5.1 $y[n] = Q[x[n]] = x[n] + e[n]$

$x[n]$: white zero-mean. $Var(x) = \sigma_x^2$

$\underline{\Delta_Q \ll \sigma_x^2}$:

$e[n]$: white. $e[n] \sim \underbrace{U[-\frac{\Delta}{2}, \frac{\Delta}{2}]}$ $corr[e,x]=0$

a). $E[e]$ $Var[e]$ $r_e[k]$

$$E[e] = \int_{-\infty}^{+\infty} e[n] p(e[n]) de[n] = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e \cdot \underline{\frac{1}{\Delta}} de$$

$$= \frac{1}{\Delta} \frac{e^2}{2} \Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{\Delta^2}{8\Delta} - \frac{\Delta^2}{8\Delta} = 0$$

$$\sigma_e^2 = E[(e - E(e))^2] = E[e^2] = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^2 \frac{1}{\Delta} de$$

$$= \frac{e^3}{3\Delta} \Big|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{\Delta^3}{24\Delta} + \frac{\Delta^3}{24\Delta} = \underline{\frac{\Delta^2}{12}}$$

$$r_e[k] = E\{e[n]e[n-k]\} = \begin{cases} \sigma_e^2 & k=0 \\ 0 & o.w. \end{cases}$$

① definition of white noise

② PSD

b). SNR $\quad y[n] = Q[x[n]] = \underbrace{x[n]}_{y_x} + \underbrace{e[n]}_{y_e}$

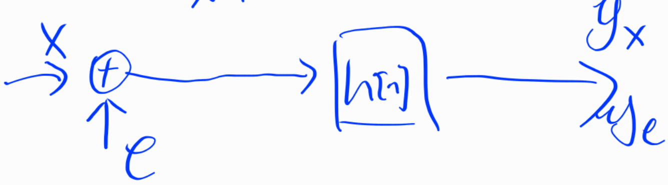
$$SNR = \frac{\text{Power}[x]}{\text{Power}[e]} = \frac{\left\{\frac{1}{T}\overbrace{\int_T [|x|^2]}^{\text{energy}}\right\} \text{power}}{\left\{\frac{1}{T}\int_T [|e|^2]\right\}}$$

x, e both continuous $\sim \underbrace{U[-\frac{\Delta}{2}, \frac{\Delta}{2}]}$

$$= \frac{E[x^2]}{E|e^2|} = \frac{\sigma_x^2}{\sigma_e^2}$$

$x[n]$ $n = 0, 1, 2$

c). $\underset{\uparrow x+e}{y[n]} * h[n]$: $h[n] = \begin{cases} 0 & n < 0 \\ \frac{1}{2}(a^n + (-a)^n) & n \geq 0 \end{cases} = \begin{cases} 0 & n \text{ odd} \\ a^n & n \text{ even} \end{cases}$



$y_x = x * h$, $y_e = e * h$

$\underline{E\{y_x^2\}}$ $\quad \underline{E\{y_e^2\}}$ $\quad E\{y_e\}$

$$E\{y_x^2\} = E\{(x * h)^2\} = E_n\left\{\left(\sum_m h[m] \cdot x[n-m]\right)^2\right\}$$

$$= E\left\{\left(\sum_m h[m] \cdot x[n-m]\right)\left(\sum_k h[k] x[n-k]\right)\right\}$$

$$= E\left\{\sum_m \sum_k h[m] h[k] x[n-m] x[n-k]\right\}$$

$$= \sum_m \sum_k h[m] h[k] \underbrace{E\{x[n-m] x[n-k]\}}_{r_x[m-k]}$$

$$= \underbrace{\sum_n h^2[n]}\, \sigma_x^2 \qquad r_x[m-k] = \begin{cases} \sigma_x^2 & m = k \\ 0 & o.w \end{cases}$$

$$E\{y_e^2\} = \sum_n h^2[n]\,\sigma_e^2$$

$$SNR = \frac{E\{y_x^2\}}{E\{y_e^2\}} = \frac{\sigma_x^2}{\sigma_e^2}$$

$$E\{y_e\} = \underbrace{E\{h[n] * e[n]\}}_{\triangle} = h\,E\{e[n]\} = 0$$

$$E\{x * y\} = E\{x\} + E\{y\}$$

random varbles .

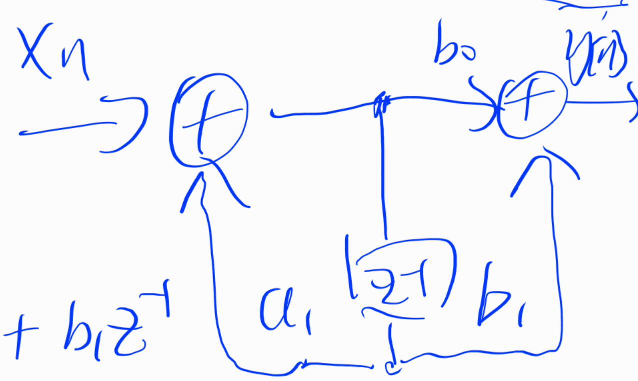
$$5.2\ \text{I}:\ y[n] = y_x + y_{e_1} + y_{e_2} + y_{e_3}$$

$$y_{e_2} = e_2, \quad y_{e_3} = e_2$$

$$y_{e_1} = h[n] * e_1[n] \qquad \boxed{E\{y_{e_1}^2\} = \sum h^2[n]\,\sigma_e^2}$$

$$z^{-1} \rightarrow h[n]:\ \times z$$

$$H(z) = \frac{0.3 + 0.2z^{-1}}{1 - 0.5z^{-1}}$$

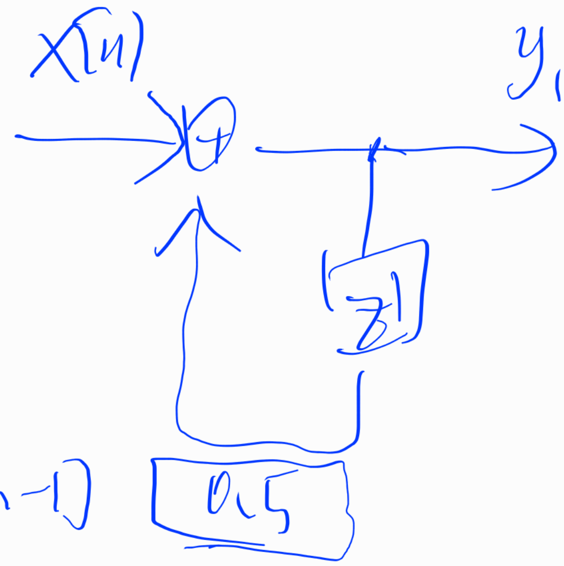


$$H = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

I: $y[n] = 0.3\left(x[n] + \frac{0.5}{0.3} y[n-1]\right)$

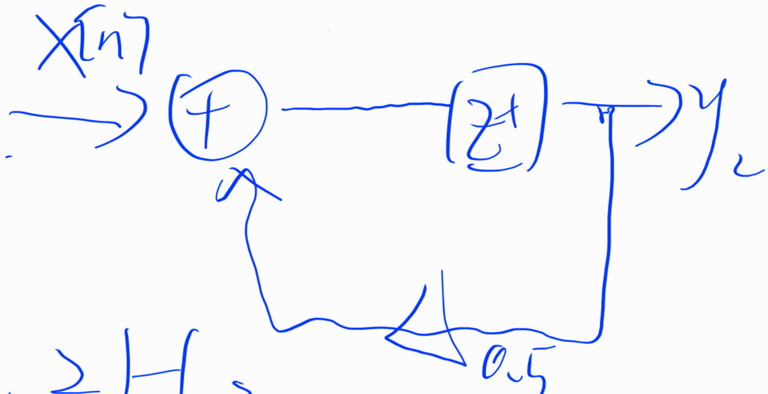
$+ 0.2x[n-1]$

$y_1[n] = x[n] + 0.5 y_1[n-1]$



$H_1 = \frac{1}{1-0.5z^{-1}}$

$y_2[n] = x[n-1] + 0.5 y_2[n-1]$



$H_2 = \frac{z^{-1}}{1-0.5z^{-1}}$

$H = 0.3H_1 + 0.2H_2$

$= \frac{0.3+0.2z^{-1}}{1-0.5z^{-1}}$

or by define $y[n] = 0.3y_1 + 0.2y_2$

$y[n] = 0.3y_1[n] + 0.2y_2[n]$

$= 0.3x[n] + 0.2x[n-1] + 0.5[0.3y_1[n] + 0.2y_2[n]]$

$= 0.3x[n] + 0.2x[n-1] + 0.5y[n-1]$

z-transfor pair = $\frac{1}{1-az^{-1}} \overset{z}{\Longleftrightarrow} a^n u[n]$

$$H = \frac{0.3+0.2z^{-1}}{1-0.5z^{-1}} = \frac{0.3}{1-0.5z^{-1}} + \frac{0.2}{1-0.5z^{-1}} z^{-1}$$

$\downarrow$ I z trans $\qquad\qquad$ $\downarrow$ I z trans $\qquad\qquad$ $\downarrow$

$$0.3(0.5)^n u[n] \qquad 0.2(0.5)^{n-1} u[n-1]$$

$$h[n] = \underbrace{0.3(0.5)^n u[n]}_{n=0,1,2,3} + 0.2\underbrace{(0.5)^{n-1} u[n-1]}_{n=1,2,3,4}. \quad n=0,1,2,3$$

$$\sum_{n=0}^{\infty} h^2[n] = 0.3^2 + \sum_{n=1}^{\infty}\left(0.3(0.5)^n + 0.2(0.5)^{n-1}\right)^2$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$= 0.3^2 + \sum_{n=1}^{\infty}\left(0.3(0.5)^n + \frac{0.2}{0.5}(0.5)^n\right)^2$$

0.4

$$= 0.3^2 + \sum_{n=1}^{\infty}\left(0.7\,(0.5)^n\right)^2$$

$$= 0.3^2 + 0.49\sum_{n=1}^{\infty}(0.5)^{2n}$$

$$= 0.3^2 \underline{-0.49} + 0.49\sum_{n=0}^{\infty}(0.25)^n$$

$$= -0.4 + 0.49\frac{1}{1-0.25}$$

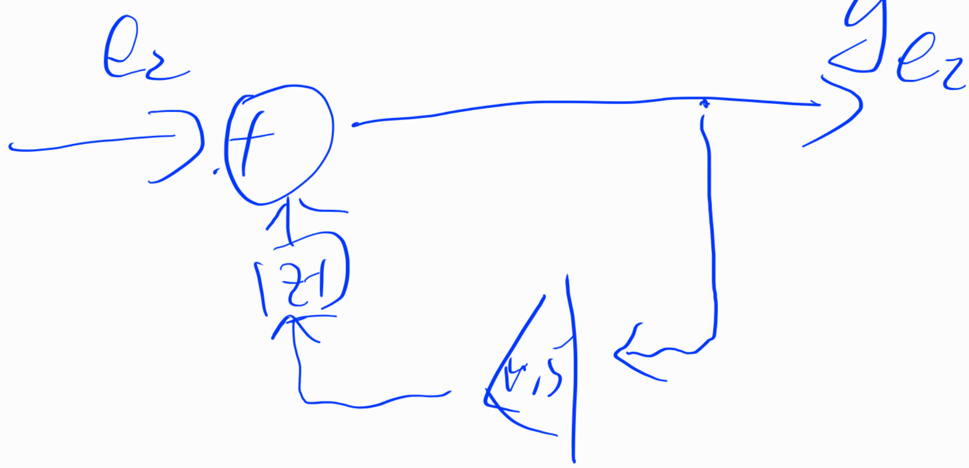
$$\approx 0.25333$$

$$E[y_{e_1}^2] = \sum h[n]\,\sigma_{e_1}^2 = 0.25333\,\sigma_{e_1}^2$$

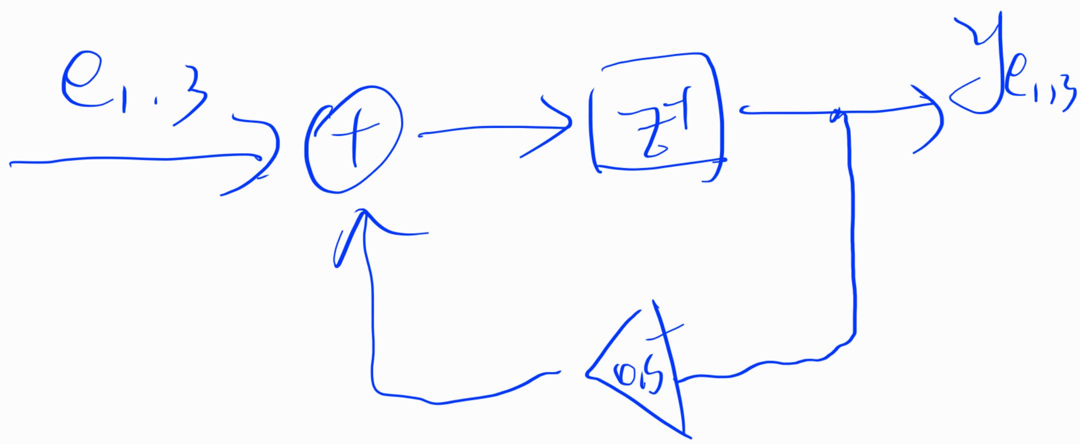
$$E\{y_{e_1}^2\} + E\{y_{e_2}^2\} + E\{y_{e_3}^2\}$$

$$= (1 + 1 + 0.5333)\,\sigma_e^2$$

II: h_{e_2}:



$h_{e_{1,3}}$



$$E\{y_e^2\} = 4\,\sigma_e^2$$