EQ2330 – Image and Video Processing

Exercise #5: Unitary Transforms and Multiresolution Processing

Most of the problems are taken from R. C. Gonzales and R.E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. Consider a zero-mean Gaussian-distributed random vector with covariance matrix

$$E[XX^T] = \begin{bmatrix} 4 & \sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}.$$

Find the Karhunen-Loeve transform.

2. **Problem 7.9**

(a) Compute the Haar transform of 2×2 image:

$$\mathbf{F} = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}.$$

(b) The inverse Haar transform is $\mathbf{F} = \mathbf{H}^T\mathbf{T}\mathbf{H}$, where $\mathbf{T}$ is the Haar transform of $\mathbf{F}$ and $\mathbf{H}^T$ is the matrix inverse of $\mathbf{H}$. Show that $\mathbf{H}_2^{-1} = \mathbf{H}_2^T$ and use it to compute the inverse Haar transform of the result in (a).

3. **Exam March 2010** *Unitary Transforms*

Unitary transforms are commonly used in image coding since they have many useful properties. A feature of unitary transforms is that they rotate the signal space.

(a) Consider a 2×2 transform rotating the signal space by an angle θ. The transform is represented by the following rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Show that the rotating transform is unitary.

(b) The Haar transform is often used in the context of image coding. The main feature of the Haar transform is that its basis functions are the simplest orthonormal wavelets. The 2×2 Haar transformation matrix is given by:

$$\mathbf{H}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Determine a rotation angle θ corresponding to the given Haar transform.

(c) The DCT is another unitary transform commonly used in image coding. A $N \times N$ DCT transform matrix is given by

$$A = \begin{bmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{bmatrix},$$

where

$$a_{kl} = \begin{cases} \sqrt{\frac{1}{N}} & k = 1,\ 1 \le l \le N \\ -\sqrt{\frac{2}{N}} \cos\left(\frac{(2(l-1)+1)(k-1)\pi}{2N}\right) & 2 \le k \le N,\ 1 \le l \le N \end{cases}.$$

Consider the DCT for $N = 2$ and determine the corresponding rotation angle?

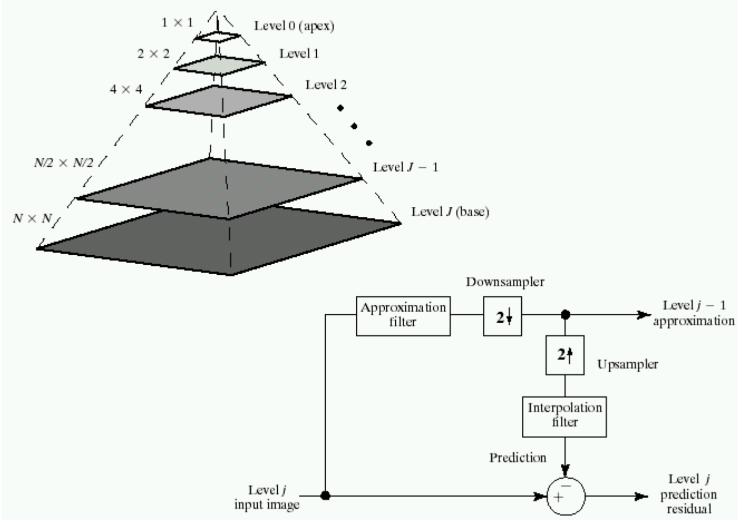
(d) The KLT is a transform that diagonalizes a covariance matrix. Suppose that a covariance matrix is given by

$$C = \frac{1}{4} \begin{bmatrix} 5 & \sqrt{3} \\ \sqrt{3} & 7 \end{bmatrix}.$$

Find the KLT that diagonalizes C. Determine the corresponding rotation angle. *Hint: To save time on computations, use the information that one eigenvalue of the covariance matrix is 1.*

(e) What is the main difference in how the rotation is performed for the KLT and the DCT?

4. **Problem 7.1**
Design a system for decoding the prediction residual pyramid generated by the encoder of Fig. 7.2(b) and draw its block diagram. Assume there is no quantization error introduced by the encoder.



a b

FIGURE 7.2 (a) A pyramidal image structure and (b) system block diagram for creating it.

5. **Problem 7.2**
Construct a fully populated approximation pyramid and corresponding prediction residual pyramid for the image

$$f(x,y) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

Use 2×2 block neighborhood averaging for the approximation filter in Fig. 7.2(b) and assume the interpolation filter is omitted.