SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 99 **Digital Signalbehandling**, 2E1340

Final Examination 2006–06–05, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage directly after the exam.

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1. As we have seen during this course, for example in the homework projects, we may sometimes want to handle digital signals with different sampling frequencies. In the first project many of you used linear interpolation to increase the rate of a signal by a factor $I = 2$. Sometimes we are interested in upsampling using greater values than 2.

a) Describe how to do linear interpolation with an upsampling factor $I = 3$. Use a direct implementation of the general interpolation framework that we use in the course. The solution should include a block diagram and specify all filters involved. (3p)

b) We have seen that a direct implementation of upsampling will result in a large number of multiplications by zeros. Describe a more efficient implementation of the solution from a). (3p)

c) Now, consider linear interpolation for a general upsampling factor $I = k$. Derive a general expression for the filter used in the direct implementation. (4p)

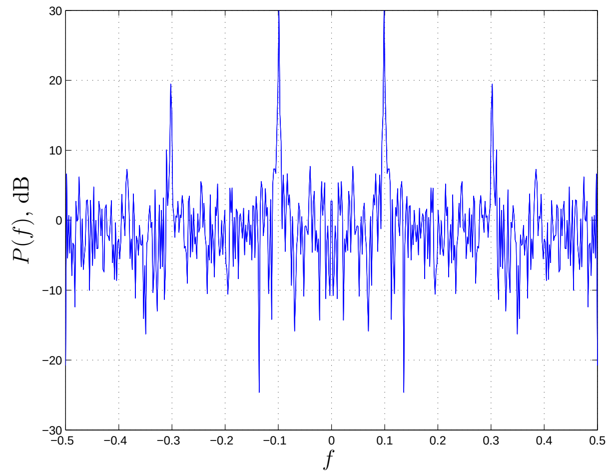

Figure 1: Periodogram of the sampled input signal $x(t)$. Note that the power is plotted in dB.

2. The previous exam included a question related to Figure 1, which shows the periodogram of a signal with two sinusoids with normalized frequencies $f_1 = 0.3$ and $f_2 = 0.9$, embedded in white noise. The periodogram was calculated from 512 samples, using an FFT without any zero padding.

a) Sketch approximately what the figure would have looked like if instead we had used the Bartlett method with a segment length of 128 samples. Make sure to include a scale for both axes and indicate clearly the height and location of the peaks. (5p)

b) Explain how the figure would have changed if we had calculated the periodogram using zero-padding and an FFT of length 2048. (3p)

c) List two other methods that can be used to estimate the frequencies and powers of the sinusoids and the noise power. For each method, write one or two lines explaining why to use (or not to use) the method for this particular signal and problem. (2p)

3. A digital filter will be implemented on a DSP. The filter is an FIR-filter but the number of filter coefficients (denoted by P) and their values can be changed online. The system is to operate using the sampling frequency 16kHz. The delay in the system (that is not due to the delay of the filter itself) may not exceed 50ms. Floating point arithmetics is used on a 32 bit system. Two alternatives for the implementation are investigated; direct implementation in the time domain using convolution or filtering using the FFT. As is customary, the system will operate using the input/output buffer technique. Motivate your answers carefully!

a) What is the maximum size of the input/output buffers? (3p)
Assume in the following that the buffer size was selected to be 200 *samples.*

b) If the FFT implementation is selected, what is the minimum size of the FFT that must be calculated for each processed buffer? (3p)

c) Assume that the total number of samples to be processed is 2000. Derive expressions for the total number of (real valued) multiplications required by each of the two methods. Include the number of multiplications from the moment the filter coefficients are available. (4p)

4. In the following question, assume that the operations are implemented using fixed point arithmetics, leading to round-off errors after all multiplications.

a) A 2nd order FIR filter with input signal $x(n)$ and output $y(n)$ can be implemented as two cascaded first order filters, i.e.

$$\begin{aligned} u(n) &= x(n) * f(n) \\ y(n) &= u(n) * g(n) \end{aligned}$$

where the transfer function of the filters is given by

$$\begin{aligned} F(z) &= c(1 - az^{-1}) \\ G(z) &= 1 - bz^{-1} \,, \end{aligned}$$

for some coefficients a, b and c. For a given filter with zeros z_1 and z_2, determine how to choose the coefficients a and b such that the power of the quantization noise at the output is minimized. (4p)

b) Similarly, a 2nd order all-pole filter with input $x(n)$ and output $y(n)$ can be implemented as

$$\begin{aligned} u(n) &= x(n) * h(n) \\ y(n) &= u(n) * k(n) \end{aligned}$$

where the transfer function of the filters is given by

$$\begin{aligned} H(z) &= \frac{C}{1 - Az^{-1}} \\ K(z) &= \frac{1}{1 - Bz^{-1}} \end{aligned}$$

For a given filter with poles p_1 and p_2, determine how to choose the coefficients A and B such that the power of the quantization noise at the output is minimized. (6p)

5. In wireless communications, it is possible to improve the system performance by using several antenna elements instead of a single antenna, at the receiver and/or transmitter. Here, we consider a situation where the receiver is equipped with two antenna elements and we have a single antenna at the transmitter. We assume that the transmitted signal is narrow-band and we use a complex valued so-called base-band model to represent the signals. The only thing you need to know about this

representation (which was illustrated in the first set of computer exercises) is that signals are represented as complex valued signals and that the propagation from the transmitter to each of the receive antennas is described as a multiplication by a complex number. So, if the transmitted signal is $x(n)$, the signal received at antenna element i, $i = 1, 2$ is

$$y_i(n) = h_i x(n) + \nu_i(n),$$

where h_i is the influence of the radio channel to receive antenna i and $\nu_i(n)$ is additive white Gaussian noise with $\mathrm{E}[|\nu_i(n)|^2] = \sigma^2$, which is independent from antenna to antenna. It may be convenient to use vector notation,

$$\underbrace{\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix}}_{\mathbf{y}(n)} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}_{\mathbf{h}} x(n) + \underbrace{\begin{bmatrix} \nu_1(n) \\ \nu_2(n) \end{bmatrix}}_{\boldsymbol{\nu}(n)} ,$$

i.e. $\mathbf{y}(n) = \mathbf{h}x(n) + \boldsymbol{\nu}(n)$. Note that $\mathrm{E}[\boldsymbol{\nu}(n)\boldsymbol{\nu}^H(n)] = \sigma^2\mathbf{I}$.

Assume that the data signal we want to transmit is $s(n)$. Before the signal is actually transmitted, it is amplified by a (complex valued) factor b, so $x(n) = bs(n)$. The receiver forms a linear combination of the signals from the two antenna elements, to try to reconstruct the original data as well as possible, forming

$$\hat{s}(n) = a_1^* y_1(n) + a_2^* y_2(n) = \underbrace{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}^H}_{\mathbf{a}^H} \underbrace{\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix}}_{\mathbf{y}(n)} = \mathbf{a}^H \mathbf{y}(n) \,.$$

This is often called beamforming. The complex valued weighting coefficients $\mathbf{a}$ have been defined in this way with a complex conjugation, just for notational convenience.

If the radio channel $\mathbf{h}$ and noise power σ^2 is known both at the transmitter and receiver, a natural design criterion to determine $\mathbf{a}$ and b is to minimize the mean square error between $\hat{s}(n)$ and $s(n)$.

a) Assume that $s(n)$ is a stationary stochastic process with $\mathrm{E}[|s(n)|^2] = 1$. Determine an expression for the mean square error (MSE) between $\hat{s}(n)$ and $s(n)$. (3p)

b) Assume that we have already calculated the receiver beamformer so that $\mathbf{a}$ is known. Determine the value of b that minimizes the MSE. (3p)

c) Similarly, assume that we already have a given value for b (not necessarily the one you obtained above) and determine the value of the vector $\mathbf{a}$ that minimizes the MSE. (4p)

Good luck!