EQ2330 Image and Video Processing
Tutorial #1: Point Operations

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Reminder I

- Before tutorial:
  - Solve preparation assignment
  - Submit solutions via Canvas
- During tutorial:
  - Peer correction
  - Solve exercise problems
- After the tutorial:
  - Submit corrected solutions.
- If you miss a session, write me an email first and then submit your solution + correction in two days.

Reminder II

- Project group
  - In group of 2-3 students
  - Send group information before Monday 2nd Nov
- 1st Project
  - Publish after grouping is finished (3rd Nov).

Recap

Histogram Equalization

- Find a non-linear transform T: $g = T(f)$
  - f: intensity of image pixels (0 $\rightarrow$ black, 1 $\rightarrow$ white)
  - g: intensity of transformed image pixels
  - T: applied to each pixel of the input image $f(x, y)$
- Such that the output image $g(x, y)$ results in a uniform distribution of grey levels in the entire grey level range, i.e.

$$p_g = const.$$

Recap

Histogram Equalization for Continuous Case

- Assume $f \in [0,1]$, $g \in [0,1]$, $T(\cdot)$ is monotonically increasing
- $$g = T(f) = \int_0^f p_f(\alpha) d\alpha \text{ for } f \in [0,1]$$
  $$\frac{dg}{df} = p_f(f)$$
  $$p_g(g) = [p_f(f)\frac{df}{dg}]_{f=T^{-1}(g)} = [p_f(f)\frac{1}{p_f(f)}]_{f=T^{-1}(g)} = 1.$$

Recap

Histogram Equalization for Discrete Case

- Discrete values $f_0, f_1, \cdots$, with probability $p_0 = \frac{n_0}{n}$, $p_1 = \frac{n_1}{n}$, $\cdots$
- Discrete approximation of $g = T(f) = \int_0^f p_f(\alpha)d\alpha$ is $g_k = T(f_k) = \sum_{i=0}^{k} p_i$
- Resulting values $g_k \in [0, 1]$ needs to be scaled and rounded appropriately.

Continuous V.S. Discrete Histogram Equalization

Continuous:

- Transform: taking integral of original pdf and map to new intensity.
- Always result in uniform histogram.

Discrete:

- Transform: taking summation of original possibilities values and map to new intensity.
- Does not necessarily result in uniform histogram.

Exercise #1:

Prob 2.7 & 2.8:

known: $i(x,y) = 255 e^{-[(x-x_0)^2+(y-y_0)^2]}$

$r(x,y) = 1$

False conturing:

intensity function:

$f(x,y) = i(x,y)\, r(x,y) = 255 e^{-[(x-x_0)^2+(y-y_0)^2]}$

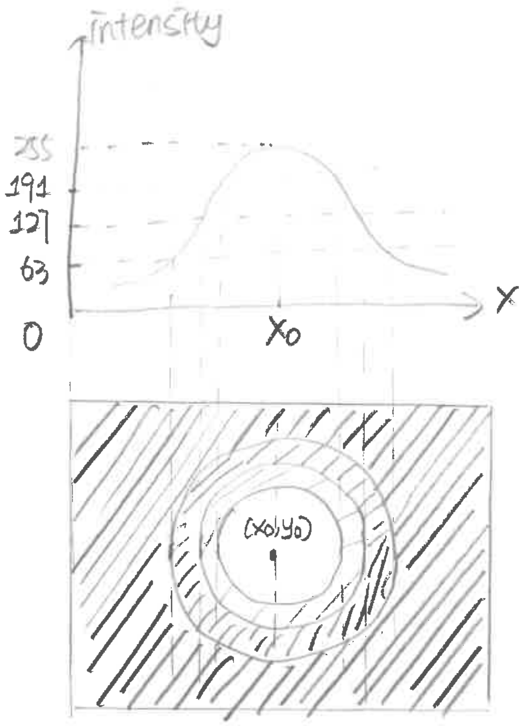
digitalized with k bits, then step size of discrete intensity

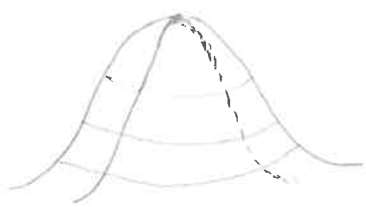
is $\Delta G = \frac{255+1}{2^k}$

Set $\Delta G = 8$, then $k=5$

when $k \le 5$, there would be false contouring.

sketch $k=2$: 0 – black , 255 – black





Prob 3.6:

$$\boxed{r} \xrightarrow[\text{equal.}]{\text{histo.}} \boxed{s} \xrightarrow[\text{equal.}]{\text{histo.}} \boxed{v}$$

show $s = v$

First Pass:

n: # pixels

n_{r_j} = # pixels with intensity r_j

Apply histogram equalization transform:

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_{r_j}}{n} = \frac{1}{n} \sum_{j=0}^{k} n_{r_j}$$

i.e., intensity r_k is mapped to s_k $\Rightarrow$

occurance of s_k in image s = occurace of r_k in r

Second Pass:

Apply histogram equalization transform:

$$v_k = T(s_k) = \sum_{j=0}^{k} \frac{n_{s_j}}{n} = \frac{1}{n} \sum_{j=0}^{k} n_{r_j} = s_k$$

$\Rightarrow$ second pass results in the same image as first pass.

Prob 3.10:

$$P_r(r) = -2r+2 \quad , \quad r \in [0,1]$$
$$P_z(z) = 2r \quad , \quad r \in [0,1]$$

Idea: Do histogram equalization to r, z to obtain uniformly distributed s, v

$$s = T(r) = \int_0^r P_r(w)\,dw = \int_0^r -2w+2\,dw = 2w - w^2 \Big|_0^r = 2r - r^2$$
$$v = T(z) = \int_0^z P_z(w)\,dw = \int_0^z 2w\,dw = w^2 \Big|_0^z = z^2$$

$$\Rightarrow \quad z^2 = 2r - r^2 \quad \Rightarrow \quad z = \sqrt{2r - r^2}$$

Prob 5:

① Mostly black.

② $\circ\ h(x) \geq 0 \Rightarrow \dfrac{e^{1-x} - 1}{a} \geq 0 \ , \ \forall\ x \in [0,1]$

$$\Rightarrow \quad a > 0$$

$\circ \int h(x)\,dx = 1 \Rightarrow \int_0^1 \dfrac{e^{1-x} - 1}{a}\,dx = 1$

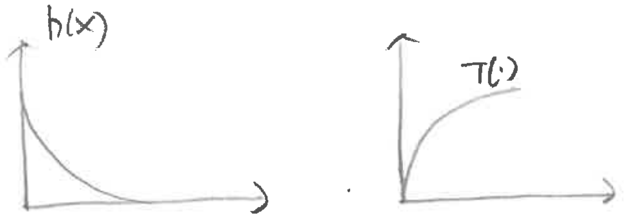
$$\Rightarrow \frac{-e^{1-x} - x}{a} \Big|_0^1 = \frac{(-e^0 - 1) - (-e - 0)}{a} = \frac{e-2}{a} = 1$$

$$\Rightarrow a = e - 2.$$

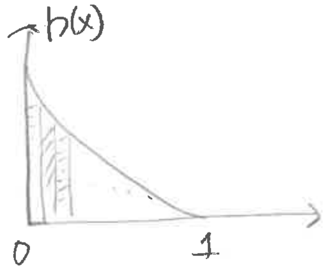
③ histogram equalization:

$$y = T(x) = \int_0^x h(z)\,dz = \frac{-e^{1-z} - z}{e-2} \Big|_0^x = \frac{e - e^{1-x} - x}{e-2}$$

④



⑤



After digitalization of x =

Histogram Equalization Transformation: