SIGNALBEHANDLING
Institutionen för Signaler, Sensorer & System

E 90 **Digital Signalbehandling**, 2E1340

Final Examination 2003–04–30, 08.00–13.00

Literature: Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
On popular demand: Copies of the slides, distributed with the exam.
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Josefsson: formel- och tabellsamling i matematik
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solution (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. a) Figure 1 shows the implementation of a filter $h(n)$. For what values of the parameter α is the filter stable?. (3p)

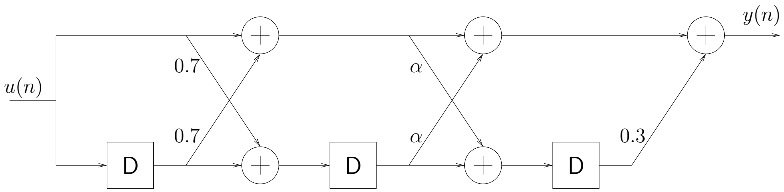

Figure 1: $h(n)$

b) Figure 2 shows the implementation of a filter $g(n)$. For what values of the parameter β is the **inverse** of the filter $g(n)$ stable?. (3p)

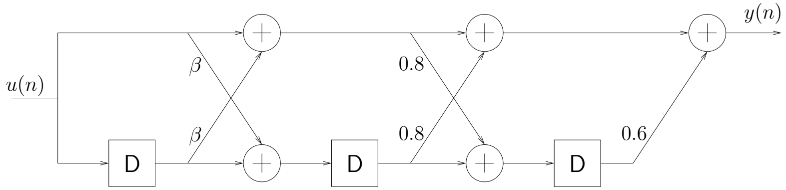

Figure 2: $g(n)$

c) A signal is sampled at $F_s = 16$kHz and 1500 samples are stored in a buffer. The data is then downsampled by a factor 3 and a Periodogram is calculated from the resulting 500 samples.

What is the main frequency of the original signal if the Periodogram has a peak at $k = 200$? The Periodogram is calculated using zero-padding and an FFT of length 1024. (4p)

2. A sensor produces a signal $u(n)$ with limited amplitude $|u(n)| \leq 5$. The signal is sampled, quantized and sent through the digital filter shown in Figure 3. All calculations are performed with fixed point numbers using 10 bits (including the sign bit) and the results of the multiplications are rounded to the closest number that can be represented. It can be shown that the output signal (and all intermediate values) is guaranteed to stay in the range $[-30, 30]$.

Calculate the quantization error at the output $y(n)$. Notice that the implementation should be done such that no overflow occurs during the filtering. (10p)

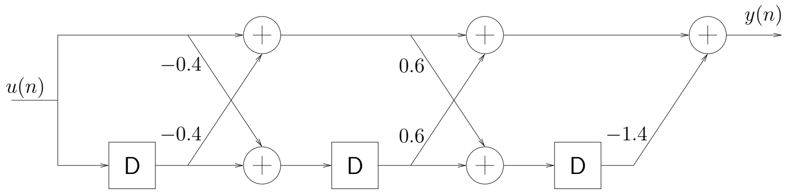

Figure 3: Fixed point filter

3. In this problem, we will study the basic principle of so-called multi-carrier techniques for digital communication over a radio link or cable.

Let $x(n)$ denote the time discrete signal to be transmitted. Then, we can approximately describe the received signal after down-conversion and sampling, $r(n)$, using the following model,

$$r(n) = h(n) * x(n) = \sum_{k=0}^{L} h(k)x(n-k)$$

Here, the communication channel (cable, radio link or whatever) is described as an FIR filter $h(k)$ of order L. A realistic channel model should also include additive noise, but for simplicity we ignore the noise here.

In multicarrier systems, data is transmitted in blocks, where each block contains M values of a data stream $d(n)$. However, the transmitted block also contains a so-called cyclic prefix. This means that the last values in the block are duplicated at the beginning of the block. Consider for example the first block of data $\{d(0), d(1), \ldots, d(M-1)\}$. The corresponding block to be transmitted, including the cyclic prefix, is then

$$x(n) = \{\underbrace{d(M-K), d(M-K+1), \ldots d(M-1)}_{\text{cyclic prefix}}, \underbrace{d(0), d(1), \ldots, d(M-1)}_{\text{original data}}\} .$$

At the receiver, the initial K values are dismissed and the following M values are saved in a buffer $y(n) = \{r(K+1), r(K+2), \ldots, r(K+M)\}$.

Show that the original data stream can be recovered using the following algorithm if the cyclic prefix is long enough.

- $Y(k) = \mathrm{FFT}[y(n)]$
- $\hat{D}(k) = Y(k)/H(k)$
- $\hat{d}(n) = \mathrm{IFFT}[\hat{D}(k)]$

Determine how the length of the cyclic prefix, K, should be chosen such that $\hat{d}(n) = d(n)$ holds for all $n = 1, \ldots, M$. The answer should be given in terms of L, the length of the channel, as defined above.

Exactly how should $H(k)$ be calculated from $h(n)$? (10p)

4. During the course, you have learned how to process long data sequences by using the FFT and block-wise processing. You have also had the opportunity to study the use of filter banks in signal compression. Now, it is time to look at an analogy between a two-channel filter bank and a two-point DFT (a basic building block in the radix-2 FFT algorithm).

Block-wise processing can be seen as serial to parallel conversion, followed by parallel processing, and finally parallel to serial conversion. This scheme is depicted in Figure 4 for a 2-point DFT system.

a) Find expressions for the filters $H_0(z)$, $H_1(z)$, $F_0(z)$ and $F_1(z)$ in Figure 5, such that the analysis and synthesis filter banks are equivalent to the 2-point DFT and IDFT in Figure 4. (7p)

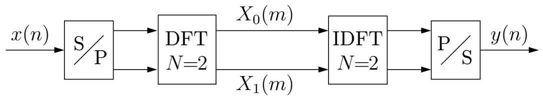

Figure 4: DFT system

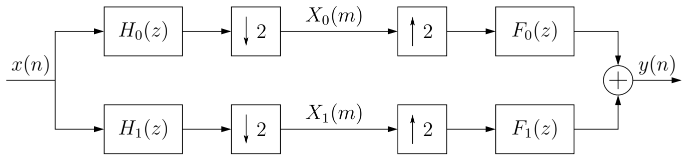

Figure 5: Filter bank system

b) From Figure 4 it is clearly seen that $y(n)$ is simply a delayed version of $x(n)$. What is the total delay of the two-channel filter bank system, and how does it relate to the DFT system? (3p)

5. a) A former student in Digital Signal Processing has implemented a routine to calculate the periodogram of a signal. A few months later, he discovers that he actually needed a so-called modified periodogram, where the data is windowed before it is Fourier transformed:

$$\hat{P}_{xx}^{M}(f) = \frac{1}{N}\left|\sum_{n=0}^{N-1} w(n)x(n)e^{-j2\pi fn}\right|^2 , \tag{1}$$

where the window $w(n)$ is selected to reduce the leakage in the spectral estimate. It is easy for him to modify the program to incorporate the windowing. However, the question is how he easiest can handle all the data he has collected and stored using the first version of the system. Unfortunately, he has only stored the FFT of the original data, not the time-domain data itself.

Show how to calculate the modified periodogram (1) at the frequencies $f = k/N$, $k = 0, 1, \ldots, N-1$, if you are given the N-point FFT of $x(n)$ and the N-point FFT of the window function $w(n)$. The solution should not involve any FFT or IFFT operations. (7p)

b) Would it be possible to calculate the modified periodogram if only the ordinary periodogram

$$\hat{P}_{xx}(f) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-j2\pi fn}\right|^2$$

was available, and not the FFT of $x(n)$?

If so, show how. If not, explain why. (3p)

Good luck!