KTH, INFORMATION SCIENCE AND ENGINEERING

SCHOOL OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

Digital Signal Processing   EQ2300 / 2E1340

Final Examination 2018–04–03,   14:00-19:00

Literature:

- Jaldén: *Summary notes for EQ2300* (30 pages printed material).
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH.*
- *One A4 of your own notes.* You may write on both sides, and it does not have to be hand written, but cannot contain full solutions to tutorial problems or previous exam problems.
- An unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. We begin with a few mixed shorter questions, remember to motivate your answers...

a) Propose values so for a and b such that the 5 point discrete Fourier transform $X[k] = \mathcal{F}_5\{x[n]\}$, where

$$x[n] = \{\underset{\uparrow}{3},\, 2,\, 1,\, a,\, b\}\,,$$

satisfies the following properties.

i. $X[0] = 0$ (1p)

ii. $X[2] = 0$ (1p)

iii. $X[k]$ is real valued for all k (2p)

b) We have to estimate the power spectrum $P_x(\nu)$ of a stochastic process $x[n]$, of which we have $N = 1000$ samples. We need to have a resolution of at least $\Delta\nu \leq 0.01$, and wish to have as low variance as possible.

i. How many blocks can you divide the data into if you use Bartlett's method, and what will then the variance of the spectrum estimate be? (2p)

ii. How many blocks can you divide the data into if you use Welch's method with a Bartlett window, and what will then the variance of the spectrum estimate be? (2p)

iii. Which of Bartlett's method or Welch's method gives you the best variance of the two? (2p)

2. Computing spectrum estimates by hand is hard work, but sometimes this needs to be done for exam points and in order to show that you understand the procedure. Thus consider the relatively short data set

$$x[n] = \{\underset{\uparrow}{1}, -1, 1, -1, 1, 1, -1, -1, 1, -1\}\,.$$

Let $\hat{P}_x(\nu)$ be the Blackman-Tukey spectrum estimate calculated from the data, with max-lag 2 and window

$$w[n] = \begin{cases} \frac{3-|n|}{3} & |n| \leq 2 \\ 0 & |n| > 3 \end{cases}.$$

a) Compute $\hat{P}_x(\nu)$ for all ν, i.e., give a functional expression for $\hat{P}_x(\nu)$, simplified as far as possible. (6p)

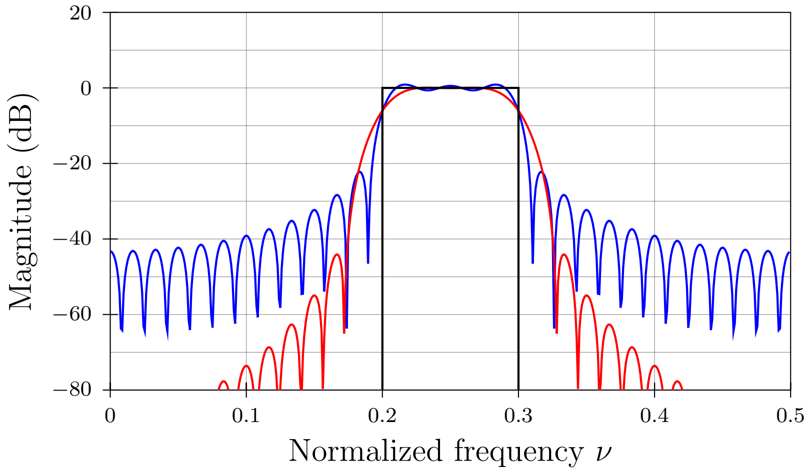
b) Does the data contain more power in the low or high frequencies? (2p)

c) Compute

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \hat{P}_x(\nu) d\nu$$

and provide some intuition behind the result. (2p)

3. In this problem, you should design a $N = 61$ tap Type I FIR filter, that should approximate a bandpass filter with lower normalized cutoff frequency $\nu_\mathrm{L} = 0.2$ and upper cutoff frequency $\nu_\mathrm{H} = 0.3$, shown below in black. Your filter should satisfy $h[n] = 0$ when $n < 0$ and $n \leq N$. You try the method of windows for the filter design, and obtain the following two frequency responses when trying a rectangular window, and a Hann window.



a) Which frequency response is which, i.e., does the red or the blue curve correspond to the design with a rectangular window and which one is the design with a Hann window? (2p)

b) Obtain the impulse response $h_\mathrm{R}[n]$ when using the rectangular window for the FIR filter design. (6p)

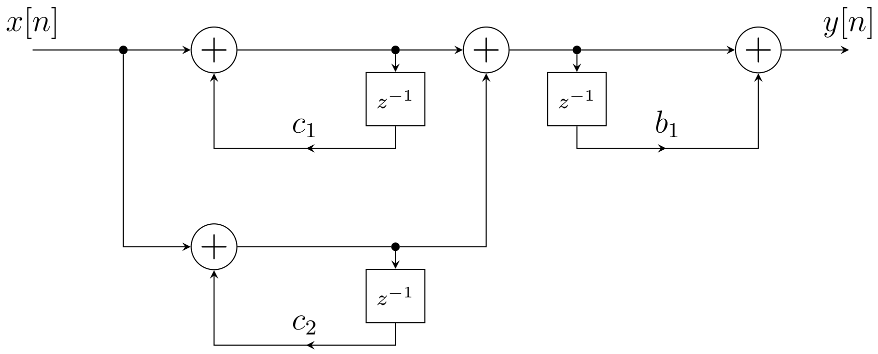
c) Obtain the impulse response $h_\mathrm{H}[n]$ when using the Hann window for the design. For this part, you may be helped by knowing that the Hann window is given as follows. (2p)

$$w_\mathrm{H}[n] = \sin^2\left(\frac{\pi n}{N-1}\right) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right]$$

4. You have to realize a digital filter with the transfer function

$$H(z) = \frac{z^2 + 0.4z - 0.3}{z^2 - 0.6z + 0.08}$$

in fixed point arithmetics, using a $B + 1$ signed magnitude representation of the range $[-1, 1]$. You decide to do so using the circuit below, and you can assume that no overflows occur.

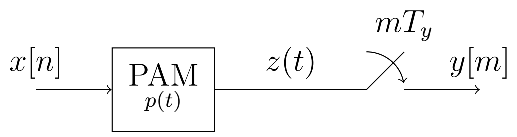


a) Determine the constants b_1, c_1, and c_2 so that the circuit implements the given transfer function. (2p)

b) Draw the additive quantization noise model for the fixed point implementation, i.e., indicate in the schematic where quantization noise is introduced. (1p)

c) Compute the total power of the quantization noise at the output $y[n]$. (7p)

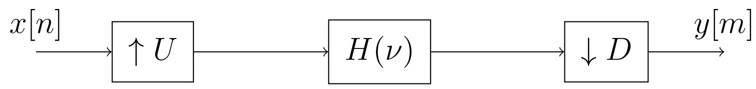
5. You need to convert the sample rate of a signal sampled at 16 kHz to one sampled at 24 kHz. One method to do so is to convert the signal back into an analog signal, and resample it using the new sample rate, i.e., to use the following idealized circuit.



The PAM puls $p(t)$ is assumed ideal and matched to the sample rate of $x[n]$, i.e., it has Fourier transform

$$P(f) = \begin{cases} T_x & |f| \leq \frac{1}{2T_x} \\ 0 & |f| > \frac{1}{2T_x} \end{cases} .$$

In the above, $F_x = T_x^{-1} = 16\,\text{kHz}$ and $F_y = T_y^{-1} = 24\,\text{kHz}$. Another, usually better, option is to do the conversion fully in the digital domain. This can be accomplished with a circuit of the following form, where $y[m]$ should be the same as above.



a) What are the smallest (integer) values of U and D which can be used in the all digital implementation, and how should you choose $H(\nu)$? (4p)

b) Determine the impulse response $h[n]$ of the ideal discrete time filter. (2p)

c) In order to efficiently implement the filter, you truncate $h[n]$ to $n \in [-40, 40]$, i.e., you use in place of $h[n]$ a FIR filter $h'[n]$ where $h'[n] = h[n]$ for $|n| \leq 40$ and $h'[n] = 0$ when $|n| > 40$. When testing the system with a zero mean unit variance white noise test signal $x[n]$ you get the spectrum below for $y[m]$ when using a really good spectrum estimator. What should the power spectrum of $y[n]$ be in the ideal case (assuming $h[n]$ was used), what is the cause of the high frequency artifacts that you observe in practice, and what could you do to remove them while still having an efficiently implementable filter? (4p)