KTH, INFORMATION SCIENCE AND ENGINEERING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing   EQ2300 / 2E1340

Final Examination 2017–04–12, 14:00-19:00

Literature:

- Jaldén: *Summary notes for EQ2300* (30 page printed material).
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH.*
- *One A4 of your own notes.* You may write on both sides, and it does not have to be hand written, but cannot contain full solutions to tutorial problems or previous exam problems.
- An unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. As usual, we begin with a few mixed shorter questions...

a) Compute $\{3,\ 2,\ 1,\ 0\} \text{\textcircled{4}} \{0,\ 1,\ 2,\ 3\}$. (2p)

b) Which ones of the following signals have real valued 5-point discrete Fourier transforms? (There could be more than one) (3p)

i) $x[n] = \{\underset{\uparrow}{1},\ 1,\ 1,\ 1,\ 1\}$

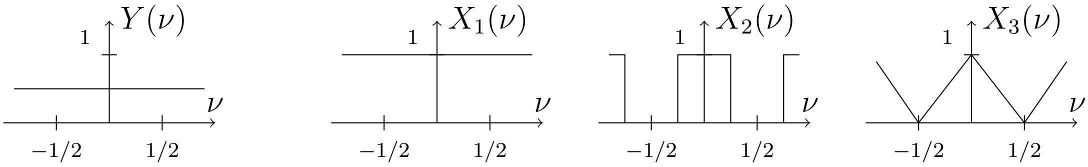
ii) $x[n] = \{\underset{\uparrow}{1},\ 1,\ 1,\ 0,\ 0\}$

iii) $x[n] = \{\underset{\uparrow}{1},\ 1,\ 0,\ 1,\ 1\}$

iv) $x[n] = \{\underset{\uparrow}{1},\ 1,\ 0,\ 0,\ 1\}$

v) $x[n] = \{\underset{\uparrow}{1},\ 0,\ 0,\ 0,\ 0\}$

c) Assume that $y[n] = x[2n]$, where the discrete-time Fourier transform $Y(\nu)$ of $y[n]$ is shown below. Which signal $x_i[n]$ where $i \in \{1,\ 2,\ 3\}$ *can not* satisfy $x[n] = x_i[n]$, if their respective discrete-time Fourier transforms $X_i(\nu)$ are as shown below? (2p)



d) Two filters A and B have their respective transfer functions given by

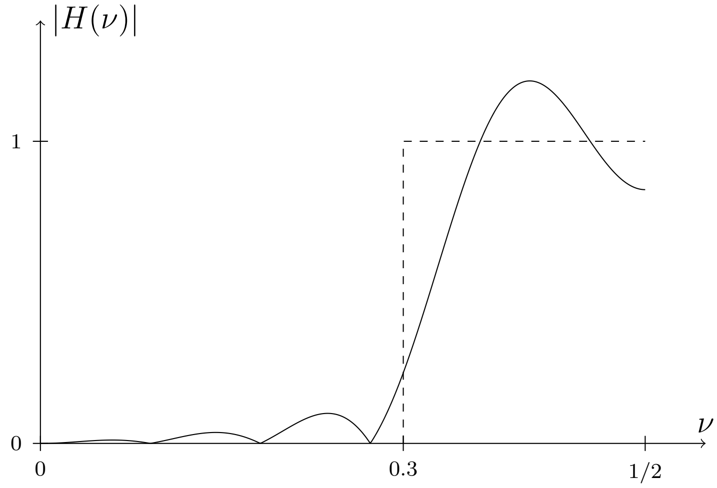
$$H_{\mathrm{A}}(z) = \frac{z^2}{z^2 - z + a} \quad \text{and} \quad H_{\mathrm{B}}(z) = \frac{z^2}{z^2 - a}$$

where $a \approx \frac{1}{4}$. Which filter's poles are most sensitive to small changes in the coefficient a around the nominal value $\frac{1}{4}$, and why? (3p)

2. A Type I linear phase FIR filter has been designed to approximate the ideal high-pass filter with cut-off frequency $\nu_c = 0.3$, i.e., the magnitude of the ideal frequency response satisfies

$$|H_I(\nu)| = \begin{cases} 0 & |\nu| < 0.3 \\ 1 & |\nu| \geq 0.3 \end{cases},$$

for $\nu \in [-\frac{1}{2}, \frac{1}{2}]$. The filter design was based on the frequency sampling approach, with uniform sampling across the frequency domain. The resulting magnitude of the frequency response $H(\nu)$ of the designed filter is shown below, together with the magnitude of the ideal frequency response $H_I(\nu)$ as a dashed line.



The resulting finite length impulse response satisfies $h[n] = 0$, for $n < 0$ and $n \geq N$ where N is the length of the $h[n]$.

a) Based on the information above, what is the filter length N? Give a numerical value. (2p)

b) Show that the filter's impulse response can be written on the form

$$h[n] = \frac{1}{11}\left[\cos\left(\pi\frac{8(n-5)}{11}\right) - \cos\left(\pi\frac{10(n-5))}{11}\right)\right],$$

for $n = 0, \ldots, N-1$. (4p)

c) Give an expression for the obtained frequency response $H(\nu)$, i.e., the frequency respons for which $|H(\nu)|$ is shown above. The closed form analytical expressions for $h[n]$ will most likely be quite messy, so to avoid principally straightforward but error prone calculations, you may and are encouraged to give your answer as a frequency domain (circular) convolution between simpler functions. (4p)

3. The following piece of Matlab code is a (modified) part of the function `plotspectrum`, which was supplied with the course project. The code estimates the power spectral density of a signal $y[n]$ from samples contained in the vector `y`, and stores the result in `Py`.

```
y = y(:); % Input, in column form
N = length(y); % Data length
M = 2048; % Number of FFT coefficicents
L = 512; % Length of window (L < M and L < N)
D = floor(L*0.5); % 50% overlap
w = bartlett(L); % Window - sidelobe supression (-27 dB)

% Compute number of full segments
K = floor((N-L)/D)+1;

% Window-normalization
w = w(:); % Column form
U = 1/L*sum(abs(w).^2); % Normalization

% Average of modified periodograms
Py = zeros(M,1);
for n=1:D:(N-L+1)
    x = [y(n:n+L-1).*w; zeros(M-L,1)]; % Window and zero-pad
    Py = Py + 1/(K*L*U)*abs(fft(x)).^2; % Add contribution
end;
```

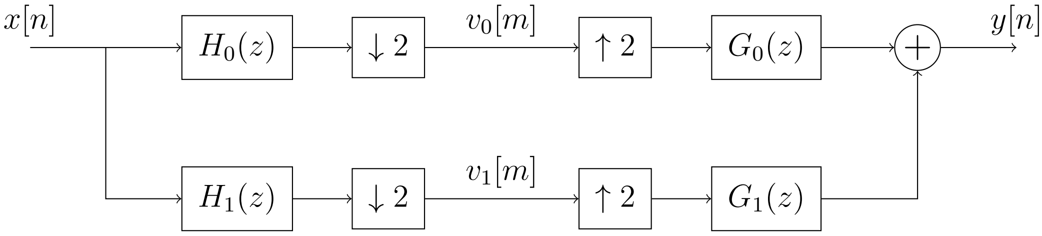
a) Which spectrum estimation method is actually implemented? Motivate you answer based on the code above for full credits. (2p)

b) What it the frequency resolution of the spectrum estimator, i.e., the closest distance between two frequencies ν_1 and ν_2 where $y[n] = \cos(2\pi\nu_1 + \varphi_1) + \cos(2\pi\nu_2 + \varphi_2)$ yield a spectrum with two distinct main peaks? (2p)

c) Let $\hat{P}_y(\nu)$ be the estimate of $P_y(\nu)$, the true power spectrum of $y[n]$, for all normalized frequencies ν. Which values of $\hat{P}_y(\nu)$ are actually stored in `Py`? (2p)

d) If we require that the variance of the spectrum estimate $\hat{P}_y(\nu)$ is at most $\frac{1}{10}P_y^2(\nu)$, what is then the minimum length `N` of `y`? (4p)

4. Consider the filterbank of the form



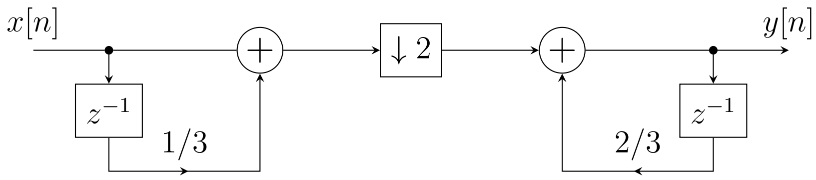
where $H_0(z) = \frac{1}{4}(1 + 3z^{-1} + 3z^{-2} + z^{-3})$ and $H_1(z) = \frac{1}{4}(-1 - 3z^{-1} + 3z^{-2} + z^{-3})$.

a) Show that the filterbank, like the one in the project, is *not* a QMF filterbank, i.e., there is no filter $H(z)$ so that $H_0(z) = H(z)$ and $H_1(z) = H(-z)$. (1p)

b) Explain how one can see that $H_0(z)$ and $H_1(z)$ are linear phase FIR filters, and specify the type (Type I, II, III, or IV) of the filters. (2p)

c) Propose filters $G_0(z)$ and $G_1(z)$ that with this choice of $H_0(z)$ and $H_1(z)$ yield perfect reconstruction, i.e., ensures that $y[n] = x[n - l]$ for some l, and specify the value of l. In this problem, it is important that you motivate your answer by proving that your filters do in fact yield perfect reconstruction. (7p)

5. The multirate circuit shown below is implemented in fixed point arithmetics, with a $B + 1$ signed magnitude representation of the range $(-1, 1)$. You may assume that the signals are scaled so that there is no overflow in the circuit.



a) Compute the power of the quantization noise, due to the fixed point implementation, at the output of the circuit. (6p)

b) Sketch the power spectral density of the quantization noise at the output. (4p)