Exercise # 9.

Video Compression. Exam March 2010

(a) Energy of $f_n(x,y)$: $\sum_{x=0}^{3}\sum_{y=0}^{3} f_n^2(x,y)$

Energy of three frames are: 199, 174, 342.

(b) DC coefficients: $F_n(0,0)=\frac{1}{2}\cdot\frac{1}{2}\sum_{x=0}^{3}\sum_{y=0}^{3} f_n(x,y)$

DC coefficients of three frames are 10.75, 10.5, 14.5,

with energy $10.75^2=115.56$, $10.5^2=110.25$, $14.5^2=210.25$.

(c) Consider $F_{DC}(u,v)=\begin{cases} F(u,v), & (u,v)=(0,0) \\ 0, & \text{otherwise} \end{cases}$

Let $\bar{f}_n(x,y)$ denotes the average pixel value of $f_n(x,y)$. The reconstruction is:

$\hat{f}_n(x,y)=\frac{1}{2}\cdot\frac{1}{2}\cos 0\cdot\cos 0\cdot F_n(0,0)=\frac{1}{4}F_n(0,0)=\frac{1}{16}\sum_{x=0}^{3}\sum_{y=0}^{3} f_n(x,y)=\bar{f}_n(x,y)$

The distortion between $f_n(x,y)$ and $\hat{f}_n(x,y)$ is:

$$\sum_{x,y}\left(f_n(x,y)-\hat{f}_n(x,y)\right)^2=\sum_{x,y}\left(f_n(x,y)-\bar{f}_n(x,y)\right)$$

$$=\sum_{x,y} f_n^2(x,y)-2\bar{f}_n(x,y)\sum_{x,y} f_n(x,y)+16\bar{f}_n^2(x,y)$$

$$=\sum_{x,y} f_n^2(x,y)-2\bar{f}_n(x,y)\cdot 16\,\bar{f}_n(x,y)+16\bar{f}_n^2(x,y)$$

$$=\sum_{x,y} f_n^2(x,y)-16\bar{f}_n^2(x,y)$$

$$=\sum_{x,y} f_n^2(x,y)-F_n^2(0,0)$$

Therefore, the distortion for three frames are

$119-115.56=83.44$, $174-110.25=63.75$, $342-210.25=131.75$

Method 2: $\sum_{x,y}(f_n(x,y)-\hat{f}_n(x,y))^2=\sum_{u,v}(F_n(u,v)-F_{DC}(u,v))^2=\sum_{(u,v)\neq(0,0)} F_n^2(u,v)$

$$=\sum_{u,v} F_n^2(u,v)-F_n^2(0,0)$$

$$=\sum_{x,y} f_n(x,y)-F_n^2(0,0).$$

(d) The DC value is quantized to 10.

If we use entropy code, we need $-\log_2 P_r(F_1(0,0)=10)=-\log_2 0.1\approx 3.32$ bits

(e) The problem can be formulated as follows (take copy mode as an example)

$$f \to \underbrace{\boxed{DCT} \to F \to \boxed{\text{keep only DC value}}}_{\text{distortion 1}} \to F_{DC} \to \underbrace{\boxed{Q} \to \hat{F}_{DC} \to \boxed{\text{copy mode}}}_{\text{distortion 2.}} \to \tilde{F}_{DC} \to \boxed{IDCT} \to \hat{f}$$

frame 1: $D_{1,f_1} = E[f_1] - E[F_{DC}]$, $D_{2,f_1} = (F_{DC} - \tilde{F}_{DC})^2$.

where we use $E[\cdot]$ to denote energy function.

Lagrangian $J_n = D_n + \lambda R_n$

copy mode: $J_2^{copy} = D_2^{copy} + \lambda R_2^{copy}$, where $R_2^{copy} = 1$. (1 bit to store mode)

frame 2 copy $\hat{F}_{DC}$ from frame 1, which is 10 (according to (d)).

Therefore $D_{2,f_2} = (10.5 - 10)^2 = 0.25$

$D_2^{copy} = 63.75 + 0.25 = 64$

$\Rightarrow J_2^{copy} = 64 + 2 \times 1 = 66$

coding mode: $R_2^{code} = -\log_2(\Pr(F(0,0) = 10)) + \underset{\text{1 bit to Store mode}}{1} = 4.32$ bits.

$D_2^{code} = 64$ (same as D_2^{copy})

$\Rightarrow J_2^{code} = 64 + 2 \times 4.32 = 72.64$

Therefore we choose copy mode to transmit frame 2 using 1 bit.

(f) $J_3 = D_3 + \lambda R_3$

copy mode: $D_3^{copy} = 131.75 + (14.5 - 10)^2 = 152$

$R_3^{copy} = 1$ (1 bit to store mode)

$J_3^{copy} = 152 + 2 \times 1 = 154$

coding mode: $D_3^{code} = 131.75 + (14.5 - 14)^2 = 132$

$R_3^{code} = -\log_2(\Pr(F(0,0) = 14)) + 1 = -\log_2 0.1 + 1 = 4.32$ bits

$J_3^{code} = 132 + 2 \times 4.32 = 140.64$

Therefore we choose code mode to transmit frame 3 using 4.32 bits.