SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 84 **Digital Signalbehandling**, 2E1340

Final Examination 2001–04–18, 09.00–13.00

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Formelsamling i Kretsteori/Signalteori, KTH
Unprogrammed pocket calculator.

Notice: At most one problem should be treated per page.
Motivate each step in the solution.
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. a) A real valued signal $x(t) = \cos(2\pi F t)$, with $F = 12\,800$Hz is sampled with sampling frequency 8 000Hz. Determine the normalized frequency f of the sampled signal ($f \in [0, 0.5]$).

(i) 0.2
(ii) 0.4
(iii) 0.6
(iv) 1.2
(v) 1.6

(3p)

b) For the evaluation of a radio communications system, we want to measure the channel between a transmit antenna and a receive antenna. The channel is modeled as a linear time-invariant filter. To do so, we transmit a known signal and estimate the impulse response of the channel from the data measured at the receiver side.

Suppose we want to save the sampled impulse response (which is complex valued) in the frequency domain, so we use the FFT to transform the estimated impulse response. The frequency bandwidth (see Fig. 1) of the transmitted signal is 200 MHz and the excess delay window (i.e. the whole sampling time for the impulse response) is 0.3 μs. Assume the sampling frequency is 1000 MHz, which means that we have 300 samples for the whole impulse response. In order to save space, we only want to keep the FFT values corresponding to the 200 MHz band used in the measurements. If we use a 300-point FFT, how many points in the frequency domain should we keep in order to cover the whole 200 MHz bandwidth?

(i) 30
(ii) 60
(iii) 100
(iv) 120
(v) 200

(3p)

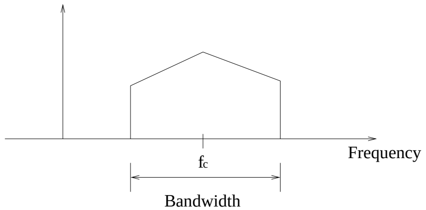

Figure 1: Measurement Bandwidth

c) Continuing the above question, if we have the data saved in the frequency domain and want to restore the impulse response, we use the IFFT to transform the data back to the time domain. Suppose we have the data saved as 97 FFT values and use the following three methods to recover the time-domain impulse response

(i) 97-point IFFT
(ii) 1000-point IFFT
(iii) Multiplication by a Hamming window followed by a 1000-point IFFT.

The results are shown in Fig. 2, find the approach corresponding to each plot. (4p)

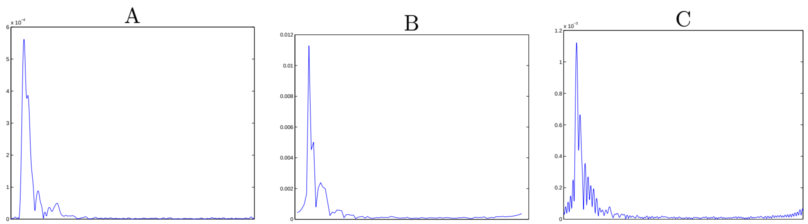

Figure 2: Magnitude of restored impulse responses

2. A former student in signal processing wanted to implement a filter to remove a high frequency disturbance from a low frequency signal. In his first solution, the signal was segmented into blocks of 64 samples. Each block $x_i(n)$ was transformed into the frequency domain using a 64 point FFT. In order to save computations, he didn't multiply the frequency domain signal with a filter, but simply set all FFT values $X_i(k)$ to zero, except for $X_i((-1)_{\text{ mod } 64})$, $X_i(0)$ and $X_i(1)$ which were left untouched. Finally he calculated the inverse FFT of each segment and sent them to the output, one after each other.

a) Determine the approximate bandwidth of this filter. (2p)

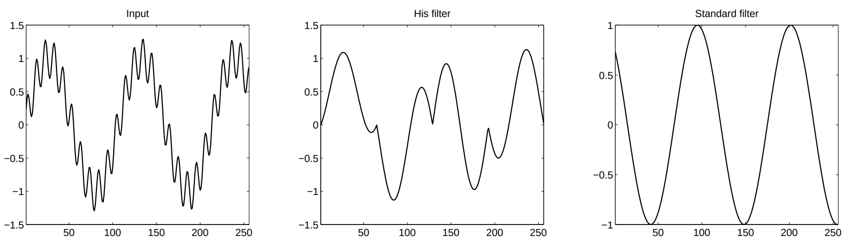

Figure 3: Input signal and the output of the two filter implementations.

b) Figure 3 shows the input signal, the output of this implementation and the output of a normal filter with the same bandwidth. Explain why the implementation does not work as he expected. (4p)

c) Compare the number of (real valued) multiplications needed for his solution, with a standard time-domain implementation and a proper overlap-save implementation. Assume that the filter used in the time-domain and overlap-save methods, is an FIR filter with 10 coefficients and that a 64 point FFT is used also for the overlap-save method. (4p)

3. In the first laboratory exercise in the course, we illustrated the phenomenon of aliasing by listening to a sinus tone with varying frequency. The implementation of this system is illustrated in Figure 4. For simplicity, we assume that the pulse shape $p(t)$ in the reconstruction (pulse amplitude modulation, PAM) is matched to the sampling frequency.

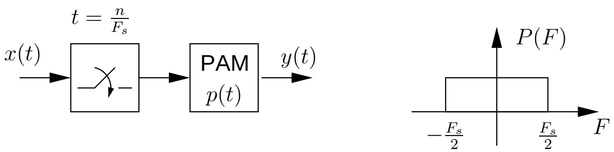

Figure 4: Desired system

However, in the implementation of this exercise, we experienced a problem; all modern DSP equipment performs antialias filtering before the sampling, which means that the aliasing is removed before we get a chance to listen to it. Thus, we had to simulate the desired system by a digital implementation described in Figure 5, where the actual sampling and reconstruction is done at $4F_s$ (the reconstruction filter $q(t)$ looks like $p(t)$ but with a cut-off frequency at $2F_s$).

Determine the filters $F(f)$, $G(f)$ and $H(f)$ such that the two implementations in Figures 4 and 5 produce the same output, at least for all frequencies up to $2F_s$.

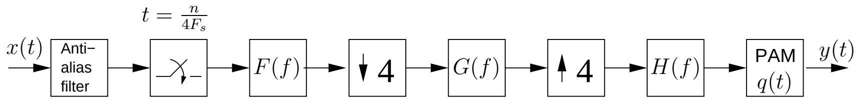

Figure 5: Actual implementation

(10p)

4. In the circuit in Figure 6, fix-point arithmetics with n bits plus sign bit is used. The results of the multiplications are rounded, giving quantization noise. No round-off errors occur due to overflow.

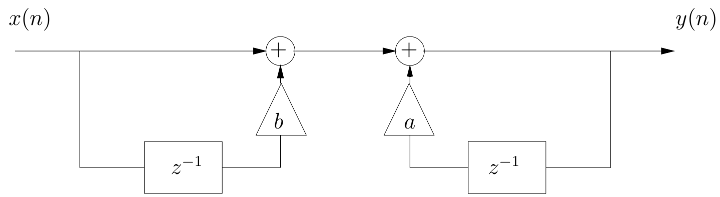

Figure 6: Fix-point circuit

a) Calculate an expression for the variance of the quantization noise at the output. (5p)

b) Sketch the spectral density of the quantization noise at the output. Assume $a = 0.7$ and $b = 1.3$.

(5p)

5. From a stationary time-discrete signal $x(n)$, we have estimated the following auto-correlation sequence $r_x(k) = \mathrm{E}[x(n)x^*(n-k)]$:

k	$r_x(k)$
0	75.5
1	18.8
2	-63.4
3	-48.9
4	37.0

a) Estimate the parameters a_1, a_2 and σ_e^2 in the following AR-like model of the signal,

$$x(n) + a_1 x(n-2) + a_2 x(n-4) = e(n) ,$$

where $e(n)$ is white noise with $\mathrm{E}[|e(n)|^2] = \sigma_e^2$.
Note the difference from the standard AR model!

(5p)

b) Based on the answer in a) or some other parametric method, determine the main frequency of the signal $x(n)$. (5p)