SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 81 **Digital Signalbehandling**, 2E1340

Final Examination 2000–04–26, 0900–1300

Literature:	Proakis, Manolakis: Digital Signal Processing Josefsson: formel- och tabellsamling i matematik Beta – Mathematics Handbook Formelsamling i Kretsteori/Signalteori, KTH Unprogrammed pocket calculator.
Notice:	At most one problem should be treated per page. Motivate each step in the solution. Write your name and *personnummer* on each page. Write the number of solution pages on the cover page. The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.
Contact:	Björn Ottersten, Signalbehandling, 790 72 39,
Results:	Will be posted within three working weeks Osquldas väg 10, floor 2.
Solutions:	Will be available on the course homepage.

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1. For the questions in this exercise, just provide an answer. No specific motivations or calculations are necessary.

a) Assume that the stochastic process $x(n)$ is stationary. Which of the following processes is/are stationary?

- $y_1(n) = \begin{cases} x(k) & n = 5k \\ 0 & \text{otherwise} \end{cases}$
- $y_2(n) = \begin{cases} x(k) & n = 5k + \phi \\ 0 & \text{otherwise} \end{cases}$
- $y_3(n) = x(5n)$
- $y_4(n) = x(5n + \phi)$

ϕ is a stochastic variable, independent of $x(n)$, with probability $\Pr[\phi = 0] = \Pr[\phi = 1] = \Pr[\phi = 2] = \Pr[\phi = 3] = \Pr[\phi = 4] = \frac{1}{5}$. (6p)

b) Let $Y(k) = \{\underset{\uparrow}{1}, 2+i, 3, 4-i, 5\}$, determine

$$X(k) = \text{IFFT}\{\text{IFFT}\{Y(k)\}\}$$

(using 5 points IFFT). (4p)

2.

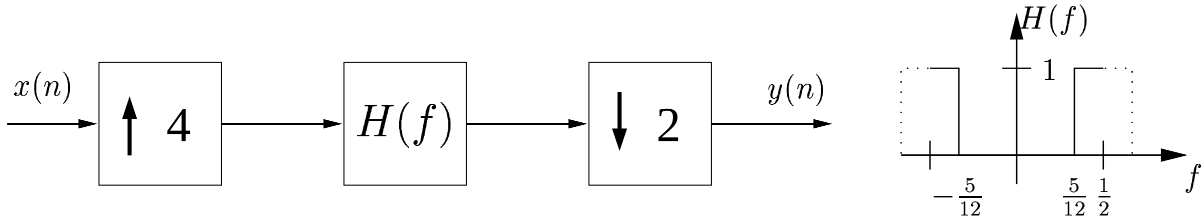
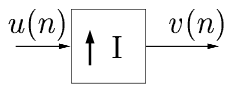
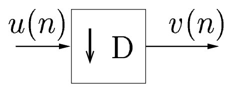


Figure 1:

a) Describe in figures (for some properly chosen example signal) and a formula, the relationship between the time discrete signals $x(n)$ and $y(n)$ in Figure 1. You may give the answer in terms of $X(f)$ and $Y(f)$, the Fourier transforms of the signals. (5p)

b) Describe an alternative implementation of the system in Figure 1. Your implementation should use a minimal number of the following building blocks:

- Ideal **lowpass** filters.
- Interpolators, $v(n) = \begin{cases} u(k) & n = kI \\ 0 & \text{otherwise} \end{cases}$ 

- Decimators, $v(n) = u(nD)$ 

No other building blocks may be used. (5p)

3. We want to design a system that can compress recorded music. The idea is that the music normally consists of only a few frequencies, which can be described in a very compact way.

The strategy is to first divide the sampled recording into segments of length N. For each segment $y(n)$, we do:

(i) Calculate the FFT $Y(k) = \text{DFT}\{y(n)\}$ of length N.

(ii) Set all $Y(k)$ that have $|Y(k)| < \gamma$ to zero (γ is a threshold value which will be described below).

(iii) Describe each non-zero $Y(k)$ by its index k and value, using
- $\log_2(N)$ bits for the index k and
- 16 bits for $Y(k)$ (8 bits for the real and 8 bits for the imaginary part).

The threshold value γ is determined such that only a few values $Y(k)$ have to be transmitted. For simplicity, we assume here that $\gamma = \frac{\sin(0.5\pi)}{0.5\pi} \cdot (\max[|Y(k)|])$.

a) Assume that the music is played by a single flute, which at each time instance can be well approximated by two pure sinus signals, well separated in frequency (the main tone and the first overtone). Assume also for simplicity that both sinus signals have the same power. Determine the compression rate as the number of bits per input sample. Note that the compression rate will be a function of N, the length of each segment, both because of the windowing caused by the segmentations but also since the number of input samples per tone will change. (5p)

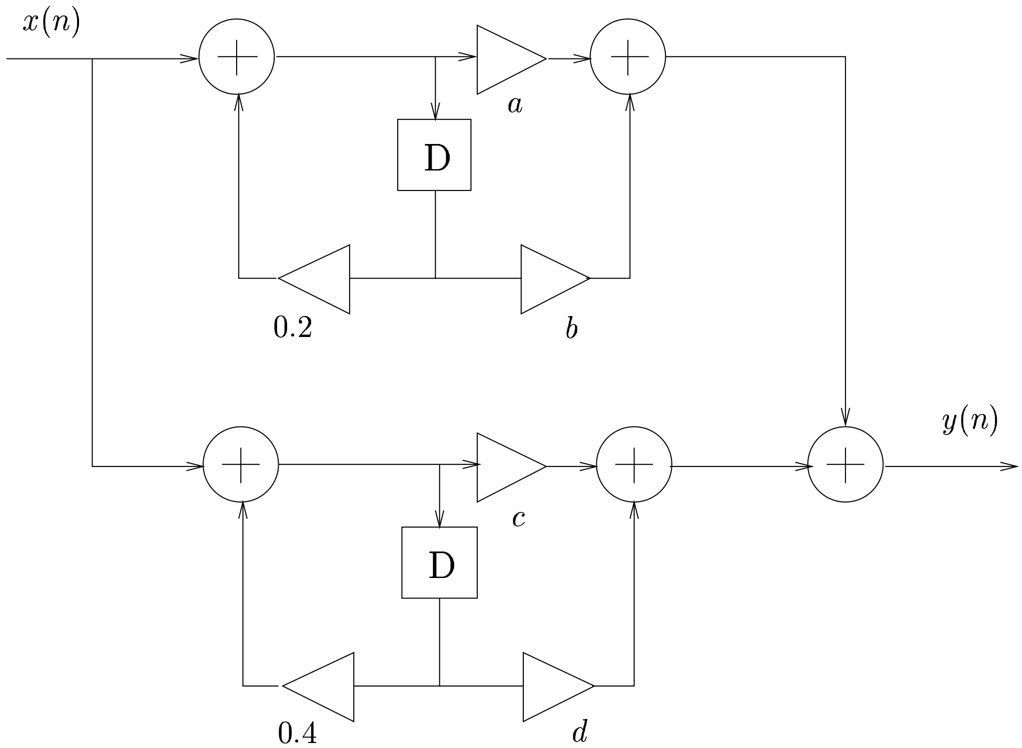
b) Explain how to determine N. What is the advantage of have a short and a long segment size, respectively? (2p)

c) Explain how to modify the algorithm to use zero-padding, that is, to use more frequency values than time samples. Give a list of advantages and disadvantages of this modification. (3p)

4. We wish to implement the following transfer function

$$H(z) = \frac{(1+0.4z^{-1})(1+0.2z^{-1})}{(1-0.2z^{-1})(1-0.4z^{-1})}$$

as two first order filters in series, see Figure 4.



Fixed point arithmetic is used and round-off error occurs due to the multiplications. There is no overflow. By letting a or c be equal to zero, multiplications (and thereby round-off error) can be avoided. Which choice provides the smallest round-off error variance on the output? (10p)

5. Bits are stored on a magnetic medium by recording a pulse, $\pm p(t)$, where the sign of the pulse represents a "1" or "0". When reading the stored data, the pulse is measured and a discrete time signal is formed which can be modeled as

$$y(t) = ap(t) + n(t), \quad t = 0, \dots, N-1$$

where a is the amplitude of the signal and $n(t)$ is zero-mean white noise with variance σ^2. We normalize the pulse so that

$$\sum_{t=0}^{N-1} p^2(t) = N$$

By estimating the amplitude a from the measurements $y(t)$ and looking at the sign, an estimate of the stored bit is obtained.

a) Derive the least squares estimate $\hat{a}$ of a. (2p)

b) Compute the expected value (mean) and variance of $\hat{a}$. (5p)

c) Assume that the noise is Gaussian, is it in principle possible to compute the probability of making a mistake? You do not have to do the calculations but show why its possible or why it is not possible. (3p)