EQ2330 – Image and Video Processing

Exercise 8: Compression

Unless stated otherwise, the problems are from on R. C. Gonzales and R. E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 8.11, modified** The rate distortion function of a zero-memory Gaussian source of arbitrary mean and variance σ^2 with respect to the mean-square error criterion is
$$R(D) = \begin{cases} \frac{1}{2}\log\frac{\sigma^2}{D} & 0 \leq D \leq \sigma^2 \\ 0 & D \geq \sigma^2. \end{cases} \tag{1}$$
   (a) Plot this function.
   (b) What is D_{max}?
   (c) Show that increasing/decreasing 1 bit decreases/increases the distortion by 6 dB.
   (d) If distortion of no more than 75% of the source's variance is allowed, what is the maximum compression that can be achieved.
   (e) Two independent zero-memory Gaussian sources with variances σ_1^2 and σ_2^2 are to be quantized. The total distortion is defined as the sum of their individual mean-square errors. The total rate is defined as the sum of their individual rates, say R_1 and R_2. Given a total rate budget $\bar{R}$, find the R_1 and R_2 that minimize the total distortion.

2. **Uniform quantization** A random variable has the following probability density function
$$f_X(x) = \begin{cases} a & -1.5 \leq x < -0.5 \\ 2a & -0.5 \leq x < 0.5 \\ 4a & 0.5 < x \leq 1.5. \end{cases} \tag{2}$$
   It is quantized by
$$Q_X(x) = \begin{cases} -1 & -\infty < x < -0.5 \\ 0 & -0.5 \leq x < 0.5 \\ 1 & 0.5 \leq x < \infty. \end{cases} \tag{3}$$

(a) What is a?

(b) Plot the quantizer function.

(c) Calculate the mean-square error of the quantization on the source.

(d) Redo the calculation with another random variable, which has this probability density function

$$f_X(x) = 2, -0.25 \leq x \leq 0.25. \tag{4}$$

3. **Problem 8.21, modified** Derive the 1-bit Lloyd-Max cell centroids and quantization thresholds for the uniform distribution density function

$$p(s) = \begin{cases} \frac{1}{2A} & -A \leq s \leq A \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

4. **Lloyd-Max Scalar Quantizer** You are supposed to design a 2-bit scalar quantizer that uses a squared error criterion. Consider the experimental data $\{0, 0, 1, 2, 2, 6, 6, 6, 8, 8, 8\}$. You use as initial condition for your codebook $\{0, 1, 2, 3\}$.

(a) Apply the Lloyd algorithm to train the above codebook with the given data.

(b) In what sense is the solution of the Lloyd algorithm optimal?

(c) Compute the total error for the solution you found above. Is the initial condition a good one and why? Can you define a better one?