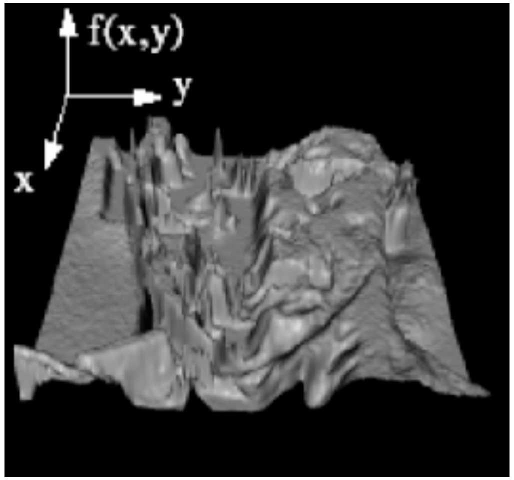
Digital Images

- What is an image?
- Digital images and pixels
- Image size and resolution
- Color components
- Number of gray levels
- Brightness discrimination experiment
- Weber's Law

What is an Image?

- Ideally, a 2-dimensional **light intensity function f(x,y),** where x and y are spatial coordinates, and f at (x,y) is related to the brightness or color of the image at that point.
- In practice, most images are defined over a **rectangle**.
- **Continuous in amplitude** ("continuous-tone")
- **Continuous in space**: no pixels!

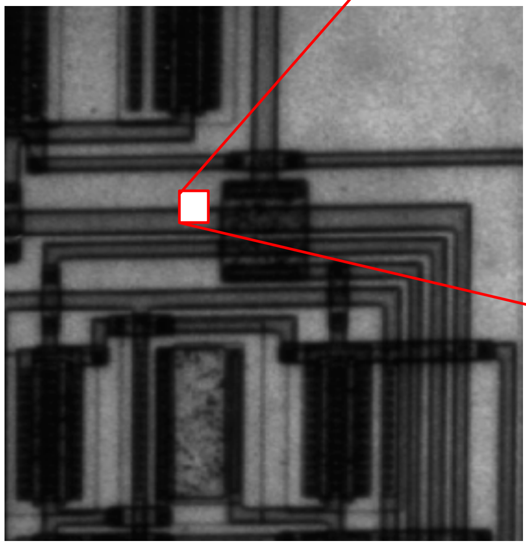


Digital Images and Pixels

- A **digital image** is the representation of a continuous image f(x,y) by a **2-d array of discrete samples**. The **amplitude of each sample is quantized** to be represented by a finite number of bits.
- Each element of the 2-d array of samples is called a **pixel** or **pel** (from “picture element”)
- Pixels are point samples, without extent.
- A **pixel is not:**
  - Round, square, or rectangular
  - An element of an image sensor
  - An element of a display

A Digital Image is Represented by Numbers

272 pixels



280 pixels

128	125	107	105	110	118	116	114	110
121	122	115	108	106	107	116	116	107
110	114	112	107	105	103	106	106	100
100	96	100	99	94	94	101	101	89
85	82	81	80	76	75	80	82	72
58	58	56	54	53	52	51	49	45
41	41	41	39	39	38	36	35	33
43	43	42	43	41	41	41	43	40
60	60	59	59	60	59	59	58	56

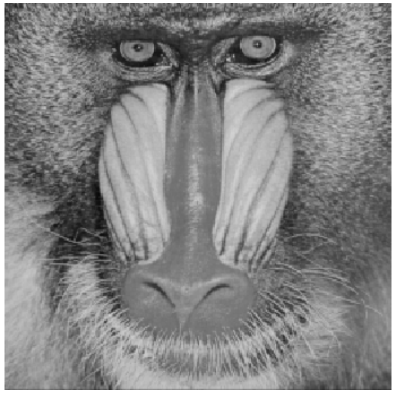
- Pixel = “picture element”
- Represents brightness at one point

A Digital Image Represented by a Matrix

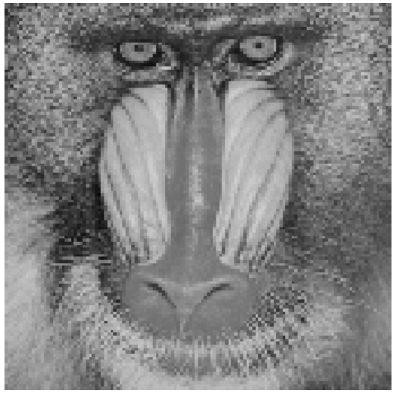
$$
\begin{array}{cl}
\xrightarrow{\quad\quad} x & \\
f = \begin{bmatrix} f(0,0) & f(1,0) & \cdots & f(W-1,0) \\ f(0,1) & f(1,1) & \cdots & f(W-1,1) \\ \vdots & \vdots & \ddots & \vdots \\ f(0,H-1) & f(1,H-1) & \cdots & f(W-1,H-1) \end{bmatrix} & \Bigg\downarrow y
\end{array}
$$

- The pixel values f(x,y) are sorted into the matrix in “natural” order, with x corresponding to the column and y to the row index. Matlab uses this convention. This results in $f(x,y)=f_{yx}$, where f_{yx} denotes an individual element in common matrix notation.
- For a color image, f might be one of the components.

Image Size and Resolution

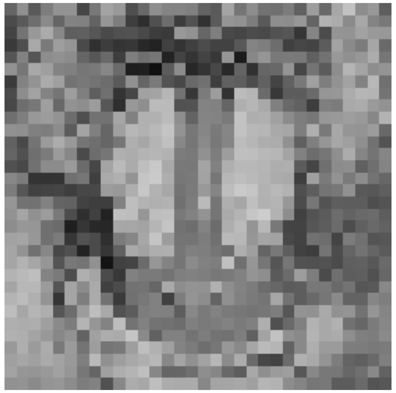


256x256



n=2

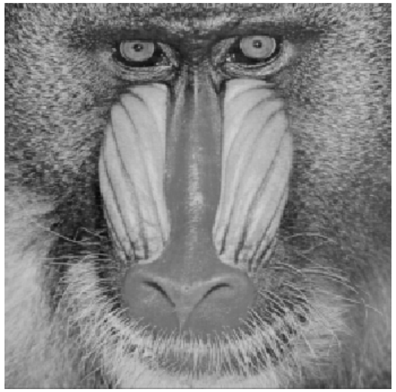
n=4

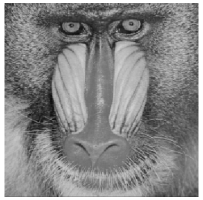


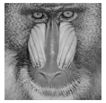
n=8

- These images were produced by simply picking every n-th sample horizontally and vertically and replicating that value nxn times.
- We can do better
  - Prefiltering before subsampling to avoid aliasing
  - Smooth interpolation

Images of Different Size

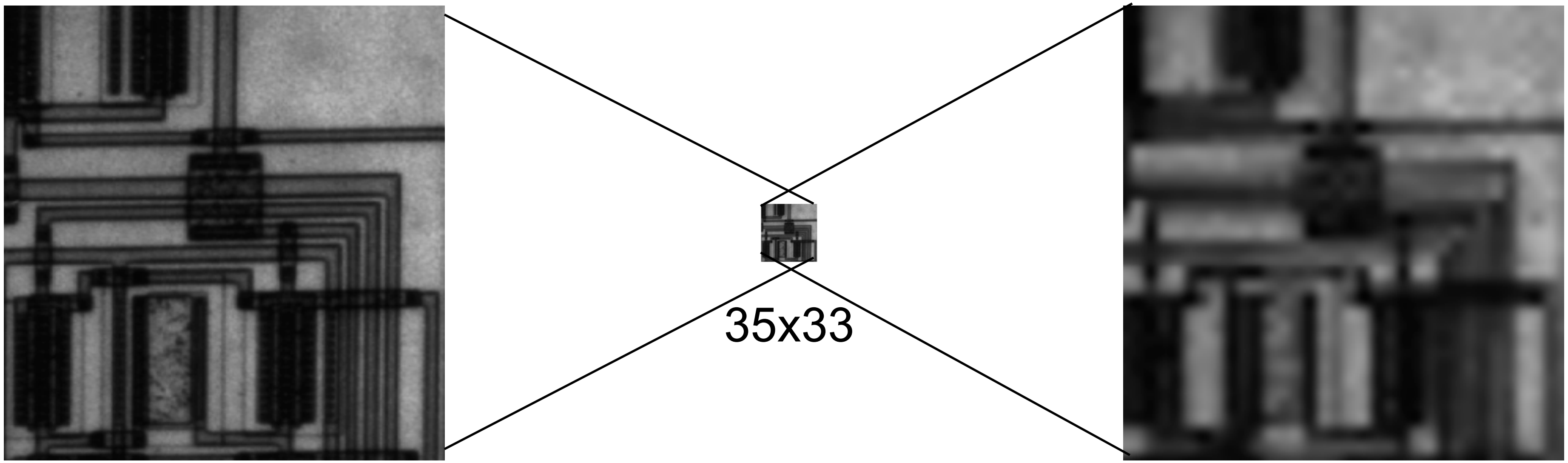

256x256


128x128


64x64


32x32

Fewer Pixels Mean Lower Spatial Resolution



35x33

280x272

35x33
Interpolated to
280x272

Color Components

- Color images typically represented by three values per sample location, for red, green and blue primary components

$$x_R(x,y), \quad x_G(x,y), \quad x_B(x,y)$$

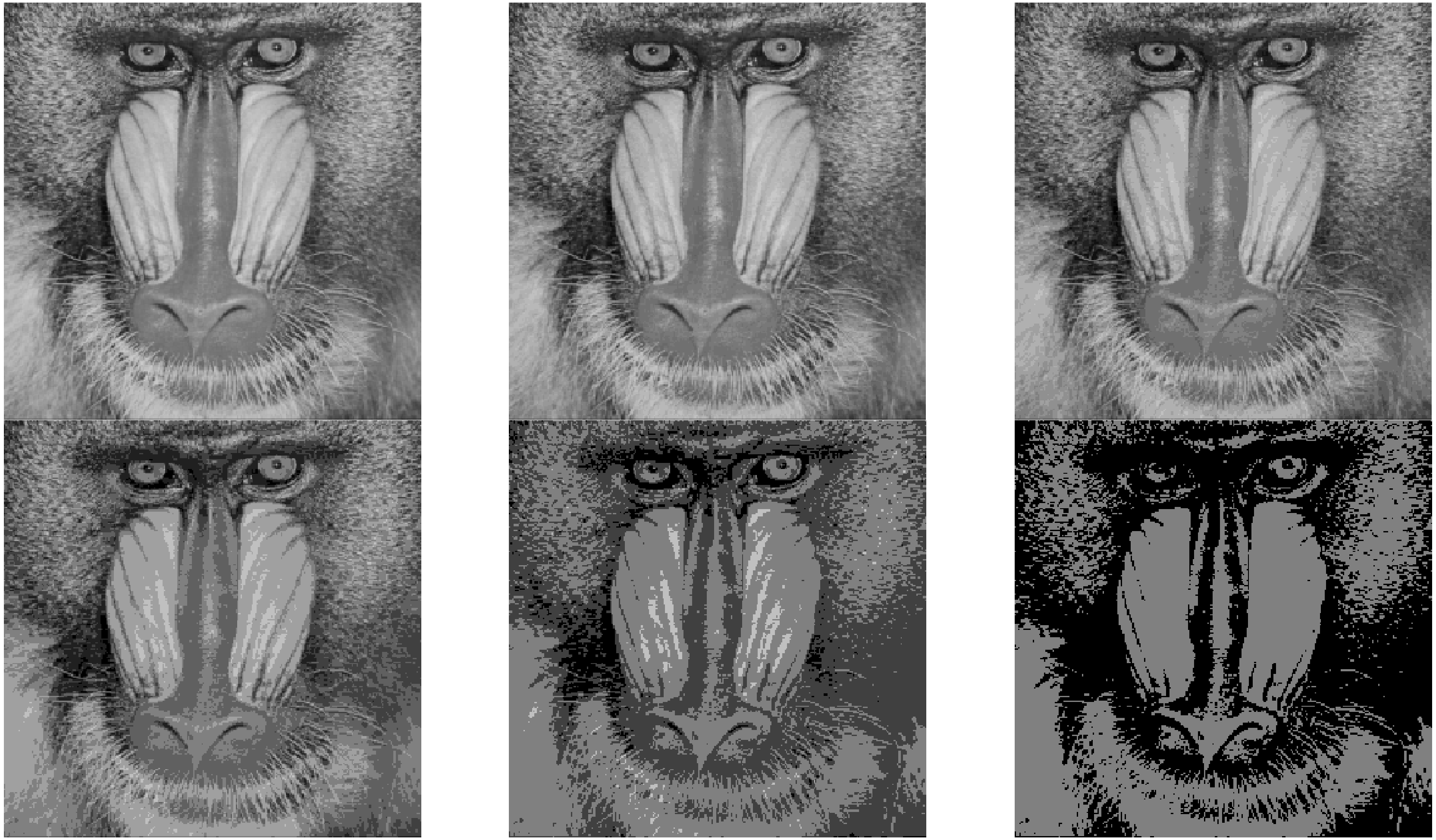
- General multi-component image

$$x_c(x,y), \quad c = 1, 2, \ldots, C$$

- Examples:
  - Color printing: cyan, magenta, yellow, black dyes, sometimes more
  - Hyperspectral satellite imaging: 100s of channels

Different Number of Gray Levels

256 32 16



8 *“Contouring”* 4 2

How Many Gray Levels are Required?

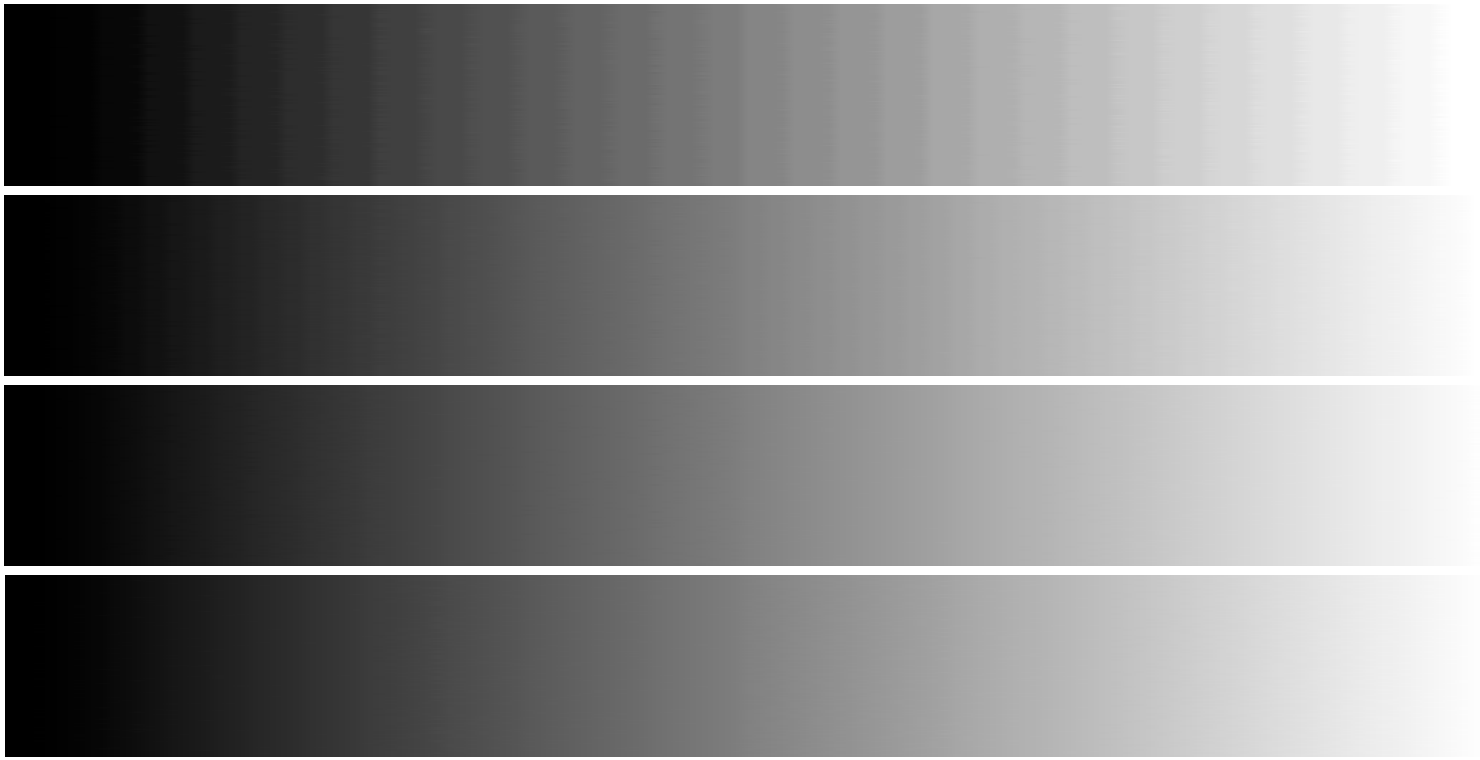
- Contouring is most visible for a ramp

32 levels

64 levels

128 levels

256 levels



- Digital images typically are quantized to 256 gray levels.

Storage Requirements for Digital Images

- Image WxH pixels, 2^B gray levels, c color components

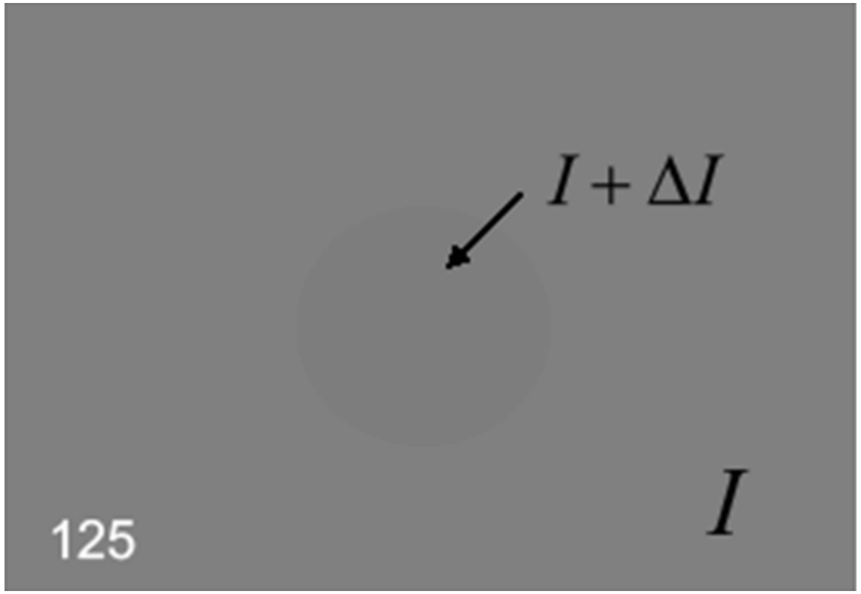
$$\text{Size} = W \times H \times B \times c$$

  - Example: W = H = 512, B=8, c=1 (i.e., monochrome)
    Size = 2,097,152 bits (or 256 kByte)
  - Example: WxH = 1280x1024, B=8, c=3 (24 bit RGB image)
    Size = 31,457,280 bits (or 3.75 MByte)

- Much less with (lossy) compression!

Brightness Discrimination Experiment

- Can you see the circle?



Note: I is luminance, measured in cd/m^2

- Visibility threshold

$$\frac{\Delta I}{I} \approx const. \approx 1 \ldots 2\%$$

"Weber fraction"
"Weber's Law"

Contrast with 8 Bits According to Weber's Law

- Assume that the luminance difference between two successive representative levels is just at visibility threshold

$$\frac{I_{max}}{I_{min}} = (1 + const.)^{255}$$

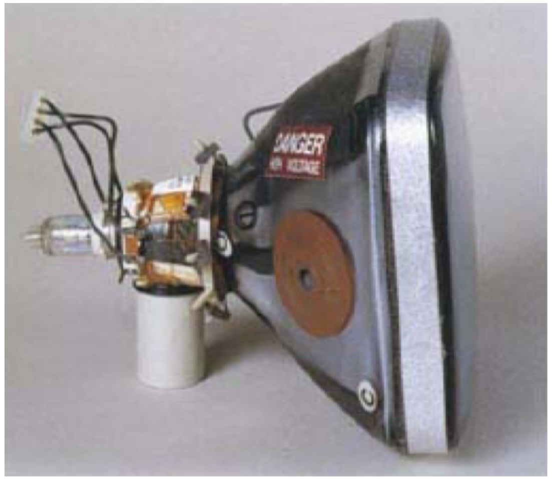
- For

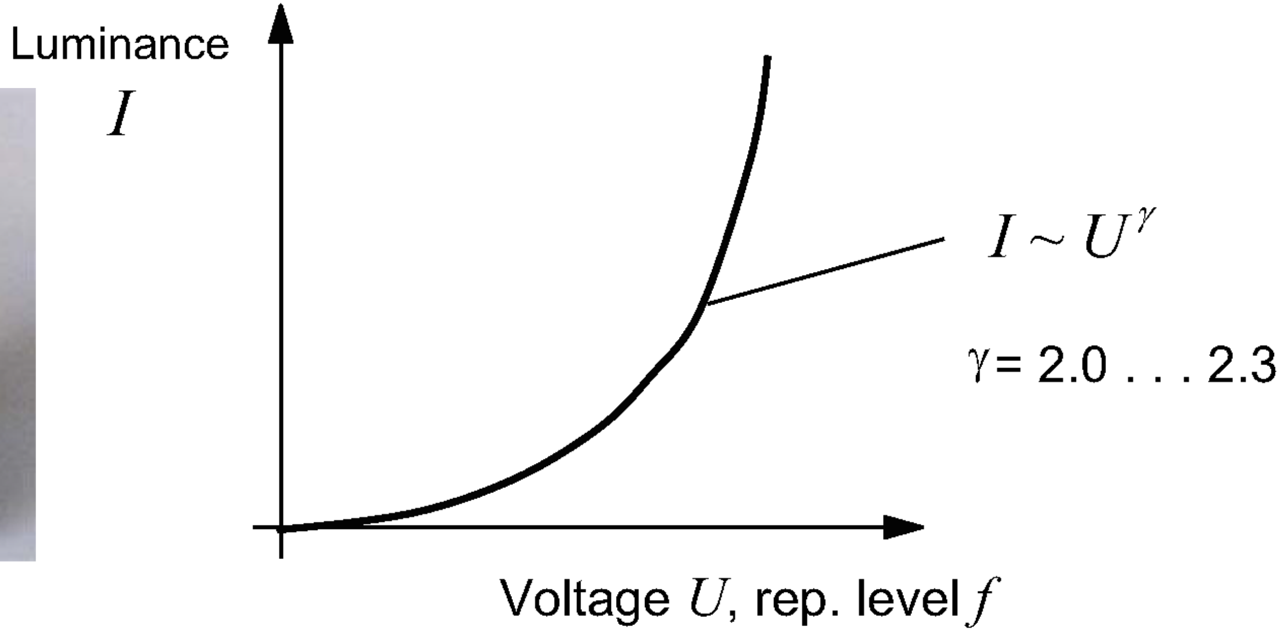
$$const. = 0.01 \ldots 0.02 \qquad \frac{I_{max}}{I_{min}} = 13 \ldots 156$$

- Typical display contrast
  - Cathode ray tube 100:1
  - Print on paper 10:1
- Suggests uniform quantization in the log(I) domain

Gamma Characteristic

- Cathode ray tubes (CRT) are nonlinear

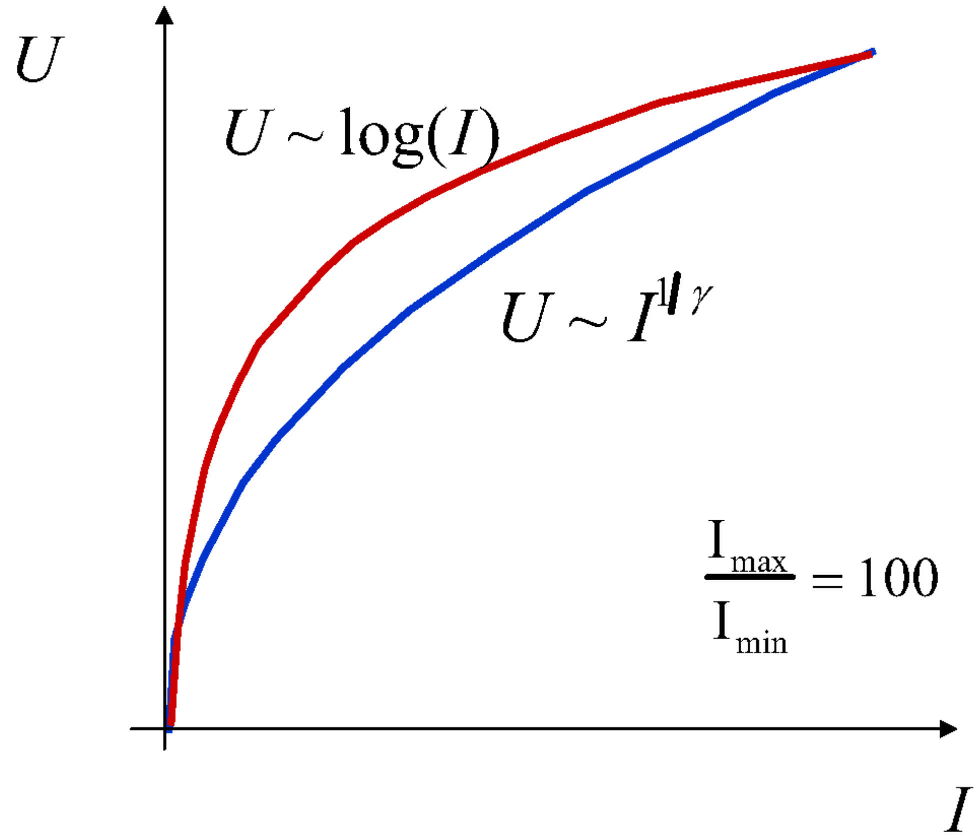




- Cameras contain γ–predistortion circuit

$$U \sim I^{\frac{1}{\gamma}}$$

Log vs. γ-predistortion



- Similar enough for most practical applications