# SIGNALBEHANDLING
## Institutionen för Signaler, Sensorer & System

E 98 **Digital Signalbehandling**, 2E1340

Final Examination 2005–12–19, 14.00–19.00

**Literature:** Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
*Copies of the slides*
*Beta – Mathematics Handbook*
*Collection of Formulas in Signal Processing, KTH*
Unprogrammed pocket calculator.

**Notice:** Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

**Contact:** Mats Bengtsson, Signal Processing, 08-790 84 63,

**Results:** Will be posted within three working weeks at Osquldas väg 10, floor 3.

**Solutions:** Will be available on the course homepage directly after the exam.

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**1.** A signal $x(t)$ consisting of two sinusoids in measurement noise is sampled giving us the discrete signal $x[n]$. The periodogram of $x[n]$ is given in Figure 1. Assume that it is known that the amplitude and frequency of the second sinusoid is an integer multiple of the amplitude and frequency of the first, i.e. that

$$x[n] = a_1 \sin(2\pi f_1 n) + a_2 \sin(2\pi f_2 n) + w(n) ,$$

where

$$a_2 = K a_1$$
$$f_2 = K f_1$$

for some integer K. Furthermore we know that a total number of N=512 samples were used.
Determine the amplitudes $a_1$ and $a_2$ as well as the frequencies $f_1$ and $f_2$ and the noise power $\sigma_w^2 = \mathrm{E}[w^2(n)]$. (10p)



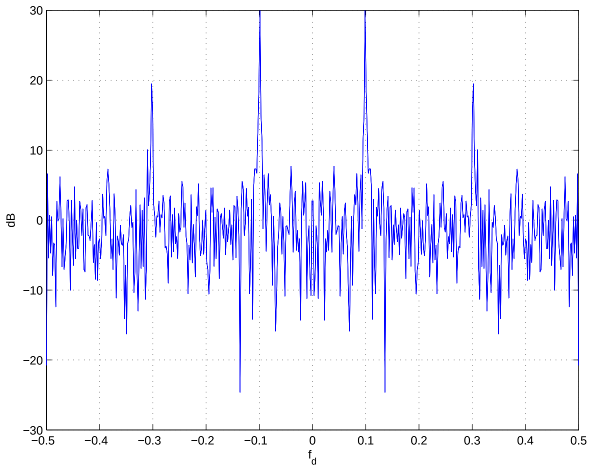


Figure 1: Periodogram of the sampled input signal $x(t)$. Note that the power is plotted in dB.

**2.** In the first project assignment this year, the most common solutions were the following

- Using plain downsampling, $z(n) = y(2n)$ to reduce the data rate of a signal $y(n)$ by 2.
- Using plain upsampling, $w(n) = \{z(0), 0, z(1), 0, z(2), 0, \dots\}$, to recover the original data rate.
- Using $\hat{P}_{xx}(f) = \left|\sum_{n=0}^{N-1} x(n) e^{-j2\pi f n}\right|^2$ to estimate the spectral density of a signal.

a) Which (none, one or several) of these solutions would you classify as
- Wrong?
- Correct, but not the best option? (2p)

b) Propose better solutions for the three subproblems. (4p)

c) Describe how these better solutions would influence the output $w(n)$ if the input signal $y(n)$ is
- a sinusoid with frequency $f < 1/4$.
- a sinusoid with frequency $f > 1/4$.

as compared to using the naïve solutions described at the top of the problem. (4p)

As in all exam problems, an answer without motivation will not give any points.

**3.** A digital filter with $z$-transform

$$H(z) = \frac{1}{A(z)},$$

where

$$A(z) = 1 - a_1 z^{-1} - a_2 z^{-2} - \ldots - a_L z^{-L}$$

is to be implemented in a system that represents numbers (in the range [-1,1[) using $b$ bits excluding the sign bit. This filter is implemented as

$$y(k) = x(k) + a_1 y(k-1) + \ldots + a_L y(k-L).$$

a) Sketch a filter structure that would implement the above filter. (2p)

b) Assume that the input signal is white noise with variance $\sigma^2$. Calculate the signal to quantization noise ratio (SQNR) as a function of $b$. (3p)

c) Now assume that the input signal is colored noise with spectral density $P_{xx}(f)$ and that the filter is designed as an ideal whitening filter for $x(n)$. Find an expression for the SQNR, expressed as a function of $P_{xx}(f)$.
(A whitening filter for a signal $x(n)$ is a filter designed such that the output $y(n)$ is white noise if $x(n)$ is the input.) (3p)

d) What happens if $H(z)$ has poles close to the unit circle, so that the filter is close to being unstable? Consider the same situation as in c). (2p)

**4.** As we have seen in Project 2, there are several possibilities to find filters that provide perfect reconstruction when used in filter banks. It turns out that it is even possible to add further symmetry constraints between the filters and still find solutions.

As you know, a direct implementation of the synthesis part of a filter bank is given by

$$y(n) = f_0(n) * u_0(n) + f_1(n) * u_1(n),$$

where $y(n)$ is the reconstructed signal at the output and $u_0(n)$ and $u_1(n)$ are upsampled versions of the two input signals $v_0(n)$ and $v_1(n)$:

$$u_k(n) = \begin{cases} v_k(n/2) & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \qquad k = 1, 2$$

a) Assume that the two filters $f_0(n)$ and $f_1(n)$ are related by $F_1(f) = F_0(f - 1/2)$. Show that it is possible to implement the synthesis part of the filter bank as

$$\begin{cases} y(2n) = p_0(n) * \Big(v_0(n) + v_1(n)\Big) \\ y(2n+1) = p_1(n) * \Big(v_0(n) - v_1(n)\Big) \end{cases}$$

where $*$ denotes linear convolution, and describe how to obtain the filters $p_0(n)$ and $p_1(n)$ from $f_0(n)$ and $f_1(n)$. (7p)

b) If the filters $f_0(n)$ and $f_1(n)$ have length $L$, determine the computational complexity of the two solutions, expressed in terms of the number of multiplications for each output value.
Which implementation is the best? (3p)



**5.** Often when doing measurements on weak signals, the power supply of the measurement equipment adds disturbances with 50Hz frequency. One way to remove this so-called power hum is to measure the 50Hz signal also directly at the power supply and subtract a filtered version of it, see Figure 2.

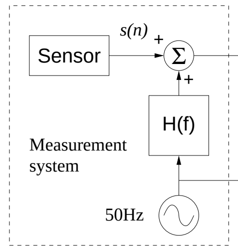


Figure 2: Measurement system.

a) View the signal $s(n)$ from the signal as a Gaussian stochastic process and assume that the filter $H(f)$, which models the influence of the 50Hz power signal on the measured signal $y(n)$, is given. Determine the coefficients of the filter $g(n) = \{g_0, g_1\}$ such that the power of the output signal $x(n)$ is minimized. The answer should be expressed in terms of the filter $H(f)$.
Motivate also why this is a good design choice for $g(n)$. (6p)

b) In practice, $H(f)$ is unknown. Describe how to determine $g(n)$ based on $N$ measured samples of $y(n)$ and the corresponding samples of $w(n)$. (4p)

***Good luck!***

- Don't forget to fill in the course evaluation form! Follow the link on "Latest News" at the course WWW page.
- Don't forget to pick up your graded project report for Project 2 from STEX.