SIGNAL PROCESSING

DEPARTMENT OF ELECTRICAL ENGINEERING

E 103 **Digital Signalbehandling** EQ2300/ 2E1340

Final Examination 2008–06–02, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or
Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides and two lecture hand outs
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Answers without motivation / justification carry no rewards
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Bhavani Shankar, Signal Processing, 08-790 84 35

Results: Will be reported within three working weeks at "My pages".

Solutions: Will be available on the course homepage after the exam.

Good luck!

1. (a) Consider a length three sequence $\{x(0), 0, x(2)\}$ and define

$$X(k) = \sum_{n=0}^{2} x(n) e^{\frac{-j2\pi nk}{4}}, k = 0, 1, 2, 3$$

to be the 4 point DFT of $\{x(0), 0, x(2)\}$. Ideally, we need at least 3 DFT points to recover $\{x(0), 0, x(2)\}$. The following exercise tries to reduce this number by exploiting the fact that the second entry of the given sequence is 0.

- Given $\{X(0), X(1)\}$, can you recover $\{x(0), x(2)\}$? Justify your answer. (2p)
- Repeat the earlier exercise if $\{X(0), X(2)\}$ is given instead. (2p)

(b) Let $X(z)$ and $Y(z)$ denote the $z-$transforms of $\{\cdots, x(-2), x(-1), \underset{\uparrow}{x(0)}, x(1), x(2), \cdots\}$ and $\{\cdots, y(-2), y(-1), \underset{\uparrow}{y(0)}, y(1), y(2), \cdots\}$ respectively. Determine the sequence (in terms of $x(n)$ and $y(n)$) whose $z-$transform is $X(z^{-2}) + z^{-1}Y(-z^2)$. (3p)

(c) Let $r(k), k = 0, 1, \ldots, 127$ be a valid autocorrelation sequence and it is desired to obtain the corresponding power spectral density using a 1024 point FFT computation block. Explain the steps involved. (3p)

2. Consider the following signal model,

$$y(n) = \sqrt{2}\sin(\omega_0 n + \phi)$$

where ϕ is assumed to be distributed uniformly in $[-\pi, \pi]$. We are interested in finding a second order linear predictor for $y(n)$. Let the predicted value of $y(n)$ be $\widehat{y}(n)$, where,

$$\widehat{y}(n) = a_1 y(n-1) + a_2 y(n-2)$$

The values of a_1, a_2 are chosen to minimize the modelling error,

$$\zeta = E[(\widehat{y}(n) - y(n))^2]$$

(a) Obtain a closed-form expression for the auto-correlation sequence $r_{yy}(k) = E\{y(n+k)[y(n)]^*\}$.

(b) Using the true auto-correlation lags, determine a_1 and a_2 that minimize the modelling error ζ given above. Find the resulting modelling error.

(c) In the example, we chose a second order predictor. Does increasing the model order help us to improve the model accuracy?

(10p)

Hint : $\cos(A+B) = \cos A \cos B - \sin A \sin B$, $\sin(A+B) = \sin A \cos B + \cos A \sin B$, $\sin(A-B) = \sin A \cos B - \cos A \sin B$.

PSfrag replacements

I
D
$x(n)$
$y(n)$
$z(n)$

3. (a) Consider the two systems given in figure 1.

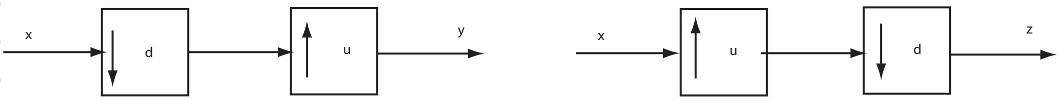

Figure 1: Interchange of Decimators and Interpolators.

Determine if these systems are equivalent for

- $D = 3, I = 3$
- $D = 3, I = 2$

(5p)

(b) Determine the two component polyphase decomposition of $H(z)$ in closed-form, where, (5p)

$$H(z) = \frac{1}{(1 + 0.1z^{-1} - 0.72z^{-2})}.$$

4. We wish to investigate the frequency content of a measured signal $x(n)$. To that end, we conduct measurements at M distinct occasions. In each occasion, we collect N samples of the signal. The data collected at ith measurement ($i \in [1\ M]$) is

$$\boldsymbol{x}_i = [x((i-1)N+1), x((i-1)N+2), \cdots, x((i-1)N+N)]$$

At the ith measurement, the power spectrum of $x(n)$ is estimated as

$$\hat{F}_i(f) = \alpha\hat{F}_{i-1}(f) + (1-\alpha)\hat{P}_i(f), \qquad 0 < \alpha < 1$$

where $\hat{P}_i(f) = \frac{1}{N}\left|\sum_{l=1}^{N} x((i-1)N+l)e^{-j(l-1)2\pi f}\right|^2$ and $\hat{F}_0(f) = 0$. We evaluate the properties of this method in the following exercises.

(a) How does the resolution of $\hat{F}_i(f)$ change with i? Motivate your answer. (2p)

(b) Assume that $x(n)$ is a white Gaussian noise process with zero mean and variance σ^2. It is then well known that, $E(\hat{P}_i(f)) = \sigma^2$ and Variance$(\hat{P}_i(f)) = \sigma^4$. Using these, determine the bias and variance of $\hat{F}_2(f)$. (2+3p)
Note: Bias of $\hat{F}_i(f)$ is $\sigma^2 - E(\hat{F}_i(f))$.

(c) Evaluate the bias and variance of $\hat{F}_2(f)$ for $\alpha = 0.3, 0.6, 0.9$. Discuss the effect of α on the bias and variance of the proposed estimator. (3p)

5. Consider the implementation of the filter

$$H(z) = \frac{0.2}{(1 - 0.75z^{-1})(1 - 0.95z^{-1})}$$

using binary fixed-point arithmetic with round-off. Assume that each multiplication causes round-off error and additions do not cause overflow.

(a) Determine the order of implementing the multiplier (multiplication by 0.2) and the two first order sections ($\frac{1}{1-0.75z^{-1}}$, $\frac{1}{1-0.95z^{-1}}$) in cascade so as to minimize the output quantization noise. What is the resulting quantization noise at the output? (5p)

Hint: The impulse response of $H(z)$ is $h(n) = 0.95^{n+1} - 0.75^{n+1}$.

(b) Decompose $H(z)$ as

$$H(z) = \frac{\alpha}{1 - 0.75z^{-1}} + \frac{\beta}{1 - 0.95z^{-1}}$$

for appropriate α and β. Determine the minimum output quantization noise when this parallel structure is implemented. Is this implementation better than the cascade implementation? (5p)