EQ2330 – Image and Video Processing

Solution #10

Solution

1. The unconstrained optimization problem is

$$\min_{R_i, i \in \{1,2,\ldots,I\}} \sum_{i=1}^{I} D_i(R_i) + \lambda(\sum_{i=1}^{I} R_i - \bar{R}), \tag{1}$$

where λ is a constant (the Lagrange multiplier).

2. Differentiating the objective function in (1) gives the condition

$$-\lambda = \frac{\partial D_i}{\partial R_i} = \frac{\partial D_j}{\partial R_j}, \tag{2}$$

for all $i, j \in \{1, 2, \ldots, I\}$.

3. Figure 1 illustrates how the rate allocated to R_1 and R_2 depends on the shape of D_1 and D_2. For $\bar{R} = 1$, λ is chosen to 0.5, and the result is $R_1 = 0.6$ and $R_2 = 0.4$. For $\bar{R} = 1.68$, λ is chosen to -0.154, and the result is $R_1 = R_2 = 0.84$ bits.

4. Equation (1) can be rewritten to

$$\sum_{i=1}^{I} \min_{R_i} D_i(R_i) + \lambda(R_i - \bar{R}), \tag{3}$$

which makes explicit that R_i and R_j can be optimized independently after λ has been chosen. The choice of λ will affect the total rate. The algorithm is shown in Table 1. It will converge since the objective function in (1) is convex, being the sum of convex functions.

5. The unconstrained optimization problem is

$$\min_{r_v, r_e} D_i(r_v, r_e) + \mu(r_v + r_e - R_i), \tag{4}$$

where μ is a constant (the Lagrange multiplier).

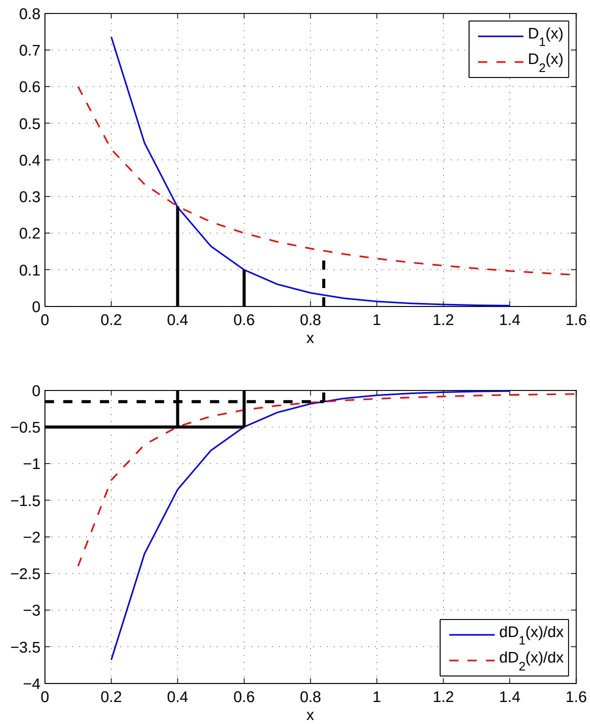

Figure 1: Solution for example $D_1(x)$, $D_2(x)$, $\frac{dD_1(x)}{dx}$ and $\frac{dD_2(x)}{dx}$.

Step 1.	Initiate the constant λ.
Step 2.	Optimize R_i for each $i \in \{1, 2, \ldots, I\}$ independently, such that the cost $D_i(R_i) + \lambda R_i$ is minimized. A one-dimensional search can be used for the optimization.
Step 3.	Calculate the total rate $\sum_{i=1}^{I} R_i$. If the constraint is not satisfied or the total rate is too small, adjust λ and continue with Step 2.

Table 1: Rate-constrained rate-allocation optimization for the sum-distortion criterion.

6. Differentiating the objective function in (4) gives the condition

$$-\mu = \frac{\partial D_i}{\partial r_v} = \frac{\partial D_i}{\partial r_e}. \tag{5}$$

7. Here, we cannot rewrite the optimization objective such that r_v and r_e are separated. Hence, the optimization must be performed jointly over r_v and r_e. The algorithm then consists of a two-dimensional search over the set of allowed values for r_v and r_e.