frac{1}{2} e^{-j\frac{\pi}{8}n} H(-\nu_1) + \frac{1}{2} e^{j\frac{\pi}{8}n} H(\nu_1)$$

where $H(\nu)$ is the frequency response of the filter $h[n]$. (5p)

(*ii*) Calculate the spectrum of $y[n]$ (draw this if you like). (5p)

2. Let $x[n] = A \sin(2\pi\nu_0 n + \phi)$ where $A > 0$, $\nu_0 \in [-\frac{1}{2}, \frac{1}{2})$ and ϕ is uniformly distributed over $[-\pi, \pi)$ (this is the so-called harmonic process). The process is fed into the system shown below



which is implemented in fixed point arithmetic using $(b+1)$ bits, of which one bit is a sign bit, and where all signals have an amplitude in $[-1, 1]$. If $\nu_0 = \frac{1}{3}$, calculate the signal to quantization noise ratio (SQNR) at the output $y[n]$ as a function of A and b (i.e., the ratio of the power of the signal divided by the power of the noise). (10p)

3. Let $x[n]$, $n = 0, \ldots, N$, be a set of data samples obtained from a real valued zero mean wide sense stationary stochastic process. The Blackman-Tukey spectral estimator is given by

$$\hat{P}_{\mathrm{BT}}(\nu) = \sum_{k=-M}^{M} w[k]\hat{r}_x[k]e^{-j2\pi\nu k},$$

where $w[k]$ is a symmetric window function for which $w[k] = 0$ for $|k| > M$, where $0 < M \ll N$, and where $\hat{r}_x[k]$ is a biased ACF estimate given by

$$\hat{r}_x[k] = \begin{cases} \frac{1}{N}\sum_{n=0}^{N-|k|} x[n]x[n+|k|] & |k| < N \\ 0 & |k| \geq N \end{cases}.$$

For a Gaussian stochastic process, the so called maximum likelihood estimate of the power of the process is given by

$$\hat{P}_{\mathrm{ML}} = \frac{1}{N}\sum_{n=0}^{N} x^2[n],$$

but one can also in principle obtain an estimate of the power from the power spectral estimate by integrating over the normalized frequencies as

$$\hat{P}_{\mathrm{BT}} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \hat{P}_{\mathrm{BT}}(\nu)d\nu.$$

a) Prove that as long as $w[0] = 1$ the two estimates of the power are the same, or in other words, that $\hat{P}_{\mathrm{ML}} = \hat{P}_{\mathrm{BT}}$. (6p)

b) If $w[k] = 1$ for $|k| \leq M$ and $w[k] = 0$ otherwise, will you be more or less likely to be able to resolve two nearby frequencies in the spectrum using the Blackman-Tukey spectral estimator if you increase M? Motivate your answer. (2p)

c) If $w[k] = 1$ for $|k| \leq M$ and $w[k] = 0$ otherwise, what will happen with the variance of the Blackman-Tukey spectral estimator if you increase M? Motivate your answer. (2p)

4. We are interested in filtering a signal $x[n]$ of length L, for $x[n] = 0$ when $n < 0$ and $n \geq L$, with a zero group delay *non-causal* symmetric FIR filter $h[n]$ where $h[n] = 0$ for $|n| > M$ and $h[n] = h[-n]$. This can be done using an overlap save type algorithm with N-point DFTs according to

1: $n_0 \leftarrow n_{\text{start}}$
2: **while** $n_0 < n_{\text{stop}}$ **do**
3: $\quad x_{\text{B}}[n] \leftarrow x[n + n_0] \quad$ for $n = 0, \ldots, N-1$
4: $\quad X_{\text{B}}[k] \leftarrow \mathcal{F}_N\{x_{\text{B}}[n]\}$
5: $\quad Y_{\text{B}}[k] \leftarrow Z[k] X_{\text{B}}[k] \quad$ for $k = 0, \ldots, N-1$
6: $\quad y_{\text{B}}[n] \leftarrow \mathcal{F}_N^{-1}\{Y_{\text{B}}[k]\}$
7: $\quad y[n + n_0 - n_{\text{shift}}] \leftarrow y_{\text{B}}[n] \quad$ for $n = n_1, \ldots, N-1$
8: $\quad n_0 \leftarrow n_0 + n_{\text{add}}$
9: **end while**

where $\mathcal{F}_N$ and $\mathcal{F}_N^{-1}$ denotes N-point DFT and inverse DFT respectively.

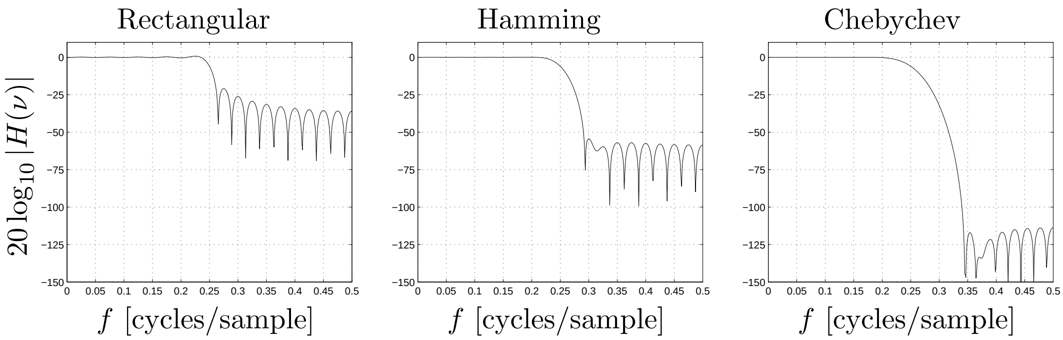
a) Explain how you could select $Z[k]$ on line 5 for $k = 0, \ldots, N-1$, n_1 on line 7, n_{add} on line 8, and n_{shift} on line 7 so that the algorithm will be able to correctly compute all non-zero values of output signal

$$y[n] = h[n] * x[n] = \sum_{m=-M}^{M} h[m] x[n-m]\,.$$

(8p)

b) Assuming that you use the FFT algorithm to efficiently compute the DFT and the inverse DFT, and that $L \gg N$, what would the computational complexity then be in terms of the number of complex valued multiplications per sample? (2p)

5. The figures below show three low pass (LP) filters designed to have a (normalized) cutoff frequency at $\nu_c = 0.25$. The three different filters are Type I causal linear phase FIR filters with $N = 41$ taps, and are designed using the window method with a rectangular window, a Hamming window, and a Chebyshev window repectively.



Your task is to design, also using the window method, a causal linear phase high pass (HP) FIR filter of length $N = 41$ with cutoff frequency $\nu_c = 0.25$. The design specification states that the filter must have a suppression (attenuation) in the stop band of more than 50 dB for all normalized frequencies up to $\nu_s = 0.2$.

a) Which window do you choose and why? Motivate your answer carefully. (2p)

b) Compute the values for the non-zero filter coefficients $h[n]$ for $n = 0, \ldots, N-1$. You may use $w[n]$ for $n = 0, \ldots, N-1$ to denote the chosen window function and can express your answer in terms of $w[n]$ and suitable trigonometric functions and expressions. (6p)

c) If a stop band suppression of 100 dB was required for all normalized frequencies in the range [0, 0.2], explain how you would have to change the design so that you could satisfy this new requirement. A qualitative argument is sufficient, you do not need to give any explicit numbers. (2p)