KTH, SIGNAL PROCESSING LAB

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300/ 2E1340

Final Examination 2012–06–08, 8.00–13.00

Literature:

- Hayes: *Statistical Digital Signal Processing and Modeling*
  or
  Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson: *Complementary Reading in Digital Signal Processing*
- Begtsson and Jaldén: *Summary slides*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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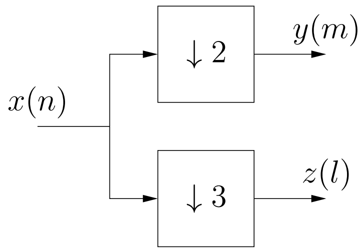
Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. A sinus signal $x(n) = \sin(2\pi f_x n + \phi_x)$ is downsampled to sinus signals $y(m) = \sin(2\pi f_y m + \phi_y)$ and $z(l) = \sin(2\pi f_z l + \phi_l)$ as shown in the figure below.



The normalized frequencies of $y(m)$ and $z(l)$ are obtained as $f_y = 0.4$ and $f_z = 0.1$. What is the normalized frequency of $x(n)$, i.e., f_x? (10p)

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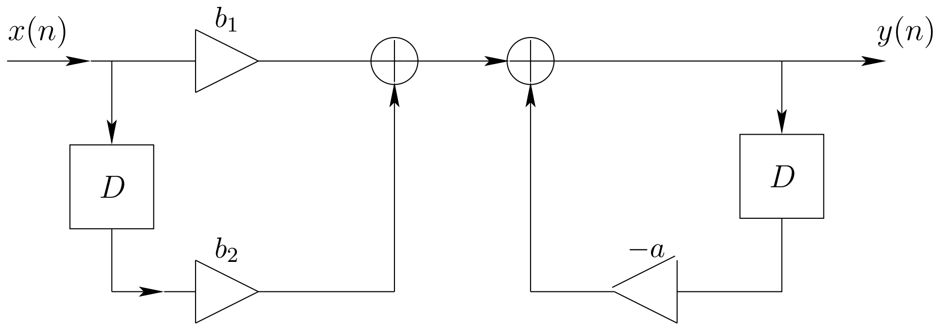
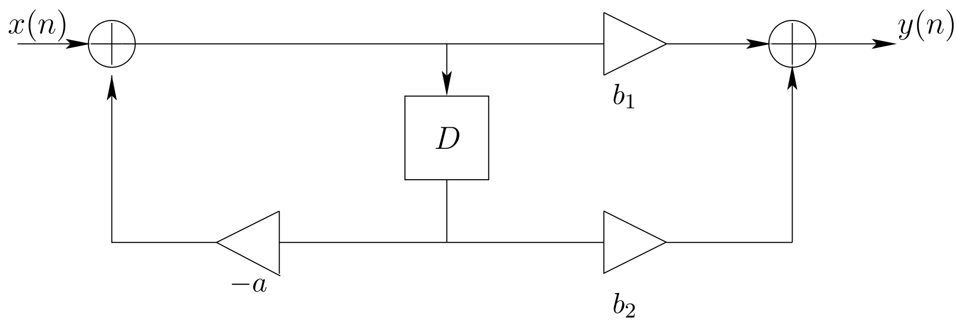
2. The next figure shows two filters implemented using binary fixed-point arithmetic with round-off:



(I)



(II)

Let $a = \cos\theta$, $b_1 = 2$ and $b_2 = 1 + 2\cos\theta$ where $\theta \in (0, 2\pi)$. Calculate the round-off noise variance at the output of both filters. For which values of θ is (I) giving smaller round-off noise variance at the output than (II) and vice-versa? (10p)

3. Assume that $x(n)$ is known to be an autoregressive process of order 2, i.e., AR(2), which is generated by filtering unit variance white noise $w(n)$ with the all-pole filter

$$H(z) = \frac{b(0)}{1 + \sum_{k=1}^{2} a(k) z^{-k}},$$

where $|b(0)|^2 = r_x(0) + \sum_{k=1}^{2} a(k) r_x(k)$.

(a) Given the following measured data,

$$x(0) = 1 \quad x(1) = 2 \quad x(2) = 4 \quad x(3) = 1 \quad x(4) = 0 \quad x(5) = 3,$$

determine (i.e., estimate) the poles of the filter $H(z)$. (8p)

(b) Is the filter $H(z)$ stable?. (2p)

4. Assume that you have a circuit that can compute an $M = 2^q$ point discrete Fourier transform (DFT) efficiently in hardware, where q is an integer. The circuit in question can be thought of as a (black) box that takes a set of values $\{x(0), \ldots, x(M-1)\}$ as input and outputs $\{X_M(0), \ldots, X_M(M-1)\}$ where

$$X_M(k) = \sum_{n=0}^{M-1} x(n) e^{-2\pi \frac{kn}{M}} \quad k = 0, \ldots, M-1\,.$$

However, suppose that what you really wish to compute is an $N = 2^p$ point DFT where p is another integer, i.e., given $\{y(0), \ldots, y(N-1)\}$ you wish to compute

$$Y_N(k) = \sum_{n=0}^{N-1} y(n) e^{-2\pi \frac{kn}{N}} \quad k = 0, \ldots, N-1\,.$$

(a) In the case where $p > q$ ($N > M$), suggest a computationally efficient way of computing the N point DFT by applying the circuit many times. How many times do you have to apply the circuit, and how many extra multiplications (i.e., apart from those that may be computed inside the circuit) would you have to use? Motivate your answers. (6p)

(b) In the case where $p < q$ ($N < M$), suggest a way to compute the N point DFT by using the circuit only once. Motivate your answers. (4p)

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5. We have a sequence $x(n)$, $n = 1, ..., N$, $N = 500$, which is a sinusoidal signal disturbed by some white noise and possibly by another sinusoidal interference. We write $x(n)$ in the following form:

$$x(n) = A_0 cos(2\pi f_0 n + \phi_0) + A_1 cos(2\pi f_1 n + \phi_1) + v(n),$$

where A_0 is the signal amplitude and A_1 is the interference amplitude (=0 if it doesn't exit), and $v(n)$ is white noise with power σ_v^2. Assume the signal power is stronger than interference ($A_0 > A_1$).

We plot the power spectrum of $x(n)$ using the periodogram (Figure 1 on next page) and Bartlett's method (Figure 2 on next page). For the bartlett's method, we divide the sequence into $L = 10$ segments.

In the figures the first two highest peaks are located and the corresponding power spectrum values (in linear scale, i.e., not in dB) are given.

$f_{p0} = 0.2$, $f_{p1} = 0.21$

$P_1(f_{p0}) = 125$, $P_1(f_{p1}) = 31.2$

$f_{b0} = 0.2$, $f_{b1} = 0.17$

$P_2(f_{b0}) = 13.6$, $P_3(f_{b1}) = 0.6$

Besides, the average power of the sequence is $P = \frac{1}{N}\sum_{n=1}^{N} x^2(n) = 0.635$.

(a) What is the signal frequency f_0? Does there exist a second sinusoidal signal, i.e., is $A_1 \neq 0$? If yes, identify its frequency f_1. (Explain which figure you use to draw your conclusions and *why*) (4p)

(b) Estimate A_0, A_1(if exists) and σ_v^2. (6p)

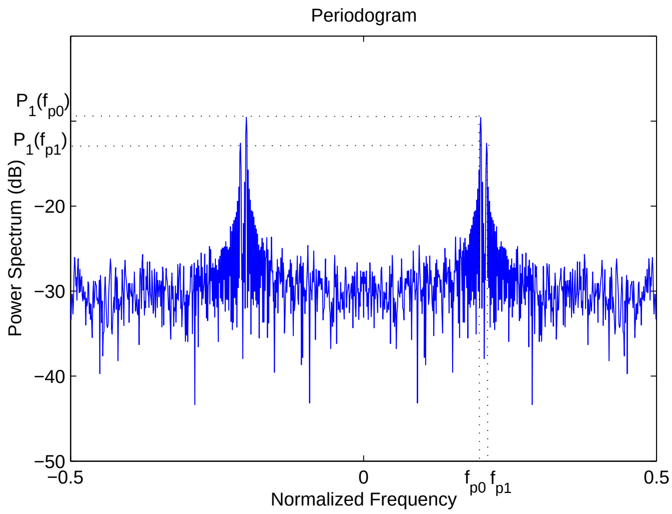

Figure 1: The periodogram of x(n).

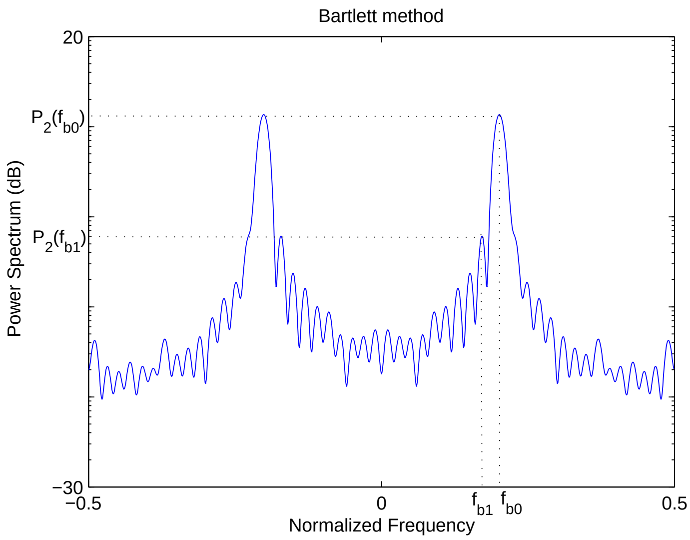

Figure 2: The averaged periodogram of x(n) using Bartlett's method.