KTH, SIGNAL PROCESSING LAB

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300/ 2E1340

Final Examination 2010–06–03, 14.00–19.00

Literature:

- Hayes: *Statistical Digital Signal Processing and Modeling*
  or
  Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson: *Complementary Reading in Digital Signal Processing*
- Begtsson and Jaldén: *Summary slides*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

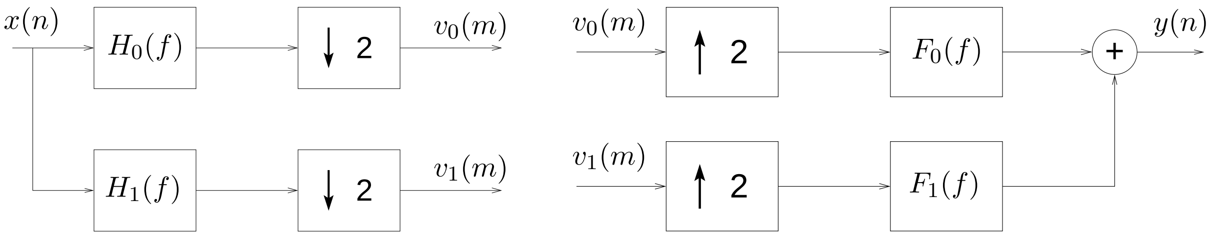
Contact: Joakim Jaldén, Signal Processing, 08-790 77 88

Results: Will be reported within three working weeks on “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

1.

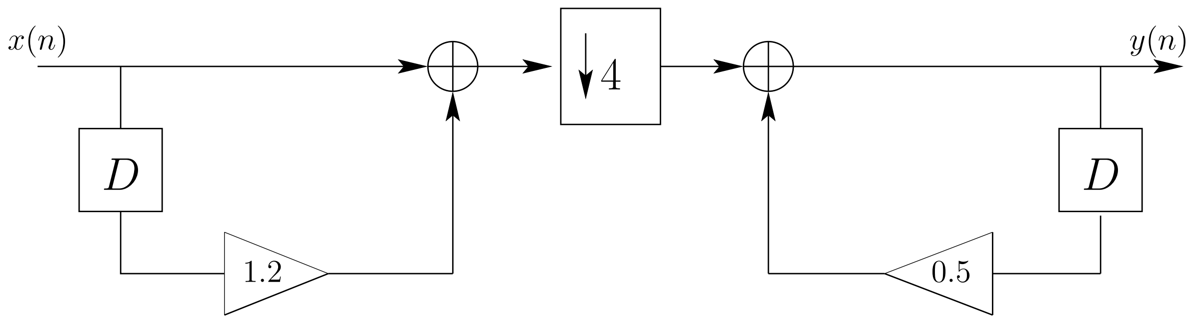


A multi-rate reconstructing filter bank is implemented as shown above. The filter $H_0(f)$ has an impulse response given by

$$h_0(n) = \{\underset{\uparrow}{1}, \tfrac{1}{2}\}$$

Obtain *causal* 2-tap FIR filters $h_1(n)$, $f_0(n)$, and $f_1(n)$ such that perfect reconstruction is achieved, i.e., $y(n) = x(n-L)$ for some delay $L \geq 0$, and specify the value of L. Design the filter $h_1(n)$ such that $H_1(f=0) = 0$, i.e., $H_1(f)$ has a high pass characteristic with zero gain for constant signals. (10p)

2.

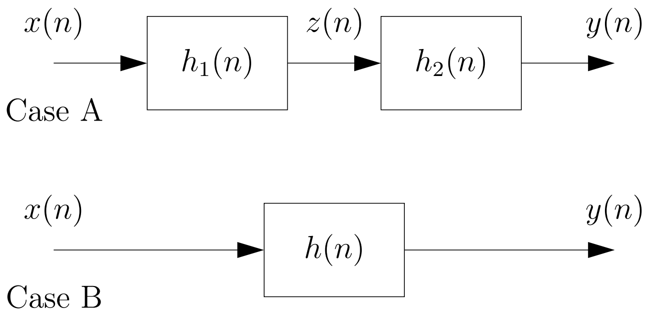


The multi-rate circuit in the figure uses fixed-point arithmetics (for signals in the range $(-1, 1)$) with $B+1$ bits including the sign bit. The results of the multiplications are rounded-off which give rise to quantization noise. No round-of errors occur due to overflow.

a) Compute the variance of the quantization noise at the output. Express your answer as a function of B. (6p)

b) Find the spectral density of the quantization noise at the output, and sketch it roughly for $B = 2$ and $B = 3$. (4p)

3.



Consider the systems shown above where $h_1(n)$ and $h_2(n)$ are FIR filters of length L_1 and L_2, respectively. The FIR filter in Case B is $h(n) = h_1(n) * h_2(n)$ and thus the two systems perform the same overall filtering operation. However, in Case A, the input $x(n)$ is first filtered to yield $z(n)$ and then filtered again to yield $y(n)$, while in Case B only the one filter is used to yield $y(n)$. We assume throughout that $x(n)$ is a long sequence such that edge effects may be neglected.

(a) Determine the number of complex multiplications per sample required for computing $y(n)$ by a direct implementation of the filters in Case A. (1p)

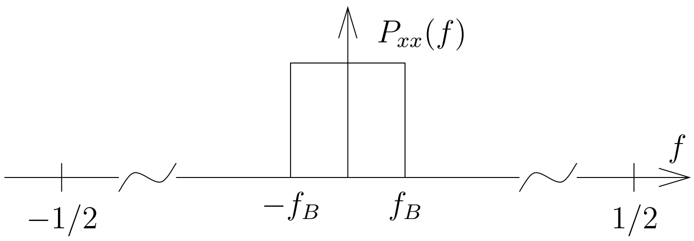
(b) Determine the number of complex multiplications per sample required for computing $y(n)$ by a direct implementation of the filter in Case B. Express the length of $h(n)$, L, as a function of L_1 and L_2. (1p)

(c) Determine the number of complex multiplications per sample if we, in Case A, choose the overlap-save method with an N_1-point FFT for the first filter $h_1(n)$, and an N_2-point FFT for the second filter $h_2(n)$. (2p)

(d) Determine the number of complex multiplications per sample if we, in Case B, choose the overlap-save method with an N-point FFT for implementing the filter $h(n)$. (2p)

(e) Which method, (a),(b),(c) or (d), is the most efficient if $L_1 = 5$ and $L_2 = 10$? Hint: In answering the question you should consider the optimal lengths for all the FFTs involved. (4p)

4. Sometimes we wish to determine the bandwith of a slowly varying stochastic process $x(n)$. An example is when we wish to determine the doppler bandwith, and the speed, of a mobile terminal in a wireless communication system.



Suppose that we have the zero-mean wide-sense-stationary discrete-time stochastic process $x(n)$ with a power spectrum (illustrated above) given by

$$P_{xx}(f) = \begin{cases} \frac{1}{2}P/f_B & |f| \leq f_B \\ 0 & f_B \leq |f| \leq \frac{1}{2} \end{cases} .$$

We do not know f_B exactly, but we do know that $0 < f_B < 10^{-3}$. We choose to compute an estimate $\hat{P}_{xx}(f)$ of $P_{xx}(f)$ from the data $x(n)$, plot $\hat{P}_{xx}(f)$, and read the value of f_B from the plot (compare to the figure above).

(a) Suppose that $\hat{P}_{xx}(f)$ is obtained by Bartlett's method with a window length of L. Suggest an appropriate value for L so that we can clearly read the value of f_B from our plot of $\hat{P}_{xx}(f)$. Remember to properly motivate your choice of L (no points for an answer without motivation). You may assume that the number of data samples is large, i.e., $N \gg L$. Hint: Think about what would happen if you pick L too small? (6p)

(b) If we wish to reduce the value of L, and the length of the FFTs used in Bartlett's method, we could downsample $x(n)$ before estimating the spectrum. How large could we choose the downsampling factor D without loosing the possibility to estimate f_B from the spectrum of the downsampled signal $y(n) = x(nD)$. (4pt)

---

5. For the all pole transfer function

$$H(z) = \frac{1}{1 + \alpha z^{-1} + 0.4z^{-2} + 0.6z^{-3}}$$

where α is a real valued parameter:

(a) Sketch the lattice filter implementation of the FIR filter $H^{-1}(z)$ and compute the corresponding reflection coefficients. (5p)

(b) Specify for which values of α the filter $H(z)$ is stable. Hint: You may assume that α is real valued. (5p)