SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 92 **Digital Signalbehandling**, 2E1340

Final Examination 2003–12–18, 08.00–13.00

Literature: Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. Assume that we have two signals of length 4, $x(n) = \{\underset{\uparrow}{x_0}, x_1, x_2, x_3\}$ and $y(n) = \{\underset{\uparrow}{y_0}, y_1, y_2, y_3\}$ and that we know the result of the following linear convolution,

$$\{\underset{\uparrow}{x_0}, x_1, x_2, x_3, x_0, x_1, x_2, x_3\} * \{\underset{\uparrow}{y_0}, y_1, y_2, y_3\} = \{\underset{\uparrow}{6}, 7, 14, 28, -6, 36, -7, 28, -12, 29, -21\} .$$

a) Determine the linear convolution, $x(n) * y(n)$. (3p)

b) Determine the circular convolution, $x(n) \textcircled{4} y(n)$. (3p)

c) In general, describe how to determine both the linear convolution, $x(n) * y(n)$ and the circular convolution, $x(n) \textcircled{N} y(n)$ of two signals $x(n)$ and $y(n)$ of length N, based on the result $z(n)$ of the linear convolution

$$z(n) = \{\underset{\uparrow}{x_0}, x_1, \ldots, x_{N-1}, x_0, x_1, \ldots, x_{N-1}\} * \{\underset{\uparrow}{y_0}, y_1, \ldots, y_{N-1}\} .$$

(4p)

2. If you play one octave of notes on a piano, starting from the middle C, you will hear tones with the following frequencies (with note names within parenthesis), 262Hz (C), 277Hz (C$^\sharp$), 294Hz (D), 311Hz (D$^\sharp$), 330Hz (E), 349Hz (F), 370Hz (F$^\sharp$), 392Hz (G), 415Hz (G$^\sharp$), 440Hz (A), 466Hz (A$^\sharp$), 494Hz (B) and 523Hz (C). We want to use non-parametric spectrum estimation to determine which of these 13 tones is/are played. Therefore, we sample the signal from a microphone using a sampling frequency of $F_s = 8000$Hz.

a) If only one tone is played, is it possible to determine the tone if we use 128 samples and calculate a periodogram using the FFT algorithm without any zero-padding? (1p)

b) If more than one tone is played simultaneously, is it possible to distinguish all the tones using the same method? (1p)

c) Same question as in 2a) but using zero-padding and an FFT of length 2048? (1p)

d) Same question as in 2b) but using zero-padding and an FFT of length 2048? (1p)

e) Same question as in 2a) but using 2048 samples of data? (1p)

f) Same question as in 2b) but using 2048 samples of data? (1p)

g) Same question as in 2a) but using 2048 samples of data, the Bartlett method with 10 segments of data and sufficient zero-padding? (2p)

h) Same question as in 2b) but using 2048 samples of data, the Bartlett method with 10 segments of data and sufficient zero-padding? (2p)

3. Figure 1 shows the implementation of a filter (D denotes the delay operator). Assume that fixed point arithmetics with B bits (excluding the sign bit) is used and that the results of the multiplications are rounded (the signals are scaled such that no overflow will occur). Determine the power of the quantization noise at the output. Use the following values: $a = 2$, $b = 1$, $c = 1$, $d = 0.5$ and $e = -0.1$.

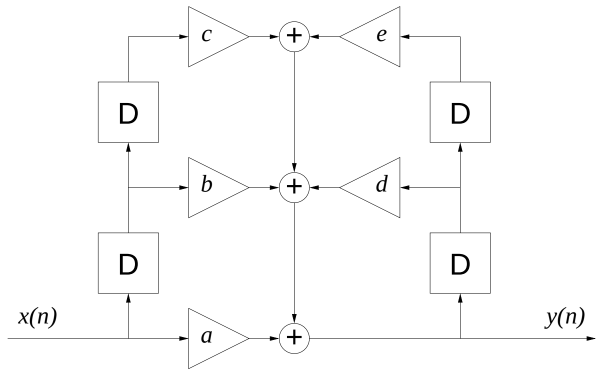

Figure 1: Filter

(10p)

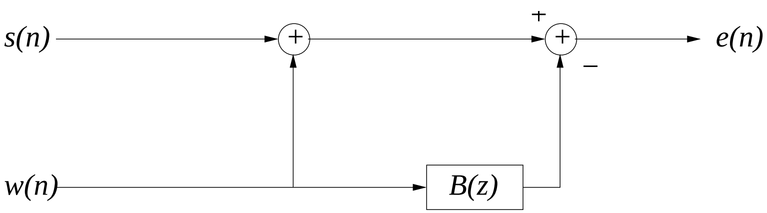

Figure 2: Filtering system

4. In the system of Figure 2, we want to determine the filter $B(z)$ such that the MSE (=the power of the error signal $e(n)$) is minimized. We know that the two input signals $s(n)$ and $w(n)$ are uncorrelated. We know also that the autocorrelation function of $s(n)$ is $r_{ss}(k) = 3^{-|k|}$.

a) Assume that $w(n)$ is a white noise process with variance 1, and that the filter is of the form $B(z) = b_0 + b_1 z^{-1}$. Find the MMSE solution to b_0 and b_1.

b) Now assume that $w(n)$ is colored noise with power spectral density $P_w(e^{jw}) = 3 + 2\cos(w)$, and that $B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$. determine the MMSE solution of b_0, b_1, b_2 and b_3.

For each of the subproblems above, you will receive 3p for an intuitive solution without any mathematical details, 3p for a strict mathematical solution and the full 5p if you provide both the mathematical details as well as an intuitive explanation of the result.

5. Figure 3 shows the power spectral density of a sampled signal $x(n)$ when the sampling is done using an ideal sampling device. However, in practice the sampling device can only provide a limited resolution, say 12 bits including a sign bit, which will add quantization noise to the signal.

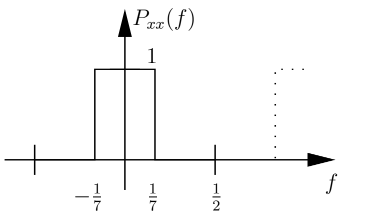

Figure 3: Power spectral density of $x(n)$.

a) Determine the signal to quantization noise ratio in the sampled signal. (4p)

b) In order to save memory requirements, the signal is decimated by a factor $D = 2$. Assume that the decimation filter is an ideal low-pass filter with cut-off frequency at $f = 1/4$ and that the filtering is performed with full precision calculations. Determine the signal to quantization noise ratio in the decimated signal. (6p)

Good luck!

Don't forget to fill in the course evaluation form! Follow the link on "Latest News" at the course WWW page.