SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 100 **Digital Signalbehandling**, 2E1340

Final Examination 2006–12–19, 08.00–13.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signal Processing, 08-790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage after the exam.

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1. The filter in Figure 1 is implemented with b-bits fix-point arithmetic, including the sign bit (representing values between -1 and 1). The additions do not cause any overflow, but all multiplications are rounded to b bits. Note that the structure differs somewhat from a standard lattice filter. The lattice coefficients are

$$k_1 = -\frac{5}{3},\ k_2 = \frac{1}{3},\ k_3 = -\frac{3}{5}$$

a) What is the round-off noise variance at the output of the lattice when $b = 5$? (7p)

b) Is this filter stable? (3p)

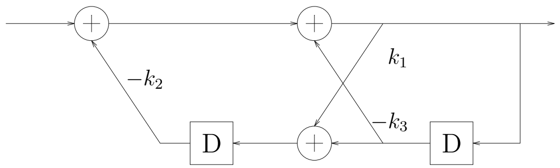

Figure 1: Non-standard lattice structure.

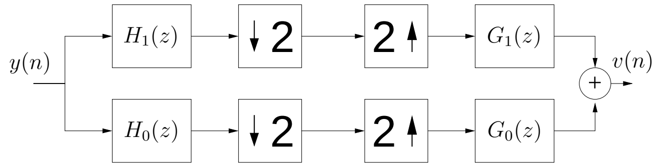

Figure 2: Filterbank

2. Figure 2 shows a standard filter bank. Assume that the four filters are given by

$$\begin{aligned} H_0(z) &= 1 + z^{-1} \\ H_1(z) &= 1 - z^{-1} \\ G_0(z) &= a + bz^{-1} \\ G_1(z) &= c + dz^{-1} \end{aligned}$$

where a, b, c and d are some real valued filter coefficients to be determined.

For which values of L is it possible to get perfect reconstruction with delay L, i.e. $v(n) = y(n - L)$? Determine the corresponding filters $G_0(z)$ and $G_1(z)$. (10p)

3. For most radio communication devices, there are regulations on the power that may be transmitted, both in the allocated frequency band and in adjacent frequencies. Such regulations are often expressed in terms of a *frequency mask* where the power spectral density of the transmitted signal must stay below this mask for all frequencies. Figure 3 shows a simplified version of such a mask for a time discrete sampled version of the transmitted signal (downconverted to base band, which means that the center frequency of the signal is 0 instead of the carrier frequency of the radio signal).

We have measured 3 different signals from different radio transmitters and handed them to 3 different students in some course on introductory signal processing. Each measured signal was 2048 samples long.

Each student used a different method to extract some kind of information about the power spectral density of his/her signal, as can be seen in Figures 4–6.

In each of the three cases, you should try to determine if the corresponding signal fulfils the spectrum mask, and provide one of the following answers:

- Yes, the power spectral density of the signal is below the mask, with high probability.
- No, the power spectral density of the signal is not entirely below the mask, with high probability.
- The figure does not provide sufficient information.

Note that the motivation of each answer is more important than the answer itself.

a) In Figure 4, the student has plotted the absolute value of the DFT of signal 1.

b) In Figure 5, the student has plotted the real part of the DFT of signal 2.

c) Figure 6, the student has plotted the Bartlett spectrum estimate of signal 3 as a function of f. The samples were divided into 16 segments. (10p)

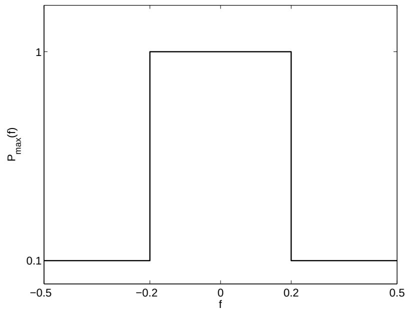

Figure 3: Spectrum mask.

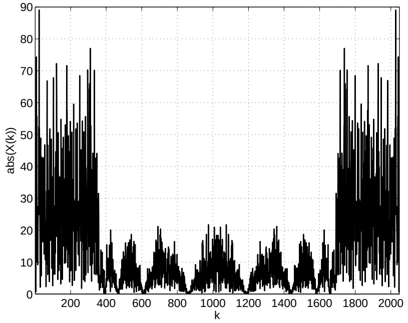

Figure 4: Absolute value of the FFT, signal 1.

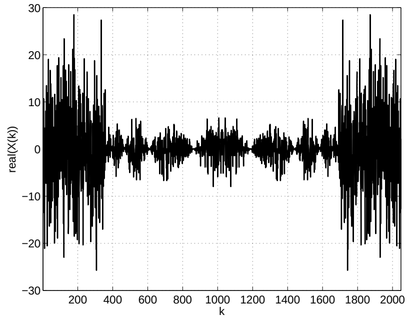

Figure 5: Real part of the FFT, signal 2.

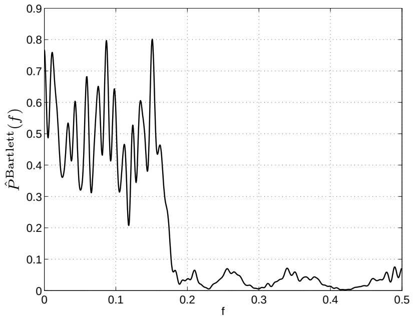

Figure 6: Bartlett estimate, signal 3.

4. Consider the transmission of a signal $s(n)$ (with $\mathrm{E}[s(n)] = 0$) over a communication channel which suffers from so-called Inter-symbol Interference, which means that each sample of the received signal is influenced not only by the current transmitted symbol but also by earlier symbols. The transmitted signal $s(n)$ is assumed to be white with power spectral density $P_s(z) = \sigma_s^2$. The signal received at the destination is given by

$$y(n) = s(n) + s(n-1) + e(n)$$

where $e(n)$ is additive white noise with variance σ_e^2 and is uncorrelated with $s(n)$.

a) Design a two-tap FIR filter of the form

$$\hat{s}(n) = ay(n) + by(n-1)$$

that estimates the transmitted signal, such that the mean square error is minimized. (7p)

b) Which values do the filter coefficients converge to when $\sigma_s^2/\sigma_e^2 \to \infty$? Discuss the result. (3p)

5. As we have seen in the first computer exercise, it is sometimes useful to be able to process complex valued signals. In general, the power spectrum of a complex valued signal is not symmetric around zero and it may be convenient to consider the frequency band $0 \leq f < 1$ as the interesting part of the spectrum of a time discrete signal (for example in so-called single-sideband modulation used in certain communication systems).

Design a system for rate conversion by 3/4 of a time discrete complex valued signal, that preserves the spectral shape of the frequency band $0 \leq f < f_{\max}$ of the original signal, where $f_{\max}$ is chosen to be as large as possible.

Illustrate how your system works by sketching the Fourier transform (DTFT) of the output of each upsampler, downwsampler and filter in your system, if the input signal has the DTFT shown in Figure 7. (10p)

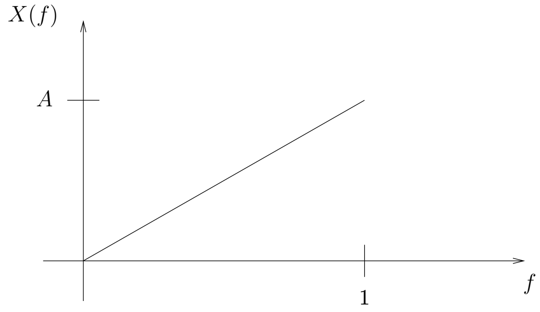

Figure 7: Fourier transform of the input signal.

Good luck!

- Don't forget to fill in the course evaluation form! Follow the link at the course WWW page.
- Don't forget to pick up your graded project report for Project 2 from STEX.