Image and Video Processing

Exercise 4: Image Restoration

The problems are taken from R. C. Gonzales and R.E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 5.16** Consider a linear, shift-invariant image degradation system with impulse response

   $$h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}.$$

   Suppose that the input to the system is an image consisting of a line of infinitesimal width located at $x = a$, and modelled by $f(x, y) = \delta(x - a)$, where *delta* is the impulse function. Assuming no noise, what is the output image $g(x, y)$?

2. **Problem 5.18** Consider the problem of image blurring caused by uniform acceleration in the x-direction. If the image is at rest at time $t = 0$ and accelerates with a uniform acceleration $x_0(t) = at^2/2$ for a time T, find the blurring function $H(u, v)$. You may assume that shutter opening and closing times are negligible.

3. **Problem 5.21** A certain X-ray imaging geometry produces a blurring degradation that can be modelled as the convolution of the sensed image with the spatial, circularly symmetric function

   $$h(x, y) = [(x^2 + y^2 - 2\sigma^2)/\sigma^4]e^{-\frac{x^2+y^2}{2\sigma^2}}.$$

   Assuming continuous variables, show that the degradation in the frequency domain is given by the expression

   $$H(u, v) = -8\pi^3\sigma^2(u^2 + v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}.$$

4. **Problem 5.22** Using the transfer function

   $$H(u, v) = -2\pi\sigma^2(u^2 + v^2)e^{-2\pi^2\sigma^2(u^2+v^2)},$$

   give the expression for a Wiener filter, assuming that the ratio of power spectra of the noise and undergraded signal is constant.

5. **Problem 5.23** Using the transfer function

$$H(u,v) = -2\pi\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)},$$

give the resulting expression for the constrained least squares filter.

6. **Problem 5.24** Assume that the model in Figure 1 is linear and shift-invariant and that the noise and image are uncorrelated. Show that:

$$|G(u,v)|^2 = |H(u,v)|^2|F(u,v)|^2 + |N(u,v)|^2.$$

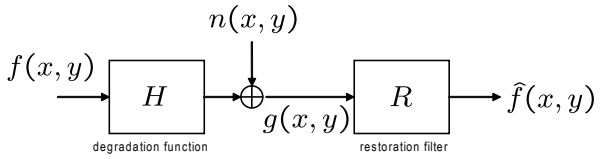

Figure 1: System block diagram for Problem 6.

7. **Problem 5.27** A professor of archeology doing research on currency exchange practices during the Roman Empire recently became aware that four Roman coins crucial to his research are listed in the holdings of the British Museum in London. Unfortunately, he was told after arriving there that the coins recently had been stolen. Further research on his part revealed that the museum keeps photographs of every item for which it is responsible. Unfortunately, the photos of the coins in question are blurred to the point where the date and other small markings are not readable. The cause of the blurring was the camera being out of focus when the pictures were taken. As an image processing expert and friend of the professor, you are asked as a favor to determine whether computer processing can be utilized to restore the images to the point where the professor can read the markings. You are told that the original camera used to take the photos is still available, as are other representative coins of the same era. Propose a step-by-step solution to this problem.