KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2017–01–11, 8:00-13:00

Literature:

- Jaldén: *Summary notes for EQ2300* (30 page printed material).
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH.*
- *One A4 of your own notes.* You may write on both sides, and it does not have to be hand written, but cannot contain full solutions to tutorial problems or previous exam problems.
- An unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. As usual, we begin with a few mixed shorter questions...

a) Match the 8-point sequences $x_{\mathrm{A}}[n]$ to $x_{\mathrm{D}}[n]$ shown on the following page to their corresponding 8-point DFTs $X_{\mathrm{I}}[k]$ to $X_{\mathrm{IV}}[k]$. (4p)

b) Which one of the following filters does not have linear phase, and why? (2p)

i) $h[n] = 1 - \frac{1}{2}z^{-1} + z^{-2}$

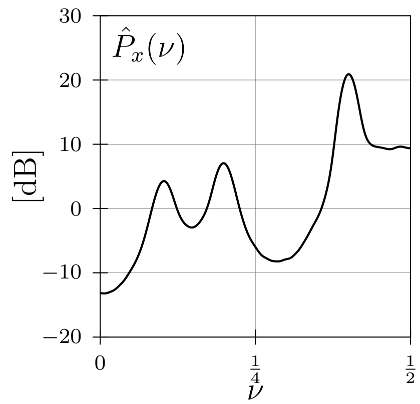
ii) $h[n] = z^{-5}$

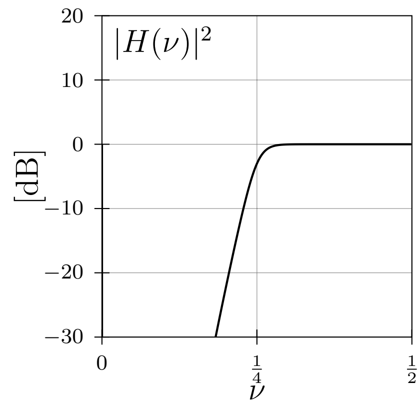
iii) $h[n] = 1 + \frac{1}{2}z^{-1}$

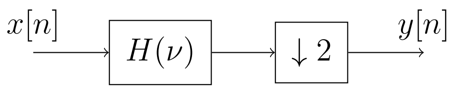
iv) $h[n] = 2z^{-1} - 2z^{-3}$

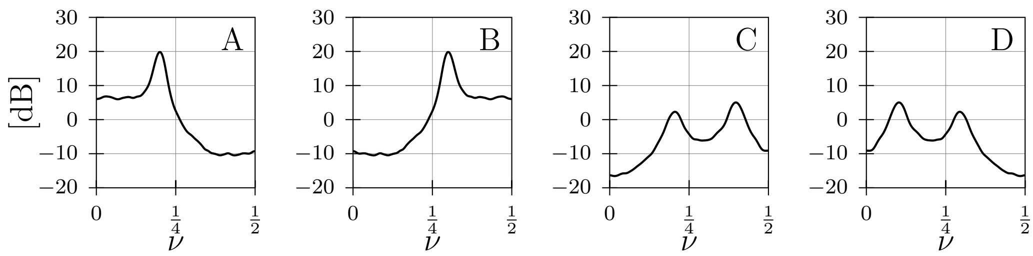
c) When designing a low pass filter using the windowing method, what are the two main differences in the resulting filter when using a Hamming window in place of the triangular (Bartlett) window. (2p)

d) A stochastic process $x[n]$ with the power spectrum shown below is filtered using an 8th order Butterworth high pass filter with frequency response $H(\nu)$ shown below, and then down sampled by a factor of 2. Which one of the spectra A to D is the power spectrum of the output $y[n]$? Motivate your answer. (2p)

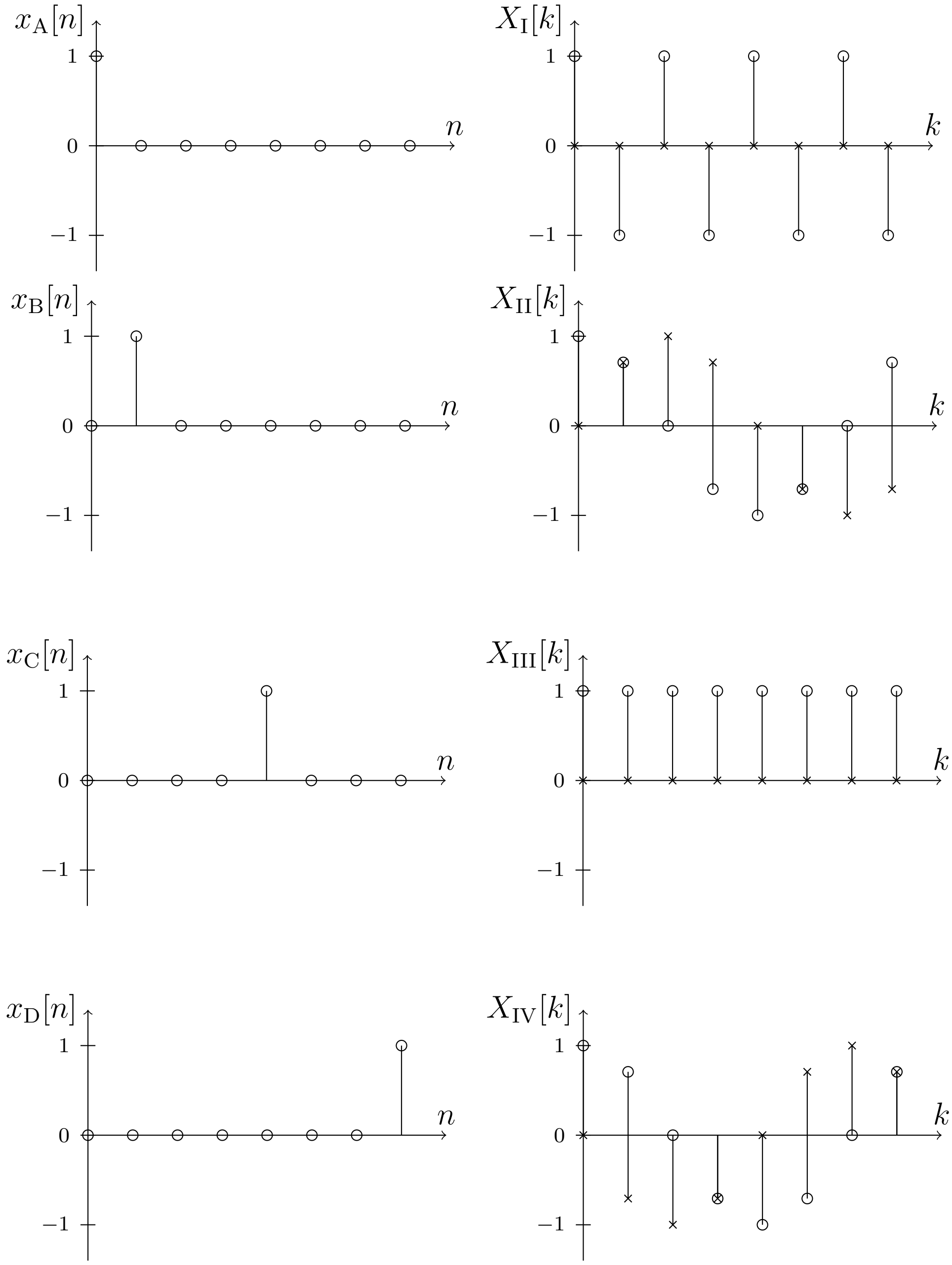








In the figures below, the real part of complex valued quantities is denoted by $\circ$, and the imaginary part is denoted by $\times$.



2. Consider the following Matlab code that implements the overlap-add filtering method. The code can efficiently compute $y[n] = h[n]*x[n]$ where $x[n]$ denotes the input signal, and $y[n]$ denotes the output signal.

```
H = fft(h,N);
y = zeros(1,(B-1)*L+N);
for b=1:B
    xB = x((b-1)*L+1:(b-1)*L+L);
    XB = fft(xB,N);
    YB = H.*XB;
    yB = ifft(YB,N);
    y((b-1)*L+1:(b-1)*L+N) = y((b-1)*L+1:(b-1)*L+N) + yB;
end
```

Note that the naming convention of the constants may differ from those used in the summary notes, and your course book.

a) Assume that the vector `h`, containing the impulse response of the filter, has length 15. Suggest appropriate (good) values for the constants `N` and `L` in the code, and remember to motivate your choice. (2p)

b) Assume that the vector `x` containing the input signal $x[n]$ has length 1000. Given your answer to part a), what would be the smallest value for `B` that allows us to compute all non-zero values of $y[n]$? What is the total number of complex-valued multiplications required to compute $y[n]$? (3p)

c) Compare the overlap-add implementation to computing $y[n]$ using a standard convolution in the time-domain. How many complex valued multiplications would be needed to compute all non-zero values of $y[n]$ using the standard convolution, and how does it compare to your answer in b). (2p)

d) Compare the overlap-add implementation to the quick and dirty FFT-based implementation shown below

```
y = ifft(fft(h,M).*fft(x,M));
```

What is the smallest value for `M` on the form 2^p that would correctly compute all non-zero values of $y[n]$ (stored in `y`)? How many complex valued multiplications would this implementation require? Is it better or worse than b)? (3p)

3. Consider the classical sinus-in-noise model of a stochastic process, given by

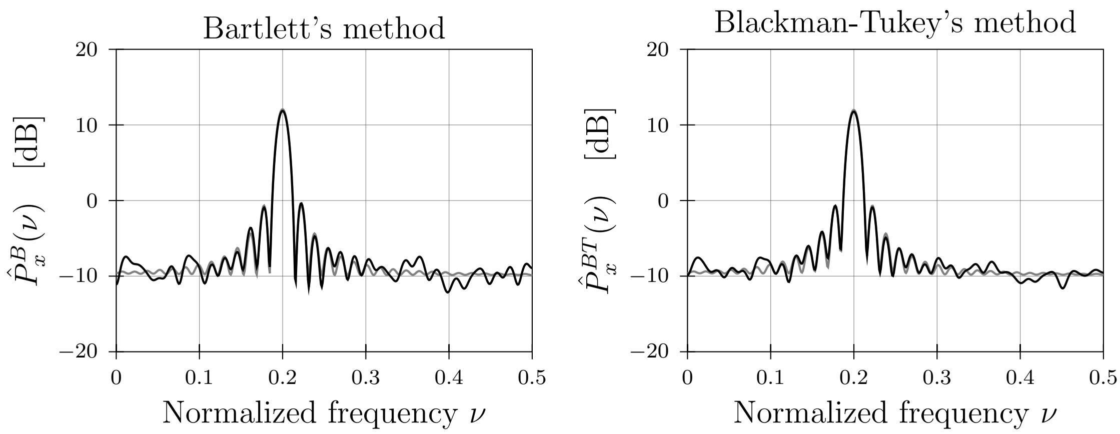
$$x[n] = A\sin(2\pi\nu_0 n + \phi_0) + w[n]$$

where A is the sinus amplitude, where ν_0 is the normalized frequency of the sinusoid, where ϕ_0 is random phase that is uniformly distributed over $[0, 2\pi]$, and where $w[n]$ is zero mean Gaussian white noise of power σ^2. Given $N = 1024$ consecutive samples of $x[n]$, the power spectrum $P_x(\nu)$ of $x[n]$ is estimated using:

i) Bartlett's method with $K = 16$ blocks of length $L = 64$; and

ii) Blackman-Tukey's method with a maximum lag of $M = 64$ and a Bartlett (triangular) window.

The results of the spectrum estimates are shown in the figures below. The respective spectrum estimates $\hat{P}_x(\nu)$ for a single realization are shown by the solid black lines, and the mean values $\mathrm{E}\{\hat{P}_x(\nu)\}$ of the estimates are shown as reference using solid gray lines in the background. In the figures we can see the sinusoid at $\nu_0 \approx 0.2$, spectral leakage around this frequency, and that $\sigma^2 \approx 0.1$ by reading the spectrum estimate at frequencies far from ν_0, e.g., at $\nu = 0.5$.



a) The two estimates look very similar. Establish that this is no coincidence by proving that $\mathrm{E}\{\hat{P}_x^B(\nu)\} \approx \mathrm{E}\{\hat{P}_x^{BT}(\nu)\}$, i.e., that the mean values of the two estimators are approximately the same for the chosen parameters. (5p)

b) What is the variance of Bartlett's estimate $\hat{P}_x^B(\nu)$ at frequencies where the spectrum estimate is dominated by the noise term, e.g., at $\nu = 0.5$? (2p)

c) What is the variance of Blackman-Tukey's estimate $\hat{P}_x^{BT}(\nu)$ at frequencies where the spectrum estimate is dominated by the noise term, e.g., at $\nu = 0.5$? (2p)

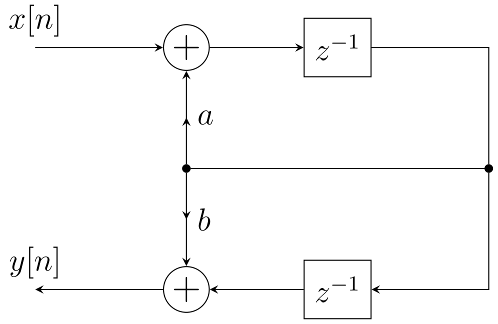
d) Which estimate is better, and why? (1p)

Hint: Parts b), c) and d) can be solved without solving part a) first. For part a) it may help to recall that the Barlett (triangular) window is given by

$$w[n] = \begin{cases} \frac{M-|n|}{M} & |n| < M \\ 0 & |n| \geq M \end{cases}$$

where M is the maximum lag of Blackman-Tukey's method.

4. In this problem, we will consider the circuit shown below, where both a and b are real valued, and where $|a| < 1$ and $|b| < 1$.



a) Obtain the transfer function

$$H(z) = \frac{Y(z)}{X(z)}$$

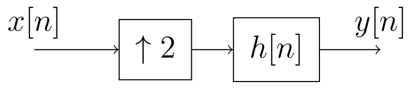
of the circuit. (2p)

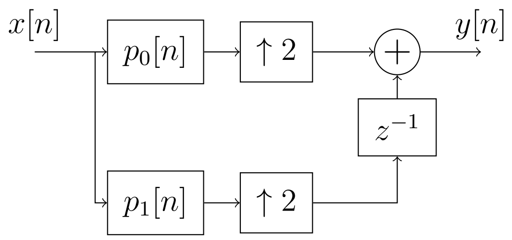
b) Obtain the impulse response $h[n]$, for which $H(z) = \mathcal{Z}\{h[n]\}$. (2p)

c) Assume that the circuit is implemented using fixed point arithmetics, using a $B+1$ bit signed magnitude representation of the range $(-1, 1)$. Draw the circuit diagram for the equivalent additive quantization noise model. (1p)

d) Compute the total power of the quantization noise at the output of the circuit for the fixed point implementation, expressed in terms of a, b and B, and simplify the answer as much as possible. (5p)

5. An interpolation circuit, as well as its polyphase implementation, are shown in the figures below. Recall that the polyphase components are $p_k[n] = h[2n+k]$ for $k = 0, 1$.





In the ideal (or proper) case, we would have

$$H(\nu) = \begin{cases} 2 & |\nu| \le \frac{1}{4} \\ 0 & \frac{1}{4} < |\nu| \le \frac{1}{2} \end{cases} \quad \text{and} \quad h[n] = \begin{cases} 1 & n = 0 \\ \frac{\sin(\pi n/2)}{\pi n/2} & n \neq 0 \end{cases}.$$

a) Show that $p_0[n] = \delta[n]$ in the ideal case, and that this implies that $y[2n] = x[n]$ for all $n \in \mathbb{Z}$. (2p)

b) For an arbitrary $h[n]$, not necessarily ideal, obtain an expression for $P_0(\nu) = \mathcal{F}\{p_0[n]\}$ in terms of the frequency response $H(\nu) = \mathcal{F}\{h[n]\}$. Verify your answer by checking (possibly by drawing) that $P_0(\nu) = 1 = \mathcal{F}\{\delta[n]\}$ if $H(\nu)$ is ideal. (2p)

c) For an arbitrary $h[n]$, not necessarily ideal, obtain an expression for $P_1(\nu) = \mathcal{F}\{p_1[n]\}$ in terms of the frequency response $H(\nu) = \mathcal{F}\{h[n]\}$. (3p)

d) Draw the magnitude and phase of $P_1(\nu)$ as functions of ν in the ideal case. Can we view $p_1[n]$ as a discrete time equivalent of a time-shift by *half* a sample? (3p)