

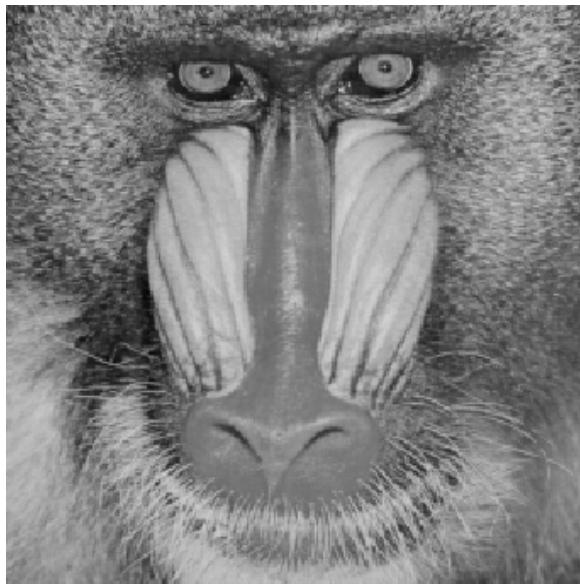
Point Operations on Images

- Intensity scaling
- Gamma adjustment
- Histogram equalization
- Image averaging
- Image subtraction



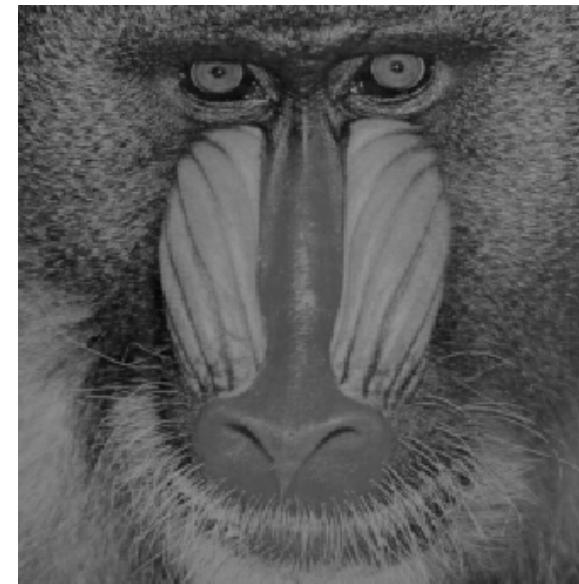
Intensity Scaling

Original image



$$f(x, y)$$

Scaled image



$$a \cdot f(x, y)$$

Scaling in the γ -domain is equivalent to scaling in the linear luminance domain

$$I \sim [a \cdot f(x, y)]^\gamma = a^\gamma \cdot [f(x, y)]^\gamma$$



Adjusting γ

Original image



γ increased by 50%

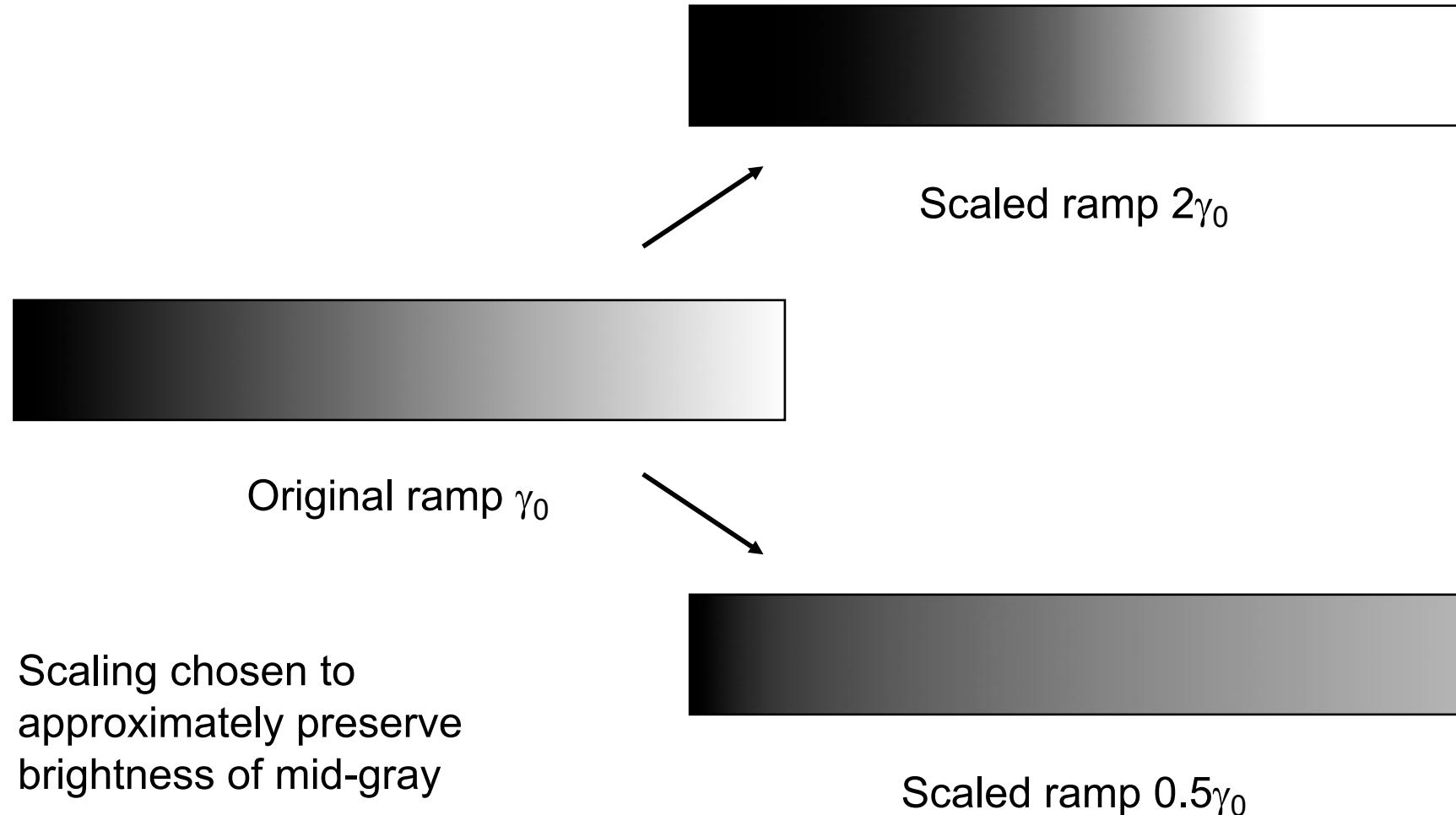


$$f(x, y)$$

$$a \cdot [f(x, y)]^\gamma \text{ with } \gamma = 1.5$$



Changing Gradation by γ -Adjustment

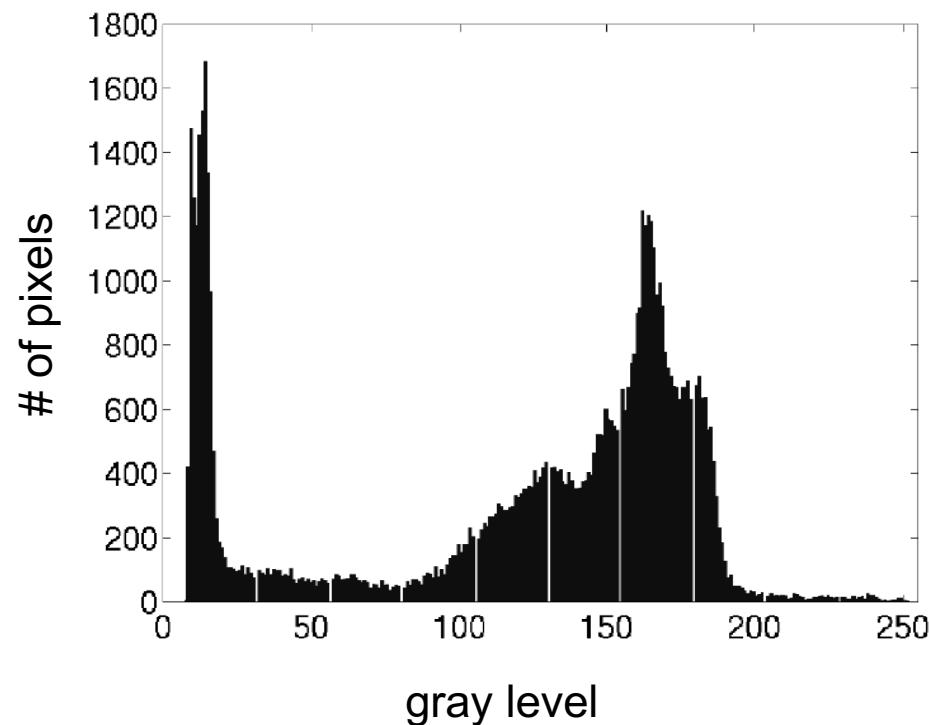


Histograms

- Distribution of gray-levels can be judged by measuring a histogram:
 - For B -bit image, initialize 2^B counters n_i , $i = 1, \dots, 2^B$, with 0
 - Loop over all pixels x, y
 - When encountering gray level $f(x, y) = i$, increment counter n_i
- Histogram can be interpreted as an estimate of the probability density function (pdf) of an underlying random process.
- You can also use fewer (i.e., larger) bins to trade off amplitude resolution against sample size.



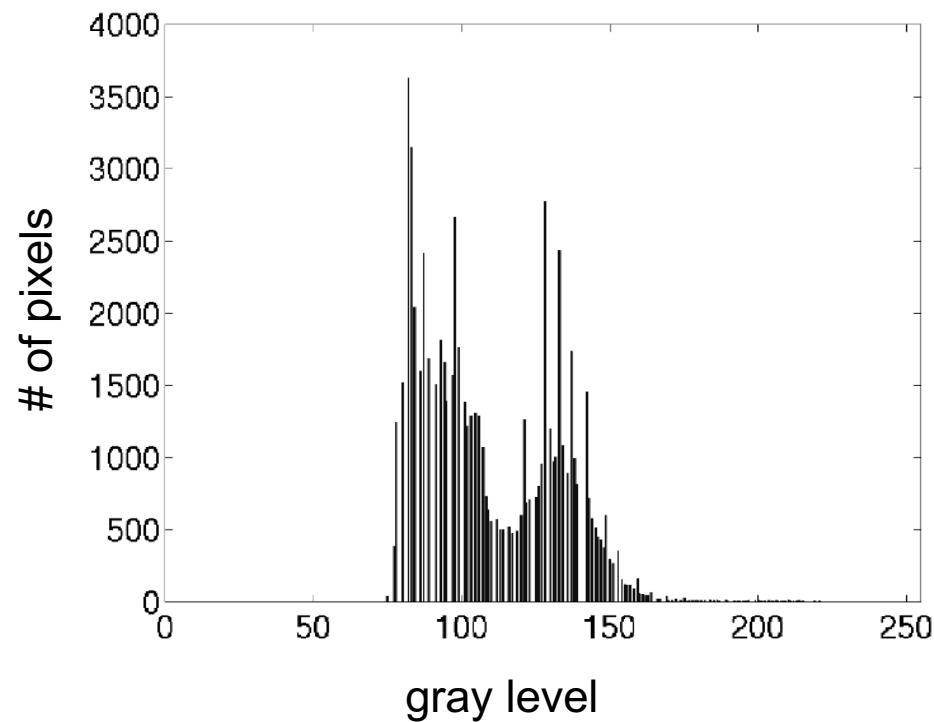
Example Histogram



Cameraman



Example Histogram



Pout



Histogram Equalization

- Idea: find a non-linear transformation

$$g = T(f)$$

to be applied to each pixel of the input image $f(x,y)$, such that a uniform distribution of gray levels in the entire range results for the output image $g(x,y)$.

- Analyze ideal, continuous case first, assuming

- $0 \leq f \leq 1$ and $0 \leq g \leq 1$
- $T(f)$ is strictly monotonically increasing, hence, there exists

$$f = T^{-1}(g) \quad \text{for} \quad 0 \leq g \leq 1$$

- Goal: pdf $p_g(g) = \text{const.}$ over the range



Histogram Equalization for Continuous Case

- From basic probability theory:

$$p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)}$$

- Consider the transformation function

$$g = T(f) = \int_0^f p_f(\alpha) d\alpha \quad \text{for } 0 \leq f \leq 1$$

- Then . . .

$$\frac{dg}{df} = p_f(f)$$

$$p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \quad 0 \leq g \leq 1$$



Histogram Equalization for Discrete Case

- Now, f only assumes discrete amplitude values f_0, f_1, \dots, f_{L-1} with probabilities

$$p_0 = \frac{n_0}{n} \quad p_1 = \frac{n_1}{n} \quad \dots \quad p_{L-1} = \frac{n_{L-1}}{n}$$

- Discrete approximation of $g = T(f) = \int_0^f p_f(\alpha) d\alpha$

$$g_k = T(f_k) = \sum_{i=0}^k p_i$$

- The resulting values g_k are in the range $[0,1]$ and need to be scaled and rounded appropriately.



Histogram Equalization Example



Original image

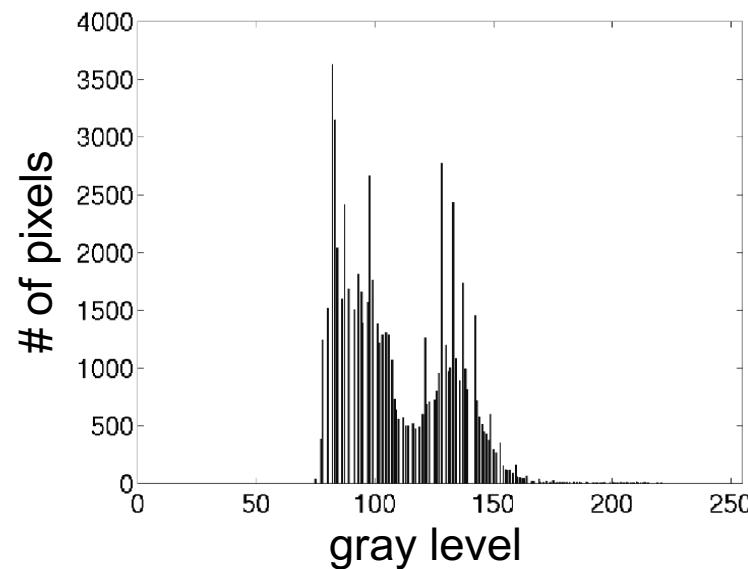


After histogram equalization

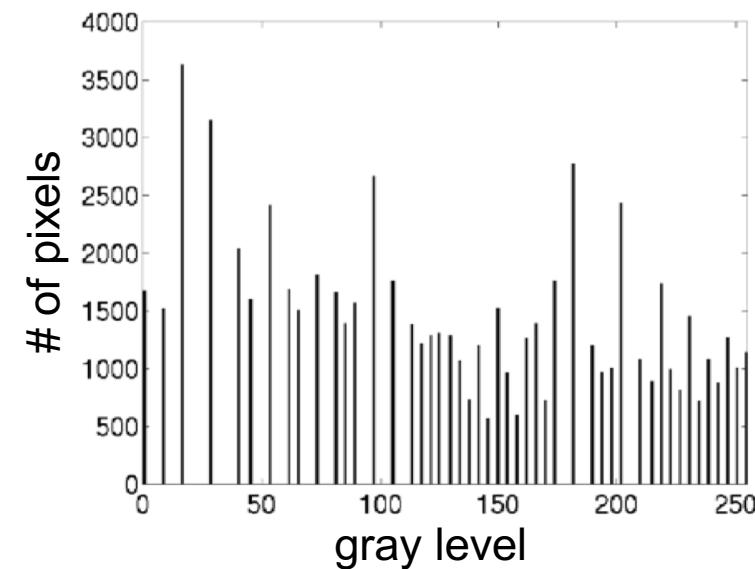


Histogram Equalization Example

Original image



After histogram equalization



Histogram Equalization Example



Cameraman
original image

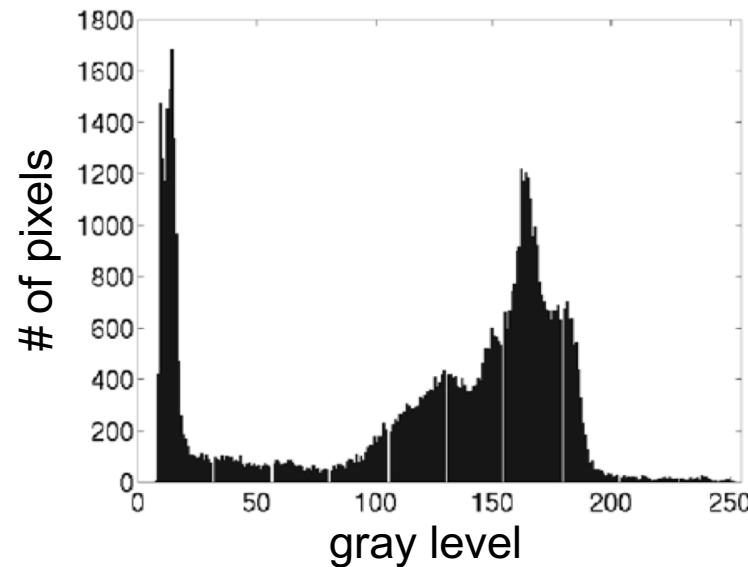


Cameraman
after histogram equalization

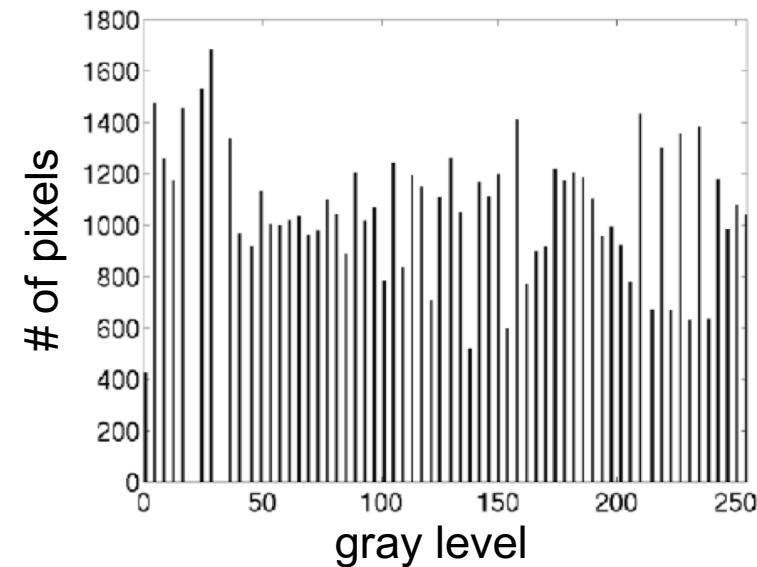


Histogram Equalization Example

Original image



After histogram equalization



Histogram Equalization Example



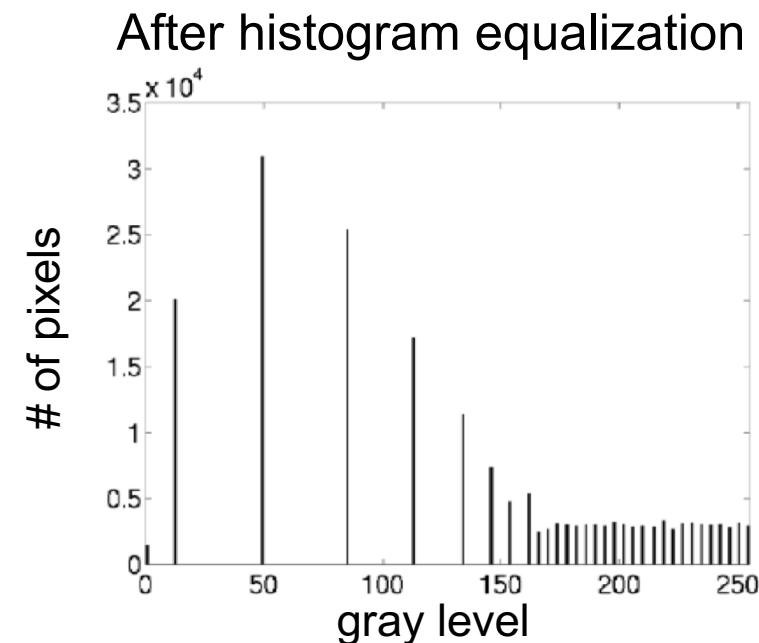
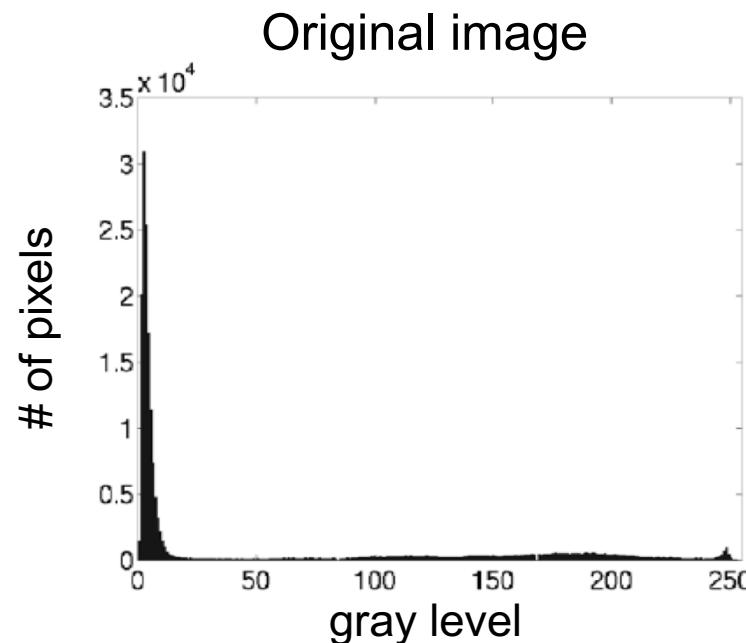
Moon
original image



Moon
after histogram equalization



Histogram Equalization Example

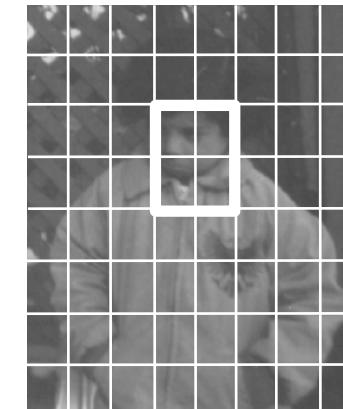


Adaptive Histogram Equalization

- Apply histogram equalization based on a histogram obtained from a portion of the image



Sliding window approach
different histogram (and mapping)
for each pixel



Tiling approach
subdivide into overlapping regions,
mitigate blocking effect by smooth
blending between neighboring tiles

- Must limit contrast expansion in flat regions of the image, e.g. by clipping individual histogram values to a maximum



Adaptive Histogram Equalization



Original



Global histogram



Tiling
8x8 histograms



Tiling
32x32 histogram

Adaptive Histogram Equalization



Original



Global histogram



Adaptive histogram
8x8 tiles

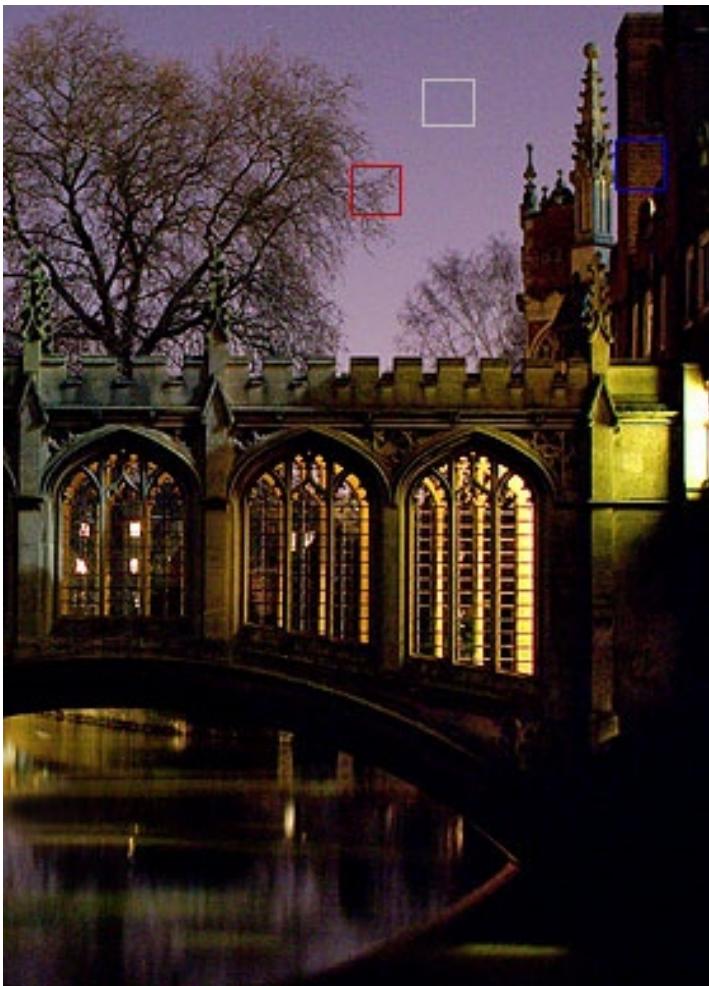


Point Operations Between Images

- Image averaging for noise reduction
- Image subtraction for change detection
- Accurate alignment is always a requirement



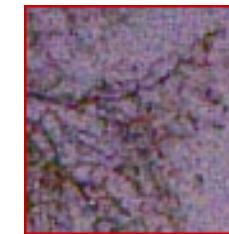
Image Averaging for Noise Reduction



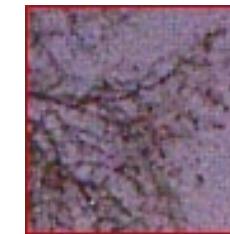
1 image



2 image



4 image



<http://www.cambridgeincolour.com/tutorials/image-averaging-noise.htm>



Image Averaging for Noise Reduction

- Take N aligned images $f_1(x,y), f_2(x,y), \dots, f_N(x,y)$

- Average image: $\overline{f(x,y)} = \frac{1}{N} \sum_{i=1}^N f_i(x,y)$

- Mean squared error vs. noise-free image g :

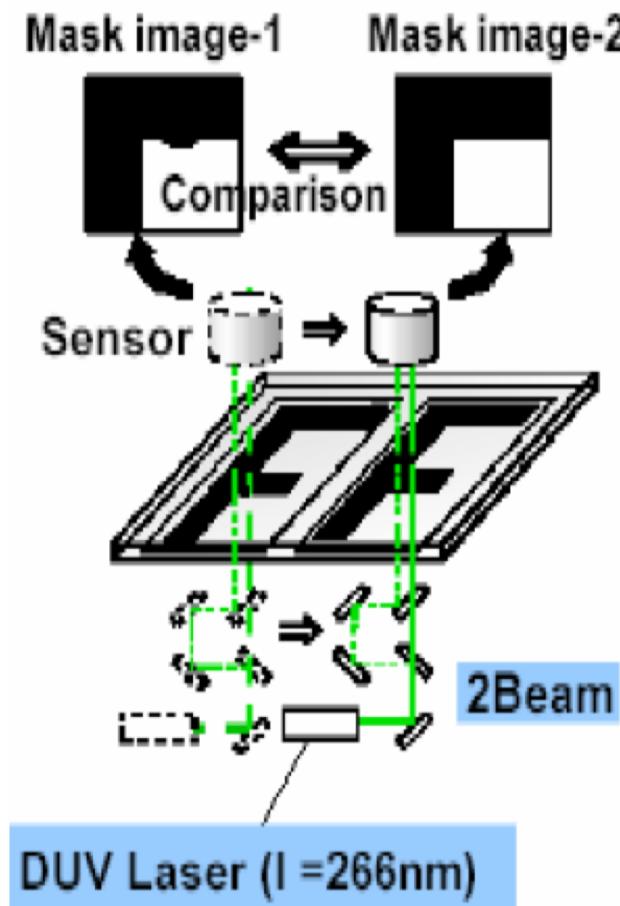
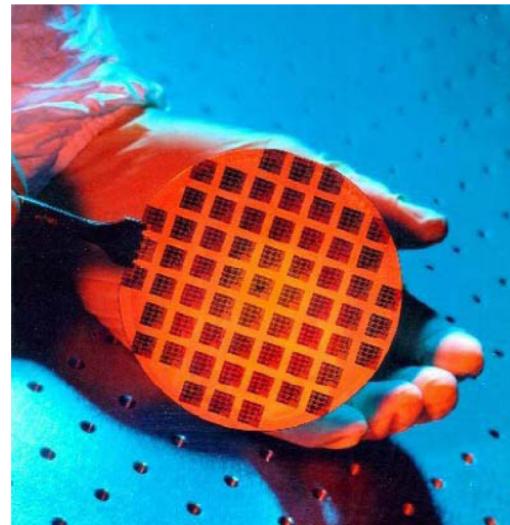
$$\begin{aligned} E \left\{ [\bar{f} - g]^2 \right\} &= E \left\{ \left[\left(\frac{1}{N} \sum_{i=1}^N g + n_i \right) - g \right]^2 \right\} = E \left\{ \left[\frac{1}{N} \sum_{i=1}^N n_i \right]^2 \right\} \\ &= \frac{1}{N^2} \sum_{i=1}^N E \{ n_i^2 \} = \frac{1}{N} E \{ n^2 \} \end{aligned}$$

↑ ↗
provided $E \{ n_i n_j \} = 0 \quad \forall i, j$ and $E \{ n_i^2 \} = E \{ n^2 \} \quad \forall i$

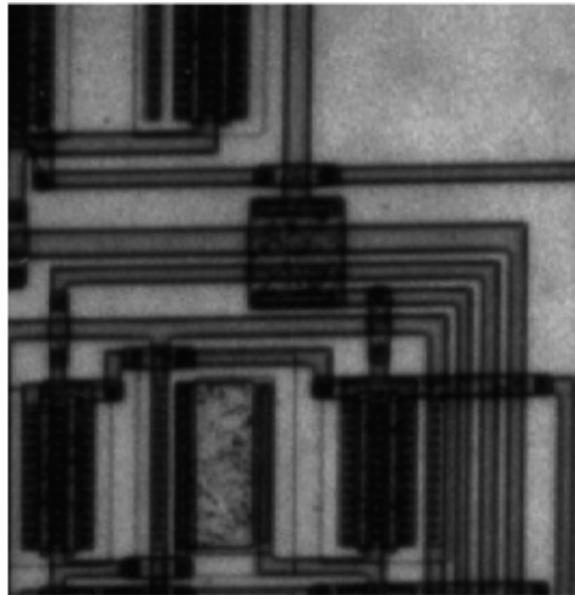


Image Subtraction

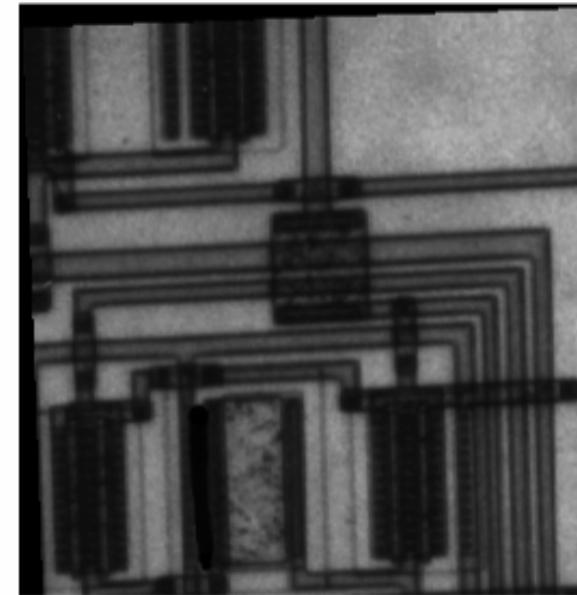
- Find differences/changes between 2 mostly identical images
- Example from IC manufacturing:
defect detection in photomasks
by die-to-die comparison



Where is the Defect?



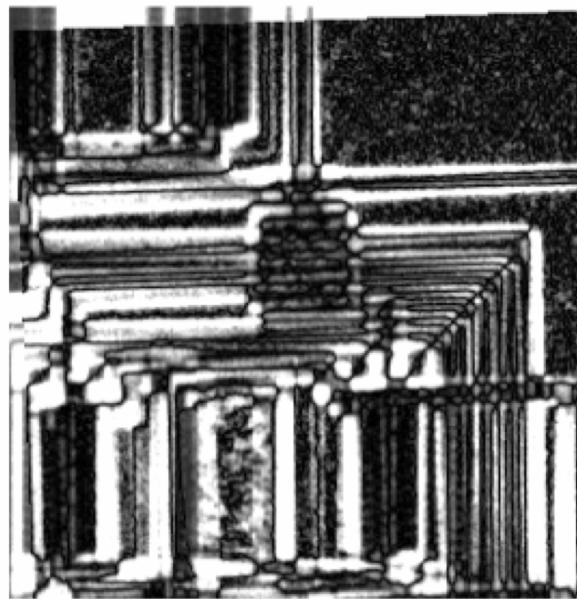
No defect



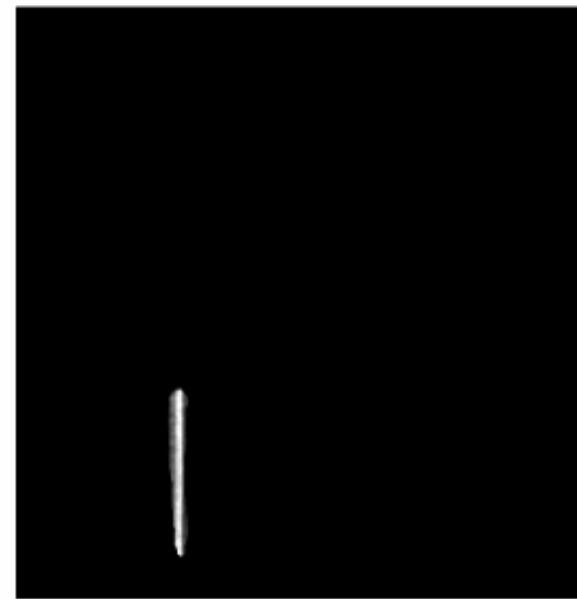
With defect



Absolute Difference Between Two Images

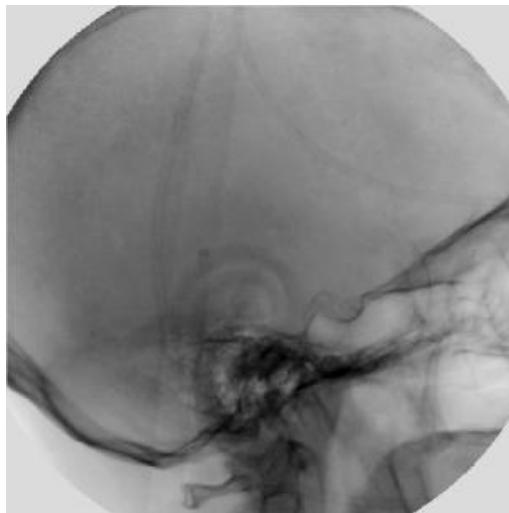


w/o alignment

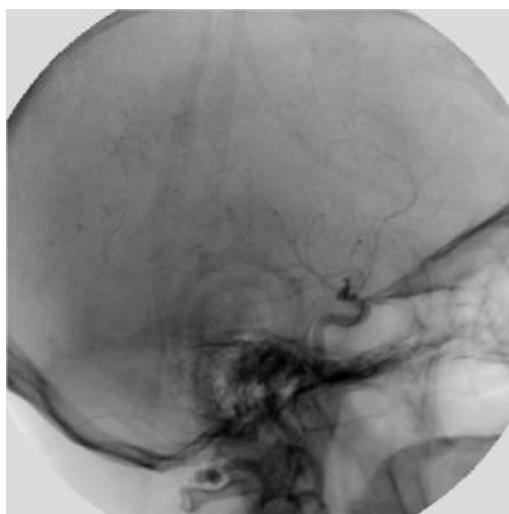


with alignment

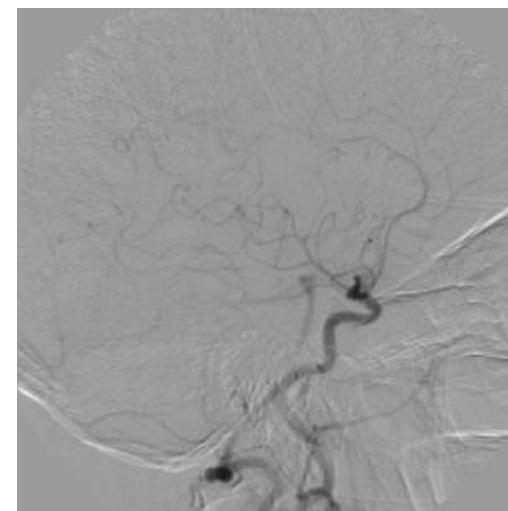
Digital Subtraction Angiography



pre-contrast mask image



subtraction



angiographic image

contrast
enhancement

