SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 82 **Digital Signalbehandling**, 2E1340

Final Examination 2000–09–01, 1400–1800

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Formelsamling i Kretsteori/Signalteori, KTH
Unprogrammed pocket calculator.

Notice: At most one problem should be treated per page.
Motivate each step in the solution.
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

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Results: Will be posted within three working weeks Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. In each of the three following questions, one of the following answers is correct:

i) $x = [1, 2, 3, 4]$
ii) $x = [4, 3, 2, 1]$
iii) $x = [1, 4, 3, 2]$
iv) $x = [1, -4i, -3, 2i]$
v) $x = [1, -2i, 3, 4i]$

a) The function `conj` in MATLAB, returns the complex conjugate of the input. What is the vector `x` if

```
>> ifft(conj(fft(x)))
```

returns $[1, 2, 3, 4]$? (3p)

b) Assume that the function `cyclicreverse` gives:

$$\texttt{cyclicreverse}([x_0, x_1, x_2, \ldots, x_{n-1}, x_n]) = [x_0, x_n, x_{n-1}, \ldots, x_2, x_1]$$

What is the vector `x` if

```
>> ifft(cyclicreverse(fft(x)))
```

returns $[1, 2, 3, 4]$? (3p)

c) The function `fliplr` in MATLAB, reverses the order of the input vector, thus

$$\texttt{fliplr([}x_1, x_2, \ldots, x_{n-1}, x_n\texttt{])} = \texttt{[}x_n, x_{n-1}, \ldots, x_2, x_1\texttt{]}$$

What is the vector `x` if

```
>> ifft(fliplr(fft(x)))
```

returns $[1, 2, 3, 4]$? (4p)

2.

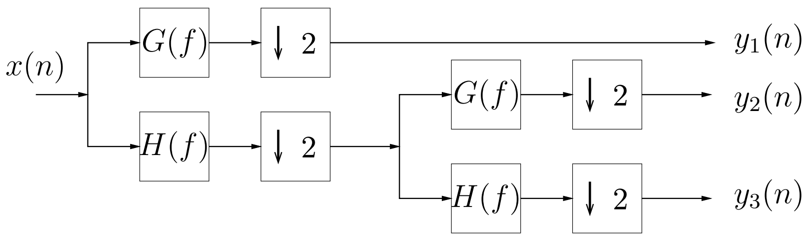
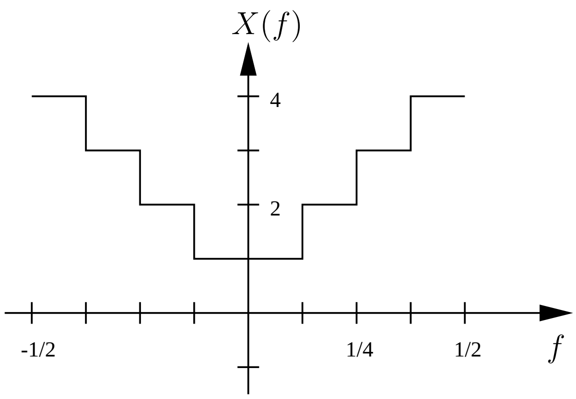
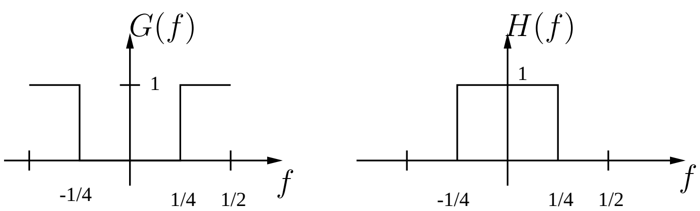


Figure 1: The system.

Determine the spectra $|Y_1(f)|$, $|Y_2(f)|$ and $|Y_3(f)|$ of the three output signals in Figure 1 if the time discrete input signal $x(n)$ has the following spectrum:



and the filters $G(f)$ and $H(f)$ have the following transfer functions:



(10p)

3. a) A signal $x(t)$ is sampled at $F_s = 16\text{kHz}$ and the periodogram is calculated using an $N = 64$ point FFT. The plots in Figure 2 shows the resulting periodograms for two sinus signals with frequency $F = 4000\text{Hz}$ and $F = 4125\text{Hz}$, respectively. Both signals have the same amplitude. The frequency axis of the periodogram has been scaled to correspond to the normalized frequency.

Determine which frequency was used in which plot and explain carefully why the plots look so different for the two frequencies.

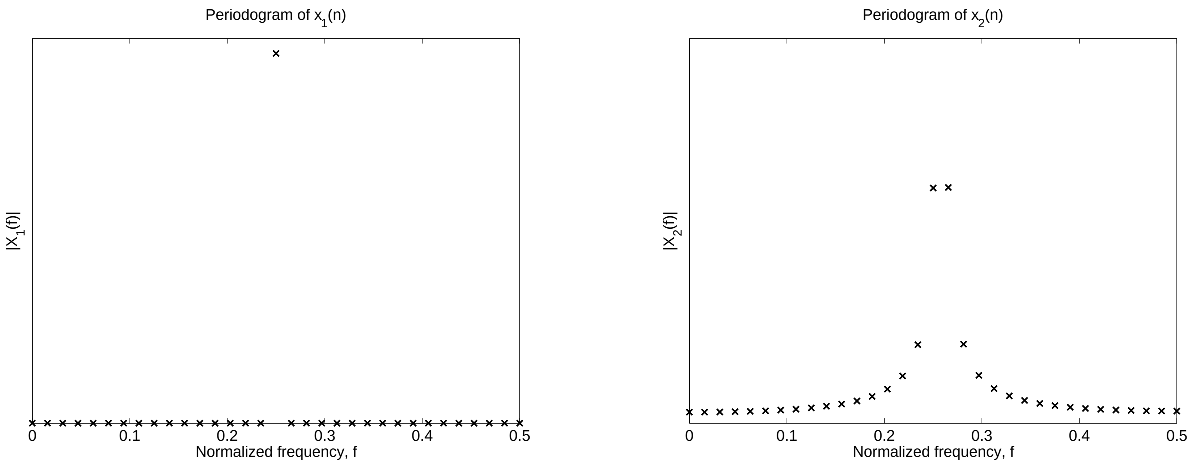


Figure 2: Periodograms for the two frequencies.

(5p)

b) In a second experiment, the signal is sampled at $F_s = 10\text{kHz}$. Decide which of the following signals produces the periodogram shown in Figure 3 and determine the corresponding frequencies $F_1, F_2, \ldots$ Motivate your answer carefully.

- $x_1(t) = \sin(2\pi F_1 t) + \sin(2\pi F_2 t)$
- $x_2(t) = \sin(2\pi F_1 t) + \sin(2\pi F_2 t) + \sin(2\pi f_3 t)$
- $x_3(t) = \sin(2\pi F_1 t) + e^{j2\pi F_2 t}$
- $x_4(t) = \sin(2\pi F_1 t) + e^{j2\pi F_2 t} + e^{j2\pi f_3 t}$

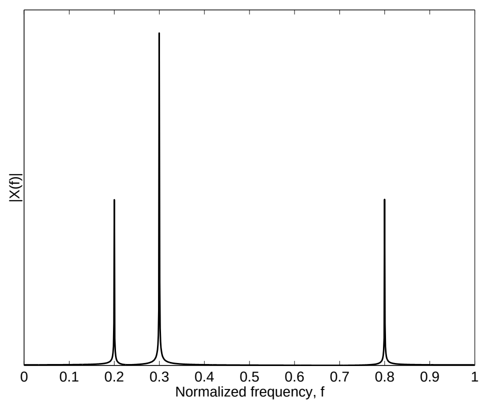


Figure 3: Periodogram for the signal.

(5p)

4. In the receiver of a simple speech decoder an IIR lattice filter, as in Figure 4, is implemented with fix-point arithmetics. From the speech coder a residual, used as input to the filter, and two reflection coefficients, $k_1 = \frac{2}{11}$ and $k_2 = -\frac{5}{16}$, are sent. In the filter, data is represented with b bits plus a sign bit and quantization noise is created at the multiplicators. No overflow arise in the additions.

To obtain sufficient quality on the filter output the Signal-to-Noise-Ratio, due to quantization noise, after the filter has to be 10 dB. Determine the necessary number of bits, b, to obtain sufficient output quality. If infinite precision is used in the filter, the output signal power, $P_s = 0.001$.

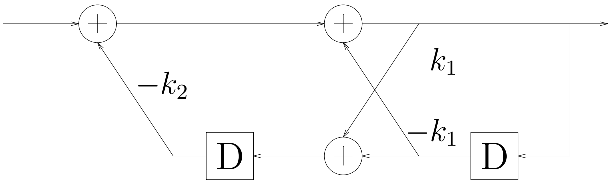


Figure 4: IIR lattice filter.

(10p)

5. Recall the direction of arrival (DOA) estimation problem in a recent project assignment. The measurement vector is modeled as

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j2\pi f\tau_d} \\ \vdots \\ e^{j(m-1)2\pi f\tau_d} \end{bmatrix} s(t) + \boldsymbol{n}(t) = \boldsymbol{a}(\theta)s(t) + \boldsymbol{n}(t) \ ,$$

where f is the center frequency of the emitter signal $s(t)$, m is the number of sensors. The DOA, θ is related to the propagation delay, τ_d through

$$\sin\theta = \frac{c\tau_d}{\Delta}$$

where c is the velocity of propagation and Δ is the spacing between the sensors. The measurement vector $\boldsymbol{x}(t)$ is sampled at $t = 1, \ldots, N$ and $\boldsymbol{n}(t)$ is additive measurement noise. (Matrix expressions are allowed, in fact encouraged, in the answers to the problems below.)

a) In the project assignment, you constructed an estimator for the DOA. Assume therefore that you know that the signal is arriving from $\theta = \theta_0$. Find the least squares estimate (by completing the squares), $\hat{s}(t)$, of $s(t)$, $t = 1, \ldots, N$ by minimizing

$$\sum_{t=1}^{N} \|\boldsymbol{x}(t) - \boldsymbol{a}(\theta_0)s(t)\|^2 \ .$$

Note that $s(t)$ may be complex valued. (2p)

b) Now consider the case when there are two signals present from two different directions.

$$\begin{aligned}\boldsymbol{x}(t) &= \boldsymbol{a}(\theta_1)s_1(t) + \boldsymbol{a}(\theta_2)s_2(t) + \boldsymbol{n}(t) \\ &= \begin{bmatrix}\boldsymbol{a}(\theta_1) & \boldsymbol{a}(\theta_2)\end{bmatrix}\begin{bmatrix}s_1(t) \\ s_2(t)\end{bmatrix} + \boldsymbol{n}(t) = \boldsymbol{A}(\theta_1, \theta_2)\boldsymbol{s}(t) + \boldsymbol{n}(t)\ .\end{aligned}$$

You know θ_1 and θ_2. Find the least squares estimate, $\hat{\boldsymbol{s}}(t)$, of $\boldsymbol{s}(t),\ t = 1, \ldots, N$ in the same manner as above. Use the fact that

$$(\boldsymbol{A}^*\boldsymbol{A})^{-1}\boldsymbol{A}^*\boldsymbol{A} = \boldsymbol{I} \ \text{ and } \ ((\boldsymbol{A}^*\boldsymbol{A})^{-1})^* = (\boldsymbol{A}^*\boldsymbol{A})^{-1}$$

(4p)

c) Compute the expected value and covariance matrix of $\hat{\boldsymbol{s}}(t)$ and comment on your results. Assume that $\boldsymbol{n}(t)$ is zero-mean with covariance $\mathrm{E}\{\boldsymbol{n}(t)\boldsymbol{n}^*(t)\} = \sigma^2\boldsymbol{I}$. If you have not solved part b), use your result in part a). (4p)