EQ2330 – Image and Video Processing

Exercise #6: Wavelets

Unless stated otherwise, the problems are from R. C. Gonzales and R. E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 7.4**
   Prove that the following filters from Table 7.1 form perfect reconstruction filter banks:

   (a) Quadrature mirror filter (QMF)

   (b) Orthonormal filter

2. **Problem 7.7**
   Draw a two-dimensional four-band filter bank decoder to reconstruct input $x(m, n)$ in Fig. 7.5.

Filter	QMF	CQF	Orthonormal
$H_0(z)$	$H_0^2(z) - H_0^2(-z) = 2$	$H_0(z)H_0(z^{-1}) + H_0^2(-z)H_0(-z^{-1}) = 2$	$G_0(z^{-1})$
$H_1(z)$	$H_0(-z)$	$z^{-1}H_0(-z^{-1})$	$G_1(z^{-1})$
$G_0(z)$	$H_0(z)$	$H_0(z^{-1})$	$G_0(z)G_0(z^{-1}) + G_0(-z)G_0(-z^{-1}) = 2$
$G_1(z)$	$-H_0(-z)$	$zH_0(-z)$	$-z^{-2K+1}G_0(-z^{-1})$

TABLE 7.1 Perfect reconstruction filter families.

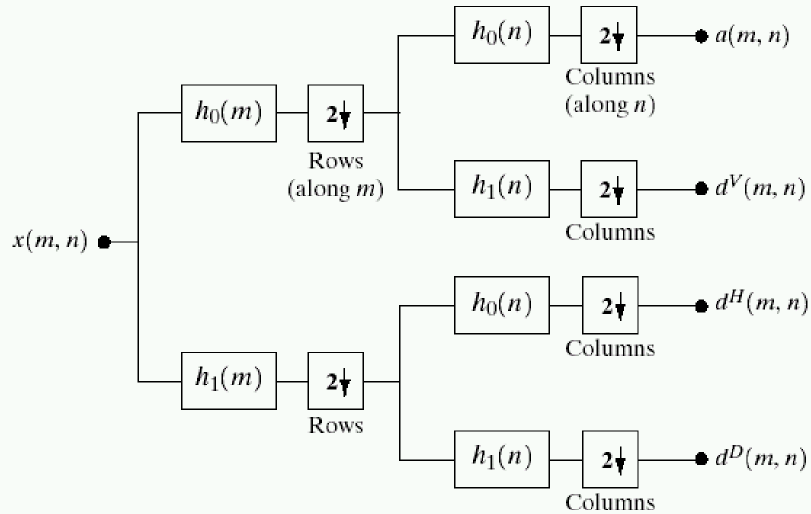

FIGURE 7.5 A two-dimensional, four-band filter bank for subband image coding.

3. **Exam, March 2011**:*Multiresolution Processing*

   Consider the following lifting implementation for multiresolution image processing.

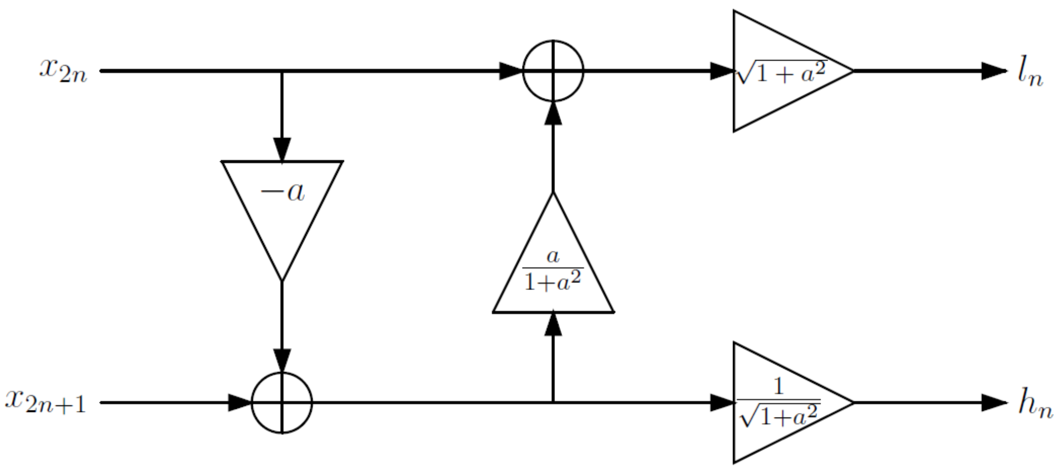

Figure 1: Lifting implementation for multiresolution image processing.

   (a) Is the resulting multiresolution representation critically sampled? Explain.

   (b) Determine the transform matrix T for any lifting implementation with parameter a such that

$$\begin{bmatrix} l_n \\ h_n \end{bmatrix} = T \begin{bmatrix} x_{2n} \\ x_{2n+1} \end{bmatrix}. \tag{1}$$

   Show that this is an orthonormal wavelet for any $a > 0$.

   (c) Even and odd samples x_{2n} and x_{2n+1} are correlated as given by the covariance matrix

$$C = \frac{1}{5} \begin{bmatrix} 14 & -18 \\ -18 & 41 \end{bmatrix}. \tag{2}$$

Determine the parameter $a > 0$ of the lifting implementation such that the even and odd samples are decorrelated by the lifting implementation. *Hint: One eigenvalue of the covariance matrix is* $\lambda_1 = 1$.

(d) If we choose $a = 1$ for the lifting implementation, we obain the well-known Haar wavelet. Can the Haar wavelet achieve a better energy compaction for signals with the covariance matrix C as given in (2)? Explain your answer and start by clarifying the term “energy compaction”.

(e) Construct the synthesis filters in the lifting implementation that allow for perfect reconstruction for any $a > 0$. *Hint: Use the advantage of the lifting implementation.*