# SIGNALBEHANDLING

## Institutionen för Signaler, Sensorer & System

**E 93 Digital Signalbehandling, 2E1340**

Final Examination 2004–04–23, 08.00–13.00

**Literature:** Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

**Notice:** Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

**Contact:** Mats Bengtsson, Signalbehandling, 08-790 84 63,

**Results:** Will be posted within three working weeks at Osquldas väg 10, floor 2.

**Solutions:** Will be available on the course homepage.

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**1.** A discrete time random process containing a real-valued sine wave can be expressed by

$$x(n) = A\cos(2\pi f_0 n + \theta),$$

where both the amplitude $A$ and the frequency $f_0$ are deterministic and unknown parameters. The phase, $\theta$, is a random variable, uniformly distributed over $[0, 2\pi)$.

Assume that we are given the following three estimated values of the autocorrelation sequence $r_{xx}(k) = \mathrm{E}[x(n)x^*(n-k)]$,

$$\hat{r}_{xx}(0) = \sqrt{2}, \qquad \hat{r}_{xx}(1) = 1, \qquad \hat{r}_{xx}(2) = 0\ .$$

a) Define an $N$th order predictor as

$$\hat{x}(n) = -a_N(1)x(n-1) - a_N(2)x(n-2) - \cdots - a_N(N)x(n-N).$$

Determine numerical values of the optimum predictor coefficients for the 1st, 2nd and 3rd order predictors and the corresponding minimum mean square error (MMSE) $\epsilon_N$. (5p)

b) Determine an estimate of $r_{xx}(3)$. (3p)

c) Determine the order of this process. (2p)



**2.** In this problem, we use the same signal and the same estimates of $\hat{r}_{xx}(k)$ as in problem 1.

a) Calculate a mathematical expression for the periodogram of $x(n)$ based on the given estimates $\hat{r}_{xx}(k)$ and sketch the periodogram in a figure. (3p)

b) Is it possible to determine the frequency $f_0$ using this periodogram? (2p)

c) Is it possible to determine the frequency if the Blackman-Tukey method is used instead of the periodogram? (2p)

d) Estimate the frequency using the estimated AR model in problem 1. If you could not solve that problem, give the solution in terms of general AR model parameters, $a_N(k)$. (3p)

**3.** a) Which of the following pieces of code is the best implementation if you want to reduce the rate of a signal $\{x(0), \ldots, x(N-1)\}$ by a factor 2? Assume that $y(n)$ is the output signal and that $\{h(0), h(1), \ldots, h(M)\}$ is the impulse response of a time-discrete low-pass filter with a cut-off frequency of $f_c \approx 1/4$.

Impl. 1:

```
for m=0:N/2-1
  y(m)=0;
  for k=0:min(M,2*m)
    y(m)=y(m)+x(2*m-k)*h(k)
  end
end
```

Impl. 2:

```
for m=0:N/2-1
  y(m)=0;
  for k=0:min(M/2,2*m)
    y(m)=y(m)+x(2*m-k)*h(2*k)
  end
end
```

Impl. 3:

```
for m=0:N/2-1
  y(m)=0;
  for k=0:min(M,m)
    y(m)=y(m)+x(2*(m-k))*h(k)
  end
end
```

Impl. 4:

```
for m=0:N/2-1
  y(m)=0;
  for k=0:min(M/2,m)
    y(m)=y(m)+x(2*(m-k))*h(2*k)
  end
end
```

(4p)

b) If the decimation should be done in a "real-time" system, with a limited time delay from input to output, data will be collected in buffers of length $N$. While one input buffer is filled with data, the program processes the previous input buffer and stores the result in an output buffer. Explain in words (and figures) how the implementation in a) should be modified to work well in such a system (no program code is necessary). (3p)

c) Assume that the input signal $x(n)$ is obtained by sampling a sinus signal with frequency $F = 440$Hz using a sampling frequency of $F_s = 8000$Hz. Then, the rate is reduced by 2 using the algorithm in a). If we calculate the periodogram from 512 of the resulting samples $y(m)$, using an FFT of length 512, what is the number (=the frequency index) of the two FFT values with highest amplitude in the periodogram?

(3p)

**4.** A normal music CD contains digital audio signals sampled at 44.1kHz. Prior to sampling, the anti-alias filter of the recording device limits the bandwidth of the signal to 20kHz. This gives a small margin to the Nyquist frequency, and since the equipment at the recording studio is expensive, we can assume that the filter is an ideal low-pass filter.

A (cheap) CD-player converts the digital audio to an analog signal using a simple zero-order-hold D/A-converter. The resulting signal looks like a staircase, and has spectral images at high frequencies. To improve the sound quality, the spectral images can be removed using an additional analog low-pass filter. A problem is that the small margin to the Nyquist frequency makes suitable analog filters difficult (and expensive) to build (see Figure 1).

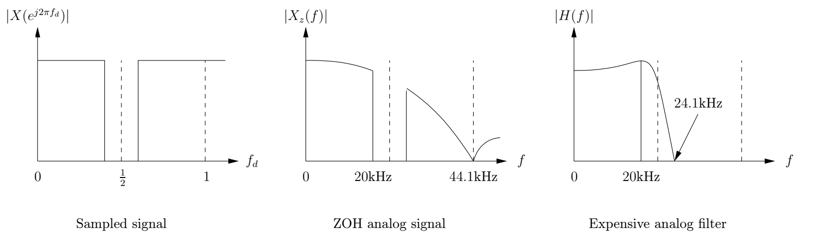


Figure 1: Zero order hold D/A conversion of CD audio signal

Figure 2 shows a frequency mask and an example of a filter that satisfies it. Analog filters that are designed to satisfy this performance are cheap and easy to build.

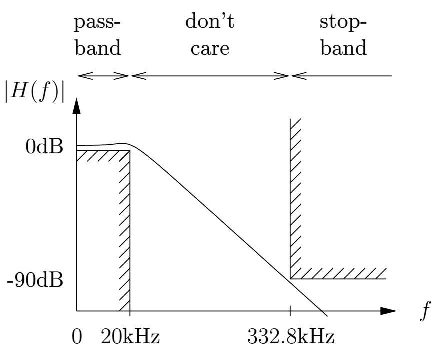


Figure 2: Frequency mask of cheap analog filter

Assume that you can use a zero order hold circuit with higher sampling frequency. Suggest a procedure using digital signal processing, that makes it possible to use a cheap analog filter that satisfies the frequency mask of Figure 2. (10p)

In order to receive the full 10p, your answer should include a block diagram of the DSP procedure, figures with spectral shapes of the output from all the important steps, and (clearly stated) values of all design parameters.



**5.** The filter in Figure 3 is implemented using fixed point arithmetics, which means that the results of the multiplications have to be rounded. Assume that $a = 0.6$, $b = -1$, $c = -0.39$ and that 15 bits plus sign bit are used to represent numbers in the range $[-1, 1]$. Calculate the average power of the quantization noise at the output. Note that the multiplication by $b = -1$ will not introduce any round-offs. (10p)

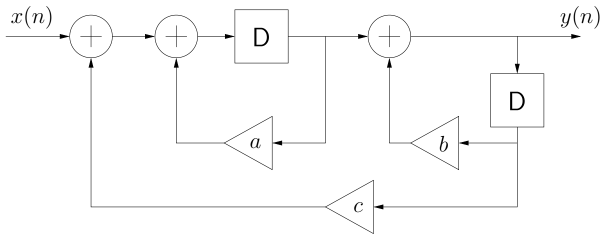


Figure 3: Filter with round-offs in the multiplications.

***Good luck!***