KTH, SIGNAL PROCESSING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2015–01–14, 08.00–13.00

Literature:

- Course text book:
  - Diniz, da Silva & Netto: *Digital Signal Processing; System Analysis and Design*

    or
  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or
  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck, and remember to answer the course survey to receive your bonus points!

1. A few quick and shorter questions. Remember to motivate your answer.

a) In AR modeling, what is the best sign of the model order N geing too high? (choose the correct option and motivate) (2p)

i) that the coefficients a_k for the highest ks are small or zero

ii) $a_N = 0$, which is the only reliable indicator of N being to high

iii) that the resulting AR filter is unstable

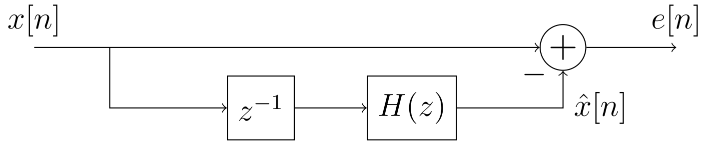
iv) that the difference between b_0^2 for the Nth order model and the $(N-1)$th order model is negligible.

Note: We assume a labeling of the coefficients in the AR model consistent with

$$y[n] + \sum_{k=1}^{N} a_k y[n-k] = b_0 w[n]$$

where $w[k]$ is the zero mean unit variance driving noise of the AR process.

b) In the circuit below, the filter $H(z) = \sum_{n=0}^{N-1} h[n]z^{-n}$ is the MMSE optimal causal length N FIR estimator (predictor) for the w.s.s. stochastic process $x[n] \in \mathbb{R}$. That is, $h[n] = 0$ when $n < 0$ and $n \geq N$ with $h[n]$ for $n = 0, \ldots, N-1$ optimized to give the smallest $E\{e^2[n]\}$ possible.



What can we say about the power of the prediction error $e[n]$? (choose the correct option and motivate) (2p)

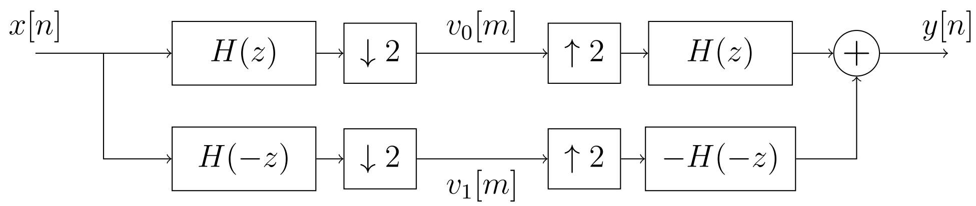
i) $E\{e^2[n]\} = E\{x^2[n]\} - E\{\hat{x}^2[n]\}$

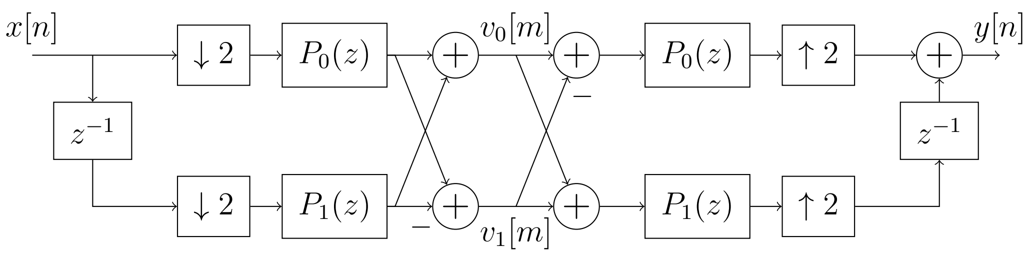
ii) $E\{e^2[n]\} = r_x[0] - \sum_{k=1}^{N} h[k-1]r_x[k]$

iii) $E\{e^2[n]\} = \sum_{k=0}^{N-1} r_x[k]h^2[k]$

Note: $r_x[k] = E\{x[n]x[n+k]\}$ is the acf of $x[n]$

c) A quadrature mirror filter (QMF) bank with perfect reconstruction is implemented in two different ways as shown below. Both implementations yield the same $v_0[m]$, $v_1[m]$, and $y[n]$ given the same $x[n]$. Give expressions for $p_0[n]$ and $p_1[n]$ in terms of the base filter $h[n]$, where $P_0(z)$, $P_1(z)$, and $H(z)$ are the respective transfer functions of the filters. (3p)





d) An implementation of an FIR filter of length M using overlap add or overlap save requires approximately

$$C = \frac{N \log_2 2N}{N - M + 1}$$

complex valued multiplications per sample if we use the length N FFT to evaluate DFTs and their inverses, and assume a long input signal. How many complex valued multiplication would be needed if we instead used the direct computation of the DFT based on

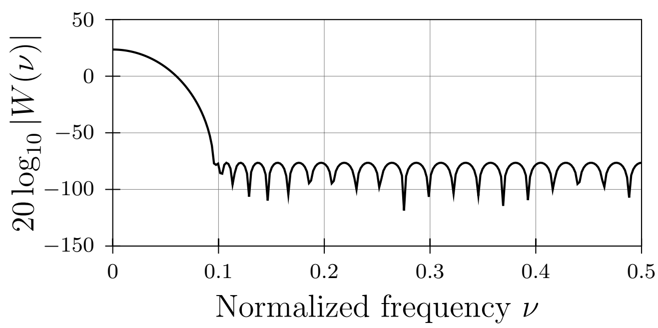
$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{-nk} \quad \text{and} \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{nk}$$

where $W_N = e^{j2\pi/N}$? You may assume that all multiplications are complex valued. That is, you do not need to separate real and complex valued multiplications. (3p)

2. A linear phase length $M = 41$ FIR filter, approximating an ideal bandpass filter $H_I(\nu)$, where

$$|H_\mathrm{I}(\nu)| = \begin{cases} 1 & 0.2 \le |\nu| \le 0.3 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } |\nu| \le 0.5,$$

is designed using the window method with a Chebyshev window $w[n] \in \mathbb{R}$. The magnitude of the DTFT of the window, $W(\nu) = \mathcal{F}\{w[n]\}$, is shown below.



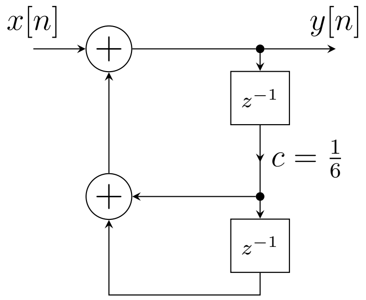
a) Sketch the expected magnitude of the frequency response of the designed filter and illustrate how this response would change if we were to increase the length of the FIR filter. The sketch does not have to be perfect but should show that you understand how the frequency response of the designed filter depends on the ideal target response and the window's properties, and to the extent possible you should appropriately label important points on both axes of the sketch. (5p)

b) Assuming that the filter is implemented causally, i.e., that $h[n] = 0$ for $n < 0$ and $n \ge M = 41$, and $x[n] = \sin(2\pi\nu_0 n + \phi_0)$ where $\nu_0 = 0.25$ is filtered by the designed filter. What would (approximately) the filtered signal be? (3p)

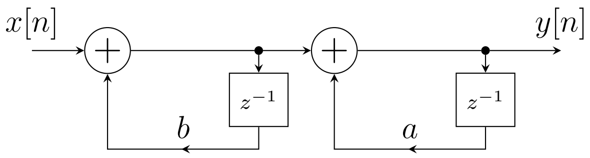
c) Assuming that the designed FIR filter is to be implemented using overlap add for a long input signal, propose a good FFT length N for the implementation and motivate your choice. Specify what the number of complex valued multiplications per signal sample will be for the implementation. (2p)

3. The filters in this problem are assumed to be implemented in fixed point using a $B+1$ bit signed magnitude representation of the range $[-1, 1]$. All internal signals can be assumed to be within the range $[-1, 1]$, i.e., no overflow occurs.

a) Calculate the power of the quantization noise at the output of the circuit below, for the given value of the multiplier $c = \frac{1}{6}$. (6p)



b) Choose the constants a and b such that the circuit below realizes the same transfer function as the circuit in part a), and calculate the power of the quantization noise at the output. There will be some freedom in choosing a and b, and you should choose a and b such that the overall power of the quantization noise at the output $y[n]$ is minimized. (3p)

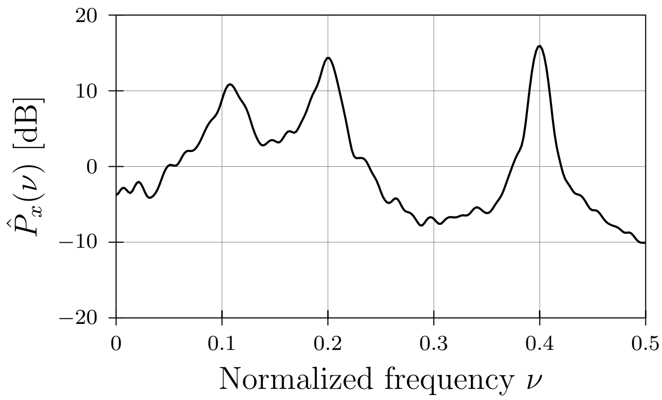


c) Which circuit [the one in part a) or the one in part b)] yields smaller overall fixed point quantization noise power at the output. (1p)

4. The Matlab spectrum estimation code below is applied to $N = 2048$ samples of a real valued signal $x[n]$ stored in the vector `x`.

```
% Assume input vector x of legnth N > L
R = 512; L = 128; D = 64; K = floor((N-L)/D+1)
w = window('hamming',L)'; U = 1/L*sum(abs(w).^2);
Ph = 0;
for k=0:K-1
    xk = x(k*D+1:k*D+L);
    Xk = fft(xk.*w,R);
    Ph = Ph + 1/(K*L*U)*abs(Xk).^2;
end;
```

The resulting spectrum estimate is stored in `Ph` and is plotted (for a limited normalized frequency range and in dB scale) in the figure below. The 3dB resolution of the spectrum estimator is $\Delta\nu_{3\text{ dB}} \approx 0.01$.



a) What type of spectrum estimator is implemented by the above code? Give its name and essential parameters. (2p)

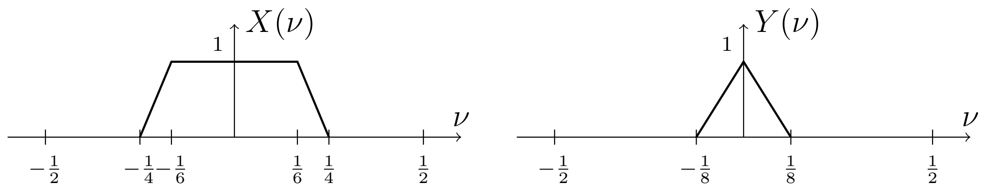
b) Since the vector `Ph` is of finite length it only contains the spectrum estimate $\hat{P}_x(\nu)$ for a finite set of normalized frequencies $\nu \in [0, 1]$. Specify what these normalized frequencies are. (1p)

c) Suppose that you needed to improve the 3 dB resolution to $\Delta\nu_{3\text{ dB}} \approx 0.005$. How would you change the code in order to accomplish this, and what effect would this have on the *variance* of the spectrum estimator? Give a quantitative answer. (3p)

d) Suppose that you needed to reduce the spectral leakage of the estimator. How would you change the code to accomplish this? (2p)

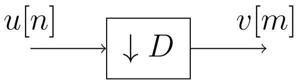
e) Assuming that the data in $x[n]$ was generated from an auto-regressive (AR) process it would make sense to use an AR-based spectrum estimator instead. What value would be an appropriate AR model order for this estimator? (2p)

5. Consider the signals $x[n]$ and $y[n]$ with DTFTs $X(\nu)$ and $Y(\nu)$ shown below.



Create a circuit that turns $x[n]$ into $y[n]$ by using the following set of components arranged in a suitable order.

- One down-sampler



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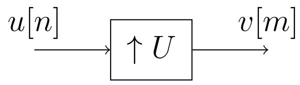
- One up-sampler



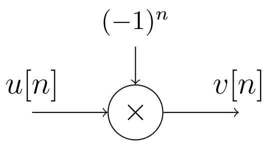
- One ideal low-pass filter of the form

$$H_{\mathrm{LP}}(\nu) = \begin{cases} c_{\mathrm{LP}} & |\nu| \le \nu_{\mathrm{LP}} \\ 0 & \nu_{\mathrm{LP}} < |\nu| \le \frac{1}{2} \end{cases}$$

- One ideal high-pass filter of the form

$$H_{\mathrm{HP}}(\nu) = \begin{cases} 0 & |\nu| \le \nu_{\mathrm{HP}} \\ c_{\mathrm{HP}} & \nu_{\mathrm{HP}} < |\nu| \le \frac{1}{2} \end{cases}$$

- One frequency shifter



Choose the order of the components, and specify the value of all constants, i.e., D, U, c_{LP}, ν_{HP}, c_{HP}, and ν_{HP}. Remember to explain why your solution will work, e.g., by drawing intermediate DTFTs. (10p)