EQ2330 – Image and Video Processing

Solution #9

Solution

1. Let the quantization cells be indexed by $i \in \{0, 1, \ldots, I-1\}$ with centroids $\hat{x}_i$. A value x is mapped to cell index i if $x \in [b_i, b_{i+1})$, where t_i are quantization thresholds. Define $t_0 = -\infty$ and $t_I = \infty$.

Step 1.	Initiate the centroids $\hat{x}_i$ arbitrarily. For example, distribute the centroids uniformly over the support of $f(x)$.
Step 2.	Update the quantization thresholds according to $$t_i = \frac{1}{2}(\hat{x}_{i-1} + \hat{x}_i), \quad i \in \{1, 2, \ldots, I-1\}.$$
Step 3.	Update the cell centroids according to $$\hat{x}_i = \frac{\int_{t_i}^{t_{i+1}} x f(x)\ \mathrm{d}x}{\int_{t_i}^{t_{i+1}} f(x)\ \mathrm{d}x}, \quad i \in \{0, 1, \ldots, I-1\}.$$
Step 4.	If expected distortion has not converged continue with Step 2.

Table 1: The Lloyd-Max Algorithm.

2. Figure shows the centroids and thresholds after 0, 1, 2, 3, 4 and 9 iterations.

3. The average codeword length is 2 bits, as implied by the name "2-bit Lloyd-Max quantizer".

4. The probability that a realization of X belongs to cell i is given by
$$p_i = \int_{t_i}^{t_{i+1}} f(x)\ \mathrm{d}x. \tag{1}$$

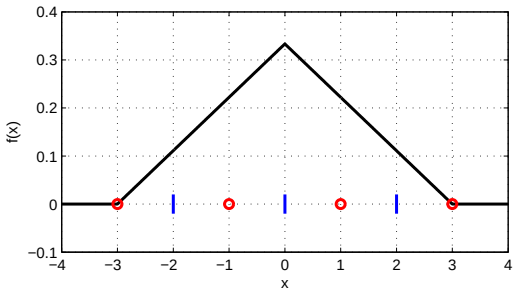
5. The variance of X is 1.5, while the variance of $\hat{X}$ is 1.36. They are not equal as the difference of X and $\hat{X}$ (the quantization noise) is not correlated with $\hat{X}$ and has non-zero variance. The difference between the variances is called the distortion.

6. The mean squared error contribution of the cells evaluates to 0.036, 0.034, 0.034, 0.036. These values show the (approximate) equidistortion principle for Lloyd-Max quantizers.

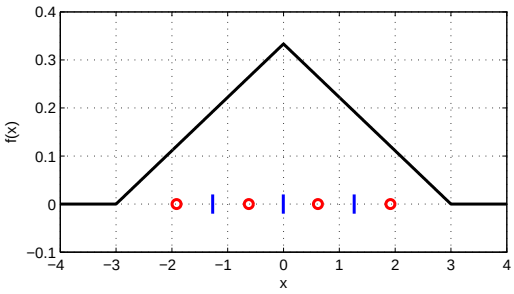
7. The entropy evaluates to 1.96 bits.

8. A Huffman code gives the average codeword length 2.0 bits.

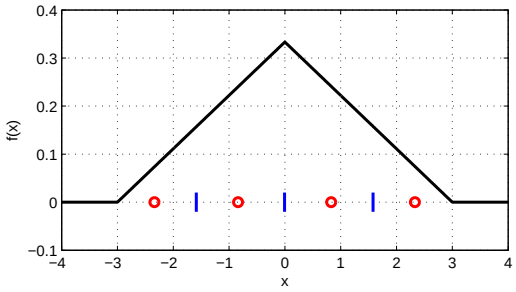
9. The Lloyd-Max algorithm for quantizer design is based on necessary conditions for rate-distortion optimality under the constraint that the same codeword length is used for all symbols. Hence, we expect better performance if the quantizer is optimized for a system with an entropy coder.



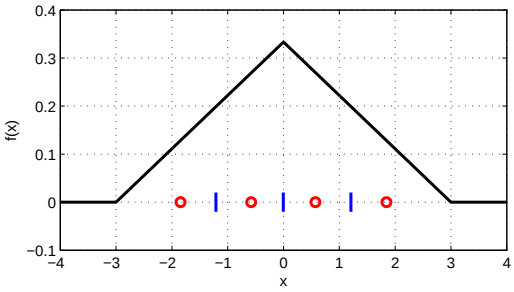
(a) 0 iterations.



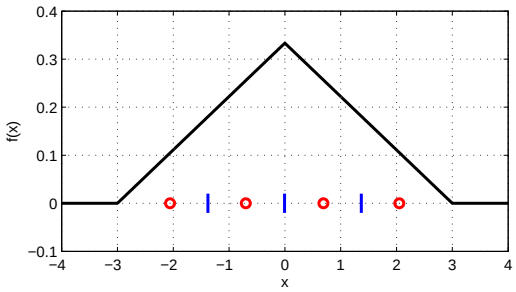
(b) 3 iterations.



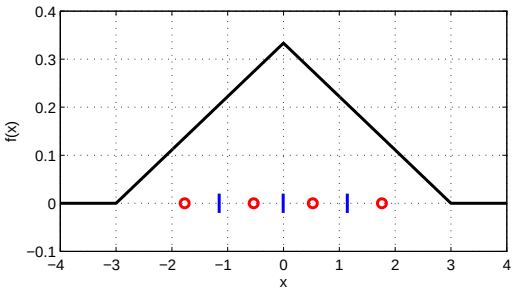
(c) 1 iteration.



(d) 4 iterations.



(e) 2 iterations.



(f) 9 iterations.