EQ2330 – Image and Video Processing

Exercise 11: Image Segmentation

Unless stated otherwise, the problems are from on R. C. Gonzales and R. E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 10.22**
   Suppose that an image has the gray-level probability density functions shown in Figure 1. Here, $p_1(z)$ corresponds to objects and $p_2(z)$ corresponds to the background. Assume that $P_1 = P_2$ and find the optimal threshold between object and background pixels.

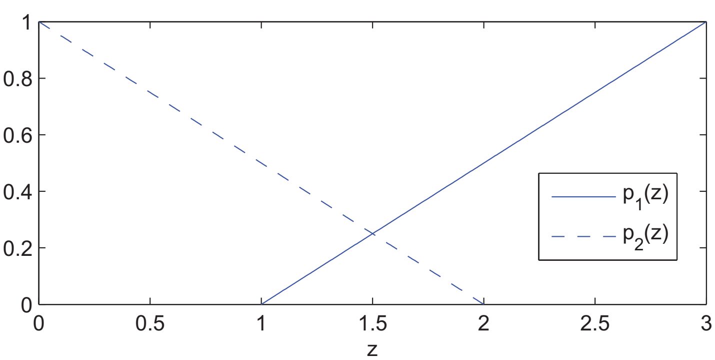

Figure 1: Image for Problem 1.

2. **Image Segmentation, Exam March 2010**
   We assume that an image is modelled by the following probability density function

$$f(x) = P_{fg} f_{fg}(x) + P_{bg} f_{bg}(x),$$

   where P_{fg} is a probability that a pixel belongs to foreground, $f_{fg}(x)$ is probability density function for pixels belonging to the foreground, P_{bg} is a probability that a pixel belongs to background, $f_{bg}(x)$ is probability density function for pixels belonging to the background. Since $f(x)$ is a valid probability density function $P_{fg} + P_{bg} = 1$. In this problem we consider *supervised thresholding*.

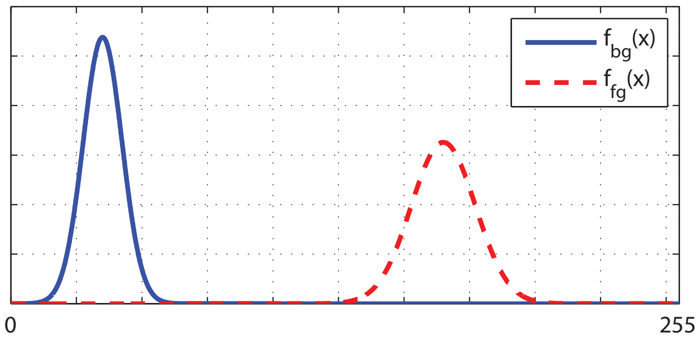

Figure 2: Gray-level probability density functions of background and foreground regions in the image.

(a) Optimal supervised thresholding aims at minimizing the probability of misclassification, which is a sum of two error terms. Describe these two terms shortly and give the corresponding expressions.

(b) Derive the expression that can be used to find an optimal threshold T that minimizes the probability of misclassification.

(c) Assume that

$$f_{fg}(x) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right),$$

$$f_{bg}(x) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right)$$

and find the optimal threshold T.

(d) Note that in the general case the solution to the problem above requires two threshold values that are needed to obtain an optimal solution. Sketch an example of a distribution $f(x)$ and mark these two thresholds. Explain how the pixel values would be classified using these two thresholds.

(e) State the condition when there is only a single optimal threshold.

(f) What is the optimal threshold if $\sigma_1 = \sigma_2$ and $P_{fg} = P_{bg}$?

3. **Histogram Equalization and Thresholding, Exam May 2009**

Consider the gray-level discrete *Flower Field* image f shown in Figure 3. The histogram of the image is given by $h(\ell)$, where $\ell \in \{0, 1, \ldots, 255\}$ is the gray-level value. $h(\ell)$ is shown in Figure 4.

You apply histogram equalization $g(x, y) = H(f(x, y))$ to adjust the contrast of the image.

(a) Give an expression for the transform $H(\cdot)$.

(b) Sketch the histogram of the equalized image g. No calculations are needed.

Figure 3: Image *Flower Field*.

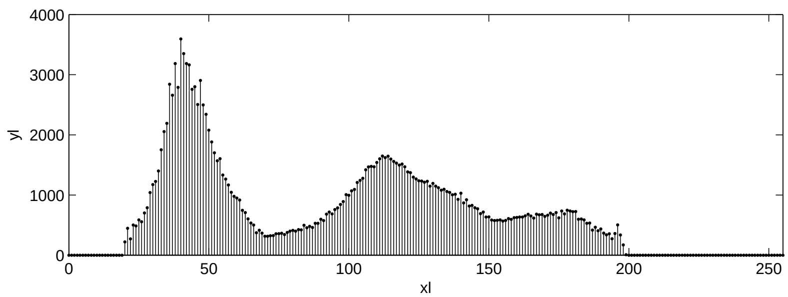

Figure 4: Histogram of *Flower Field*.

You apply thresholding with threshold $T^{(g)}$ to the equalized image to detect pixels representing the white flower petals. Let P_p and P_o be the probability of flower petal pixels and other pixels, respectively. Further, let $p_p(l)$ and $p_o(l)$ be the gray-level probability mass functions of the flower petal pixels and other pixels, respectively.

(c) What is the criterion for optimal supervised thresholding? Give a mathematical expression for that criterion.

Consider application of supervised thresholding before histogram equalization, to the image f directly, with threshold $T^{(f)}$.

(d) Argue that the optimal threshold $T^{(f)}$ for image f applied to f detects the same pixels as flower petal pixels as the optimal threshold $T^{(g)}$ for image g applied to g.

(e) Show that your argument in (d) is true. *Hint: Note that we deal with discrete distributions.*