EQ2330 – Image and Video Processing

Assignment 8

The following preparation assignment is to be solved before the next exercise session indicated by the due date of the assignment. You bring your solution to the exercise session and one of your peers will correct it during that session. After that you will discuss the correction with your peers and resolve any open questions. If necessary, the teaching assistant can help you. It is required to solve all the assignments and correct at least one peer solution of each assignment in order to pass the course.

Problem

Consider the design of a quantizer for the random variable X, with the probability density function $f(x)$ in Figure 1.

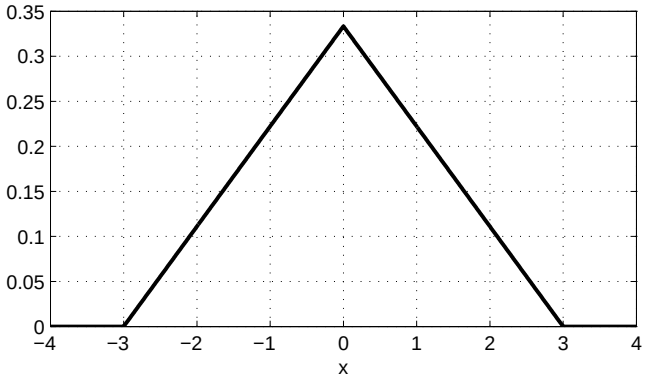

Figure 1: Probability density function $f(x)$.

1. Write down the Lloyd-Max algorithm.
2. Starting from 4 uniformly placed quantization cell centroids (reconstruction points), show 4 iterations of the Lloyd-Max algorithm for $f(x)$. For each iteration, plot a new figure with $f(x)$, the cell centroids (represented by dots on the x-axis) and the quantization thresholds (represented by vertical bars on the x-axis).
3. Let the cell indices be coded with a fixed length code. What is the average codeword length for this code? What does the term "2-bit Lloyd-Max quantizer" imply?

4. Write a formula for the probability that a realization of a random variable X will belong to cell i, where $i \in \{0, 1, 2, 3\}$.

5. Calculate the variance of the source X as well as the variance of the representative levels $\hat{X}$. Why are the variances not identical? What name is commonly used for the difference of these variances?

6. Calculate the mean squared error contribution of each cell (weighted by its probability) and compare these contributions. What do you observe?

7. The Lloyd-Max algorithm was implemented and used to train a quantizer for X. The probability of the cells after convergence evaluated to 0.19, 0.31, 0.31, 0.19. Calculate the entropy of the quantization cell indices.

8. Design a Huffman code for the quantization cell indices. What is the average codeword length of the Huffman code?

9. Using entropy coding after Lloyd-Max design has the potential of giving smaller average codeword length than the code in 3. Now assume that the entropy code design is integral part of the quantizer design. Do you expect an improved rate-distortion performance? Motivate your argument.