EQ2330 – Image and Video Processing

Solution #6

Solution

(a) Yes, two input samples $x(2n)$ and $x(2n+1)$ give two output samples $l(n)$ and $h(n)$.

(b) From the figure in the exam we get that

$$h(n) = \frac{1}{\sqrt{2}}\Big(x(2n+1) - x(2n)\Big)$$
$$l(n) = \sqrt{2}\Big(x(2n) + \frac{1}{2}\sqrt{2}h(n)\Big)$$
$$= \frac{1}{\sqrt{2}}\Big(x(2n+1) + x(2n)\Big)$$

In matrix form we have

$$\begin{bmatrix} l(n) \\ h(n) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x(2n) \\ x(2n+1) \end{bmatrix} = B \begin{bmatrix} x(2n) \\ x(2n+1) \end{bmatrix}$$

The basis vectors are $b_1 = \frac{1}{\sqrt{2}}[1 \ -1]^T$ and $b_2 = \frac{1}{\sqrt{2}}[1 \ 1]^T$.

(c) $b_1^T b_2 = 0$, $b_1^T b_1 = 1$ and $b_2^T b_2 = 1$.

(d) The lifting implementation of the synthesis filter is shown in Figure 1.

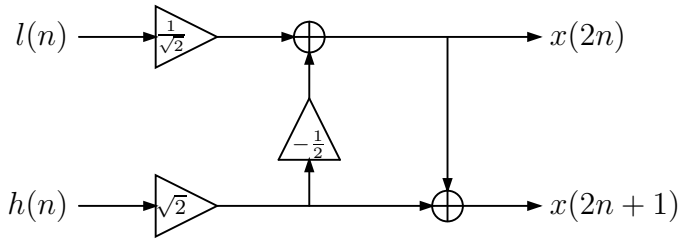

Figure 1: Lifting synthesis filter.

(e) A biorthogonal wavelet allows for perfect reconstruction, which is always achieved with a lifting implementation. As we could implement the Haar wavelet with a lifting structure, it is a biorthogonal wavelet.