SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 95 **Digital Signalbehandling**, 2E1340

Final Examination 2004–12–14, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 08-790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage directly after the exam.

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1. In the first project, you were asked to implement som kind of decimation. Most solutions proposed a simple downsampling without any filtering. However, some of you proposed the following somewhat more advanced algorithm to decimate the signal $y(n)$ into a signal $z(n)$ with half the rate:

$$z_0(n) = y(2n)$$
$$z_1(n) = y(2n+1)$$
$$z(n) = \frac{z_0(n) + z_1(n)}{2}$$

a) This algorithm can be seen as a polyphase implementation of a decimator. Draw a block diagram of this polyphase implementation and determine the polyphase filters. (3p)

b) Determine an expression for the Discrete Time Fourier Transform (DTFT) $Z(f)$ of the decimated signal as a function of the DTFT $Y(f)$ of the input. (4p)

c) Is the proposed algorithm better than using a simple downsampling (i.e. to use $z_0(n)$ directly)? Does it matter what the input signal is? (3p)

2. The filter $y(n) = x(n) + ay(n-1)$ is implemented in fixed point arithmetics, using $B = 8$ bits including the sign bit. When the input signal is $x(n) = A\sin(2\pi f_0 n + \phi)$, with $f_0 = 0.123$, the spectral density $P_{yy}(f)$ of the output is given in Figure 1, where $P_{yy}(0) \approx 1.186 \cdot 10^{-5}$ and $P_{yy}(1/2) \approx 2.812 \cdot 10^{-6}$. Determine the filter coefficient a. The signal amplitude A is set small enough to avoid overflow.

(10p)

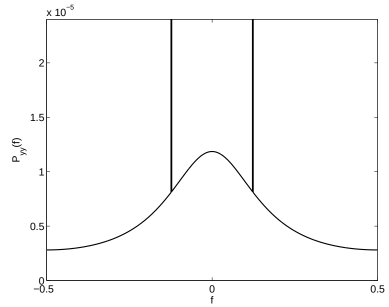

Figure 1: Spectral density $P_{yy}(f)$ of the filter output.

3. Consider the following three scenarios where we want to use some kind of spectral estimation technique. For each of the scenarios, you should recommend a good method to use. Also, for each scenario, you should mention one method that you don't recommend. The motivations for each answer should explain why the recommended method is better than the one you don't recommend.

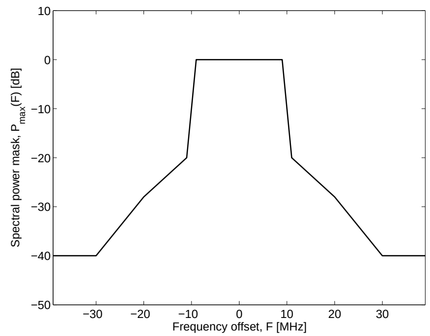

Figure 2: Spectral power mask

a) You have measured a signal $y(n)$, where you know that $y(n) = A\cos(2\pi f n + \phi) + \nu(n)$, where $\nu(n)$ is white noise. You want to determine the frequency f as accurately as possible, the other parameters are not interesting. (3p)

b) You have measured a signal $y(n)$, where you know that $y(n) = A_1\cos(2\pi f_1 n + \phi_1) + A_2\cos(2\pi f_2 n + \phi_2) + \nu(n)$, where $\nu(n)$ is white noise. You want to determine the frequencies f_1 and f_2 as accurately as possible, the other parameters are not interesting. You know also that the separation between the two frequencies may be fairly small. (3p)

c) Figure 2 shows a so-called spectral power mask from the specification of a wireless system. Such masks are used to specify limits on how much power may be transmitted outside the main bandwidth of the system. The requirement is that the spectral density of the signal transmitted from the antenna may not exceed the mask. Imagine that you work at a certification institute that should do measurements on equipment from different manufacturers. So, you collect a number of samples of the signal transmitted from the antenna, estimate the spectral density $\hat{P}(F)$ and check that $\hat{P}(F) \leq P_{\max}(F)$ for all frequencies. (In the mask, the power is specified in dB relative to the maximum of the power spectral density of the transmitted signal.) (4p)

4. Figure 3 shows an implementation of a filter in a MATLAB like programming language. For what values of the parameters `a` and `b` is the filter stable? Assume that the calculations are done with high precision floating point numbers, so there is no need to consider round-off errors or overflow. (10p)

```
function y = myfilter(x,a,b)
% Signal length:
N = length(x);
% Initialize y(n):
y(n) = zeros(N,1);
for n=1:length(x)
  z(n) = a*y(n-1) + a*y(n-2) - 0.5*y(n-3);
  y(n) = x(n) + b*x(n-1) + b*x(n-2) + 0.5*x(n-3) + z(n);
end
```

Figure 3: Filter implementation.

5. Consider the filter bank in Figure 4. We have seen in the course that it is possible to get perfect reconstruction, $y(n) = x(n - L)$ with some delay L both when using ideal low and high pass filters and when using appropriately designed FIR filters. Now, consider that we use the ideal band-pass and band-stop filters shown in Figure 5. Show that it is impossible to reconstruct the original signal $x(n)$, no matter how we choose the filters $F_0(f)$ and $F_1(f)$. (10p)

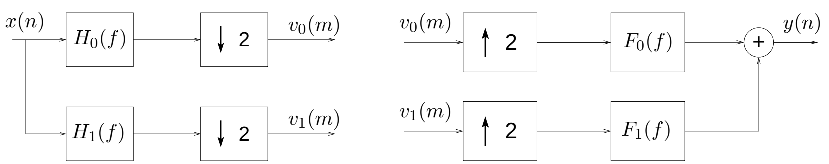

Figure 4: Filter bank.

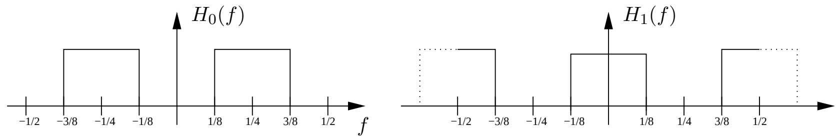

Figure 5: Band pass filter $H_0(f)$ (left figure) and band stop filter $H_1(f)$ (right figure).

Good luck!

Don't forget to fill in the course evaluation form!
Follow the link on "Latest News" at the course WWW page.