EQ2330 – Image and Video Processing

Assignment 10

The following preparation assignment is to be solved before the next exercise session indicated by the due date of the assignment. You bring your solution to the exercise session and one of your peers will correct it during that session. After that you will discuss the correction with your peers and resolve any open questions. If necessary, the teaching assistant can help you. It is required to solve all the assignments and correct at least one peer solution of each assignment in order to pass the course.

Problem

Consider the encoding of I blocks in a video frame. Let R_i denote the rate assigned to the encoding of block i. Further, let $D_i(R_i)$ denote the operational distortion-rate function for block i. We are interested in solving the following constrained optimization problem

$$\begin{array}{cl} \min\limits_{R_i, i \in \{1,2,\ldots,I\}} & \sum_{i=1}^{I} D_i(R_i) \\ \text{s.t.} & \sum_{i=1}^{I} R_i \leq \bar{R}. \end{array} \tag{1}$$

1. Write down the corresponding unconstrained optimization problem, i.e., the Lagrangian.
2. Derive the relation between $\frac{dD_i}{dR_i}$ and $\frac{dD_j}{dR_j}$ for all i and j ($i, j \in \{1, 2, \ldots, I\}$) that ensures an optimal rate allocation.
3. Figure 1 illustrates example functions $D_1(x)$ and $D_2(x)$ and their derivatives. Using your results from question 2, illustrate in the figure how the optimal rate-allocation is performed for two rate constraints: $\bar{R} = 1$ and $\bar{R} = 1.68$.
4. Write down an iterative algorithm for the practical implementation of rate-allocation. Will your algorithm converge? *Hint: Begin with simplifying the Lagrangian from question 1 by expressing the minimization as a sum of minimizations. Hint: Use the convexity of the operational distortion-rate function.*

Now consider the encoding of block i in a simple hybrid video coder, using rate R_i. Assume that block i is encoded independently of other blocks in the video frame. We are interested

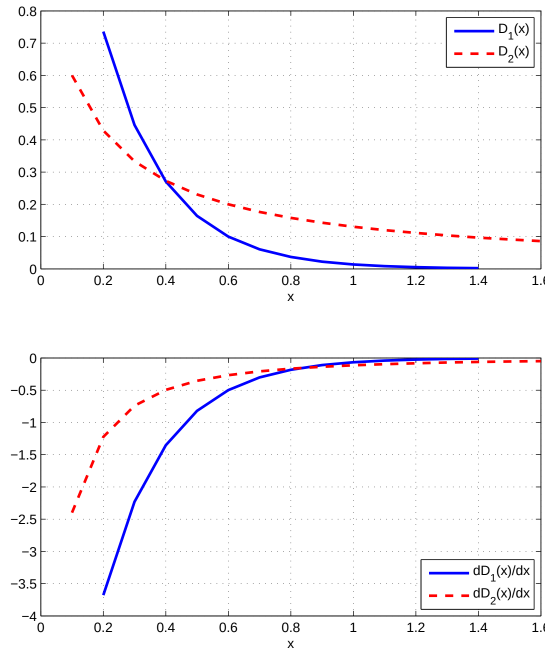

Figure 1: Example $D_1(x)$, $D_2(x)$, $\frac{dD_1(x)}{dx}$ and $\frac{dD_2(x)}{dx}$.

in finding out how much rate r_v should be allocated to code the motion vectors and how much rate r_e should be allocated to code the motion-compensated prediction error. Let D_i be a function of these two rates, according to $D_i(r_v, r_e)$. The constrained optimization problem is

$$\begin{aligned} \min_{r_v, r_e} \quad & D_i(r_v, r_e) \\ \text{s.t.} \quad & r_v + r_e \leq R_i. \end{aligned} \tag{2}$$

5. Write down the corresponding unconstrained optimization problem, i.e., the Lagrangian.
6. Derive the relation between $\frac{\partial D_i}{\partial r_v}$ and $\frac{\partial D_i}{\partial r_e}$ that ensures an optimal rate allocation.
7. Write down an algorithm for the hypothetical search for the optimal rate-allocation. How does it differ conceptually from the algorithm in question 4?