EQ2330 – Image and Video Processing

Exercise 10: Image and Video Compression

Unless stated otherwise, the problems are from on R. C. Gonzales and R. E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom *Image and Video Compression*

1. **Video Compression, Exam March 2010**
   Conditional Replenishment

   In this problem, we consider the encoding of a video signal f, with a conditional replenishment coder. The coder uses the 4×4 DCT transform. Let f be a $4 \times 4 \times 3$ signal, where each of the three frames is given by

$$f_1(x,y) = \begin{bmatrix} 5 & 3 & 8 & 1 \\ 4 & 3 & 2 & 2 \\ 2 & 6 & 1 & 0 \\ 0 & 5 & 0 & 1 \end{bmatrix}, \quad f_2(x,y) = \begin{bmatrix} 5 & 4 & 7 & 1 \\ 4 & 3 & 2 & 2 \\ 2 & 4 & 1 & 0 \\ 0 & 5 & 0 & 2 \end{bmatrix}, \quad f_3(x,y) = \begin{bmatrix} 7 & 6 & 4 & 2 \\ 4 & 6 & 0 & 9 \\ 0 & 8 & 2 & 0 \\ 4 & 2 & 0 & 4 \end{bmatrix}.$$

   The forward transform for a frame f_n is given by

$$F_n(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{3} \sum_{y=0}^{3} \cos\left(\frac{(2x+1)\pi u}{8}\right) \cos\left(\frac{(2y+1)\pi v}{8}\right) f_n(x,y),$$

   where $u, v \in \{0,1,2,3\}$, $\alpha(u=0) = 1/2$ and $\alpha(u>0) = 1/\sqrt{2}$.

   (a) (2p) Write down the formula for the energy of a 4×4 signal $f_n(x,y)$. Calculate the energy of the three frames $f_1(x,y)$, $f_2(x,y)$ and $f_3(x,y)$.

   (b) (2p) Calculate the DC value, i.e., $F_n(0,0)$, of the three frames with $n = 1,2,3$. What is the energy of the DC coefficients?

   (c) (2p) Assume now that only the DC value is used for reconstruction of each frame, i.e., all other DCT coefficients are set to zero. What is the squared-error distortion for each of the three frames $f_1(x,y)$, $f_2(x,y)$ and $f_3(x,y)$?

(d) (2p) You quantize the DC value of frame f_1 to the nearest integer that is divisible by 2. The quantized DC coefficient has the probability mass function shown in Table 1. Estimate the rate needed to encode the quantized DC coefficient of frame $f_1(x, y)$.

(e) (4p) To encode frame $f_2(x, y)$, you consider using conditional replenishment and the Lagrangian multiplier $\lambda = 2$. Calculate the Lagrangian cost for the copy mode. For the coding mode, you quantize the DC value of frame $f_2(x, y)$ to the nearest integer that is divisible by 2. Calculate the Lagrangian cost for the coding mode. Which mode do you choose? At what rate do you encode $f_2(x, y)$? *Hint: Consider the rate that is needed to signal the mode.*

(f) (3p) Encode frame $f_3(x, y)$ using the same procedure as for frame $f_2(x, y)$. Which mode do you use and at what rate do you encode $f_3(x, y)$?

z	0	2	4	6	8	10	12	14	16	18
$\Pr\{F(0,0) = z\}$	0.05	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.05

Table 1: Probability mass function for DC coefficient of $f(x, y)$.