SIGNAL PROCESSING

DEPARTMENT OF ELECTRICAL ENGINEERING

E 104 **Digital Signalbehandling** EQ2300/ 2E1340

Final Examination 2008–12–17, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or
Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides and lecture hand outs
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions) unless otherwise mentioned.
Answers without motivation / justification carry no rewards unless otherwise mentioned
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

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Results: Will be reported within three working weeks at “My pages”.

Solutions: Will be available on the course homepage after the exam.

Good luck!

- Check the course homepage for information about Project reports and grading.

1. (a) For each of the following statements, indicate your answer as **T**rue or **F**alse. You do not need to provide any motivation for questions in 1.(a). Each of the sub-problems will be graded as follows: a correct answer gives +1p, an incorrect answer gives −1p and no answer gives 0p. The minimum possible **total score** on this problem is 0, i.e. if negative it will be rounded up to zero.

i. All the eigenvalues of a Hermitian matrix are always non-negative.

ii. Let $X(k), k = 0, \ldots, N-1$ be the N point DFT of a M length sequence $\{x(n)\}$ with $N < M$. Padding $X(k)$ with $M - N$ zeros and taking IDFT of the resulting sequence does not necessarily yield $x(n)$.

iii. To ensure perfect reconstruction of a signal whose energy is contained in the frequency band $0 < f_1 < |f| < f_2$, where $f_1 < f_2$, the sampling frequency can never be less than $2f_2$.

iv. Upsampling a signal by a factor I followed by downsampling by $2I$ is equivalent to downsampling by I.

v. Unlike in the modified periodogram, it is not possible to use any window in the Blackman-Tukey method.

vi. The sub-space based Pisarenko method is limited to estimation of a single unknown frequency embedded in noise.

(b) Let $y(n)$ be a wide-sense stationary (WSS) process with autocorrelation function, $r_{yy}(m) = 9(\delta(m) - \alpha\delta(m-1) - \alpha\delta(m+1))$, where $\alpha > 0$. The maximum value of α is (2p)

i. 1

ii. $\frac{1}{2}$

iii. 2

iv. Unbounded

Motivate your answer for full credits.

(c) The continuous time signal, $v(t)$, is sampled at 1 kHz to get $v(n)$. The sampled signal is multiplied by a rectangular window of length 32 and amplitude $\frac{1}{32}$ followed by a 32 point FFT operation. The magnitude response of the FFT, $|V(k)|$, is plotted in figure 1. Determine which one of the following is the most likely input signal and why. (2p)

i. $v(t) = 31.25e^{j230\pi t}$

ii. $v(t) = 31.25e^{j250\pi t}$

iii. $v(t) = 1000e^{j230\pi t}$

iv. $v(t) = 1000e^{j250\pi t}$

2. Figures 2 and 3 show two filter implementations in a MATLAB like programming language.

(a) Determine the transfer function of filter1. (1p)

(b) Determine the real valued inputs `a` and `b` to the second filter so that the two filters are the same. (2p)

(c) When using fixed point arithmetic the multiplications give rise to quantization errors. Determine the implementation yielding the smallest quantization error, using the values for `a` and `b` found in part (b). The additions do not cause any quantization noise. (7p)

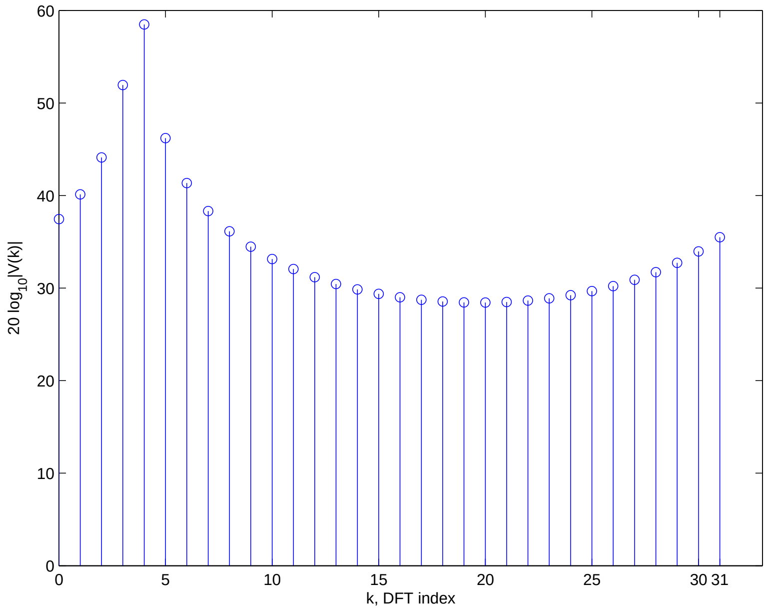

Figure 1: Discrete time Spectrum

```
function y = filter1(x)
% Signal length:
N = length(x);
% Initialize y(n):
y(n) = zeros(N,1);
for n=1:N
    y(n) = 2*x(n) + 1.1*y(n-1) - 0.3*y(n-2);
end
```

Figure 2: Filter implementation 1.

```
function y = filter2(x,a,b)
% Signal length:
N = length(x);
% Initialize y1(n),y2(n) and y(n):
y1(n) = zeros(N,1);
y2(n) = zeros(N,1);
y(n) = zeros(N,1);
for n=1:N
    y1(n) = -10*x(n) + a*y1(n-1);
    y2(n) = 12*x(n) + b*y2(n-1);
    y(n) = y1(n) + y2(n);
end
```

Figure 3: Filter implementation 2.

3. (a) Let $y(n)$ be a random signal that evolves according to,

$$y(n) = \sum_{k=1}^{N} \alpha_k y(n-k) + w(n)$$

In the above equation, $w(n)$ is an $AR(M)$ process, i. e,

$$w(n) = \sum_{k=1}^{M} \beta_k w(n-k) + e(n)$$

with $e(n)$ being a white noise sequence. Show that $y(n)$ is a $AR(M+N)$ process. (4p)

(b) Let $x(n)$ be a zero-mean real random process with autocorrelation function $r_{xx}(k)$. The different values of $r_{xx}(k)$ are given in Table 1 and it is assumed that $r_{xx}(k) = 0, |k| > 4$.

$r_{xx}(0)$	$r_{xx}(1)$	$r_{xx}(2)$	$r_{xx}(3)$	$r_{xx}(4)$
1	-0.1	0.4	-0.6	0.2

Table 1: Autocorrelation values

We are interested in finding a predictor of the form,

$$\widehat{x}(n) = \gamma x(n-L)$$

Exploiting the values of $r_{xx}(k)$ given above, determine the values of γ, L so that the mean squared error, $E\left[(x(n) - \widehat{x}(n))^2\right]$, is minimized. Also determine the prediction error of your predictor. (6p)

4. (a) Consider the problem of estimating frequencies, $\{\omega_k\}, k = 1, 2, \dots p$ from the signal of the form,

$$x(n) = \sum_{k=1}^{p} c_k e^{j\omega_k n} + w(n), \quad n = 0, 1, \dots N-1 \tag{1}$$

where $w(n)$ is a noise sequence. Assume that the number of frequencies, p, is known and that $\omega_k = \frac{2\pi n_k}{N}$, where $0 < n_1 < n_2 \dots n_p < N-1$ are distinct unknown integers. Exploit the fact that ω_k's have a special structure and develop a simple algorithm for estimating $\{\omega_k\}$ (i. e, $\{n_k\}$) and $\{c_k\}$. You may assume the noise to be negligible in comparison to $|c_k|$'s. (6p)

Note : It is necessary to exploit the fact that $\omega_k = \frac{2\pi n_k}{N}$.

(b) Let $s(n), t(n)$ be length 6 sequences and $z(n) = s(n) ⑥ t(n)$, where ⑥ denotes the 6 point circular convolution. Assume $s(n)$ is periodic with a period 3 and that $s(n), z(n)$ are known. Show that it is not possible to determine $t(n)$ completely using $s(n)$ and $z(n)$. You should explain this phenomenon by exploiting the structure of $S(k)$, the DFT of $s(n)$. (4p)

5. Consider a two channel real filter bank with analysis filters $\{h_0(n), h_1(n)\}$ and synthesis filters $\{f_0(n), f_1(n)\}$. The length of all filters is $L+1$ with L being odd. Let $r(n) = f_0(n) * f_0(-n)$ denote the linear convolution of $f_0(n)$ with $f_0(-n)$ (autocorrelation in some sense). Assume,

$$\begin{aligned} r(2k) &= \delta(k) \\ f_1(n) &= (-1)^{n+1} f_0(L-n) \\ h_i(n) &= f_i(L-n), \quad i = 0, 1 \end{aligned}$$

where $\delta(0) = 1$ and $\delta(k) = 0, k \neq 0$. The aim is to construct a perfect reconstruction filter bank using these assumptions.

(a) Determine $F_1(z), H_0(z)$ and $H_1(z)$ in terms of $F_0(z)$. (3p)

(b) Show that $r(2k) = \delta(k)$ can be equivalently written as, (3p)

$$F_0(z)F_0(z^{-1}) + F_0(-z)F_0(-z^{-1}) = 2 \tag{2}$$

Note: $r(2n)$ is the decimated version of $r(n)$ and use the $z-$ transform of $r(n)$

(c) Using parts (a), (b), (c) show that the proposed filter bank has the property of perfect reconstruction. (1p)

(d) During a design based on the earlier assumptions, a student obtains the filter $f_0(n)$ whose frequency response is sketched in figure 3. Evaluating equation (2) on the unit circle, argue that the resulting filter bank cannot be perfect reconstructing. (3p)

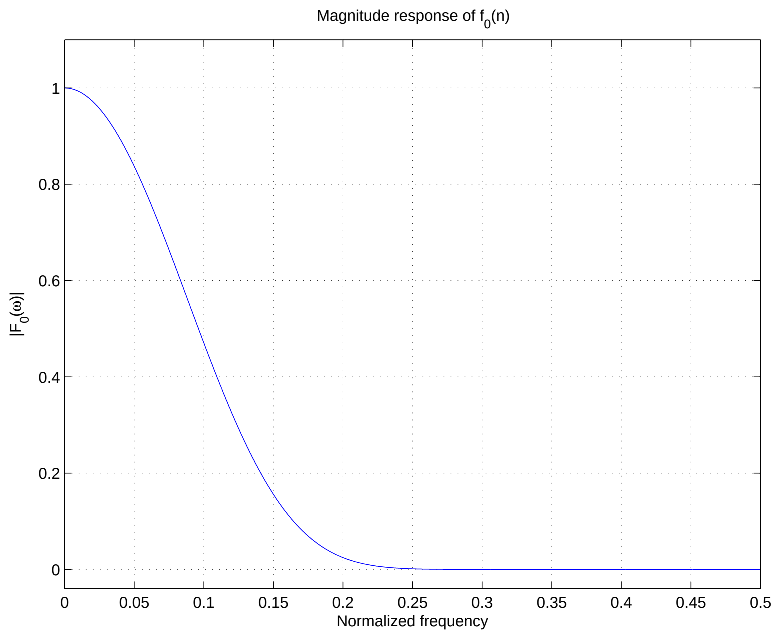

Figure 4: Magnitude response of a designed $f_0(n)$