SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 83 **Digital Signalbehandling**, 2E1340

Final Examination 2000–12–20, 0900–1300

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Formelsamling i Kretsteori/Signalteori, KTH
Unprogrammed pocket calculator.

Notice: At most one problem should be treated per page.
Motivate each step in the solution.
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

---

1. a) Which of the following signals has a real-valued DFT (of length 5)?

(i) $x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 2, 1\}$

(ii) $x_2(n) = \{\underset{\uparrow}{1}, 2, 3, -2, -1\}$

(iii) $x_3(n) = \{\underset{\uparrow}{1+i}, 2-i, 3, 2+i, 1-i\}$

(iv) $x_4(n) = \{\underset{\uparrow}{3}, 2, 1, 1, 2\}$

(v) $x_5(n) = \{\underset{\uparrow}{3}, 2, 1, -1, -2\}$

(vi) $x_6(n) = \{\underset{\uparrow}{3}, 2-i, 1+i, 1-i, 2+i\}$

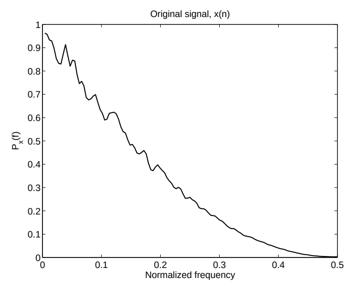
One or more options may be correct. (3p)

b) In a program for numerical data analysis, there are two commands, `y=decimate(x,D,H)` and `y=interp(x,I,H)`, that perform decimation (downsampling) by a factor `D` and interpolation (upsampling) by a factor `I`, respectively. The third argument, `H`, is optional. If it is omitted, the resampling operations are combined with lowpass filtering to preserve the spectral shape and avoid aliasing, according to the theory in *Proakis & Manolakis*, "Digital Signal Processing". If the third argument `H=1`, the decimation/interpolation is performed **without** the proper filtering.

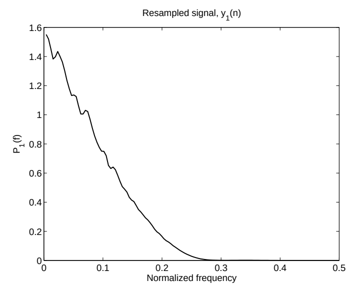
The signals $y_1(n)$, $y_2(n)$ and $y_3(n)$ are created using the commands

```
y=interp(decimate(x,3,1),5,1);
y=decimate(interp(x,5),3);
y=interp(decimate(x,3),5);
```

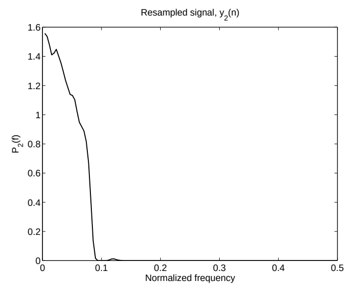
where the signal $x(n)$ has the estimated spectrogram $P_x(f)$ shown in Figure 1(a). Determine which command was used for each of the three signals. The estimated spectrograms are shown in Figure 1(b)–1(d). (3p)



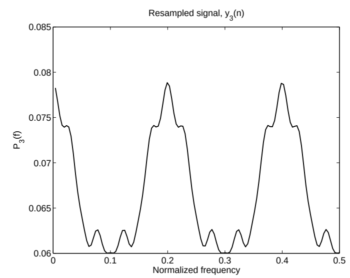
(a) Spectrogram of $x(n)$



(b) Spectrogram of $y_1(n)$



(c) Spectrogram of $y_2(n)$



(d) Spectrogram of $y_3(n)$

Figure 1: Spectrograms of the different signals.

c) Figure 2 shows four different estimates of the spectral density of a signal. The same 1024 samples of the signal have been used to calculate the four figures.

The following methods were used:

(i) Non-parametric spectral estimation using the periodogram.

(ii) Non-parametric spectral estimation using the Blackman-Tukey method with a Hamming window of length 129.

(iii) Parametric spectral estimation using an AR-model of order 3.

(iv) Parametric spectral estimation using an AR-model of order 10.

Which method was used in each figure?

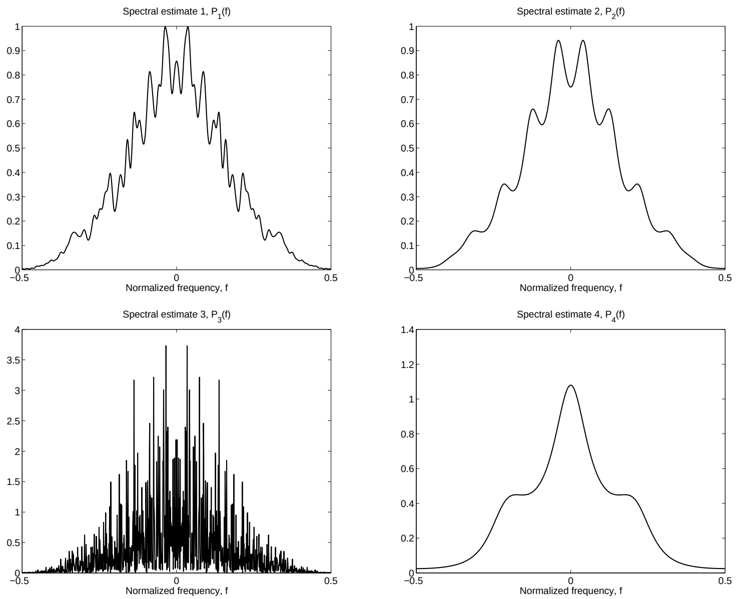


Figure 2: Four different spectral estimates of the same signal.

(4p)

2. A filter with transfer function

$$H(z) = \frac{0.04}{(1-0.9z^{-1})(1-0.8z^{-1})}$$

can be implemented in different forms. Because of round-off effects in the multiplications, the output signal gets corrupted by quantization noise. Determine which of the two implementations in Figure 3 gives the smallest quantization error on the output. The additions do not contribute to the quantization noise.

(10p)

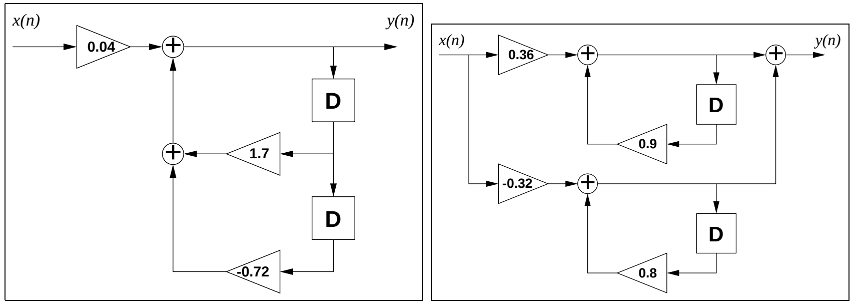


Figure 3: Alternative filter implementations.

3. A system with an input signal $u(n)$, an output signal $y(n)$ and an internal noise source $e(n)$ is described by the model

$$y(n) + a_1 y(n-1) = b_0 u(n) + b_1 u(n-1) + e(n) \ ,$$

where $e(n)$ is white noise with $\mathrm{E}[e(n)] = 0$, $\gamma_{ee}(k) = \mathrm{E}[e(n)e(n-k)] = \sigma_e^2 \delta(k)$ which is uncorrelated with the input, $\mathrm{E}[e(n)u(n-k)] = 0$.

Derive a set of equations (similar to Yule-Walker) to determine the parameters a_1, b_0, b_1 and σ_e^2 if the correlation functions $\gamma_{yy}(k) = \mathrm{E}[y(n)y(n-k)]$ and $\gamma_{uy}(k) = \mathrm{E}[u(n)y(n-k)]$ are known.

(10p)

4. a) The non-linear filter $p(n) = x^2(n) + x^2(n-1) + x^2(n-2)$ is used in a device to detect abrupt changes in the signal power of an input signal $x(n)$. In a real-time implementation, the calculations have to be performed batch-wise, i.e., the input signal is sampled and stored alternately into one of two buffers. When one input buffer is full, the corresponding output values are calculated while the second buffer is filled. Show how the calculations can be performed using the overlap-save method. As an illustration, perform all the calculations for the input signal

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, -2, 4, -1, -3, 2, 0, 2, -1, 3, -4, -2, 1\}$$

where the 15 input samples are divided into 3 segments, producing 5 new output values $p(n)$ for each segment. Assume that the signal $x(n)$ is zero for $n < 0$.

(8p)

b) Is it possible to implement this filter using the overlap-add method? (2p)

5. A major Swedish communications company is planning the installation of a new cellular system for mobile communication. One important formula when determining the number of base stations and their locations, is

$$\gamma = \frac{K}{r^{\alpha}}$$

This formula describes how the average path loss γ depends on the propagation distance r. In theory, the parameter α is 2 for free space propagation, but in practice it is often larger, depending on the propagation conditions. In order to get an accurate model, the company has performed measurements of the actual path loss in a number of cases. Since the mobile receiver was equipped with a GPS to determine its position, the distance r between transmitter and receiver for each path loss measurement, is known with good accuracy.

Describe how they can estimate the **two** model parameters K and α from a series of 100 measurements of γ and the corresponding r, using the Least Squares method. The company has access to an excellent numerical routine for numerical optimization but unfortunately, it can only handle a function of a single variable.

(10p)