SIGNALBEHANDLING

INSTITUTIONEN FÖR SIGNALER, SENSORER & SYSTEM

E 101 **Digital Signalbehandling**, 2E1340

Final Examination 2007–06–04, 14.00–19.00

Literature: Hayes: *Statistical Digital Signal Processing and Modeling*
or Proakis, Manolakis: *Digital Signal Processing*
Bengtsson: *Complementary Reading in Digital Signal Processing*
Copies of the slides
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solutions (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signal Processing, 08-790 84 63,

Results: Will be reported within three working weeks at "My pages".

Solutions: Will be available on the course homepage after the exam.

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1. In this problem, just as in all problems in the exam, an answer without any motivation will not give any points.

a) The DTFT of a complex valued signal $x(n)$ is shown in Fig. 1. Which one of $Y_1(f)$, $Y_2(f)$ or $Y_3(f)$ correspond to the DTFT of $y(m) = x(2m)$?

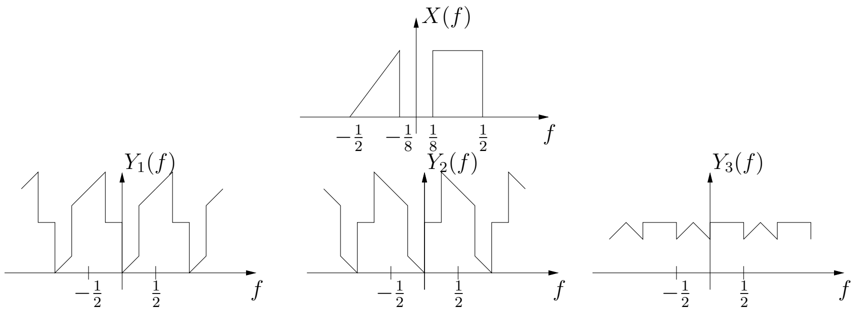

Figure 1: Input $X(f)$ and three possible output signals $Y_k(f)$.

(4p)

b) Assume that we want to estimate the power spectral density of a signal and have 16384 samples available. The sampling rate was 48kHz and we need a resolution of about 100Hz. Which of the following methods do you recommend?

i) A periodogram based on all the 16384 samples.

ii) Bartlett's method, dividing the samples into 128 segments and using zero-padding and FFTs of length 16384.

iii) Bartlett's method, dividing the samples into 16 segments and using zero-padding and FFTs of length 16384.

(4p)

c) A time signal consists of a single sinusoid with frequency 440Hz. We sample the signal using a sampling frequency of 8kHz and save 1024 samples $x(0), \ldots, x(1023)$. Let $X_{\text{ZP}}(k)$ denote the DFT of length 4096 of $x(n)$, using zero-padding. Which of the following values will be largest?

i) $|X_{\text{ZP}}(56)|$

ii) $|X_{\text{ZP}}(225)|$

iii) $|X_{\text{ZP}}(440)|$

(2p)

2. Consider a system, implemented in fixed-point arithmetics using 15 bits plus one sign bit (used to represent numbers between -1 and +1). All multiplications result in round-off errors, but there is no overflow.

a) A signal $x(n)$ passes through the filter shown in Fig. 2 with output $y(n)$. Let $q_y(n)$ denote the quantization error in $y(n)$. Determine $\mathrm{E}[q_y^2(n)]$. (5p)

b) The signal $y(n)$ described above is interpolated to double the data rate, using linear interpolation. Denote the interpolated signal by $w(m)$ and let $q_w(m)$ be the total quantization error in $w(m)$. Determine $\mathrm{E}[q_w^2(m)]$. Note that the answer may be different for odd and even values of m. (5p)

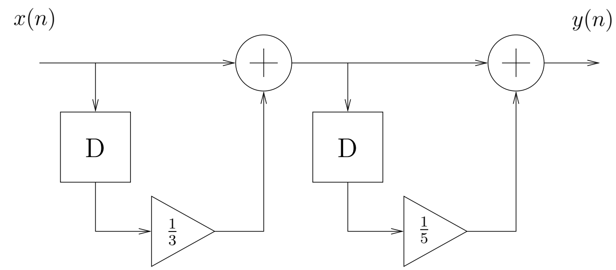

Figure 2: Filter implementation.

3. The students in a course on digital signal processing, get a homework assignment to implement the periodogram method for a signal $x(n), n = 0, 1, \ldots, N-1$. Here we will look in detail at two somewhat erroneous implementations. Both students start correctly by estimating the autocovariance using

$$\hat{r}(k) = \frac{1}{N} \sum_{n=0}^{N-1-k} x(n+k)x^*(n), \ k = 0, 1, \ldots, N-1$$

and $\hat{r}(k) = \hat{r}^*(-k)$ for $k = -N+1, \ldots, -1$. Then, they try to calculate the DFT, using a function `FFT` that works as in Matlab, i.e.

$$\texttt{FFT}([\underset{\uparrow}{y(0)}, y(1), \ldots, y(M-1)]) = [\underset{\uparrow}{Y(0)}, Y(1), \ldots, Y(M-1)]$$

where $Y(k)$ is the M-point DFT of $y(n)$.

a) One student calculates

$$\texttt{phat1} = \texttt{FFT}([\hat{r}(\underset{\uparrow}{-N}+1), \ldots, \hat{r}(-1), \hat{r}(0), \hat{r}(1) \ldots, \hat{r}(N-1)])$$

Is it possible to plot the periodogram based on this result? How? (3p)

b) The student tries to plot the periodogram using `plot(real(phat1))`. Is it possible to get any useful information from this plot, if you for example want to estimate the power of some background white noise in the signal? (2p)

c) The other student calculates

$$\texttt{phat2} = \texttt{FFT}([\underset{\uparrow}{\hat{r}(0)}, \hat{r}(1) \ldots, \hat{r}(N-1)])$$

Is it possible to plot the periodogram based on this result? How? (3p)

d) Also this student tries to plot the periodogram using `plot(real(phat2))`. Is it possible to get any useful information from this plot, if you for example want to estimate the power of some background white noise in the signal? (2p)

4. A discrete time sinusoid

$$x(n) = \cos(\omega_0 n + \phi)$$

is to be up sampled by a factor $U = 2$. The frequency $\omega_0 \in [0, \pi/2)$ radians. Fig. 3 shows a schematic view of the interpolator.

a) What is the discrete time Fourier transform of $w(m)$? Clearly state the frequencies of the sinusoidal components of the signal. (3p)

b) Sketch the frequency response of an ideal interpolation filter $H(z)$ such that

$$y(m) = \cos\left(\frac{\omega_0}{2} m + \phi\right),$$

for all frequencies $\omega_0 \in [0, \pi/2)$. (2p)

c) The ideal interpolation filter cannot be implemented in practice. Assume instead that the filter is of the form $H(z) = \alpha + \beta z^{-1} + \alpha z^{-2}$.

What values of α and β correspond to ordinary linear interpolation (possibly with an extra delay)? (2p)

d) Assume that the frequency ω_0 is fixed and known. Can you find filter coefficients α and β and an integer delay L such that

$$y(m) = \cos\left(\frac{\omega_0}{2}(m - L) + \phi\right)$$

(3p)

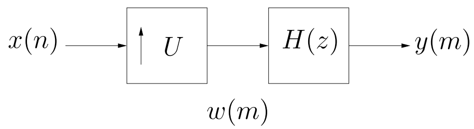

Figure 3: Schematic view of the up sampler.

5. The following equation can be used to describe communication over a radio channel.

$$y(k) = hx(k) + z(k) \qquad k = 0 \cdots M-1 \tag{1}$$

Here, $y(k)$ is the received signal, h denotes the channel attenuation, $x(k)$ is the transmitted sequence and $z(k)$ is the receiver noise, which is a zero-mean white sequence with $\mathrm{E}[|z(k)|^2] = 1$. Assume that the channel attenuation h is known to have the following statistical properties

$$E[h] = 0, \quad E[|h|^2] = \sigma^2.$$

Just as in the first set of computer exercises, so-called complex valued base band notation is used in Eq. (1), meaning that $y(k)$, h, $x(k)$ and $z(k)$ are complex valued.

When the connection is established, a known sequence of symbols is transmitted in order to be able to estimate the channel h, so we assume that the M values of $x(k)$ in Eq. (1) are known.

a) Determine the coefficients in a linear estimate of the channel attenuation (h),

$$\hat{h} = \sum_{n=0}^{M-1} w(n) y(n)$$

such that the mean square error

$$E\left[|h - \hat{h}|^2\right]$$

is minimized. (7p)

b) Find the mean square error of the estimator. (3p)

Good luck!