EQ2330 – Image and Video Processing

Solution #2

Solution

1. Averaging filter and weighted averaging filter are linear filters and can be implemented by *convolving* the image with a spatial filter mask.

   Example:

   Averaging filter $\mathbf{h}_1 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

   Weighted averaging filter $\mathbf{h}_2 = \frac{1}{12}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   A down-sampling filter (2:1 image size reduction) is specified by the following matrices $\mathbf{H}_x = \mathbf{H}_y = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{bmatrix}$. Let $\mathbf{f}$ be a 4×4 matrix representing the image. The down-sampling operation may be represented by matrix multiplication according to $\mathbf{g} = \mathbf{H_y}^T \mathbf{f} \mathbf{H_x}$. It is a linear operation.

2. The output for unweighed averaging is simply $x = \frac{1}{9}(73 + 172 + 59 + 166 + 170 + 169 + 165) \approx 108$. The downsampling operation may be implemented by using the down-sampling method from the previous problem. We use $\mathbf{H}_x$ and $\mathbf{H}_y$ to obtain the following size-reduced image (pixel values are rounded to integers)

$$\begin{bmatrix} 72 & 74 \\ 88 & 125 \end{bmatrix}. \tag{1}$$

3. The mask can be (for example) separated into two masks as

$$\frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \tag{2}$$

or

$$\frac{1}{9}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}. \tag{3}$$

It is easily seen that the pairs of $1-D$ masks give identical result as the original 2−D mask.

4. By using the original mask of size $n \times n$, we use n^2 multiplications, $n^2 - 1$ additions, and one additional multiplication for scaling. With the decomposed mask, we use $2n$ multiplications, $2(n-1)$ additions, and one additional multiplication for scaling. This gives us a reduced computational complexity of $n^2 - 2n$ multiplications (which are generally much more time consuming than additions) and $n^2 - 2n - 3$ additions.

5. A 2−D impulse response is separable if it can be written as the following product:

$$h(x, \alpha, y, \beta) = h_x(x, \alpha) h_y(y, \beta) \tag{4}$$

A 2−D mask can be separated into two 1−D masks if it can be written as an outer product of two vectors.

6. By a repetitive application of the averaging filter to an image, we obtain an image with constant pixel values. The pixel value is the same as the mean pixel value of the original image.

7. *Solution 1* We may notice that the mean value of $\mathbf{G}$ is 0. We may use the frequency argument to tell what is the mean pixel value of the result. Since mean value of $\mathbf{G}$ is 0, the DC component of its discrete Fourier spectrum is also 0. We know that convolution of image and mask in the spatial (signal) domain is equivalent to multiplication of corresponding discrete Fourier spectra. After performing such a multiplication, the DC component of the discrete Fourier spectrum of the result is 0, therefore the mean pixel value of the resulting image is also 0.
Solution 2 We may write any image as a sum of two images. In particular, the first image may have all its pixel values constant and equal to the mean of the original image, the second image can be simply the mean-removed original image. We use the argument of linearity of convolution. We apply the convolution to a sum of images. We note that it is equivalent to summing the results of convolving the filter mask and the first image convolving the filter mask with the other image. We note that the result of first convolution is an image with all pixels equal to 0 (neglecting the boundary effect). The second convolution results in an image with mean equal to zero. Therefore, the mean value of the result (after summation of the two images) is 0.