SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 96 **Digital Signalbehandling**, 2E1340

Final Examination 2005–03–31, 08.00–13.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 08-790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage directly after the exam.

The problems of this exam will all be more or less related to a certain application, namely chirp RADARs.

The general principle of a RADAR (RAdio Detection and Ranging) system is to transmit a radio signal $x(t)$ from an antenna and listen for the echo $y(t)$ that is reflected from some object. The delay between the transmitted signal and the echo is then used to calculate the distance. If, for example, there is an object at distance d [m], then the received echo is $y(t) = Ax(t-\Delta)+n(t)$, where A is some attenuation, $n(t)$ is background noise and $\Delta = 2d/c$ [s] where $c \approx 3 \cdot 10^8$ [m/s] is the speed of light. Maybe the most common kind of RADAR operation is so-called pulse RADARs, that uses short pulses. However, this requires a radio transmitter that can handle a very high peak power.

In this exam, we will study an alternative type of RADAR called **chirp RADAR**. The idea of a chirp RADAR is to transmit a signal with constant power, where the frequency is varied periodically as shown in Fig. 1, a so-called chirp signal.

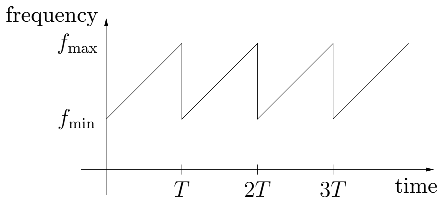

Figure 1: The instantaneous frequency as a function of time for the transmitted chirp signal $x(t)$.

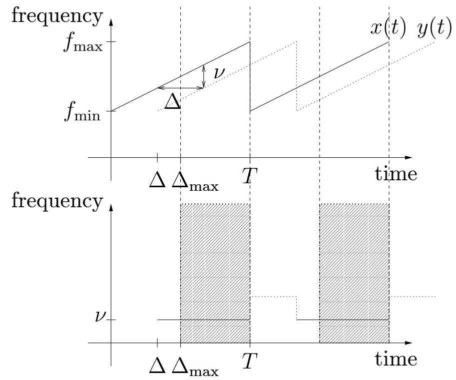

Figure 2: The upper plot shows the transmitted signal together with the received echo. The lower plot shows the difference of the two curves = the low pass frequency content of $z(f)$.

In mathematical notation, the signal is

$$x(t) = \begin{cases} \cos(2\pi(\frac{\beta}{2}t^2 + f_{\min}t)) & 0 \le t < T \\ x(t-T) & \text{otherwise} \end{cases}$$

where $\beta = (f_{\max} - f_{\min})/T$. This choice of signal makes it easy to determine the delays, since a delay of Δ will correspond to a fixed frequency difference $\nu = \Delta\beta$ as shown in Fig. 2. This is easily exploited if the received echo $y(t)$ is multiplied with the the transmitted signal $x(t)$, which gives

$$\begin{aligned} z(t) = y(t)x(t) &= A\cos(2\pi(\frac{\beta}{2}(t-\Delta)^2 + f_{\min}(t-\Delta)))\cos(2\pi(\frac{\beta}{2}t^2 + f_{\min}t)) + n'(t) \\ &= \frac{A}{2}\big(\cos(2\pi(\beta t^2 + (f_{\min} - \beta\Delta)t + \text{const.})) + \cos(2\pi\beta\Delta t + \text{const.})\big) , \end{aligned}$$

for $\Delta \le t < T$. The first term, corresponding to the sum of the frequencies in $x(t)$ and $y(t)$, is a high frequency chirp signal with lowest frequency above $2f_{\min}$ whereas the second term, corresponding to the frequency difference between $x(t)$ and $y(t)$, has a constant frequency $\nu = \Delta\beta$ as shown in Fig. 2. If the system is designed such that $2f_{\min} \gg \nu$, then the high frequency chirp can be removed using a low pass filter which leaves only a sine wave with constant frequency ν. Therefore, the full radar system can be implemented as shown in Fig. 3. Note that the signal only should be sampled during the periods marked with gray in Fig. 2, where $\Delta_{\max}$ is the delay corresponding to the largest distance that can be measured with the system. If $y(t)$ contains echoes from several targets, the spectrum will contain one peak for each target with a frequency corresponding to the distance.

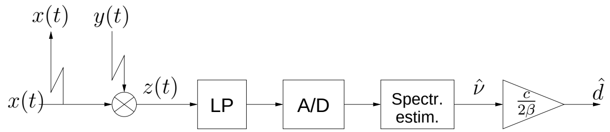

Figure 3: Chirp radar system.

1. Assume that the chirp RADAR system is designed such that $f_{\min} = 10\text{MHz}$, $f_{\max} = 50\text{MHz}$, the sweep time $T = 10\text{ms}$ and the maximum range (distance to the target) is approximately 30km corresponding to $\Delta_{\max}0.2\text{ms}$. The low pass filtered signal is sampled using a sampling Frequency of $F_s = 2\text{MHz}$.

a) Assume that the Periodogram is used for the spectral estimation and that it is calculated using an FFT without zero-padding. How accurately (in meters) can the distance to the target be determined? (3p)

b) Is it possible for this system to distinguish two targets that are separated by 10m? (4p)

c) Propose a modification of the Periodogram method that provides better estimation performance in this particular application. (3p)

2. One possible modification to the system in Fig. 3 is to do the sampling before the low pass filtering, i.e. to filter digitally. Assume that the filter is an FIR filter with 20 taps (non-zero filter coefficients) and that 8 000 samples of $z(n)$ are collected in each period of the chirp signal.

a) Find the implementation of the filter that requires the smallest computational complexity (in terms of number of multiplications) and determine the corresponding number of multiplications to filter the 8 000 samples. (4p)

b) Assume that an ordinary periodogram is used for the spectral estimation. What is the computational complexity of the periodogram calculation if 8192 frequency values are needed to obtain sufficient accuracy. (2p)

c) Is it possible to exploit that the output of the filter is directly input to the periodogram, using only a single FFT for both operations? If so, describe in detail how this combined filtering and periodogram can be implemented and calculate the total number of multiplications needed. If it is not possible, explain why. (4p)

3. Assume that the filter is implemented digitally, as in Problem 2. Assume that the signal $z(t)$ is sampled using a 12 bit A/D converter and that the filter is implemented using a direct form implementation with 16 bits fixed arithmetics, where in both cases the number of bits include the sign bit and the input signal has been scaled so that all values are in the range $[-1, 1]$. Both the A/D conversion and the multiplications will cause round-off errors.

a) Calculate the power of the quantization noise at the output of the filter if the impulse response of the filter is given by $h(n) = \{\underset{\uparrow}{.11}, .18, .21, .18, .11\}$. (7p)

b) In the text books, you can find expressions for the variance of a periodogram. Can these expressions be used to predict the variance of the periodogram, due to the quantization noise in the input? (3p)

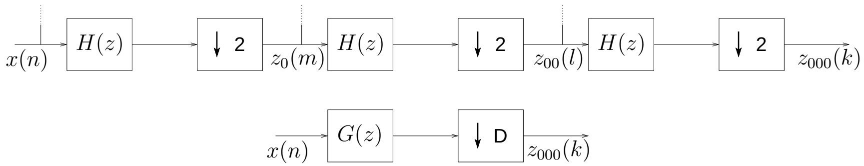

Figure 4: Low pass branch of a nested analysis filter bank (upper plot). Equivalent system (lower plot)

4. One method to design low pass (or band pass or high pass) filters is to use a filter bank structure similar to Project 2 in the course. Consider, for example, a filter bank structure composed of 3 nested analysis filter banks, where in each stage the low pass signal from the previous stage is split into a high pass and an low pass part. Fig. 4 shows the low pass part of such a nested filter bank.

a) Determine the coefficients of the filter $g(n)$ and the downsampling factor D such that the system in the lower part of Fig. 4 provides the same output as the nested filter bank in the upper part of Fig. 4, if the filter $h(n) = \{\underset{\uparrow}{1}, 2, 1\}$. (8p)

b) Determine a rough estimate of the bandwidth of the filter $G(z)$. (2p)

5. Consider a scenario where several RADARs can be used together to localize some object, as illustrated in Fig. 5. Let $\mathbf{e}_1 = [x_1, y_1]^T, \ldots \mathbf{e}_M = [x_M, y_M]^T$ denote the

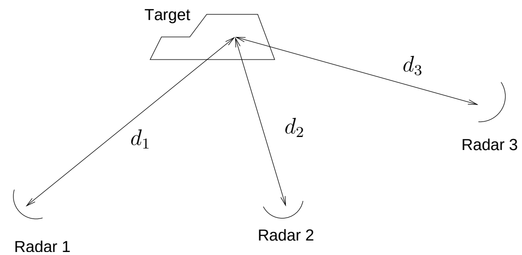

Figure 5: Localization using several RADARs.

known location of the M radars and $\hat{d}_m$, $m = 1, \ldots, M$ denote the estimated distance from radar m to the target.

a) Describe a method to determine the coordinates of the target given $\mathbf{e}_m$ and $\hat{d}_m$, $m = 1, \ldots, M$. The answer can for example be given in the form of an optimization problem where the optimal value provides the location, there is no need to describe all details of the implementation.

Full grade for this problem will only be given for solutions that relate directly to some method taught in the course, but also other solutions will be rewarded with up to 5p. (7p)

b) How many RADARs are needed to solve the problem. Is there any advantage to use even more RADARs? (3p)