SIGNALBEHANDLING

Institutionen för Signaler, Sensorer & System

E 89 **Digital Signalbehandling**, 2E1340

Final Examination 2002–12–17, 14.00–19.00

Literature: Hayes: Statistical Digital Signal Processing and Modeling
(Proakis, Manolakis: Digital Signal Processing)
Bengtsson: Complementary Reading in Digital Signal Processing
On popular demand: Copies of the slides, distributed with the exam.
Beta – Mathematics Handbook
Collection of Formulas in Signal Processing, KTH
Josefsson: formel- och tabellsamling i matematik
Unprogrammed pocket calculator.

Notice: Answer in Swedish or English.
At most one problem should be treated per page.
Motivate each step in the solution (also for the multi-choice questions).
Write your name and *personnummer* on each page.
Write the number of solution pages on the cover page.

The exam consists of five problems with a maximum of 10 points each.
For a passing grade, 24 points are normally required.

Contact: Mats Bengtsson, Signalbehandling, 790 84 63,

Results: Will be posted within three working weeks at Osquldas väg 10, floor 2.

Solutions: Will be available on the course homepage.

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1. a) The student Pelle measures M samples of a signal $x(n) = A_1 \cos(2\pi f_1 n + \phi_1) + A_2 \cos(2\pi f_2 n + \phi_2)$ and plots the periodogram of the signal, which gives the lower curve in Figure 1. When he repeats the same experiment a few hours later, he gets the upper curve in Figure 1. Obviously, something has changed in the experiment. Please help Pelle to understand what has changed, he has already made sure that the frequencies f_1 and f_2 (indicated with dashed lines in the figures) have remained the same. Which of the following parameters is most likely to have been changed?

(i) The number of samples M.
(ii) The amount of zero-padding.
(iii) One of the amplitudes A_1.
(iv) One of the phase values ϕ_1.
(v) The windowing function used in the modified Periodogram. (3p)

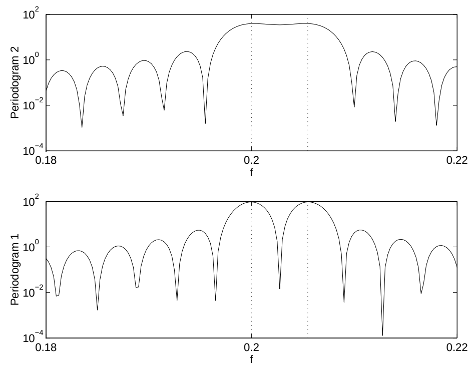

Figure 1: Periodograms, plotted in logarithmic scale.

b) A time continuous sinusoidal signal with frequency $F = 13$kHz is sampled and $N = 1024$ of the samples are used to calculate a periodogram. The periodogram, $\hat{P}_{\text{per}}(k)$, calculated using an FFT without extra zero-padding, has a peak at the DFT index $k = 841$. What sampling frequency was used?

(i) $F_s = 8\,000$Hz.
(ii) $F_s = 9\,600$Hz.
(iii) $F_s = 11\,025$Hz.
(iv) $F_s = 16\,000$Hz.
(v) $F_s = 18\,900$Hz.

(3p)

c) In the periodogram described in the previous question, it was observed that the total estimated power in the periodogram was $\hat{P}_{\text{tot}} = \sum_{k=0}^{1023} \hat{P}_{\text{per}}(k) = 17000$. Assume that the true time continuous signal was the "complex sinusoid" $x(t) = Ae^{j2\pi Ft}$. Determine the amplitude A. The measurement noise can be neglected.

(i) $A = 4.07$
(ii) $A = 5.76$
(iii) $A = 16.6$
(iv) $A = 33.2$
(v) $A = 130$

(4p)

2. In the circuit in Figure 2, fix-point arithmetics with b bits plus sign bit is used. The results of the multiplications are rounded, causing quantization noise. No round-off errors occur due to overflow.

Calculate an expression for the variance of the quantization noise at the output. Use $a = \frac{4}{5}$, $b_0 = 2$, $b_1 = -1$ and $c = \frac{1}{5}$. (10p)

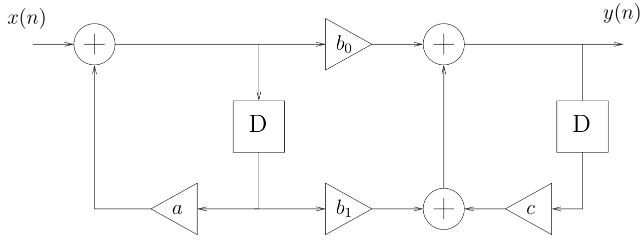

Figure 2: Fix-point circuit

3. Implementation of multipliers in hardware requires many levels of logic. In systems with decimation from very high sampling frequencies, the need for long filters with fast multipliers can become a serious bottleneck. Figure 3 shows an alternative decimation design known as a Cascaded Integrator-Comb (CIC) filter, which can be implemented in hardware using only additions and delay elements. The part of the filter on the left side of the decimation is known as an integrator, and the part on the right is called a comb-filter.

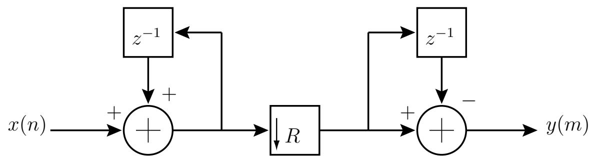

Figure 3: CIC decimation filter

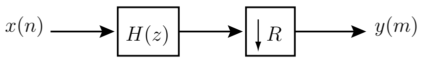

Figure 4: Equivalent decimation

a) Determine the transfer function, $H(z)$, of the anti-alias filter of the equivalent system shown in Figure 4.
Hint: The Noble identities might be useful. (6p)

b) Show that $h(n)$ is an FIR-filter, i.e $H(z) = \sum_{n=0}^{N-1} a_n z^{-n}$.
Determine a_n and N. (2p)

c) What is gained by placing the comb part of the anti-alias filter *after* the decimation? Give at least *two* examples. (2p)

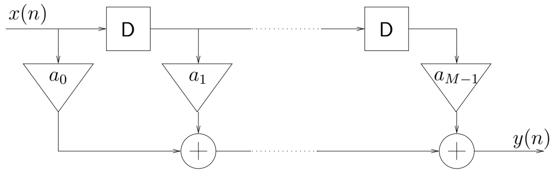

Figure 5: FIR filter

4. An FIR filter of length M, see Figure 5, is used to filter a signal $x(n)$ of length L ($n = 0, 1, \ldots, L-1$). In this specific application, the initial state of the filter is not zero, but is given by $x(-1), x(-2), \ldots, x(-M+1)$. As a replacement for the direct implementation in Figure 5, we wish to calculate the output of this filter using the following kind of FFT implementation:

- $h(n)$ contains the filter coefficients, with zeros added at the end to get total length N, $h(n) = \{\underset{\uparrow}{a_0}, a_1, \ldots, a_{M-1}, 0, \ldots, 0\}$.
- $H(k) = \text{DFT}\{h(n)\}$,
- $V(k) = \text{DFT}\{v(n)\}$,
- $W(k) = H(k)V(k)$,
- $w(n) = \text{IDFT}\{W(k)\}$.

All DFTs have length N.

Determine how the values from $x(n)$ and the initial state should be arranged in the signal $v(n)$ of length N, such that the first L values of $w(n)$ are the same as the output $y(n)$ of the FIR filter implementation in Figure 5. How should the length N be chosen? (10p)

5. In the DSP Lab, we have done an exercise on system identification, see Fig. 6. The adaptive algorithm used in the lab tries to find the filter $F(z)$ that minimizes the power $P_e = \text{E}[|e(n)|^2]$ of the error signal $e(n)$.

In this question, assume that the filter $h(n)$ is given by

$$h(n) = \begin{cases} 1 & n = 0 \\ -\alpha & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

a) If there are no constraints on the filter $F(z)$, show that P_e is minimized when $F(z)H(z) = 1$, i.e. the adaptive filter $f(n)$ will converge to the inverse of the original filter $h(n)$. What is the corresponding value of P_e? (2p)

b) Find the expression of $f(n)$ so that $f(n)$ is the inverse of $h(n)$ (2p)

c) If the filter $f(n)$ is constrained to be an FIR filter of length 2, i.e. if $f(n) = 0$ for $n > 1$, determine the coefficients in $f(n)$ that minimize P_e (i.e. the MMSE solution). (6p)

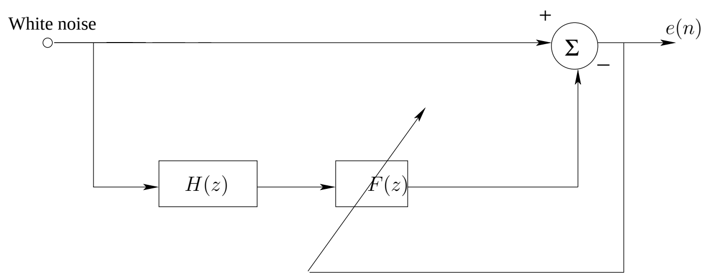

Figure 6: System Identification

Good luck!

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WWW page.