KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2013–05–30, 08.00–13.00

Literature:

- Course text book:
  - Dinis, da Silva & Netto *Digital Signal Processing; System Analysis and Design*

    or

  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or

  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

Note: In the following multiple-choice questions, just as in all other questions, be careful to motivate all answers. Answers without motivation/justification carry no rewards.

1. a) Which of the following signals is the result of the circular convolution $w(n) = \{\underset{\uparrow}{3}, 2, 1\} \textcircled{3} \{\underset{\uparrow}{3}, 0, -1\}$?

i) $w(n) = \{\underset{\uparrow}{9}, 6, 0\}$

ii) $w(n) = \{\underset{\uparrow}{9}, 6, 0, -2, -1\}$

iii) $w(n) = \{\underset{\uparrow}{8}, 3, 1\}$

iv) $w(n) = \{\underset{\uparrow}{7}, 5, 0\}$ (4p)

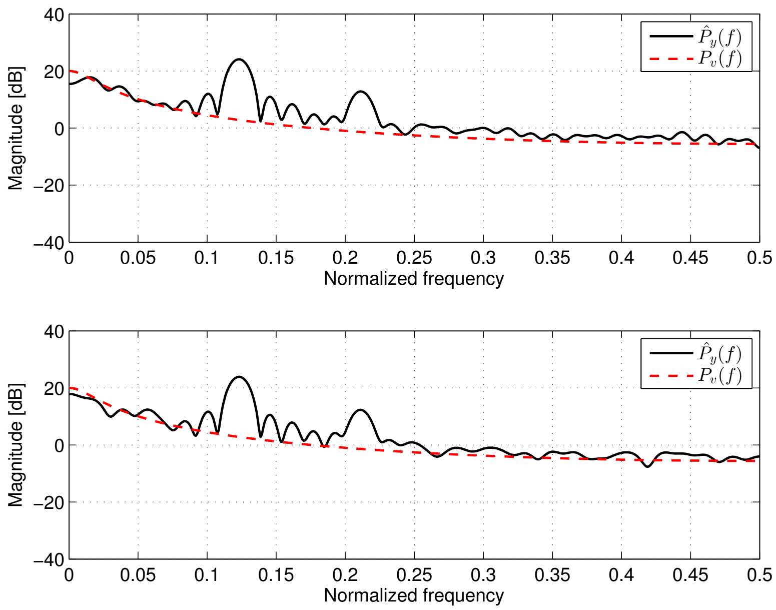


Figure 1: Power spectral estimate (black curve) of two realizations of a stochastic signal.

b) Figure 1 shows the result of a power spectral estimation method, applied on two realizations of a stochastic signal $y(n) = x(n) + v(n)$, where the additive noise $v(n)$ is independent of $x(n)$ and $v(n)$ is known to have the power spectral density shown in the red dashed curve. For $x(n)$, we only know that it consists of one or several sinusoids. Each realization of the signal contained $N = 1024$ samples. Which of the following conclusions is most reasonable?

i) The Bartlett method with $K = 16$ segments was used, since the estimates have a low variance and a low resolution.

ii) The Bartlett method with $K = 16$ segments was used, since the estimates have a high variance and a high resolution.

iii) The Bartlett method with $K = 16$ segments was used, since the estimates have a low variance and a high resolution.

iv) The Periodogram method was used, since the estimates have a low variance and a low resolution.

v) The Periodogram method was used, since the estimates have a high variance and a high resolution.

vi) The Periodogram method was used, since the estimates have a high variance and a low resolution. (3p)

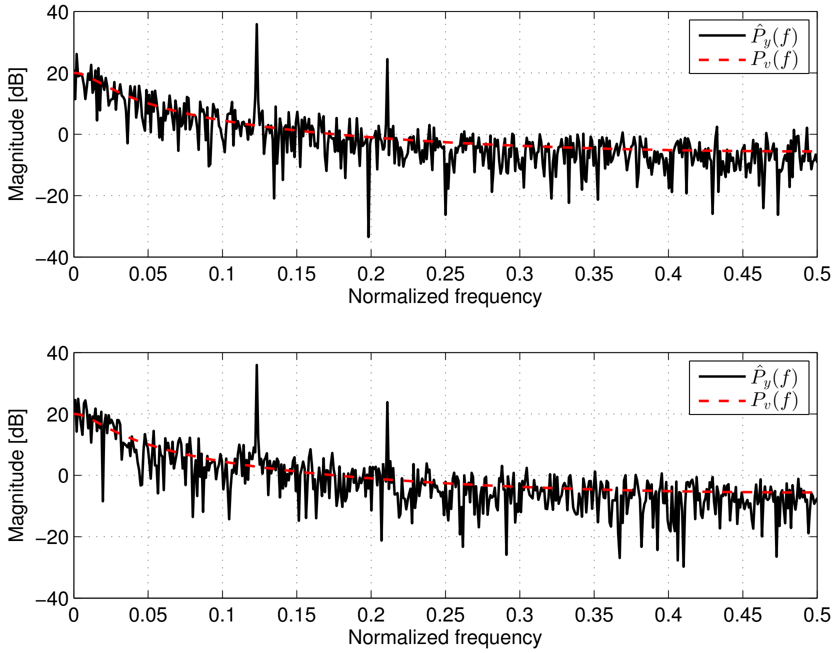


Figure 2: Another power spectral density estimate (black curve) of the two realizations of a stochastic signal.

c) In Figure 2, another spectrum estimation method was used for the same two realizations of the same stochastic signal as in b). Which of the following conclusions is most reasonable?

   i) The Bartlett method with $K = 16$ segments was used, since the estimates have a low variance and a low resolution.

   ii) The Bartlett method with $K = 16$ segments was used, since the estimates have a high variance and a high resolution.

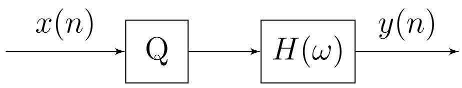
   iii) The Bartlett method with $K = 16$ segments was used, since the estimates have a low variance and a high resolution.

   iv) The Periodogram method was used, since the estimates have a low variance and a low resolution.

   v) The Periodogram method was used, since the estimates have a high variance and a high resolution.

   vi) The Periodogram method was used, since the estimates have a high variance and a low resolution. (3p)

2. You are given an AR(1) process $x(n) = 0.2x(n-1) + u(n)$, where $u(n)$ is zero-mean white with variance $\sigma_u^2 = 1$. Suppose that $x(n)$ goes through the system



where Q represents a uniform quantizer with regions Δ sufficiently small so that the usual assumptions about the quantization error to apply, and the system $H(\omega) = 1 - 0.2e^{-j\omega}$ for $-\pi \leq \omega \leq \pi$.

Find the signal to quantization noise ratio (SQNR) at the output $y(n)$ (i.e., the power due to the signal $x(n)$, divided by the quantization noise power). (10p)

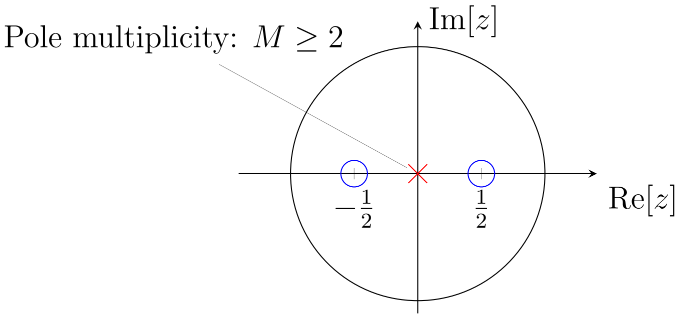


Figure 3: Zero-pole diagram.

3. Consider a filter $H(z)$ whose zero-pole plot is shown in Figure 3.

Given that $H(1) = 3/4$ and the ROC of $H(z)$ is $|z| > 0$,

a) Determine $H(z)$ and the corresponding time-domain impulse response $h(n)$. (2p)

b) Is the filter stable? (2p)

c) Is the filter causal? (2p)

d) Consider the system in Figure 4. Under what conditions is it possible to find $G(z)$ such that $r(n) = y(n)$? Determine $G(z)$ and the corresponding impulse response $g(n)$. (4p)

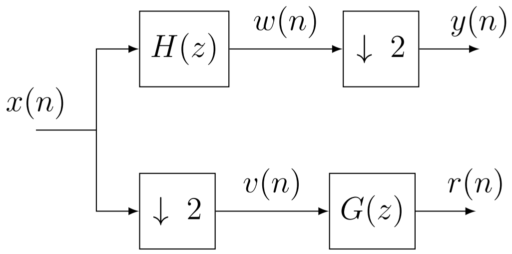


Figure 4: System with two outputs.

4. Let $x(n)$ be a *real valued sequence* of length $N = 2^L$, where L is a positive integer. Let $y(n) = x(2n) + jx(2n+1),\ 0 \leq n < M\ (M = N/2)$. Let $X(k) = DFT\{x(n)\}, 0 \leq k < N$ and $Y(k) = DFT\{y(n)\}, 0 \leq k < M$.

a) Let $y_r(n) = x(2n)$ and $y_i(n) = x(2n+1)$. Express the DFTs of length M for $y_r(n)$ and $y_i(n)$, as a function of $X(k)$. (6p)

b) Express $Y(k)$ in terms of $X(k)$. (3p)

c) If we know that $X(0) = 2$ and $X(M) = 5$, determine $Y(0)$. (1p)

5. Do you like detective stories? Often, they are fairly contrived, but may still be entertaining and mind tickling. Here comes a very contrived DSP detective task.

From different sources, you have obtained the following information about a WSS stochastic process $x(n)$.

- The real valued process
$$y(n) = x(n) + v(n),$$
has the same power as the WSS zero-mean random process $w(n)$, which is known to have the power spectral density
$$P_w(f) = \begin{cases} 2 + \sin(2\pi f), & \text{if } |f| \leq 1/4 \\ 0, & \text{if } 1/4 < |f| \leq 1/2 \end{cases}$$
- $v(n)$ is zero-mean white noise, uncorrelated with $x(n)$, with variance $\sigma_v^2 = 1/2$.
- For $x(n)$, you are only given that it is an AR(2) process.
- The following autocorrelation values have been estimated with good accuracy
$$\hat{r}_y(1) = \frac{1}{4} \quad \text{and} \quad \hat{r}_y(2) = \frac{1}{6}$$

Given these clues, provide an estimate for the power spectrum of $x(n)$. (10p)