EQ2330 – Image and Video Processing

Solution #12

Solution

1. We find the exact expressions for $f_{bg}(x)$ and $f_{fg}(x)$, by using the fact that both functions should integrate to one over their regions of support. We can notice from the plot that $f_{bg}(x)$ is given by following the equation $f_{bg}(x) = -ax + b$, since it should integrate to one over $x \in [0, 128]$, we have that $1 = \frac{1}{2}128b \Rightarrow b = \frac{1}{64}$. Knowing b we can find a by evaluating $f_{bg}(x)$ for $x = 128$. Since $f_{bg}(128) = 0 \Rightarrow a = 64 \cdot 128$ and finally

$$f_{bg}(x) = \begin{cases} \frac{1}{64} - \frac{1}{64 \cdot 128}x, & x \in [0, 128], \\ 0, & \text{otherwise} \end{cases}$$

   For $f_{bg}(x)$ it is also required to integrate to one. Using this fact we can find that

$$f_{fg}(x) = \begin{cases} \frac{1}{128}, & x \in [64, 192], \\ 0, & \text{otherwise} \end{cases}$$

2. We want to select the threshold x_T to minimize the probability of misclassification. For the limiting cases, we have

   - $P_{bg} \to 0$ we have that $x_T = 64$,
   - $P_{fg} \to 0$ we have that $x_T = 128$.

3. An optimal threshold x_T should minimize the probability of erroneous pixel classification. Considering x_T in the interval $[64, 128]$, the optimal threshold can be found by using

$$P_{bg} f_{bg}(x_T) = P_{fg} f_{fg}(x_T). \tag{1}$$

   This, in combination with $P_{bg} + P_{fg} = 1$, gives

$$x_T = 128 - 64\frac{P_{fg}}{1 - P_{fg}}, \text{ for } 0 \leq P_{fg} \leq \frac{1}{2}. \tag{2}$$

   Note that equation (2) gives x_T outside the interval $[64, 128]$ for certain values of P_{fg}. This should be commented!

4. Further steps in improving your image segmentation algorithm could include for instance local thresholding or more sophisticated segmentation techniques than thresholding.