EQ2330 – Image and Video Processing

Solution #1

Solution

Assume that the probability density function for the gray-levels is given by

$$f_X(x) = \frac{1}{|b|}\sin(a\pi x), \quad x \in [0,1] \tag{1}$$

where a and b are constants.

1. Parameters a and b that ensure that equation (1) is a pdf can be derived by using the fact that the pdf is non-negative and integrates to one over its support region, here $x \in [0,1]$. Therefore $0 < a \leq 1$, and

$$|b| = \frac{1 - cos(a\pi)}{a\pi}.$$

2. There can be many answers, but motivation should be provided. For instance we can notice that the picture contains mostly mid-shades of gray, and extreme values of pixels are rare. Therefore we can say that $a = 1$. Using the result from the previous question, we can compute the corresponding value $|b| = \frac{2}{\pi}$.

3. In this question we consider a transform function that equalizes a continuous histogram. The explicit expressions for y can be computed directly as

$$y = T(x) = \int_0^x f_X(\tau)\, d\tau = \frac{1}{|b|a\pi}\left(1 - cos(\pi a x)\right).$$

   Since the transform equalizes continuous histogram the probability density function of the processed image is given by

$$f_Y(y) = \begin{cases} 1, & \text{for } y \in [0,1], \\ 0, & \text{otherwise} \end{cases}$$

4. We find the probability mass function by integrating the pdf over the quantization cells $p_{\bar{X}}(\bar{x})$. Let $a = 1$ and $|b| = \frac{2}{\pi}$ The pmf is of form

$$\begin{aligned}
P_{\frac{1}{8}} &= \int_0^{\frac{1}{4}} \frac{\pi}{2} sin(\pi x) dx = \frac{1}{2} - \frac{1}{4}\sqrt{2}, \\
P_{\frac{3}{8}} &= \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{\pi}{2} sin(\pi x) dx = \frac{1}{4}\sqrt{2}, \\
P_{\frac{5}{8}} &= \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{\pi}{2} sin(\pi x) dx = \frac{1}{4}\sqrt{2}, \\
P_{\frac{7}{8}} &= \int_{\frac{3}{4}}^{1} \frac{\pi}{2} sin(\pi x) dx = \frac{1}{2} - \frac{1}{4}\sqrt{2}.
\end{aligned}$$

It is sufficient to compute only the first two probabilities due to symmetry and choice of uniform quanitzer.

5. Histogram equalization is applied to the quantized image, using the transform

$$z = H(\bar{x}) = \sum_{\tau \in \{0,\ldots,\bar{x}\}} p_{\bar{X}}(\tau). \tag{2}$$

Using the result from the previous question we can compute explicit expressions for the values that z can attain:

$$\begin{aligned}
H(\frac{1}{8}) &= P_{\frac{1}{8}} = \frac{1}{2} - \frac{1}{4}\sqrt{2}, \\
H(\frac{3}{8}) &= P_{\frac{1}{8}} + P_{\frac{3}{8}} = \frac{1}{2}, \\
H(\frac{5}{8}) &= P_{\frac{1}{8}} + P_{\frac{3}{8}} + P_{\frac{5}{8}} = \frac{1}{2} + \frac{1}{4}\sqrt{2}, \\
H(\frac{7}{8}) &= P_{\frac{1}{8}} + P_{\frac{3}{8}} + P_{\frac{5}{8}} + P_{\frac{7}{8}} = 1.
\end{aligned}$$

and the probability mass function $p_Z(z)$ will have the same probabilities as $p_{\bar{X}}(\bar{x})$, because we have still 4 possible values of the pixels (4 bins) and the number of pixels belonging to each bin is unchanged. Being more precisely, we can see that the values of the pixels are values are changed, but not the quantity of pixels belonging to each of the four bins. This effect happens because all the bins of the discrete histogram are occupied and the equalization function cannot create new bins. This is different than in the continuous case, where the histogram after equalization is uniform.