EQ2330 Image and Video Processing
Tutorial #2: Linear Processing

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Project Instruction

- Matlab tutorial includes basic image processing functions.
- Report template includes instructions and you should write following the template.
- Max 5 pages (Appendix not included).
- To pass, motivate your method, write report clearly, and give reasonable results and conclusions.
- Appendix: who did what. (Grades are individual.)

Project Instruction

Project 1: Image enhancement

- Spatial domain filtering
  - Histogram Equalization
  - Image denoising: mean/median filters
- Frequency domain filtering
  - Wiener filtering
  - Constrained least square filtering

Peer Correction

How?

- Correct solutions from another student.
- Hand writing or comment by inserting comment box on pdf file.

Submission

- Reply with commented pdf file on the peer-review page.

Convolution V.S. Correlation

Correlation

$$\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \tag{1}$$

Convolution

$$\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t) \tag{2}$$

Linear Operation & Linear Filtering

Linear Operator

Additivity and Homogeneity

$$H[a_i f_i(x,y) + a_j f_j(x,y)] = a_i H[f_i(x,y)] + a_j H[f_j(x,y)]. \tag{5}$$

Linear Filtering

If the operation performed on the image pixels is linear, then the filter is linear (spatial) filter.
Averaging filter, weighted averaging filter and downsampling filter are all linear filter.

DFT and DC Component

- 1-D Discrete Fourier Transform

$$F(u) = \sum_{x=-\infty}^{\infty} f(x) e^{-j2\pi ux/M}, u \in [0, M-1] \tag{7}$$

- 2-D Discrete Fourier Transform

$$F(u, v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(x, y) e^{-j2\pi(ux/M+vy/N)}, u \in [0, M-1], v \in [0, N-1] \tag{8}$$

- DC Component

$$F(0, 0) = MN \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(x, y) = MN\bar{f}(x, y) \tag{9}$$

1st & 2nd Order Derivative, Laplacian Operator

- First order derivative of a 1-D function $f(x)$ is the $\frac{\partial f}{\partial x} = f(x+1) - f(x)$.
- Second order derivative:
  $\frac{\partial^2 f}{\partial x^2} = [f(x+1) - f(x)] - [f(x) - f(x-1)] = f(x+1) + f(x-1) - 2f(x)$
- Laplacian derivative operator:
  $\Delta^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$,
  $\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$,
  $\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$,
  $\Delta^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$

Laplacian operator is linear!
Changing the order of linear operations does not change the result.
Therefore changing the order of averaging filter and Laplacian operator does not change the final result.

Exercise # 2: Problem 3.26

Unsharp Masking

- Why? To sharpen images.
- How? Subtract an unsharp version of an image from original image.
  1. Blur the original image.
  2. Subtract the blurred image from the original image, and obtain the mask:

$$g_{\text{mask}}(x, y) = f(x, y) - \bar{f}(x, y). \tag{13}$$

  3. Add mask to the original image, and obtain the sharped image:

$$g(x, y) = f(x, y) + k * g_{\text{mask}}(x, y) = 2f(x, y) - k * \bar{f}(x, y), \tag{14}$$

$k = 1$, unsharp masking
$k > 1$, highboost filtering

Exercise #2: Problem 3.27

Show that subtracting the Laplacian from an image is proportional to unsharp masking.

$$
\begin{aligned}
& f(x,y) - \Delta^2 f(x,y) \\
=& f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)] \\
=& 6f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)] \\
=& 6f(x,y) - 5\bar{f}(x,y) = 3[2f(x,y) - \frac{5}{3}\bar{f}(x,y)]
\end{aligned}
$$

If we see $k = \frac{5}{3}$ as proportional factors, then we have

$$
f(x,y) - \Delta^2 f(x,y) \sim 2f(x,y) - k * \bar{f}(x,y) \tag{15}
$$

Exercise #2: Problem 3.16

- Objective: use image averaging to reduce noise.
- Assume $g(x,y) = f(x,y) + \eta(x,y)$, where $g(x,y)$, $f(x,y)$ and $\eta(x,y)$ denote the noisy image, true image and random noise respectively.
- Also assume $\eta(x,y)$ is zero mean and σ^2 variance, and uncorrelated.
- If an image $\bar{g}(x,y)$ is produced by averaging k different images, i.e.

$$\bar{g}(x,y) = \frac{1}{k}\sum_{i=1}^{k} g_i(x,y) = \frac{1}{k}\sum_{i=1}^{k} f(x,y)+\eta_i(x,y) = f(x,y)+\frac{1}{k}\sum_{i=1}^{k}\eta_i(x,y),$$

then we have $E\{\bar{g}(x,y)\} = f(x,y)$, and

$$Var\{\bar{g}(x,y)\} = E\{[\bar{g}(x,y) - f(x,y)]^2\} = E\{[\frac{1}{k}\sum_{i=1}^{k}\eta_i(x,y)]^2\}$$

$$= \frac{1}{k^2}\sum_{i=1}^{k} E\{[\eta_i(x,y)]^2\} = \frac{1}{k}\sigma^2.$$

Exercise #2: Problem 3.16

- We want $\frac{1}{k} \leq \frac{1}{10}$ and the number of images as small as possible, therefore $k = 10$.
- The stationary time is $\frac{1}{30} * 10 = \frac{1}{3}$ second.

Thank you!