KTH, SIGNAL PROCESSING LAB

SCHOOL OF ELECTRICAL ENGINEERING

Digital Signal Processing EQ2300/ 2E1340

Final Examination 2011–06–01, 14.00–19.00

Literature:
- Hayes: *Statistical Digital Signal Processing and Modeling*
  or
  Proakis, Manolakis: *Digital Signal Processing*
- Bengtsson: *Complementary Reading in Digital Signal Processing*
- Begtsson and Jaldén: *Summary slides*
- *Beta – Mathematics Handbook*
- *Collection of Formulas in Signal Processing, KTH*
- Unprogrammed pocket calculator.

Notice:
- Answer in English or Swedish.
- At most one problem should be treated per page.
- Answers without motivation/justification carry no rewards.
- Write your name and *personnummer* on each page.
- Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks on "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

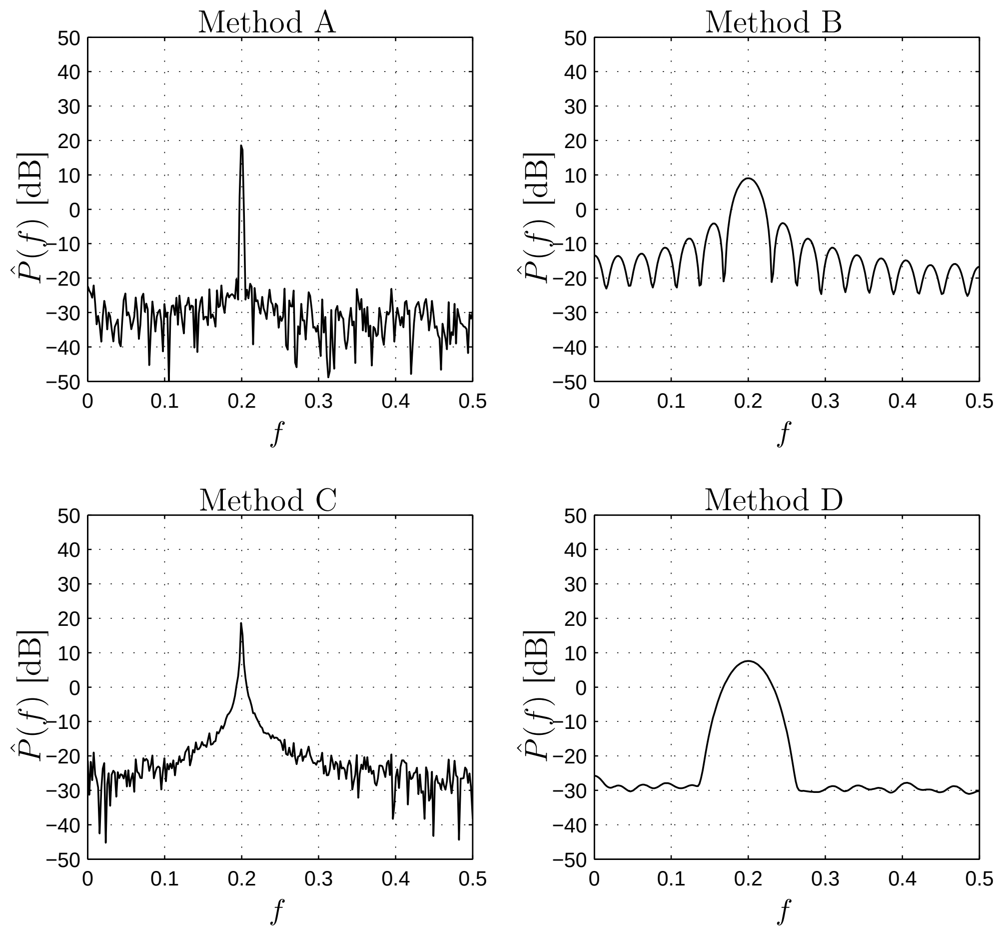
1. The power spectral density of a real valued sinusoid in white zero mean Gaussian noise is estimated using a number of nonparametric methods. There are $N = 512$ samples available for the estimation and the estimates are shown below. The four different methods used are (not in any particular order):

- The Periodogram. All $N = 512$ samples are used.
- The Modified Periodogram. All $N = 512$ samples and a Hamming window is used.
- Bartletts method (based on the Periodogram). The $N = 512$ samples are divided into segments of length $L = 32$.
- Welch method. A Hamming window is used and the $N = 512$ samples are divided into segments of length $L = 32$ with 50% overlap.

(a) Specify which plot correspond to which method, i.e., specify the estimator labeled Method A, Method B, Method C and Method D. Remember to fully motivate your answers (guessing gives no points). (6p)

(b) How many segments can be used in Bartletts method and in Welch method respectively. (2p)

(c) Argue which plot is best for estimating the noise variance and give an estimated numerical value for the noise variance. (2p)



2. A radar is used to track an airplane. The signal received at the radar is modeled as a stationary random phase complex exponential corrupted by noise according to

$$x(n) = Ae^{j(2\pi f_0 n + \phi_0)} + w(n),$$

where ϕ_0 is uniformly distributed between 0 and 2π and $w(n)$ is a zero mean white noise with unit power, i.e., $\sigma_w^2 = 1$. The first part of the expression for $x(n)$ is the signal of interest containing the received signal amplitude A and the radar carrier frequency f_0.

For the auto-correlation function of $x(n)$,

$$r_{xx}(k) = \mathbb{E}\left\{[x(n) - \mathbb{E}\{x(n)\}]\,[x(n-k) - \mathbb{E}\{x(n-k)\}]^\star\right\},$$

we assume that $r_{xx}(1)$ has been measured and found to be $r_{xx}(1) = 1 + j$ but that $r_{xx}(0)$ is unknown.

Let $r_{xx}(0) = \beta$ and show how one can apply Pisarenko method to compute the unknown value β, as well as f_0 and A, and provide the (numerical) values for these parameters. (10p)

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3. In this problem, we will design a one-step linear predictor for a wide sense stationary discrete-time stochastic process $d(n)$. We observe samples

$$x(n) = d(n) + w(n)$$

where $d(n)$ is the signal of interest, and where $w(n)$ is a zero mean additive noise with autocorrelation $r_{ww}(k) = \delta(k)$. We assume that the signal $d(n)$ is an AR process that satisfy (note the non-standard time-lag)

$$d(n) = \tfrac{1}{4}d(n-2) + v(n)$$

where $v(k)$ is a zero mean driving noise with autocorrelation $r_{vv}(k) = \delta(k)$. We also assume that $w(n)$ and $v(n)$ are uncorrelated.

(a) Design a first-order FIR linear predictor $W(z) = w(0) + w(1)z^{-1}$ for $d(n)$, i.e., a filter that computes an estimate of $d(n+1)$ from $x(n)$ and $x(n-1)$, and compute and the mean square prediction error. Specify both the filter and the estimation error. (5 pt)

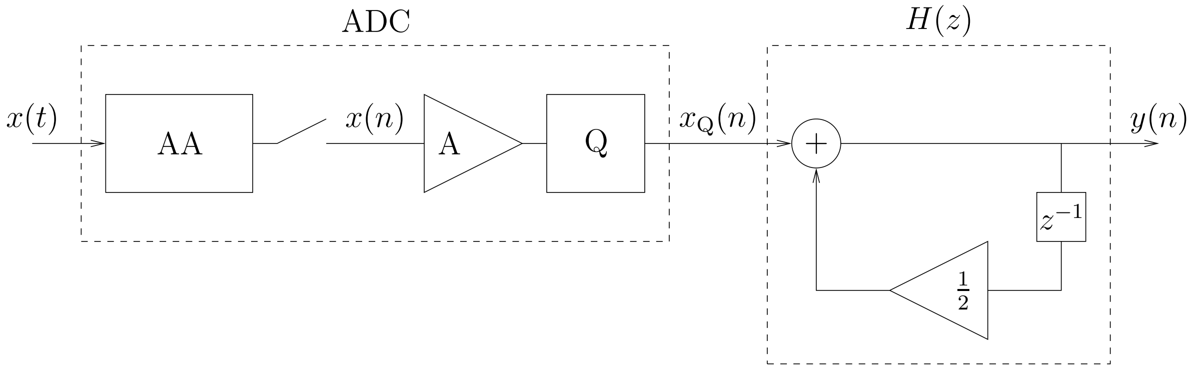
(b) Can you find a better causal filter that can further decrease the obtained mean-square error in part 3a without using more than two observed samples of the process $x(n)$? Explain your new design and compute its mean-square prediction error, or argue why it cannot be done. (5 pt)

4. In a classic digital signal processing problem, we would like to convert an analog signal $x(t)$ to the digital domain for digital processing. An analog to digital converter (ADC), shown in the figure below, is well modeled using an anti-aliasing filter (AA), a sampling device, a quantizer (Q), and an attenuator (A) used to limit the amplitude of the input signal to the operating range of the quantizer Q.

The system considered in this problem uses a 6-bit fixed point A/D converter with rounding and a dynamic range of ± 2, i.e., the quantizer uses 6 bits to uniformly quantize the range $[-2, 2]$. The filter $H(z)$, also shown below, is implemented using fixed point arithmetics with rounding, with 1 sign bit, 2 bits for the integer part and 8 bits for the fractional part. This ensures that no overflow occurs in the filter $H(z)$ for inputs $x_Q(n)$ in the range $[-2, 2]$. For simplicity, we shall assume that the sampled but un-quantized signal $x(n)$ is a zero mean *uniformly distributed* random process with autocorrelation $r_{xx}(k) = 5\delta(k)$.

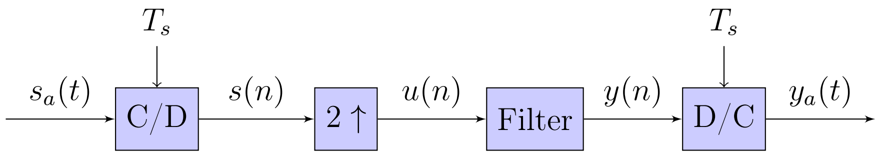
(a) What attenuation value should be applied before the quantizer to assure that it does not overflow? And for the obtained attenuation value, what is the signal to quantization noise ratio (SQNR) at the ADC output? (4p)

(b) What is the SQNR at the output of the filter due to the quantization noise introduced by the quantizer *and* the fixed point implementation of $H(z)$? (6p)

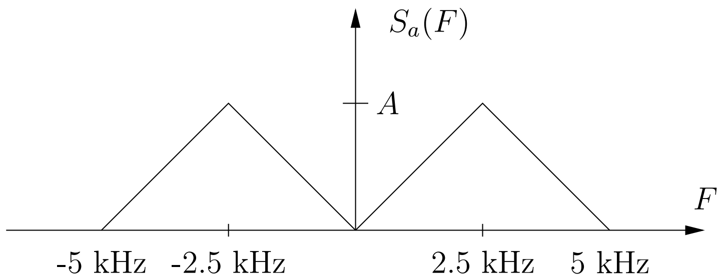


5. Suppose that we would like to slow a segment of speech to half its normal speed. The speech signal $s_a(t)$ is assumed to have no energy outside 5 kHz, and is sampled at 10 kHz, yielding the sequence $s(n) = s_a(nT_s)$.

The following system is proposed to create the slowed-down speech signal:



Assume that $S_a(F)$, the time-continuous Fourier transform of $s_a(t)$, is as shown in the following figure:



(a) Find the spectrum (time-discrete Fourier transform) of $u(n)$. You may solve the problem graphically if you wish, but you should remember to label the axes appropriately. (2p)

(b) Suppose that the discrete-time filter is described by the difference equation

$$y(n) = u(n) + \frac{1}{2}\left[u(n-1) + u(n+1)\right].$$

Find the frequency response of the filter and give an explanation of what it does with $u(n)$ (i.e., what type of filter is it?). (2p)

(c) Give an expression for the time-continuous Fourier transform $Y_a(F)$ of $y_a(t)$ in terms of $S_a(F)$. (2p)

(d) Ideally, we would like to have $y_a(t) = s_a(t/2)$ so that $y_a(t)$ is a slowed-down version of $s_a(t)$. However, this will not be the case since the filter is not ideal so we can only say that $y_a(t) \approx s_a(t/2)$. Would the approximation be better if we changed the sampling time from T_s to $2T_s$? Motivate you answer. (4p)