EQ2330 – Image and Video Processing

Exercise 7: Compression Fundamentals

Unless stated otherwise, the problems are from on R. C. Gonzales and R. E. Woods. *Digital Image Processing*, (second ed.), Prentice Hall, Upper Saddle River, New Jersey, 2002.

Problems to be solved in the classroom

1. **Problem 8.1**

   (a) Can variable-length coding procedures be used to compress a histogram equalized image with 2^n gray levels? Explain.

   (b) Can such an image contain interpixel redundancies that could be exploited for data compression.

2. **Problem 8.7** Prove that, for a zero-memory source with q symbols, the maximum value of the entropy is $\log_2 q$, which is achieved if and only if all source symbols are equiprobable. *Hint:* Consider the quantity $\log_2 q - H(\mathbf{z})$ and note the inequality $\ln x \leq x - 1$.

3. Image coding deals with compression of the information that an image holds.

   (a) Plot a histogram (not uniformly distributed) of a fictive 16×16 3-bit image.

   (b) Consider lossless encoding of that image. What can the histogram tell you about the possible performance of a lossless encoder? Calculate the entropy of the image and explain what it describes.

   (c) Plot the histogram of an image that has the highest possible value of entropy. What is this entropy?

   (d) Prove by example that there are images, with the histogram in problem 3c, that can be encoded at a lower rate than the entropy. Explain why this is so.

   (e) Describe a procedure for encoding your example image at a bit-rate lower than the entropy.

4. **Exam March 2010**: *Wavelets*

In this problem, we consider the implementation of a one-dimensional discrete wavelet transform using the Daubechies 4-tap scaling vector

$$h_\varphi = \frac{1}{4\sqrt{2}}\Big[1+\sqrt{3} \quad 3+\sqrt{3} \quad 3-\sqrt{3} \quad 1-\sqrt{3}\Big],$$

indexed by $n = 0, 1, \cdots, N-1$.

(a) Write down the wavelet vector h_ψ that is derived from the orthogonality condition

$$h_\psi(n) = (-1)^n h_\varphi(N-1-n).$$

(b) Show that h_φ and h_ψ are orthonormal vectors.

(c) Draw a block diagram of the one-dimensional two-band analysis filter bank that uses h_φ and h_ψ. Denote the input signal by x and denote the output signals by y_a and y_d.

(d) Let the length-L input signal be

$$x = 4\sqrt{2}\Big[1 \; 1 \; 1 \; 1 \; 1 \; 1\Big].$$

Use a periodic extension of x, i.e., $[x \; x \; x]$, to calculate the approximation signal y_a. You do not need to calculate the detail signal y_d. *Hint: The concatenated vector* $[y_a \; y_d]$ *should have length* L*, the same length as* x.

(e) It is not necessary to do a full periodic extension of x, only k additional samples are needed on each side of x. What is the value of k for our example? What is the value of k as a function of N?

(f) Calculate the computational complexity of the two-band analysis filter bank for a signal of length L and filters with N taps. Give the complexity as the number of multiplications needed to perform the analysis. *Hint: Use your result from question* (4e).

(g) Assume the squared error distortion measure. How much distortion do you introduce if you reconstruct x using only y_a? How much distortion do you introduce if you reconstruct x using only y_d? *Hint: Reconstruction is not necessary to answer this question.*