SIGNALBEHANDLING
INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 97 **Digital Signalbehandling**, 2E1340

Final Examination 2005–08–25, 08.00–13.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 08-790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 3.

Solutions: Will be available on the course homepage directly after the exam.

---

1. The transfer function of a system is given by the (two-sided) Z-transform

$$H(z) = \frac{(z-1/2)(z-3)}{(z-3/2)(z-1/3)}$$

Does $z = -2$ belong to a region of convergence where the system is

a) stable or unstable? (2p)

b) causal or non-causal? (2p)

Assume that the signal $x(n) = \{x(0), x(1), \ldots, x(31)\}$ is real valued.

c) What is the smallest number of values of the DFT $X(k)$ that is needed to be able to uniquely determine $x(n)$? 16, 32 or 64 values? (2p)

d) What can you say about $x(n)$ if you are told that $X(0) = 10$

- That the average of the signal values is positive?
- That the signal is symmetric around $n = 16$? (4p)

2. a) Illustrate the Blackman-Tukey method for spectral estimation by performing all the calculations on the (extremely short) data set

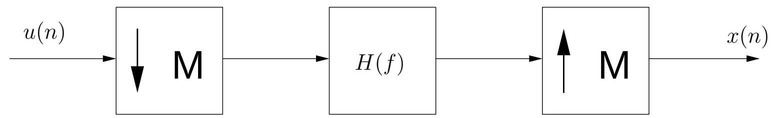
$$x(n) = \{\underset{\uparrow}{3}, 5, 2, -1, 1, -3, -2, 2, 4, 1\}$$

Use a triangular window of width 3 (i.e. that is non-zero from w(-3) through w(3)) and calculate the spectrum only in eight points. Plot the resulting spectrum estimate and do not forget to specify the scale on the frequency axis. (7p)

b) What can you say about the resolution capabilities of this spectrum estimate compared to a standard periodogram based on the 10 samples? (3p)

3. A teacher in digital signal processing has found Fig. 1 in a pile of papers. It shows parts of a solution of a problem that might be useful as an exam problem. Unfortunately, he does not have the original problem formulation and some numerical values are missing. However, the final answer, which is the Fourier transform (DTFT) of the output signal $x(n)$, contains enough information, so it is possible to reconstruct the problem formulation.

Your task is to formulate a problem (in the style of an exam problem) with the answer $X(f)$ and using the top two rows of plots from Fig. 1. (10p)



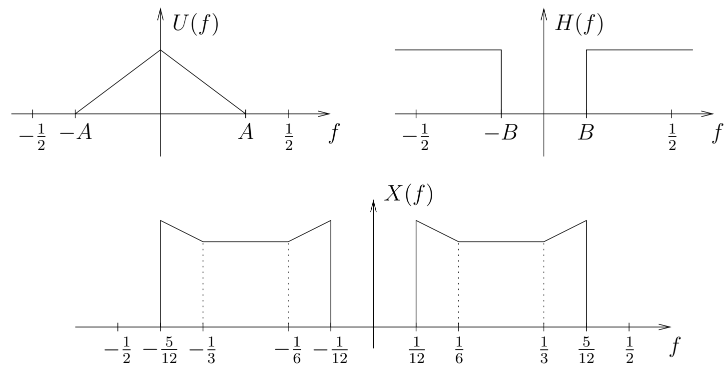


Figure 1: The available plots.

4. In wireless communication you can use several antennas at the transmitter and receiver to increase the bitrate in bits/s/Hz and/or get increased reliability. In this setup we have one transmitter and 2 receivers, a so called Single-Input-Multiple-Output (SIMO) system, see Fig. 2. To get information about the channel we transmit a training sequence, known to both the transmitter and receiver. Knowing what was transmitted and what we received we can estimate the h_1 and h_2. Unfortunately in this setup, the receiver has lost information of what the training sequence $x(n)$ looks like. We will here show that we can still make good estimates of the channel down to just a scaling factor. To start with, we know that the filters h_1 and h_2 can be well approximated by 2-tap FIR filters. Second, we make the assumption (without loss of generality) that $h_1(0) = 1$, and that the signal $x(n) = 0$ for $n < 0$.

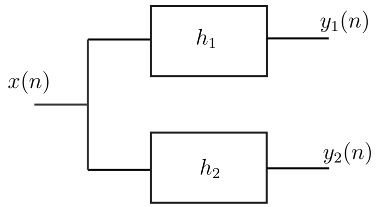


Figure 2: SIMO channel

a) Show the equality

$$y_1(n) * h_2(n) = y_2(n) * h_1(n)$$

(2p)

b) At the receiver we have the two sampled vectors y_1 and y_2

$$y_1 = \{y_1(0), y_1(1), \ldots, y_1(N)\}$$

$$y_2 = \{y_2(0), y_2(1), \ldots, y_2(N)\}$$

where N>2. Show how to estimate the FIR filter coefficients. Try to use all available samples of y_1 and y_2 to reduce the effect of noise. (8p)

5. Round-off errors can be a problem not only in fixed point implementations but also in floating point implementations. Here we consider a simple measurement system, see Fig. 3, where a signal is sampled and multiplied by π. We know that the output of the sensor varies between 2.0 and 3.0 and the digital part of the system is implemented with floating point numbers. Figure 4 shows the quantization error in $y(n)$ relative to the exact value of $\pi x(t)$.

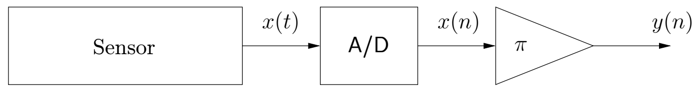


Figure 3: Measurement system

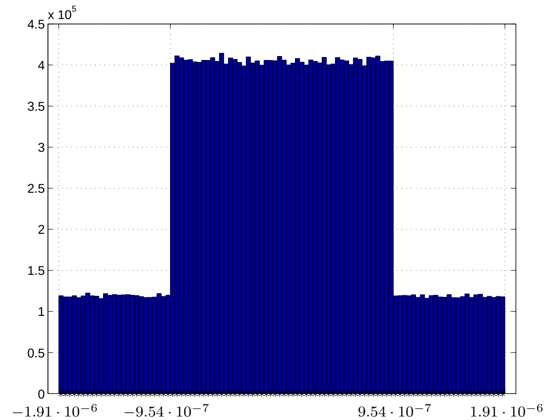


Figure 4: Probability distribution of the quantization error

a) Determine the number of bits used for the mantissa in the floating point representation of the system or motivate why it is impossible to determine the number of bits. (5p)

b) Determine the number of bits used for the exponent in the floating point representation of the system or motivate why it is impossible to determine the number of bits. (5p)