EQ2330 – Image and Video Processing

Solution #3

Solution

1. First we note that all the plots of the magnitude spectra have DC component in the center of each plot.
   Picture A: There is a slow sinusoidal change of intensity in the horizontal direction and fast sinusoidal change in the vertical direction. This corresponds to a magnitude spectrum with four spectral peaks. There are two peaks along horizontal direction and two peaks along vertical direction. Due to low frequency of changes of intensity in the horizontal direction the corresponding peaks are close together. Since the change in vertical direction is faster, the corresponding peaks are further apart. We can, therefore, match the picture A to spectrum 3.
   Picture B and Picture C: Picture B contains a centered small white square. We may consider a cross-section through the center of this square in horizontal and vertical direction. We know that cross-section should have a sinc-like spectrum. There are two spectra that fulfill the requirement. We see that picture C shows a bigger square. Since the square in picture B is small, it must have wider frequency lobes of the sinc-like spectrum, that the picture with a bigger square. You may write down an equation to see this immediately. Therefore, we match picture B to spectrum 1 and picture C to spectrum 2.
   Picture D: We match picture D to the spectrum 4. We use similar reasoning as in the case of Picture A.

2. (i) The distorting signal $q(x, y)$ is periodic with ≈ 10 periods in the horizontal direction and ≈ 20 periods in the vertical direction. The image is 512×512 pixel, so the period in horizontal direction is $T_x = 512/10 \approx 51$ pixels and the period in vertical direction is $T_y = 512/20 \approx 26$ pixels. Assume that tht signal is a sinus-like in both direction (if you assume it is not, you have to acknowledge the signal's overtones, at multiplies of the fundamental frequency). The DFT has strong components at positions corresponding to 51 pixel periodicity in horizontal direction (u) and at 26 in vertical direction v. The position of spectral peaks in the DFT spectrum would be then $\pm 512/26 \approx \pm 20$ in the v-direction and $\pm 512/51 \approx \pm 10$ in the u-direction (We use the fact that the magnitude DFT spectrum is symmetrical around the origin). See the plot in Fig. 1.

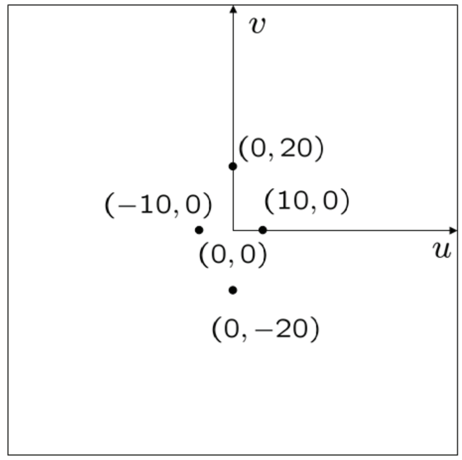

Figure 1: Magnitude of DFT of $q(x, y)$.

(ii) The distortion may be suppressed by two notch filters that filter out the sinusoids corresponding to the peaks in the DFT spectrum of the distorting signal $q(x, y)$. These filters can be defined in the frequency domain as follows:

$$H_v = \begin{cases} 0 & \text{if } v \in \text{ small circles centered around (0,-20) and (0,20)} \\ 1 & \text{otherwise} \end{cases}$$

$$H_u = \begin{cases} 0 & \text{if } u \in \text{ small circles centered around (-10,0) and (10,0)} \\ 1 & \text{otherwise} \end{cases}$$

3. (i) The DC component is located at point $(u, v) = (0, 0)$.

(ii) Since

$$f(x, y)(-1)^{x+y} = f(x, y) \exp{(j\pi(x + y))},$$

the operation is equivalent to signal modulation, therefore

$$f(x, y) \exp{(j\pi(x + y))} \leftrightarrow F(u + N/2, v + N/2).$$

If we now plot $|F(u + N/2, v + N/2)|$ we see that it is symmetrical around the center of the plot and the DC component is now moved to $(N/2, N/2)$ (the center).

(iii) The operation denoted by "?" can be implemented by taking the complex conjugate. We see it from the following

$$\begin{aligned} F(u, v)^* &= \left\{ \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \exp(-j2\pi \frac{ux + vy}{N}) \right\}^* \\ &= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \exp(-j2\pi \frac{u(-x) + v(-y)}{N}) \\ &= \mathcal{F}\{f(-x, -y)\}, \end{aligned}$$

therefore the output image is reflected around the origin. Note that images are real-valued.