SIGNAL PROCESSING

DEPARTMENT OF ELECTRICAL ENGINEERING

E 102 **Digital Signalbehandling** EQ2300/ 2E1340

Final Examination 2007–12–15, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or
Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides and two lecture hand outs
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Answers without motivation / justification carry no rewards
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks at "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

- Don't forget to fill in the course evaluation form! Follow the link at the course WWW page.
- Don't forget to pick up your graded project report for Project 2 from STEX on 21 December 2007.

1. (a) In the class, we had considered zero padding as a tool to convert circular convolution into linear convolution. We shall now consider the converse. Let $x(n)$ be a length 4 sequence (i.e, $x(n) = 0, n > 3$ or $n < 0$) and $h(n)$ is a length 3 sequence. Define sequences

$$\begin{aligned} z_1 &= \{x(3), \underset{\uparrow}{x(0)}, x(1), \ldots, x(3)\} \\ z_2 &= \{x(2), \underset{\uparrow}{x(3)}, x(0), x(1), \ldots, x(3)\} \\ z_3 &= \{x(1), \underset{\uparrow}{x(2)}, x(3), x(0), x(1), \ldots, x(3)\} \end{aligned}$$

i. Define $y(n) = z_3 * h(n)$ ($*$ denotes Linear Convolution). Show that $\{y(3), y(4), y(5), y(6)\} = x(n) ④ h(n)$ (④ denotes 4 point circular convolution). (2p)

ii. Can you extract $x(n) ④ h(n)$ from

- $z_1 * h(n)$
- $z_2 * h(n)$ (1p)

(b) If the z transform of $x(n) = \rho^n u(n), |\rho| < 1$ is $X(z) = \frac{1}{1-\rho z^{-1}}$, find the sequence whose z transform is, $Y(z) = \frac{1}{(1-\rho z^{-1})^2}$. (3p)

(c) A student is asked to verify the DFT and IDFT operations. He correctly obtains the DFT of $\{\underset{\uparrow}{a}, b, c, d, e\}$ as $\{\underset{\uparrow}{A}, B, C, D, E\}$. Instead of computing the IDFT, he computes the DFT of $\{\underset{\uparrow}{A}, B, C, D, E\}$ again. What would his answer be? (4p)

2. The received signal at a radar is given by,

$$y(n) = Ae^{j2\pi f_0 n} + w(n)$$

where $w(n)$ is a zero mean white noise with variance σ_w^2. A genie provides the following values for the auto-correlation function, $r_{yy}(k) = E\{y(n+k)[y(n)]^*\}$,

$$\begin{aligned} r_{yy}(0) &= 3.0300 \\ r_{yy}(1) &= 1.5\left(\sqrt{3} + i\right) \end{aligned}$$

From these data, determine the following using Pisarenko method,

(a) frequency of operation, f_0

(b) Amplitude, A

(c) Noise variance, σ_w^2

(10p)

3. For the following signal model,

$$y(n) = A\sin(\omega_0 n + \phi) + w(n), \qquad n = 0, 1, 2, ..., N-1$$

where $w(n)$ is a white noise, consider the problem of estimating the amplitude A and phase ϕ of the signal (ω_0 is assumed to be known). This is a nonlinear estimation problem that can be solved using a nonlinear least squares approach. However, the signal model can be reparameterized using the following trigonometric rule

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

such that the estimation problem becomes linear in two new parameters.

(a) Do the appropriate reparameterization and show how to calculate A and ϕ from the new parameters.

(b) Write the estimation problem as a linear least squares problem in the new parameters (use vectors and matrices). Find the least squares solution.

(c) Simplify the least squares solution for $\omega_0 = \pi/2$.

(10p)

4. A filter has been implemented digitally as shown in Figure 1. The discrete-time input signal $x(n)$ [Volts] is A/D converted (quantized) using quantization step size Δ [Volts]. The signal is filtered using finite precision arithmetics (the precision is Δ). Finally, the signal is D/A converted back to analog discrete-time domain, $y(n)$, again using step size Δ [Volts]. We wish to determine the unknown gain, k, and the

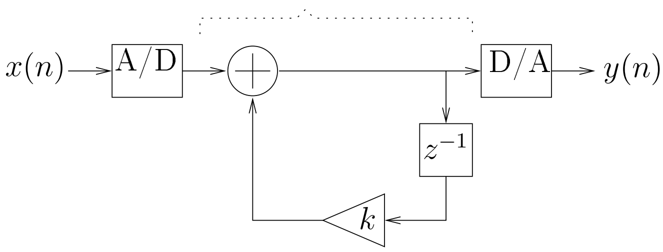

Figure 1: The digital filter.

quantization step-size, Δ, by feeding the filter with a sinusoid

$$x(n) = \cos(\omega_0 n + \phi) \quad \text{[Volts]}, \qquad n = 0, ..., N-1,$$

that has a known frequency $\omega_0 = \pi/3$ [rad/sample], and an unknown phase, ϕ, uniformly distributed in $[0, 2\pi)$. Figure 2 shows the estimated spectrum of $y(n)$ using periodogram averaging: $K = 100$ periodograms were averaged using $L = 1000$ samples for each periodogram. The total number of output samples is $N = KL$.

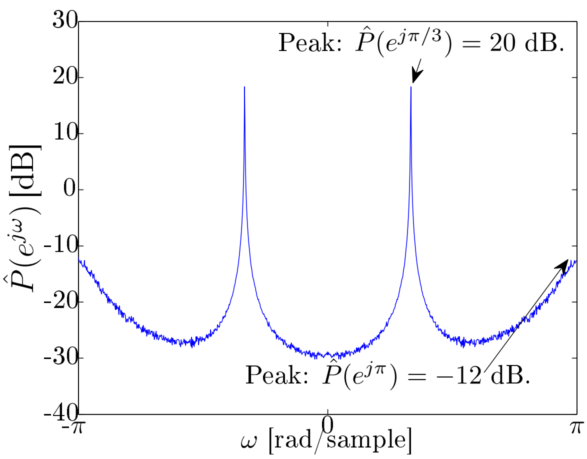

Figure 2: The averaged periodogram.

(a) Determine the filter transfer function, $H(z)$, as a function of k, disregarding the quantization effects. (1p)

(b) Show that the periodogram peak at frequency ω_0 can be approximated as

$$\hat{P}(e^{j\omega_0}) \approx \frac{L|H(e^{j\omega_0})|^2}{4}$$

Hint: Note that the quantization noise power is negligible at frequency ω_0. Further more, you may assume that $\frac{1}{L}\sum_{n=0}^{L-1} \exp(j\omega n) \approx 0$, for all $\omega \neq 0$. (3p)

(c) Using the result of part (b), obtain the value of k. (3p)

(d) Using suitable data provided in Figure 2, estimate the quantization step size, Δ. Assume that no overflow occurs in the summation. (3p)

5. Consider the system in Figure 3, where the input to the system is a sinusoidal signal with two odd overtones (harmonics). That is

$$x(n) = \sum_{k=1}^{3} A_k \sin(2\pi(2k-1)\, f_{x,0}\, n)$$

where $f_{x,0} = 1/20$. Figure 4 shows the periodogram of the output $y(m)$ calculated using $N = 10^3$ data samples.

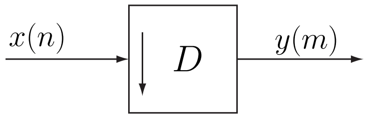

Figure 3: Downsampling

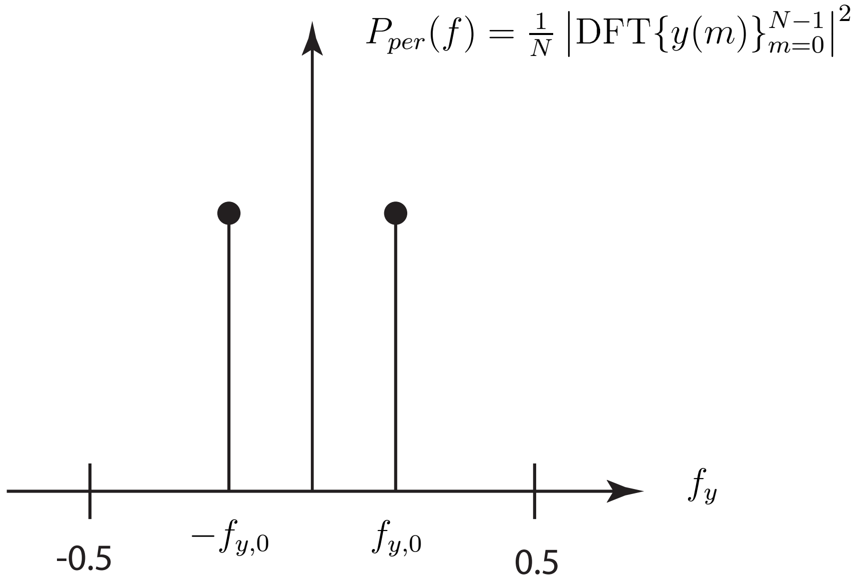

Figure 4: Periodogram of the down sampled signal.

Determine the smallest down sampling factor D that has been used in figure 3. What are the other possible values of D? (10p)

Hint: Due to aliasing caused by down sampling, all signals have the same frequency after down sampling.