SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 85 **Digital Signalbehandling**, 2E1340

Final Examination 2001–08–31, 14.00–18.00

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Formelsamling i Kretsteori/Signalteori, KTH
Unprogrammed pocket calculator.

Notice: At most one problem should be treated per page.
Motivate each step in the solution.
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. Introduce the signals

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$$
$$y(n) = \{\underset{\uparrow}{11}, 12, 13, 14, 15\}$$
$$u(n) = x(n) ⑤ y(n)$$
$$v(n) = x(n) * y(n)$$

and the corresponding column vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{u}$ and $\mathbf{v}$ ($\mathbf{x} = [1, 2, 3, 4, 5]^T$, and so on).

a) Determine the matrix $\mathbf{C}_x$ such that $\mathbf{u} = \mathbf{C}_x\mathbf{y}$. $\mathbf{C}_x$ should contain elements from $\mathbf{x}$ and $\mathbf{u} = \mathbf{C}_x\mathbf{y}$ should correspond to circular convolution with $x(n)$ for all signals $y(n)$. (3p)

b) Determine the matrix $\mathbf{L}_x$ in a similar way such that $\mathbf{v} = \mathbf{L}_x\mathbf{y}$. (3p)

c) Describe the general form of the matrices $\mathbf{C}_x$ and $\mathbf{L}_x$ that perform circular and linear convolutions, respectively, with a signal $x(n)$ of length N. (4p)

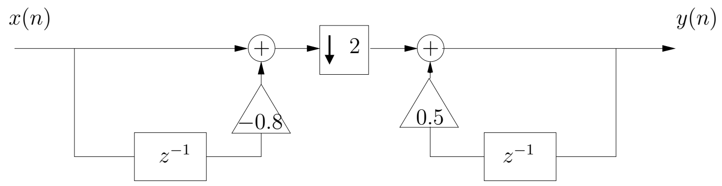

Figure 1: Fix-point circuit with decimation

2. The multirate circuit in Figure 1 uses fix-point arithmetics with b bits, excluding the sign bit. The results of the multiplications are rounded, giving quantization noise. No round-off errors occur due to overflow.

Calculate an expression for the variance of the quantization noise at the output. (10p)

3. a) Illustrate the Welch method for spectral estimation by performing all the calculations on the (extremely short) data set

$$x(n) = \{\underset{\uparrow}{5}, 2, -1, 0, 0, -4, -5, 1, 6, 2\}$$

The spectrum estimate should be calculated from data segments of length $M = 4$ with 50% overlap. Use the window $w(n) = \{1, 3, 3, 1\}$ for each segment and calculate the spectrum only in four points (no zero-padding). Plot the resulting spectrum estimate and do not forget specify the scale on the frequency axis. (7p)

b) What can you say about the resolution capabilities of this spectrum estimate compared to a standard periodogram based on the 10 samples and compared to a Bartlett spectrum estimate with the same segment length ($M = 4$)? (3p)

4. If someone had asked you to upsample a signal by a factor 2, before you took this course on Digital Signal Processing, you would probably have used linear interpolation i.e between every two samples, you would have inserted a new sample which is the average of the two surrounding old samples.

How can this simple strategy be described within the framework for interpolation given in this course? Assume the standard interpolation structure with an interpolation component followed by a filter.

a) Determine the impulse response of the filter needed in the linear interpolation upsampling circuit. (5p)

b) Determine the corresponding frequency response. (3p)

c) Compare to the ideal interpolation filter in terms of causality and spectrum properties. (2p)

Hint: The linear interpolation upsampling circuit should realize the following function:

$$y(n) = \begin{cases} x(n/2) & n \text{ even} \\ \frac{1}{2}\left[x\left(\frac{n-1}{2}\right) + x\left(\frac{n+1}{2}\right)\right] & n \text{ odd} \end{cases}$$

5. A time discrete signal is given by $y(n) = A\sin(\omega n + \phi) + v(n)$, where $v(n)$ is white noise. Assume that the angular frequency ω is known.

a) Given N snapshots of $y(n)$, $\{y(1), y(2), \ldots, y(N)\}$, derive the Least Squares estimate of the parameters A and ϕ. Note that the answer should be given in closed form (matrix expressions are OK), no numerical search techniques are needed. (6p)

b) Describe how to estimate the noise variance $\sigma_v^2 = \mathrm{E}[v^2(n)]$. (4p)