EQ2330 – Image and Video Processing

Assignment #4

The following preparation assignment is to be solved before the next exercise session indicated by the due date of the assignment. You bring your solution to the exercise session and one of your peers will correct it during that session. After that you will discuss the correction with your peers and resolve any open questions. If necessary, the teaching assistant can help you. It is required to solve all the assignments and correct at least one peer solution of each assignment in order to pass the course.

Problem

This problem discusses image enhancement by means of filtering in spatial and frequency domains. Consider the discrete image of *Cameraman* that is degraded by additive noise in Figure 1. Your task is to design an image enhancement algorithm that will reduce the noise. The original (clean) image is not available. The only information you have is that the noise is statistically independent of the original image and modelled as white Gaussian noise with a zero mean and variance $\sigma_n^2 = 25$.

Figure 1: Degraded image of *Cameraman*.

1. Propose the simplest spatial filter you may think of that can reduce the noise in the image.

2. You would like to utilize all the information you have about the noise and design an *MSE-optimal* filter. Propose a filtering technique you would use as well as a drawing of the system model. Let s denote the clean image and f the noisy observation that is degraded by additive Gaussian noise n. Denote the transfer function of the filter you use by $H(\omega_x, \omega_y)$.

3. Assume that the additive white Gaussian noise has zero mean and variance $\sigma_n^2 = 25$. Write down the autocorrelation function of that noise. Compute the power spectral density of the noise.

4. Assume that the power spectral density of the image s is $\Phi_{ss}(\omega_x, \omega_y)$. Use your system model and derive the expression for the cross spectral density $\Phi_{sf}(\omega_x \omega_y)$. How is it related to $\Phi_{ss}(\omega_x, \omega_y)$?

5. Let us define the following error signal $e = g - s$, where g is the enhanced image after filtering with the restoration filter H. What is the power spectral density of the estimation error $\Phi_{ee}(\omega_x \omega_y)$?

6. The power spectral density of the estimation error is minimized at given (ω_x, ω_y) by the following Wiener filter:

$$H(\omega_x, \omega_y) = \frac{\Phi_{sf}(\omega_x, \omega_y)}{\Phi_{ff}(\omega_x, \omega_y)}.$$

   Find a formulation of this filter in terms of $\Phi_{ss}(\omega_x, \omega_y)$ and $\Phi_{nn}(\omega_x, \omega_y)$.

7. Consider $H(\omega_x, \omega_y)$. What kind of filter is it? Compare to the simple spatial filter you proposed in the beginning.