EQ2330 – Image and Video Processing

Exercise 3: Frequency Domain Filtering

The problems are taken from R. C. Gonzales and R.E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 4.3** Let $F(u, v)$ denote the DFT of an image $f(x, y)$. we know from the discussion in Section 4.2.3 that multiplying $F(u, v)$ by a filter function $H(u, v)$ and taking the inverse Fourier transform will alter the appearance of the image, depending on the nature of the filter. Suppose that $H(u, v) = A$, a positive constant. The net effect of filtering will be to multiply the image by the same constant. Using the convolution theorem, explain mathematically why the pixels in the spatial domain representation are multiplied by the same constant.

2. **Problem 4.11** Prove the validity of the convolution theorem. For simplicity, limit your development to continuous case.

3. **Problem 4.14** Suppose that you form a lowpass spatial filter that averages the four immediate neighbors of a point (x,y) but excludes the point itself.

   (i) Find the equivalent filter $H(u, v)$ in the frequency domain.

   (ii) Show that your result is a lowpass filter.

4. **Problem 4.15** The basic approach used to approximate the discrete derivative involves taking differences of the form $f(x + 1, y) - f(x, y)$.

   (i) Obtain the filter transfer function $H(u, v)$ in the frequency domain.

   (ii) Show that $H(u, v)$ is a highpass filter.

5. **Problem 4.4** A Gaussian lowpass filter in the frequency domain has a transfer function

$$H(u, v) = Ae^{-(u^2+v^2)/2\sigma^2}.$$

Show that the corresponding filter in the spatial domain has the form

$$h(x, y) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}.$$

Hint: Treat the variables as continuous to simplify manipulations.

6. **Problem 4.5** A highpass filter has a the transfer function

$$H_{hp}(u, v) = 1 - H_{lp}(u, v),$$

where H_{lp} is the transfer function of the corresponding lowpass filter. Using the result in Problem 4.4, what is the form of the spatial domain Gaussian highpass filter?

7. **Problem 4.21** Filtering in the frequency domain requires that images need to be padded by appending zeros to the ends of rows and columns in the image (see the following image on the left, Fig. 1). Do you think it would make a difference if we centered the image and surrounded it by a border of zeros instead (see image on the right, Fig. 1), but without changing the total number of zeros used? Explain.

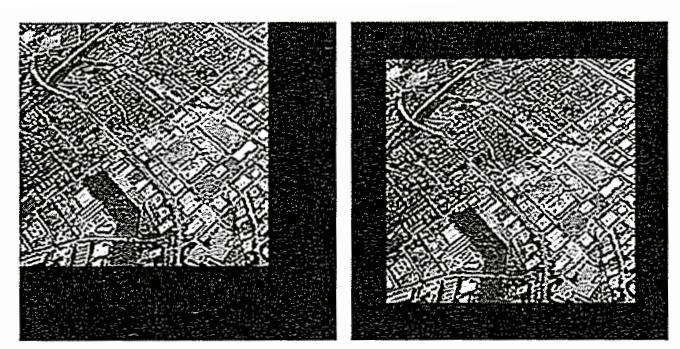

Figure 1: Pictures for Problem 7.