SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 87 **Digital Signalbehandling**, 2E1340

Final Examination 2002–04–03, 9.00–13.00

Literature: Proakis, Manolakis: Digital Signal Processing
Josefsson: formel- och tabellsamling i matematik
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English
At most one problem should be treated per page.
Motivate each step in the solution (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each. For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. a) Give an analytical expression for the output spectrum of the circuit in Figure 1. (5p)

b) Sketch the output spectrum of the circuit, if the input is a stable AR-process with one pole, $x(n) + ax(n-1) = w(n)$, where $w(n)$ is white noise and $a = -0.9$. (5p)

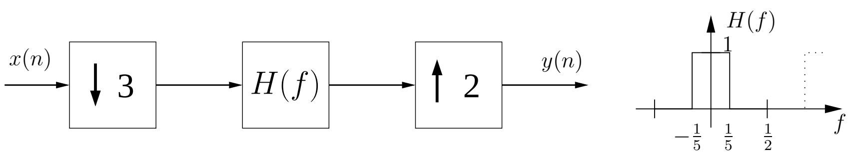

Figure 1:

2.

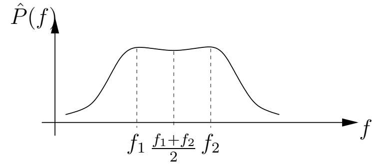

Figure 2: Spectral estimate close to the resolution limit.

Assume that we measure a signal that contains two time discrete sinusoids of the same power with frequencies f_1 and f_2. We say that a spectral estimation method is able to resolve these two frequencies if the two highest values of the estimated spectrum corresponds to the frequencies f_1 and f_2. In particular, $\hat{P}(\frac{f_1+f_2}{2}) < \hat{P}(f_1), \hat{P}(f_2)$ as illustrated in Figure 2. For DFT based methods, this resolution criterion translates into a requirement on the so-called “3dB bandwidth” of the window function $w(n)$. Unfortunately, the description in the text book is a a bit sloppy, so in this question you should clarify some details. In Figure 3, we have defined a few parameters of a typical window function $w(n)$ and its Fourier transform $W(f)$.

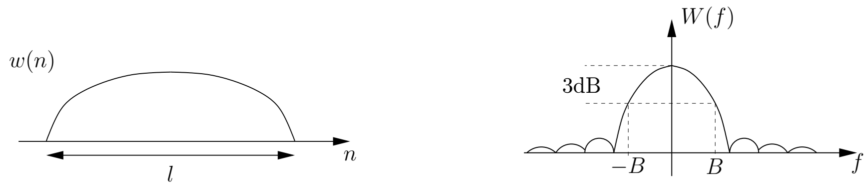

Figure 3: A typical window function in the time (left) and frequency (right) domain.

First, consider the Welch method with data segments of length M.

a) How does M relate to the window size?

i) $M = \frac{l}{2}$
ii) $M = l$
iii) $M = 2l$
iv) $M = B$
v) $M = 2B$ (2p)

b) What do we really mean by the “3dB bandwidth” of a window?

i) $|W(B)| = \frac{|W(0)|}{\sqrt{2}}$
ii) $|W(B)| = \frac{|W(0)|}{2}$
iii) $|W(B)| = \frac{|W(0)|}{4}$ (2p)

c) What is the frequency resolution of the Welch method if B is defined as in b)?

i) $|f_1 - f_2| > \frac{B}{2}$
ii) $|f_1 - f_2| > B$
iii) $|f_1 - f_2| > 2B$ (2p)

Next, consider the Blackman Tukey method based on N data samples.

d) What is the highest reasonable value of l for the Blackman Tukey method?

i) $l = \frac{N}{2}$
ii) $l = N$
iii) $l = 2N - 1$ (2p)

e) The resolution of the Blackman Tukey method is determined by the combined window function $w_{\text{comb}}(n) = w(n)(1 - \frac{|k|}{N})$, where $w(n)$ is the window chosen by the user. Define B_{comb} as the 3dB bandwidth of $w_{\text{comb}}(n)$. What is the frequency resolution?

i) $|f_1 - f_2| > \frac{B_{\text{comb}}}{2}$
ii) $|f_1 - f_2| > B_{\text{comb}}$
iii) $|f_1 - f_2| > 2B_{\text{comb}}$ (2p)

3.

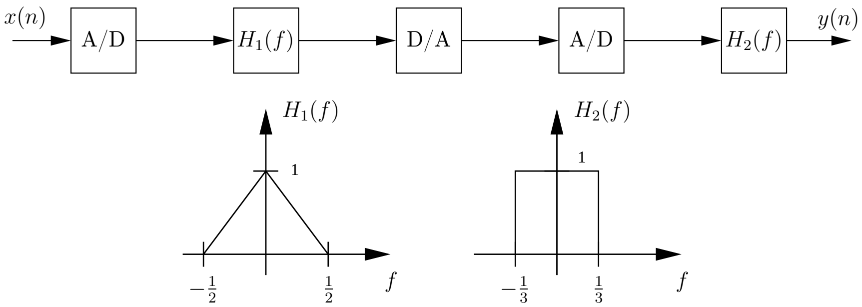

Figure 4:

A signal is sampled and quantized in a A/D converter, filtered, reconstructed to be transfered over an analog connection, sampled and quantized again and finally filtered, see Figure 4. Assume that the implemented filters have the ideal transfer functions shown in the figure and that the calculations are performed with full precision. Both the A/D converters introduce quantization errors since they use a total of b bits to represent the range [-1,1]. Determine the power of the quantization noise at the output $y(n)$. (10p)

4.

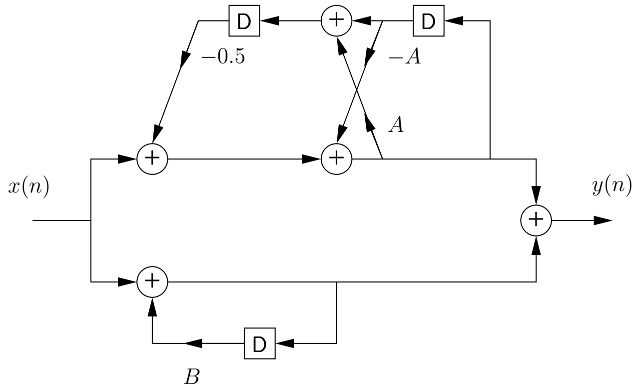

Figure 5: Filter. A constant next to an arrow denotes multiplication.

a) Determine the transfer function of the filter in Figure 5. (4p)

b) Determine for which values of A and B the filter is stable. (6p)

5. You are given N samples of a complex valued signal $x(n)$ with a single sinusoidal signal with additive noise. Assume that you already know the normalized frequency f of the signal and that f can be written $f = k/N$ where k is an integer. You want to find the complex valued amplitude α using the least squares method, i.e. the α that minimizes

$$\min_{\alpha} \sum_{n=0}^{N-1} \left| x(n) - \alpha e^{j\frac{2\pi nk}{N}} \right|^2$$

a) The LS criterion can be written in matrix form as

$$\min_{\alpha} \|\mathbf{x} - \alpha \mathbf{s}_k\|^2$$

Show this and explain how you define the vectors $\mathbf{x}$ and $\mathbf{s}_k$. (3p)

b) Let $\mathbf{W}$ denote the so-called DFT matrix of size N, i.e. the matrix such that DFT[$\mathbf{y}$] $= \mathbf{W}\mathbf{y}$ for all vectors of length N. Show that there is a simple relationship between $\|\mathbf{x} - \alpha\mathbf{s}_k\|^2$ and $\|\mathbf{W}(\mathbf{x} - \alpha\mathbf{s}_k)\|^2$. (3p)

c) Use the relationship in 5b) to show how the least squares solution of α can be found without any extra calculations if you have already calculated the DFT of $\mathbf{x}$. (4p)