KTH, INFORMATION SCIENCE AND ENGINEERING

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300 / 2E1340

Final Examination 2018–01–08, 08:00-13:00

Literature:
- Jaldén: *Summary notes for EQ2300* (30 page printed material).
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH.*
- *One A4 of your own notes.* You may write on both sides, and it does not have to be hand written, but cannot contain full solutions to tutorial problems or previous exam problems.
- An unprogrammed pocket calculator.

Notice:
- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on “My pages”.

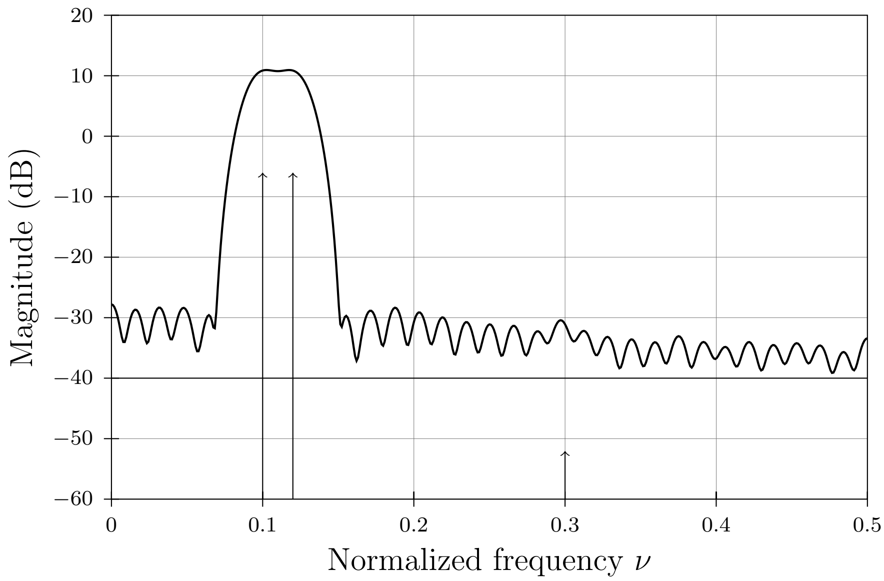
Solutions: Will be available on the course homepage after the exam.

Good luck!

1. A stochastic process has the form

$$x[n] = \sum_{k=1}^{3} A_k \sin(2\pi\nu_k n + \varphi_k) + w[n]$$

where φ_k are independent random variables, uniformly distributed over the range $[0, 2\pi)$, and where $w[n]$ is a zero mean white noise process with power $\sigma^2 = 10^{-4}$. The two first sinus components are equally strong, with $A_1 = A_2 = 1$, and close in frequency with $\nu_1 = 0.1$ and $\nu_2 = 0.12$. The third sinus component is significantly weaker, with $A_3 = 0.005$, but also better separated in frequency from the first two with $\nu_3 = 0.3$. The spectrum of $x[n]$ is estimated from $N = 1024$ samples, using Welch's method with 50% overlap and a Hamming window, resulting in the spectrum estimate shown below.



a) What is the window (block) length L used in Welch's method? Give an approximate value based on the given data and the figure. (2p)

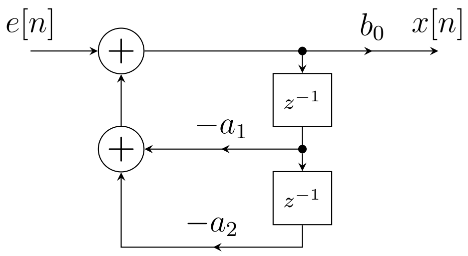
b) Assuming that the estimator uses all the data, what is then the number of blocks K that are averaged over? (2p)

c) The two main (strong) sinus components are not distinct in the spectrum estimate. Propose a change to the spectrum estimator so that the two main peaks are separated in the spectrum estimate. (1p)

d) The third (weak) sinus component is not clearly visible in the spectrum estimate. Explain why and propose a method to change the spectrum estimator so that one can see the third peak clearly in the spectrum estimate. (2p)

e) Propose another spectrum estimator that will have at least the resolution of Welch's method, but have a better variance. Give rough descriptions of how you would choose the parameters of your proposed spectrum estimator. (3p)

2. Assume that $x[n]$ is a real valued AR-2 stochastic process generated by the system below. The driving noise $e[n]$ is zero mean, white, and has unit variance. For a particular realization of the output, we learn that $x[0] = 2$, that $x[1] = 1$ and that $x[2] = 1$.



a) Propose an estimator for the auto-correlation $r_x[k] \triangleq \mathrm{E}\{x[n]x[n+k]\}$, and numerically compute the values of $r_x[k]$ for $k = 0, 1,$ and 2. (2p)

b) Compute the coefficients b_0, a_1 and a_2 such that the output $x[n]$ of the AR-2 system below has the auto-correlation that you computed above. Give numerical values for the coefficient. If you were unable to solve part a), then explain how you would compute the coefficients if you had been able to solve a). (5p)

c) Provide a well founded estimate of $x[3]$ and explain your reasoning. (3p)

3. The following piece of Matlab code contains the frequency domain processing of a real time implementation of an overlap-save implementation used for filtering a signal at a sample rate of 1 kHz.

```
% Overlap save frequency domain calculation
disp(sprintf('Start: %.3f s',cputime))
XB = fft(xB,N);
YB = H.*XB;
yB = ifft(YB,N);
disp(sprintf('Done: %.3f s',cputime))
```

The FIR filter `h` has length `M = lentgh(h)` and is used to compute the frequency domain filter coefficients according to `H = fft(h,N)` at initialization. The FFT length is `N=1024`, and `cputime` returns the time in seconds since Matlab started. There will be a continuous output of timestamps as the code is executed every time there are enough samples for processing a block in the frequency domain. A representative snapshot of the output is given as follows.

```
Start: 1489.210 s
Done: 1489.410 s
Start: 1489.710 s
Done: 1489.910 s
Start: 1490.210 s
Done: 1490.410 s
```

a) How much time (in seconds) does it take for the frequency domain processing of a block of signal samples? (1p)

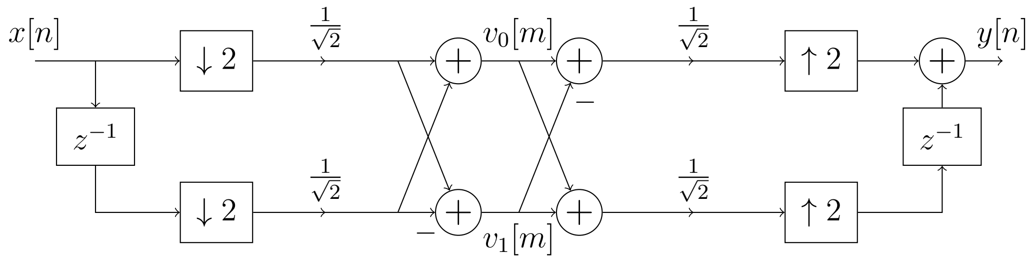
b) How much time (in seconds) does it take between successive executions of the code shown above? (1p)

c) How long is the filter, i.e., what is the value of `M`? (3p)

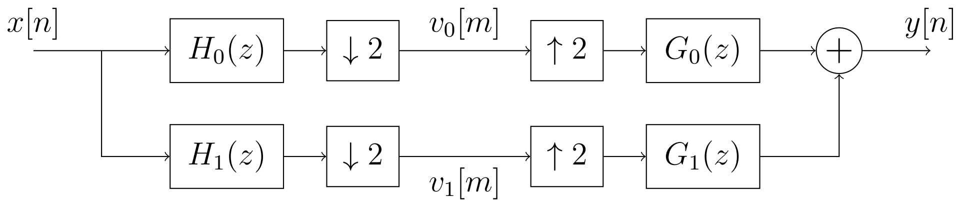
d) Assuming that the execution time is dominated by complex valued multiplications, how much time does approximately a complex valued multiplication take to perform on the platform running the code? You can use the approximation $1024 \approx 1000$ to simplify your calculations if you wish. (3p)

e) Assume now that the filter length was changed to `M=25`. What would then be the corresponding 6 lines of output if the first line was the same as it is above? (2p)

4. Consider the following quadrature mirror filter (QMF) filter bank, implemented using polyphase filters.

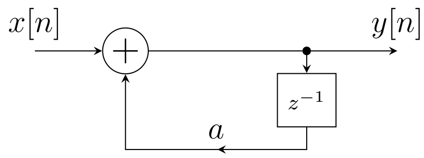


a) Find filters $H_0(z)$, $H_1(z)$, $G_0(z)$ and $G_1(z)$ so that the filter bank below becomes equivalent to the polyphase implementation, in the sense that $v_0[m]$, $v_1[m]$ and $y[n]$ are the same in both implementations. (5p)



b) Show that the the QMF filter bank provides perfect reconstruction, and give a value for L such that $y[n-L] = x[n]$. (5p)

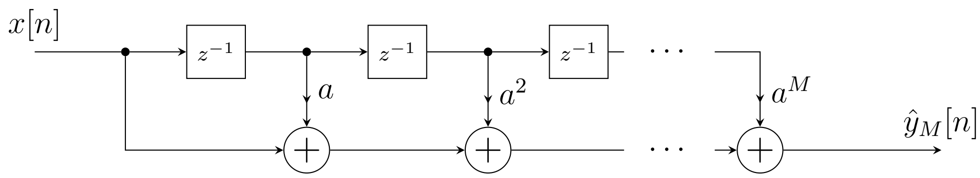
5. In this problem, we will consider the first order AR circuit shown below.



The AR circuit is known to have an infinitely impulse response (IIR) given by

$$h[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}.$$

The output $y|n]$ can however be approximately computed as $\hat{y}_M[n]$ using the Mth order FIR filter shown below.



The purpose of this question is to study the implementations when we use fixed point arithmetics. We will for simplicity assume that $x[n]$ is a white (uncorrelated) stochastic process with power σ^2, that all filters are stable, and that no overflow occurs. Assume that the both circuits are implemented using fixed point arithmetics, with a $B+1$ signed magnitude representation of the range $(-1, 1)$.

a) Compute the signal to quantization noise ratio (SQNR) at the output of the AR implementation, i.e., the ratio of the power of $y[n]$ when computed with infinite precision and the power of the quantization error at the output of the fixed point implementation of the AR circuit. (3p)

b) Compute the signal to quantization noise ratio (SQNR) at the output of the FIR approximation circuit, i.e., the ratio of the power of $\hat{y}_M[n]$ when computed with infinite precision and the power of the quantization error at the output of the fixed point implementation of the FIR circuit. (4p)

c) Compute the power spectrum $P_e(\nu)$ of the fixed point quantization noise at the outputs of both implementations. (3p)