EQ2330 – Image and Video Processing

Solution #4

Solution

1. The simplest filter that could be used for noise removal in this problem is an averaging filter. An example of such a filter is

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

   The filter works because it smoothes local variations in the image, in other words, the noise is reduced as a result of blurring.

2. The system model is depicted in Figure 1.

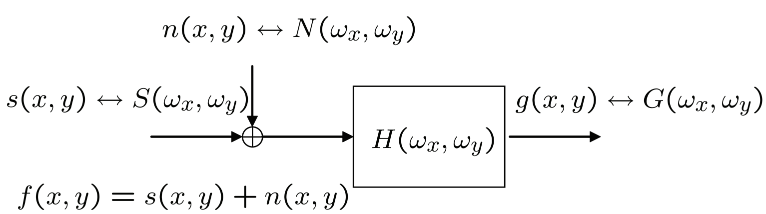

Figure 1: System model.

3. The noise is modelled as $N(x,y) \sim \mathcal{N}(0,\sigma_n^2)$. Since noise is white, the autocorrelation of that noise is given by $\phi_{nn}(x,y) = \sigma_n^2\delta(x,y)$. Then the power spectral density $\Phi_{nn}(\omega_x,\omega_y)$ is computed as a Fourier transform of $\phi_{nn}(x,y)$. Therefore,

$$\Phi_{nn}(\omega_x,\omega_y) = \sigma_n^2.$$

   The power spectral density is then constant and equal to $\Phi_{nn}(\omega_x,\omega_y) = 25$.

4. Let $\Phi_{ss}(\omega_x,\omega_y)$ be the power spectral density of s.
   The corss spectral denisty $\Phi_{sf}(\omega_x,\omega_y)$ may be easily derived using linearity of

the Fourier transform and the fact that the noise is independent from the signal.

$$\begin{aligned}
\Phi_{sf}(\omega_x,\omega_y) &= \mathcal{F}\left\{\mathbb{E}\{s[x+m,y+n]f^*[x,y]\}\right\} \\
&= \mathcal{F}\left\{\mathbb{E}\{s[x+m,y+n](s^*[x,y]+n^*[x,y])\}\right\} \\
&= \mathcal{F}\left\{\mathbb{E}\{s[x+m,y+n]s^*[x,y]\}+\mathbb{E}\{s[x+m,y+n]n^*[x,y]\}\right\} \\
&= \mathcal{F}\left\{\mathbb{E}\{s[x+m,y+n]s^*[x,y]\}\right\} \\
&= \Phi_{ss}(\omega_x,\omega_y).
\end{aligned}$$

5. Let us define the error signal $e = g - s$. Let $\Phi_{ee}(\omega_x, \omega_y)$ be the power spectral density of the error. We see that

$$\begin{aligned}
\Phi_{ee}(\omega_x,\omega_y) &= \mathcal{F}\left\{\mathbb{E}\{e[x+m,y+n]e^*[x,y]\}\right\} \\
&= \mathcal{F}\{\mathbb{E}\{(g[x+m,y+n]-s[x+m,y+n])(g^*[x,y]-s^*[x,y])\}\} \\
&= \Phi_{gg}(\omega_x,\omega_y)-\Phi_{sg}(\omega_x,\omega_y)-\Phi^*_{sg}(\omega_x,\omega_y)+\Phi_{ss}(\omega_x,\omega_y) \\
&= |H(\omega_x,\omega_y)|^2\Phi_{ff}(\omega_x,\omega_y)-H^*(\omega_x,\omega_y)\Phi_{sf}(\omega_x,\omega_y)+ \\
&\quad -H(\omega_x,\omega_y)\Phi^*_{sf}(\omega_x,\omega_y)+\Phi_{ss}(\omega_x,\omega_y).
\end{aligned}$$

6. Using the fact $\Phi_{ss}(\omega_x, \omega_y) = \Phi_{sf}(\omega_x, \omega_y)$ (shown earlier) we obtain

$$H(\omega_x,\omega_y) = \frac{\Phi_{sf}(\omega_x,\omega_y)}{\Phi_{ff}(\omega_x,\omega_y)} = \frac{\Phi_{ss}(\omega_x,\omega_y)}{\Phi_{ff}(\omega_x,\omega_y)}.$$

   It can be shown that for the system shown in the Figure 1, assuming that s and n are statistically independent, $\Phi_{ff}(\omega_x, \omega_y) = \Phi_{ss}(\omega_x, \omega_y) + \Phi_{nn}(\omega_x, \omega_y)$, therefore

$$H(\omega_x,\omega_y) = \frac{\Phi_{ss}(\omega_x,\omega_y)}{\Phi_{ss}(\omega_x,\omega_y)+\Phi_{nn}(\omega_x,\omega_y)}.$$

7. The Wiener filter should have a low-pass character in this case, assuming that the power spectral is also low-pass like. The averaging filter from the first subproblem is also a low-pass filter.