KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2016–01–16, 8:00-13:00

Literature:

- Jaldén: *Summary notes for EQ2300* (30 page printed material).
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH.*
- *One A4 of your own notes.* You may write on both sides, and it does not have to be hand written, but cannot contain full solutions to tutorial problems or previous exam problems.
- An unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. We begin with a mix of assorted DSP problems. Remember to motivate your answers to these problems as well.

a) Compute the result of the following circular convolution (2p)

$$\{3,\,0,\,2,\,0,\,1,\,0\} \textcircled{6} \{3,\,2,\,1,\,0,\,0,\,0\}\,.$$

b) The 6-point DFT is computed for a real valued input sequence and yields

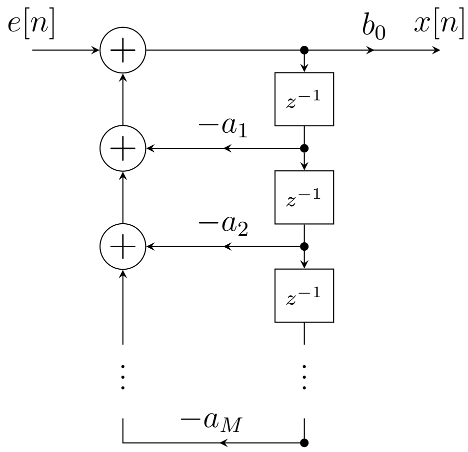
$$\mathcal{F}_6\{x[n]\} = \{2,\,1,\,j,\,0,\,?,\,?\}$$

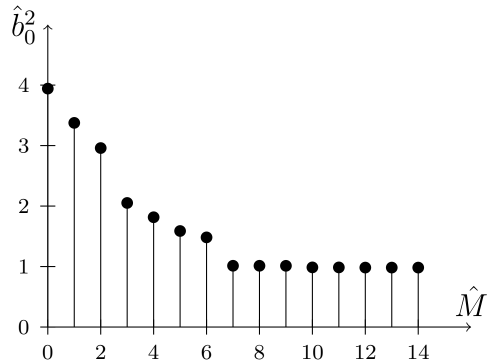
where $x[n] \in \mathbb{R}$ and $j = \sqrt{-1}$. Find the missing values denoted by "?". (2p)

c) The discrete time signal $x[n] = \sin(2\pi\nu_x n + \phi_x)$, where $\nu_x = 0.4$ and $\phi_x = 0.3$, is downsampled by a factor $D = 3$. The resulting signal has the form $y[m] = \sin(2\pi\nu_y m + \phi_y)$ where $\nu_y \in (-\frac{1}{2}, \frac{1}{2})$. What are ν_y and ϕ_y? (2p)

d) Suppose that $H(z)$ is the transfer function of a high-pass filter with impulse response $h[n]$. What kind of filter (low-pass, high-pass, band-pass, or band-stop) is the filter with transfer function $G(z) = H(-z)$, and what is its impulse response $g[n]$ expressed in terms of $h[n]$. (2p)

e) A stochastic process $x[n]$ is generated by the AR model shown below, where $e[n]$ is a zero mean unit variance driving noise, for some unknown model order M. To estimate the model order M, models of varying tentative orders $\hat{M}$ are fitted to an estimate of the acf of $x[n]$ by solving the Yule-Walker equations, and the resulting gain coefficient $\hat{b}_0$ is plotted against $\hat{M}$. What is likely the true model order M of the generating model? (2p)





2. In this problem, we will assume that we have sampled a time continuous signal $x(t)$ as $x[n] = x(nT_s)$ with a sample frequency $f_s = 1/T_s = 1\,\text{kHz}$ over one second. Our objective will be to get information about the spectral properties of the time discrete signal and relate them to those of the time continuous signal. To do so, we will process the 1000 samples in $x[n]$ for $n = 0, 1, \ldots, N-1$ with $N = 1000$ to compute estimates of the discrete time spectrum $P_x(\nu)$ of $x[n]$.

a) Suppose we use the estimate of $P_x(\nu)$ given by

$$\hat{P}_x(\nu) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x[n]e^{-j2\pi\nu n}\right|^2 .$$

What will then the spectral resolution be when expressed in terms of the normalized frequency ν? (2p)

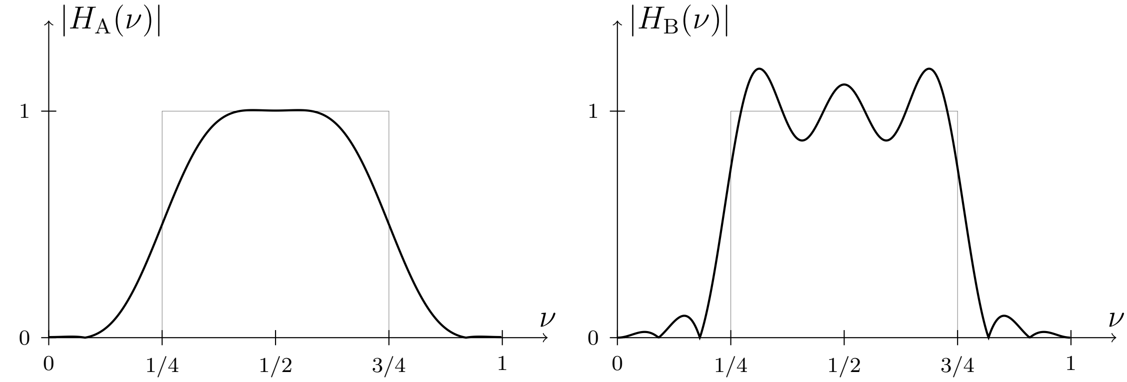
b) What will the spectral resolution of the estimator in part a) be when expressed in Hz as related to the frequencies of the original signal $x(t)$? (1p)

c) Assume that we want to evaluate the estimate $\hat{P}_x(\nu)$ at discrete frequencies $\nu = \nu_k$, for $k = 0, \ldots$ using the radix-2 FFT algorithm. What is the minimum number of zero-samples that we will need to append to the end of the signal $x[n]$ so that we can efficiently evaluate the FFT? (1p)

d) After the FFT in c) is computed, what is the spacing, measured in Hz, between the frequency-domain FFT samples ν_k, i.e., between ν_k and ν_{k+1}? (2p)

e) Assume now that, instead of the estimator in a), we use Welch's method with a triangular (Bartlett) window and 50% overlap to get a better variance of our spectral estimate than what we get with the estimator from a). If we require a minimum spectral resolution of 10 Hz, what factor of reduction in the estimator variance could we get as compared to the estimator in a)? (4p)

3. We wish to design a discrete-time unit-gain high-pass filter with normalized cutoff frequency $\nu_c = 1/4$. The filter should be a linear phase filter of Type I and order $M = 10$ meaning that the filters impulse response $h[n]$ should be symmetric in the sense that $h[n] = h[M-n]$ for $n = 0, \ldots, M$, and finite in the sense that $h[n] = 0$ for $n < 0$ and $n > M$. We consider two options to design the filter: using frequency sampling with a regular frequency grid, and using the windowing method with a Hanning (Hann) window. The resulting frequency responses of the designed filters are shown below, along with the desired ideal frequency response as the thin line.



a) Which of the frequency responses, $H_A(\nu)$ or $H_B(\nu)$, corresponds to a filter designed using frequency sampling, and which correspond to a filter designed using the windowing? Motivate you answer for full points. (2p)

b) Obtain the filter's impulse response $h_1[n]$ for $n = 0, \ldots, M$ for the frequency sampling design. (3p)

c) Obtain the filter's impulse response $h_2[n]$ for $n = 0, \ldots, M$ for the windowing design. You may be helped by knowing that the length $N = M + 1$ Hanning window is given by

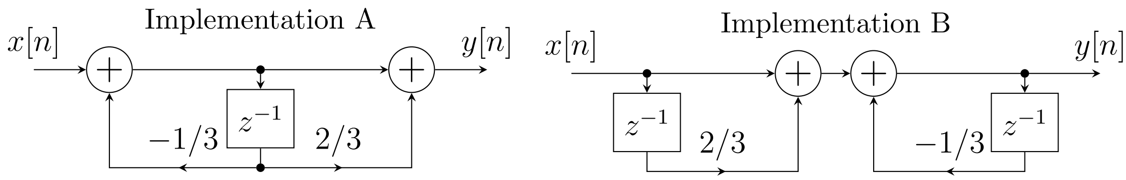
$$w[n] = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi n}{M}\right)$$

for $n = 0, \ldots, M$. (5p)

4. The two systems shown below both implement the same transfer function, namely

$$H(z) = \frac{z + 2/3}{z + 1/3},$$

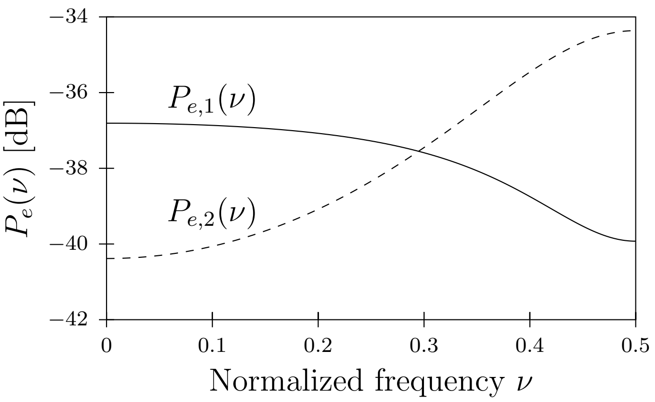
but they will have different characteristics when implemented in fixed point arithmetics.



We will in this problem assume that both implementations are realized in $B + 1$ bit signed-magnitude fixed-point over the range $(-1, 1)$. You can assume that no signal overflow occurs.

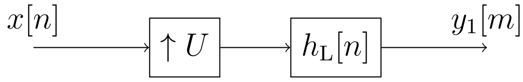
a) Draw the additive quantization noise model for both implementations, i.e., illustrate where in the systems the quantization noise appears and how you would model it under the common assumptions. (2p)

b) The following two power spectra illustrate the power spectral density of the quantization noise at the output of the system for the two fixed point implementations when $B = 5$. Which power spectrum corresponds to which implementation? Motivate your answer. (2p)



c) Compute the total power of the quantization noise in the output under both implementations. Which implementation yields the smallest quantization noise in the output? (6p)

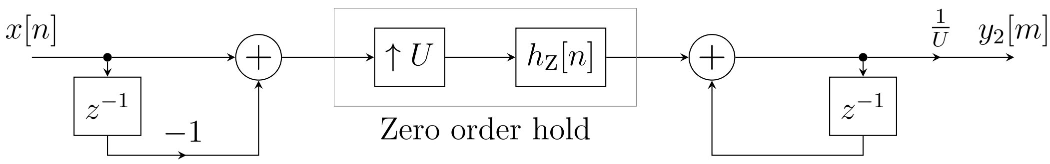
5. A straightforward linear interpolator with upsampling factor U is given by



where

$$h_L[n] \triangleq \begin{cases} \frac{U-|n|}{U} & |n| < 0 \\ 0 & |n| \geq U \end{cases}$$

is the (non-causal) linear interpolation filter. However, a direct implementation of this circuit is far from the most computationally efficient way to implement linear interpolation. In order to improve the efficiency in term of the number of multiplications carried out one can make use of a zero order hold circuit together with simple MA and AR circuits according to



where

$$h_Z[n] \triangleq \begin{cases} 1 & 0 \leq n \leq U-1 \\ 0 & \text{otherwise} \end{cases}$$

is the zero order hold filter, which together with the upsampling operation keeps the output of a sample for U time steps before the next sample arrives. By noting that multiplications with -1 is only a sign change, and that we could implement $h_Z[n]$ without any multiplications, we can implement the whole circuit using only one single multiplication per output sample given by the multiplication by $1/U$ at the end. However, as the second circuit is causal and the first one is not, the second circuit will also introduce a time delay of L samples.

Your task is to prove that the two circuits are equivalent up to the delay L, i.e., that $y_2[m] = y_1[m - L]$, and also compute the delay L explicitely. For full credits, you should prove this for all integer U, but if you get stuck in the formulas, partial credit will be given if you prove it for $U = 2$. (10p)