SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 94 **Digital Signalbehandling**, 2E1340

Final Examination 2004–08–27, 14.00–19.00

Literature: Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 08-790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage.

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1. As you know from previous math courses, the sum of two sinusoidal signals can be written also as the product of two sinusoids,

$$x(n) = \cos(2\pi f_1 n) + \cos(2\pi f_2 n) = 2\cos(2\pi f_{\text{aver}} n)\cos(2\pi f_{\text{diff}} n)\ ,$$

where $f_{\text{aver}} = (f_1 + f_2)/2$ is the average frequency and $f_{\text{diff}} = (f_1 - f_2)/2$ is half the frequency separation.

Figure 1 shows $N = 256$ samples each of three such signals, with different frequency separation. The noise in the signals is so weak that it can be neglected.

Hint for the questions below: how do f_{aver} and f_{diff} show up in the figures?

a) If you use the periodogram method to estimate the two frequencies based on $N = 256$ samples, in which of the three signals `Signal 1-3` will you be able to determine the two frequencies. (3p)

b) Answer the same question if you, instead, divide the available 256 samples into 16 segments and use the Bartlett method. (3p)

c) For the periodogram method, formulate a general rule of thumb, that says if it is possible to resolve the two frequencies or not, based on what the available signal samples look like in the time domain. (4p)

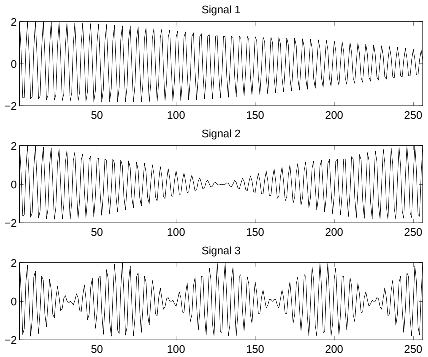

Figure 1: Three examples of the sum of two sinusoids, $x(n)$.

2. The first-order filter with transfer function

$$H(z) = \frac{1 + 0.7z^{-1}}{1 - 0.23z^{-1}}$$

can be implemented in two different ways. The implementations are mathematically equivalent, but will have different numerical properties due to finite-word-length effects.

a) Draw block diagrams of *two* implementations of the filter $H(z)$ that have *different* numerical properties. (3p)

b) Assume a fixed-point number representation using $B + 1$ bits, of which one bit is a sign bit. All multiplications result in round-off errors, but there is no overflow. Determine the power of the round-off error at the output for both implementations in a). Which implementation gives the smallest round-off noise? (7p)

3. You are given the task to implement a system that should sample a signal at 1MHz sampling frequency, filter the signal using an FIR filter with 1000 taps (filter coefficients) and send out the resulting signal. The real-time specifications of the system state that the total delay from input to output may not exceed 8ms. The DSP that you can use for the calculations can handle 100MFlops (MFlops=million floating point operations per second). Since it clearly is impossible to process the data sample by sample, you use an implementation with double input buffers and double output buffers, where one input buffer is processed while the other one is being filled. The overhead to switch between the buffers takes about 100ns.

Describe how you would design and implement the system and specify all relevant parameters, such as buffer sizes. At your disposal, you have a nice implementation of a radix-2 FFT algorithm. When calculating the computation times, you can simplify and assume that only the multiplications take any time. (10p)

4. Figure 2 shows a generalization of the standard filter banks, dividing the signal into D subbands instead of 2.

a) Derive an expression for $Y(f)$ (the DTFT of $y(n)$) in terms of $X(f)$, $H_i(f)$ and $F_i(f)$. (5p)

b) Find conditions on the filters $H_i(f)$ and $F_i(f)$ such that $y(n) = x(n - L)$ for all signals $x(n)$, i.e. conditions so the filter bank provides perfect reconstruction with a given delay L. (5p)

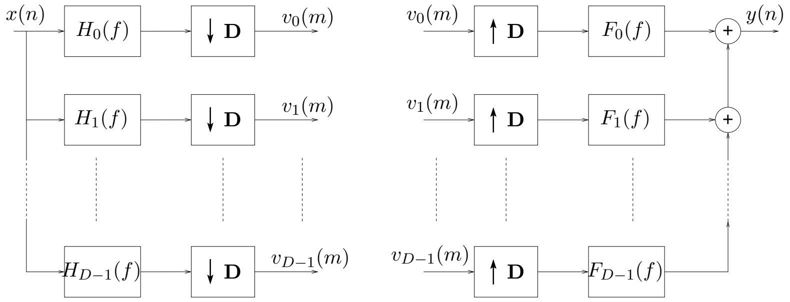

Figure 2: Generalized filter bank.

5. A common problem in image and video processing, is to locate a given object in an image, for example in sports broadcast, traffic surveillance and video coding (often the only difference between two consecutive frames of a video signal is that some objects have moved, so you can save lots of bandwidth by only storing the relative location of each object instead of the full picture). Often, not only the location but also the absolute size of the object is unknown, since the distance to the camera and the zoom of the camera lens may differ. To avoid the complications of handling the two dimension of an image, we here consider the corresponding problem for a one-dimensional signal.

Figure 5 illustrates how an "object" can be specified as a pulse shape $p(n)$. The measured signal $x(n)$ contains a copy of this pulse shape that has been stretched and moved in time. Also, the measurement contains some additional noise.

a) Formulate a mathematical model for the measured signal $x(n)$ in terms of the given pulse shape $p(n)$ and two parameters that describe the start time of the pulse and how the pulse has been stretched in time. (4p)

b) Propose a method to estimate the two unknown parameters from samples of $x(n)$. The solution can be given in the form of an optimization problem to be solved numerically, there is no need to consider the numerical implementation. (6p)

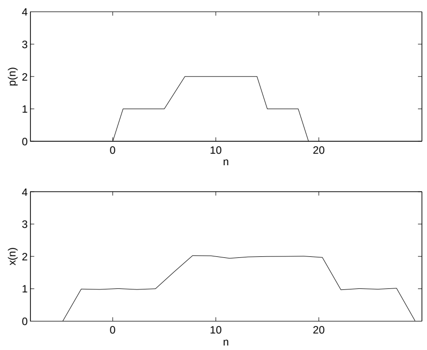

Figure 3: Template pulse shape (top) and measured signal (bottom).

Good luck!