KTH, SIGNAL PROCESSING LAB

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2012–12–13, 14.00–19.00

Literature:

- Course text book:
  - Dinis, da Silva & Netto *Digital Signal Processing; System Analysis and Design*

    or

  - Hayes: *Statistical Digital Signal Processing and Modeling* and
    Bengtsson: *Complementary Reading in Digital Signal Processing*

    or

  - Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson and Jaldén: *Summary slides*
- Tsakonas and Bengtsson: *Some Notes on Non-Parametric Spectrum Estimation*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:

- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

Note: In the following multi-choice questions, just as in all other questions, be careful to motivate all answers. Answers without motivation/justification carry no rewards.

1. a) Consider the signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ which goes through an upsampling and downsampling as shown in Fig. 1. Which of the following expressions for $y(m)$ is correct?

i) $y(m) = \{\underset{\uparrow}{1}, 0, 0, 3, 0, 0\}$

ii) $y(m) = \{\underset{\uparrow}{1}, 0, 0, 0\}$

iii) $y(m) = \{\underset{\uparrow}{1}, 0, 4\}$

iv) $y(m) = \{\underset{\uparrow}{1}, 0, 3, 0, 2, 0, 4\}$ (2p)

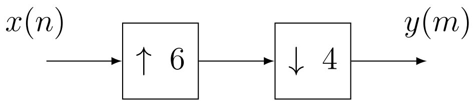

Figure 1: Signal passing through upsampling and downsampling.

b) Which of the following type of systems can have the frequency response shown in Fig. 2? Assume that the system is causal and BIBO stable.

i) An IIR system

ii) A linear phase FIR system of Type 2, i.e., with odd order and a symmetric impulse response.

iii) A linear phase FIR system of Type 3, i.e., with even order and an anti-symmetric impulse response. (3p)

c) Assume that N samples of the discrete time signal $x(n) = 4\sin(2\pi f_0 n)$ have been collected in a Matlab vector `x` and that the following commands have been used to plot the curve shown in Fig. 3.

```
M=256;
plot([0:M-1],abs(fft(x,M)).^2/N)
```

Which of the following values is the best estimate of the frequency f_0?

i) $f_0 = \frac{88}{N}$

ii) $f_0 = \frac{88M}{N}$

iii) $f_0 = \frac{88N}{M}$

iv) $f_0 = \frac{88}{M}$ (2p)

d) Based on the same figure, Fig. 3, what is the best estimate for the number of samples N?

i) $N = 8$

ii) $N = 32$

iii) $N = 64$

iv) $N = 128$ (3p)

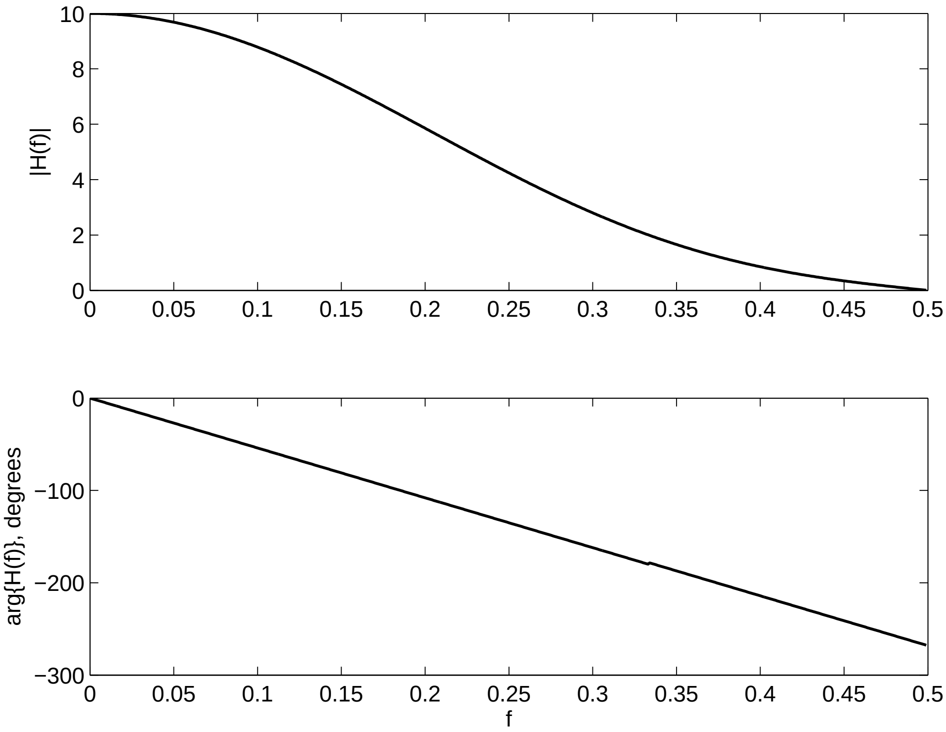

Figure 2: Bode plot of a system, showing magnitude response (upper curve) and phase response (lower curve).

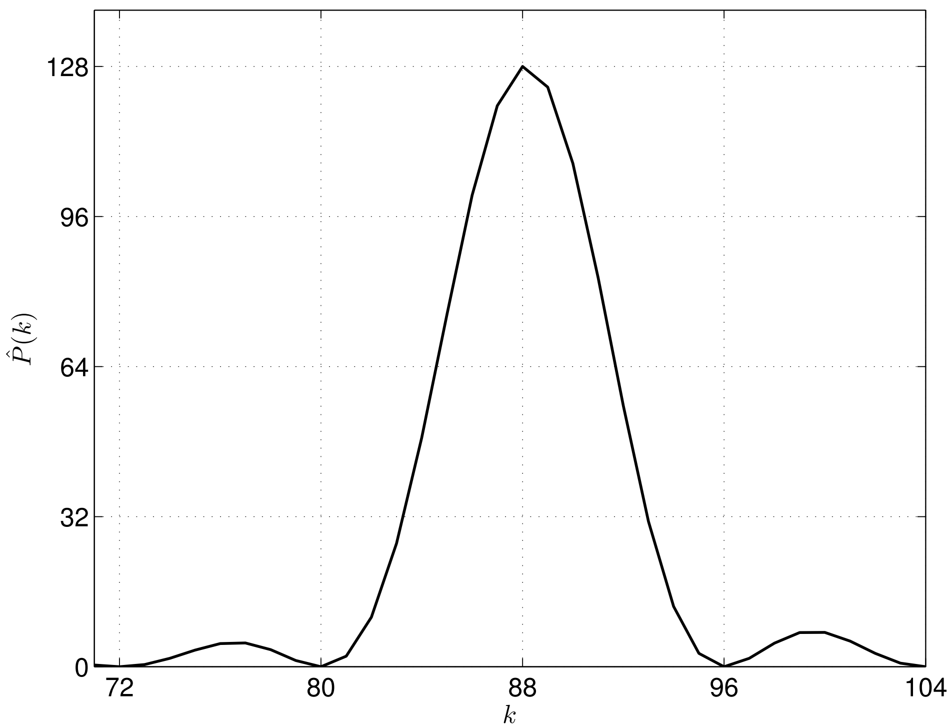

Figure 3: Matlab plot (zoomed in).

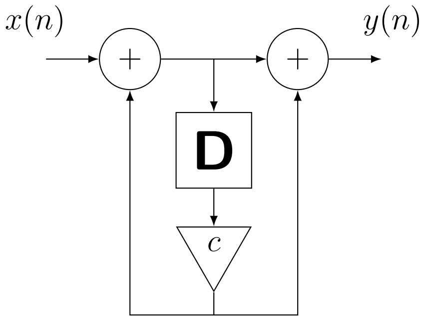

Figure 4: Filter implemented using fix point implementation.

2. Consider the filter shown in Fig. 4. Assume that the input signal $x(n)$ is a white stationary stochastic process with power spectral density $P_x(f) = 0.1$ and that the filter is implemented using a fixed point processor, using $b = 8$ bits to represent numbers in the range $[-1, 1[$.

Determine an expression for the **power spectral density** of the output signal $y(n)$. Assume for simplicity that there is no overflow and that the filter coefficient c can be represented exactly.

Do not spend time on simplifying the answer. (10p)

3. You are given the discrete time signal $x(n) = A_1 \cos(2\pi f_1 n) + A_2 \cos(2\pi f_2 n)$, with $f_1 = 1/16$, $f_2 = 1/8$ and $-\infty < n < \infty$. Assume that $x(n)$ passes through the system shown in Fig. 5,

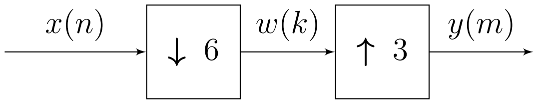

Figure 5: System with downsampling and upsampling.

Determine the discrete time Fourier transform (DTFT) of $y(m)$ in $f \in [-1/2, 1/2)$. Solve the problem graphically if you prefer, but do not forget to mention the formulas used in each step of the solution. (10p)

4. Assume that the stationary stochastic process $x(n)$ is given by the AR(1) model

$$x(n) = 0.2x(n-1) + e(n) ,$$

where $e(n)$ is a white noise process with variance $\sigma_e^2 = 1$.

The signal $x(n)$ is measured using a sensor that introduces some additional measurement noise, resulting in

$$y(n) = x(n) + w(n) ,$$

where $w(n)$ is a white noise process with variance $\sigma_w^2 = 0.1$, which is uncorrelated with $e(n)$.

a) Determine the autocorrelation function $r_{yy}(k)$ for all values of k. (2p)

b) Determine the coefficients of a 2^{nd} order autoregressive (AR(2)) model for $y(n)$. Use the true autocorrelation values $r_{yy}(k)$ (corresponding to a situation where these have been estimated with high accuracy, using a large number of samples of $y(n)$). (6p)

c) Provide an expression for a model based power spectral estimate for $y(n)$ using the AR(2) model you just derived. (2p)

5. Consider two finite-length sequences $x(n)$ and $h(n)$ for which $x(n) = 0$ outside the interval $0 \le n \le 49$ and $h(n) = 0$ outside the interval $0 \le n \le 9$.

a) What is the maximum possible number of nonzero values in the linear convolution $y(n) = x(n) * h(n)$? (2p)

b) Denote the $N = 50$ point circular convolution of $h(n)$ and $x(n)$ by

$$y_c(n) = x(n) \textcircled{N} h(n)$$

Show that for all $0 \le n \le 49$, the relationship $y_c(n) = y(n) + y(n+50)$ holds. (5p)

c) Assume now in addition that $y_c(n) = 10$, for all $0 \le n \le 49$ and that the first 5 points of the linear convolution are $y(n) = 5$, $0 \le n \le 4$. Determine as many points as possible of the linear convolution $y(n)$. (3p)